

The scalar and electromagnetic form factors of the nucleon in dispersively improved Chiral EFT

Jose Manuel Alarcón



Works done in collaboration with C. Weiss
arXiv:1707.07682



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- Extend this knowledge to wider kinematic regions.

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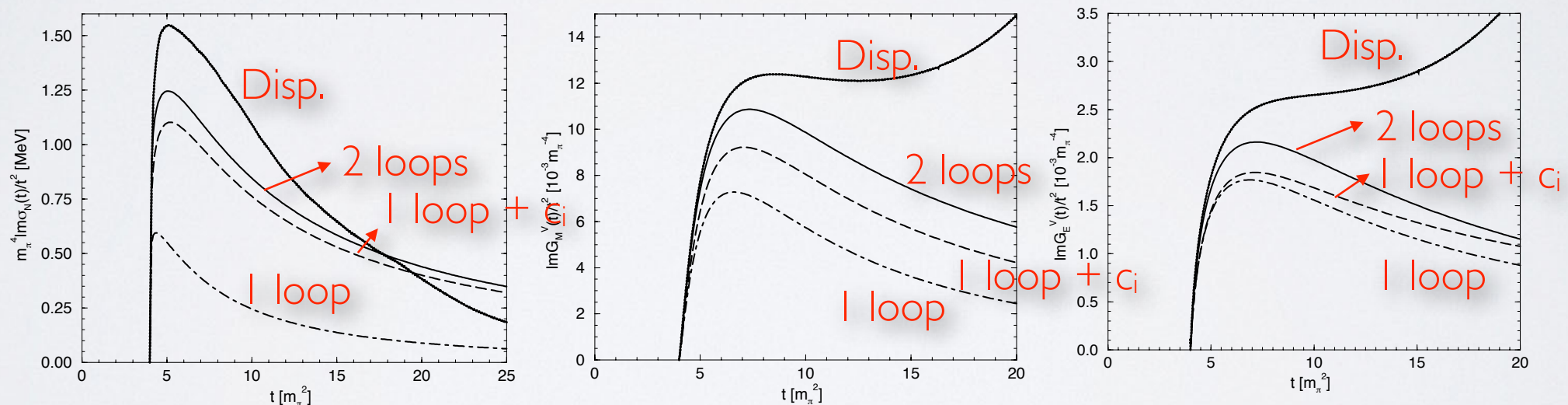
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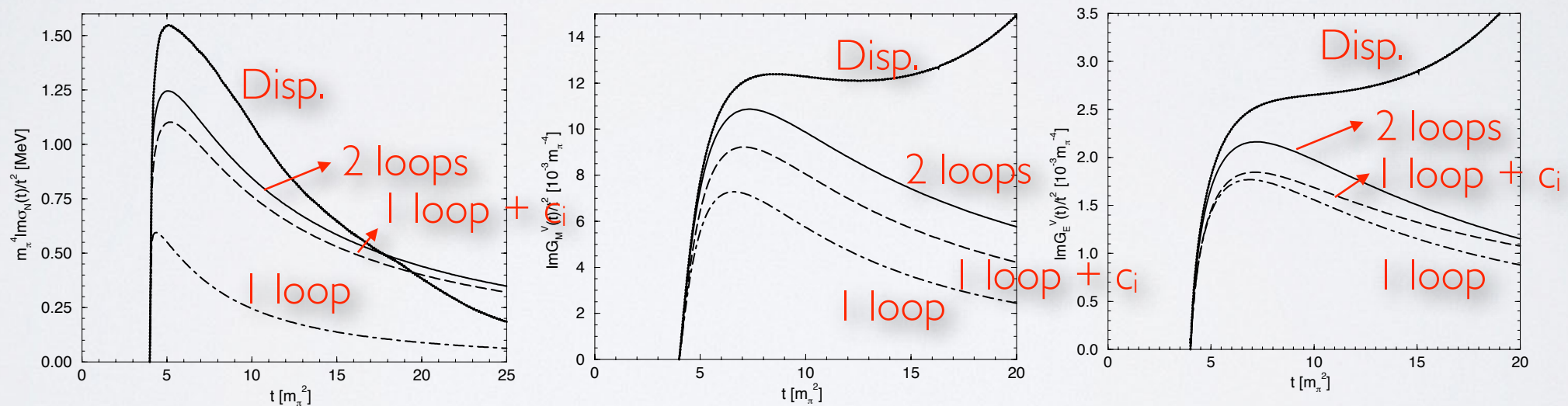
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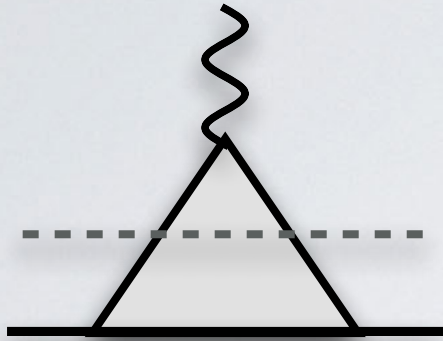


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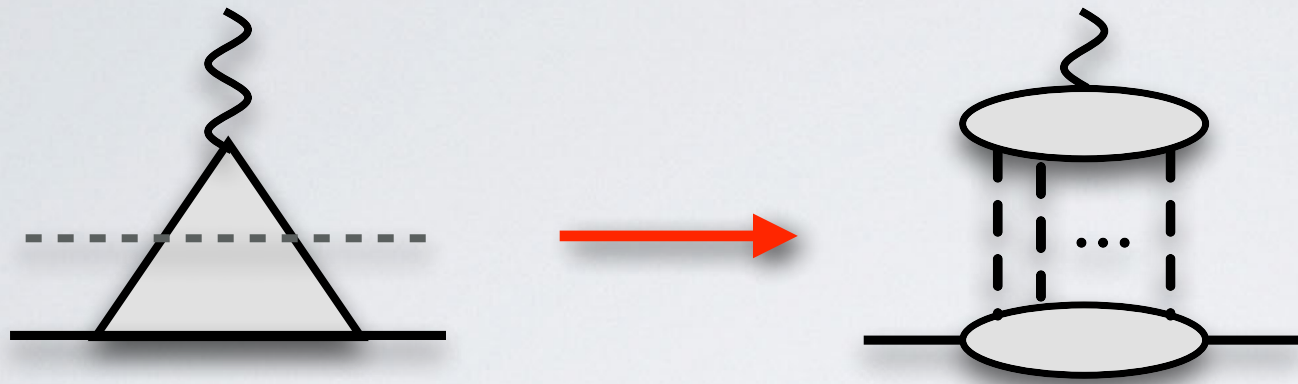
- Higher order calculations are needed (unpractical).

Dispersively Improved χ EFT
(DI χ EFT)

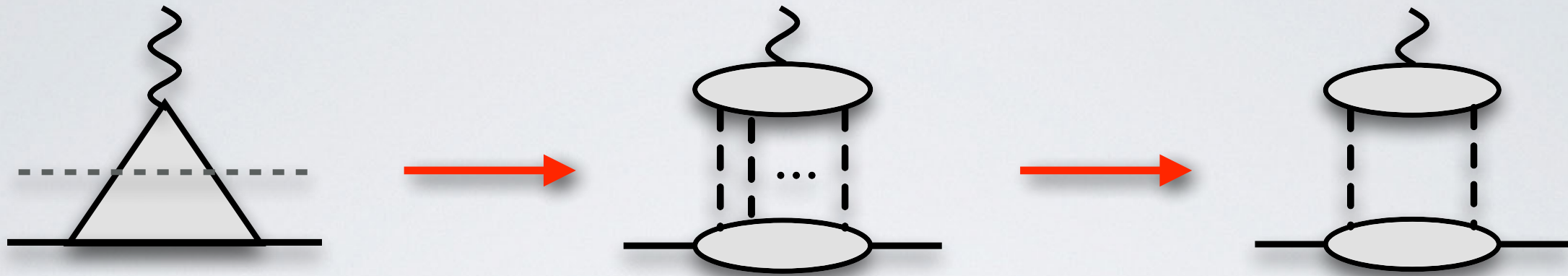
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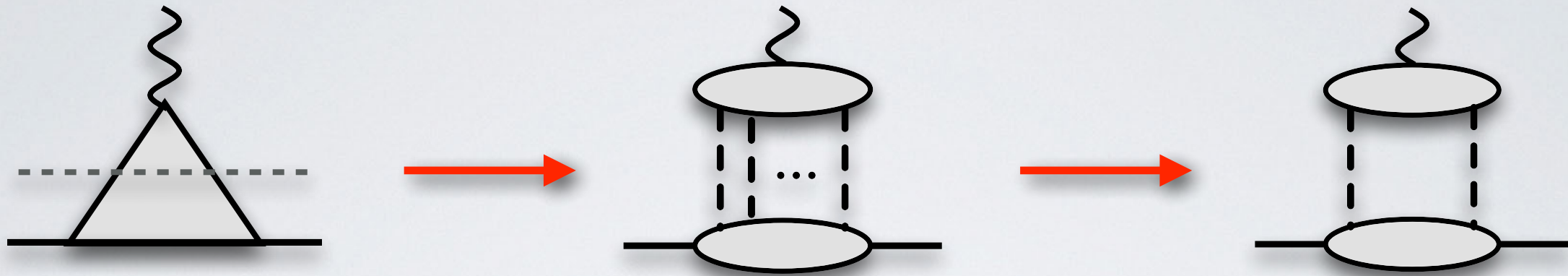
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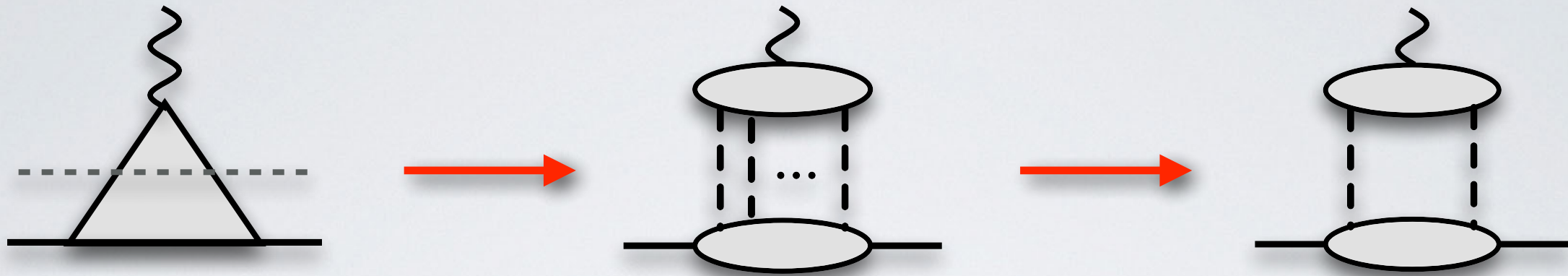
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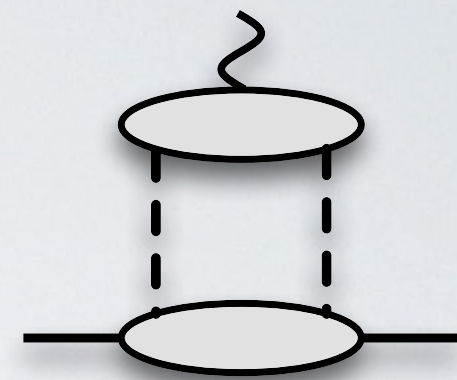
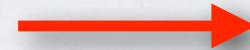
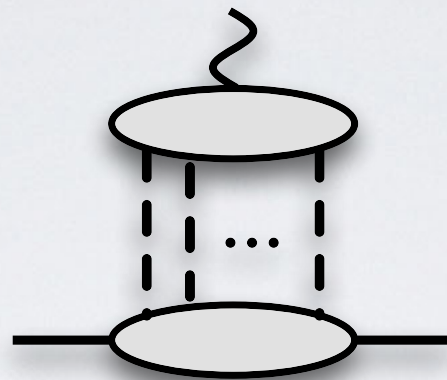
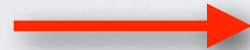
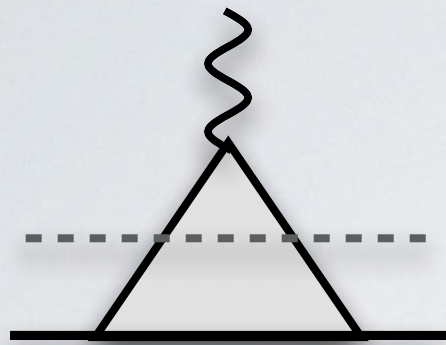
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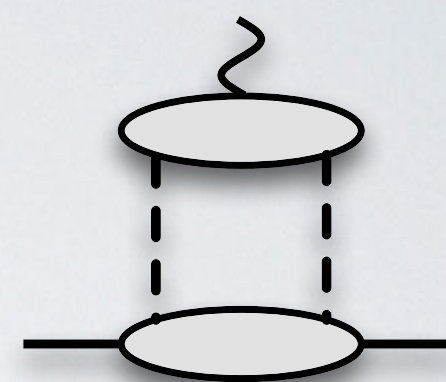
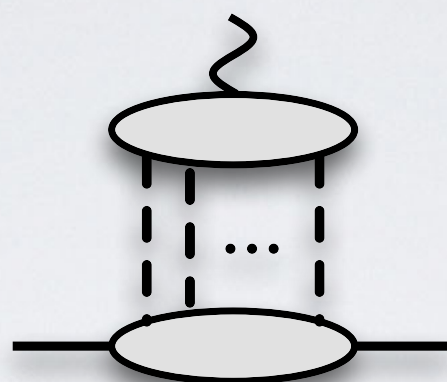
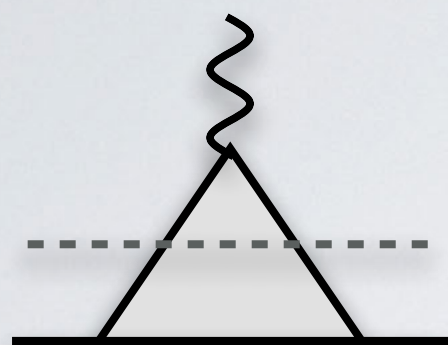
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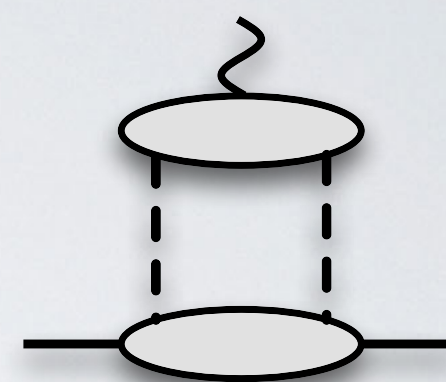
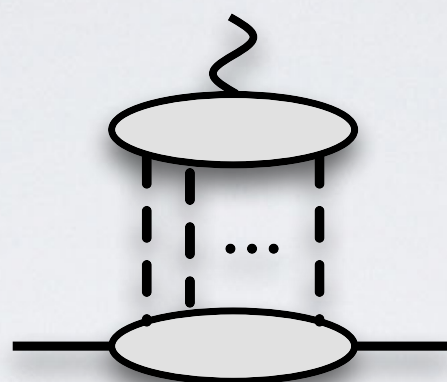
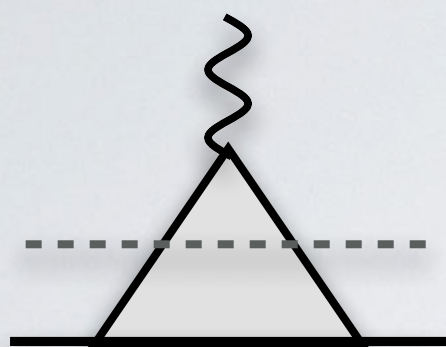
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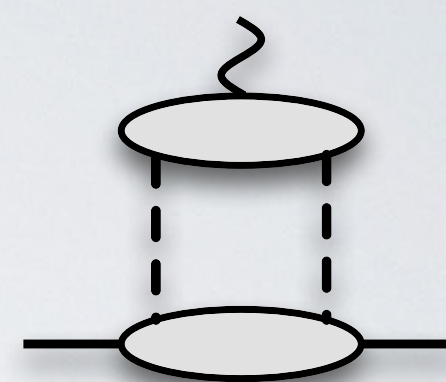
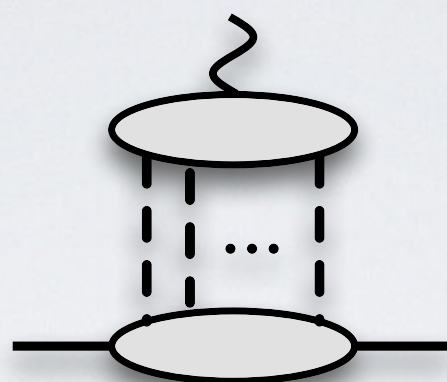
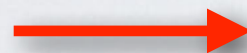
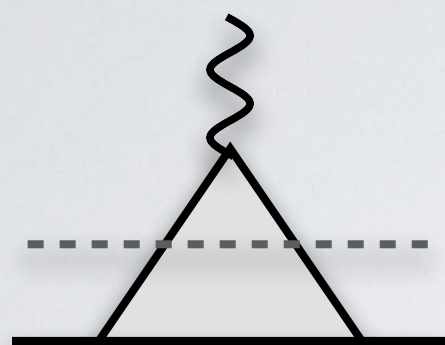
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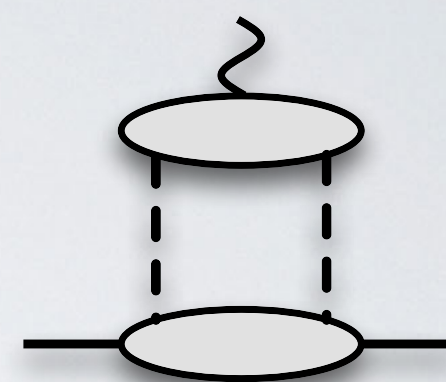
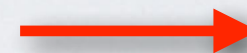
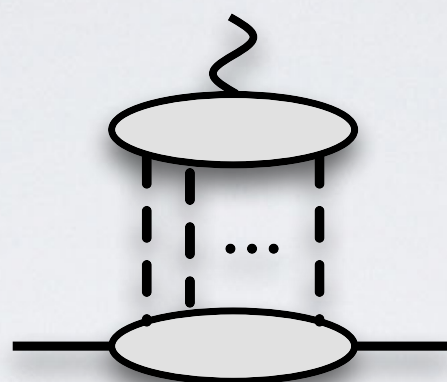
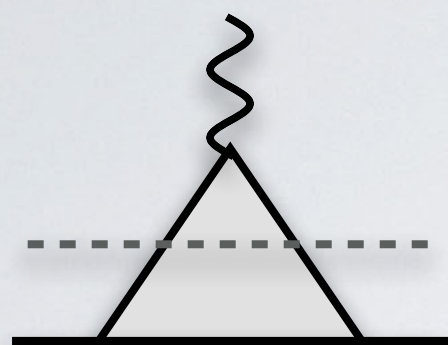


Chiral EFT



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Data/Dispersion Theory

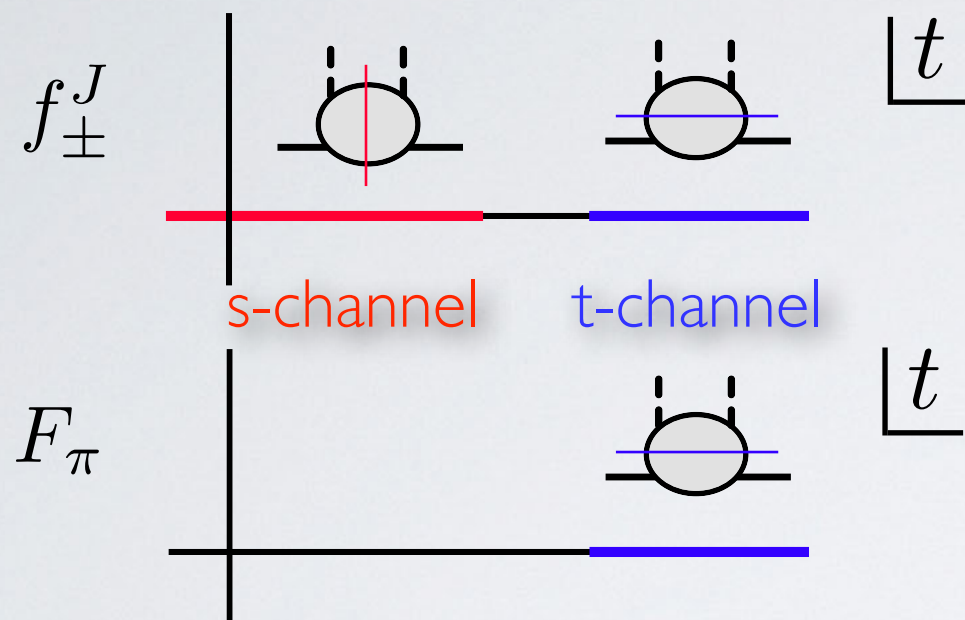
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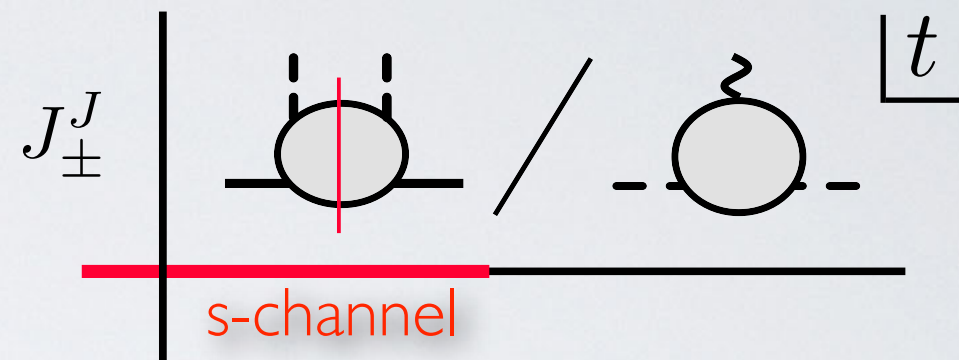
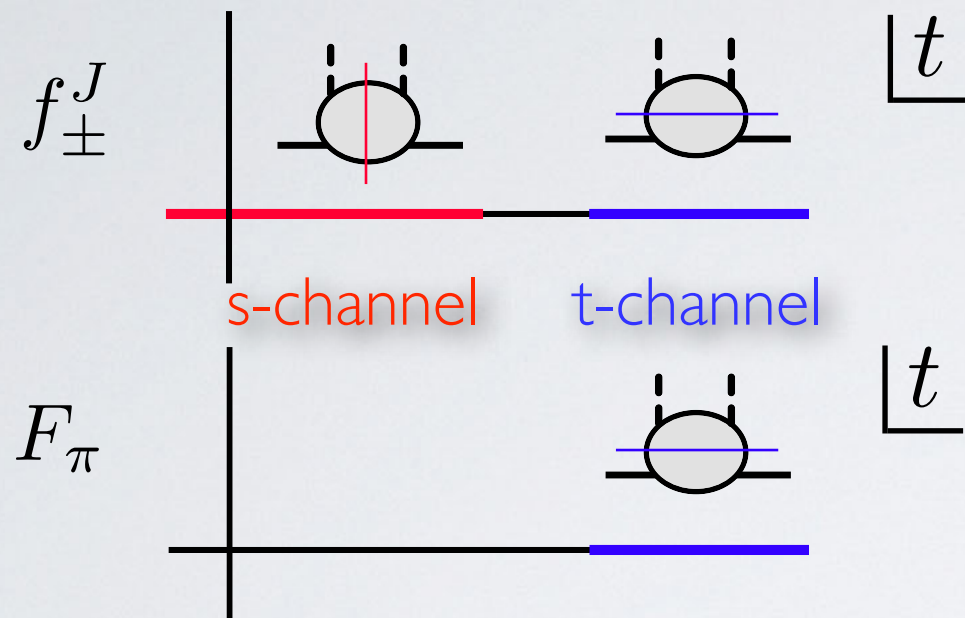
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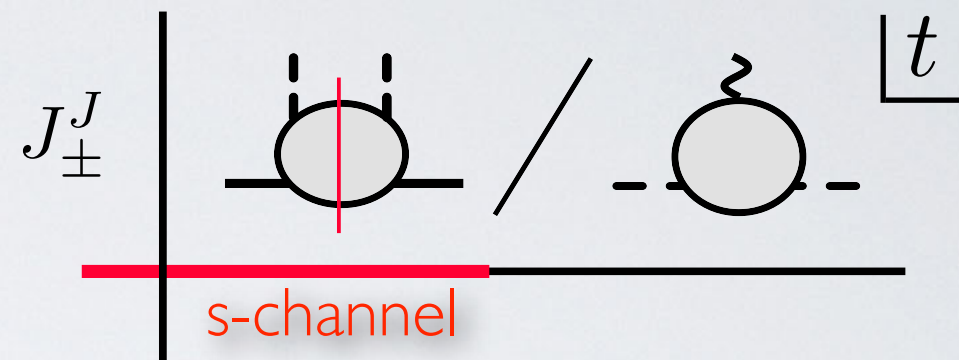
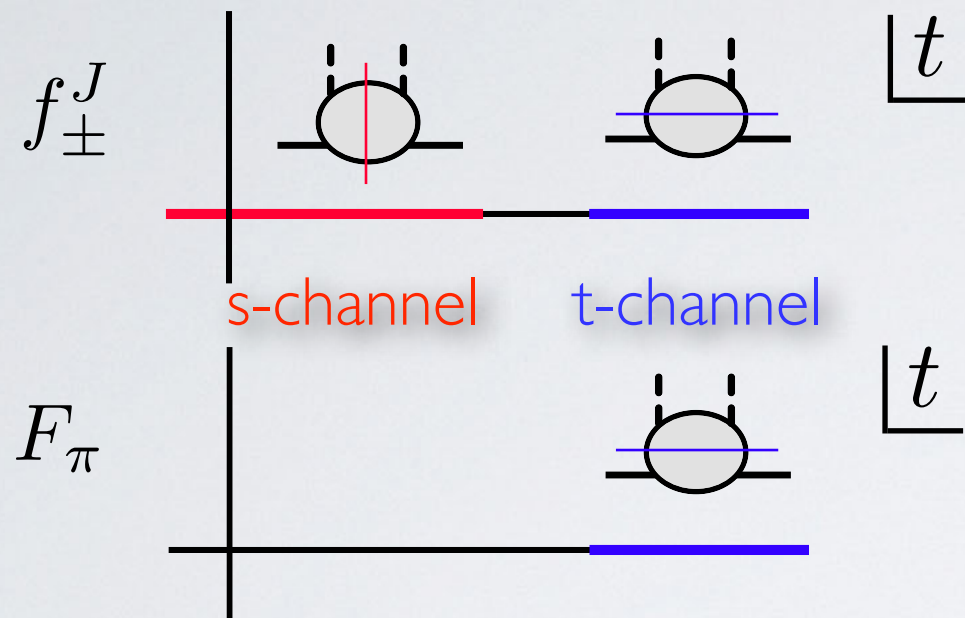
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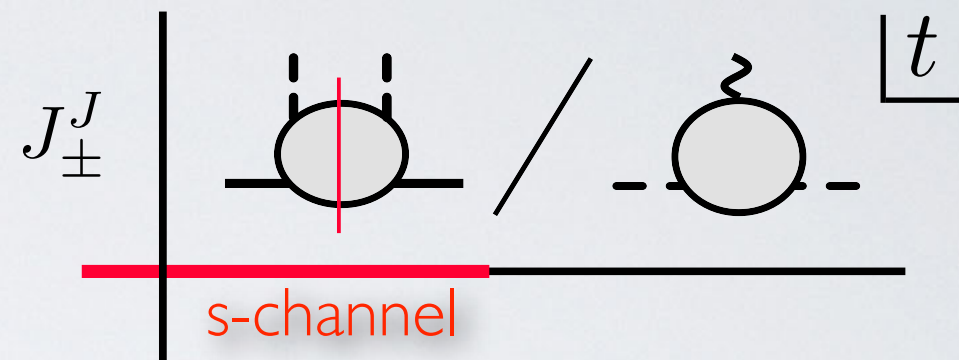
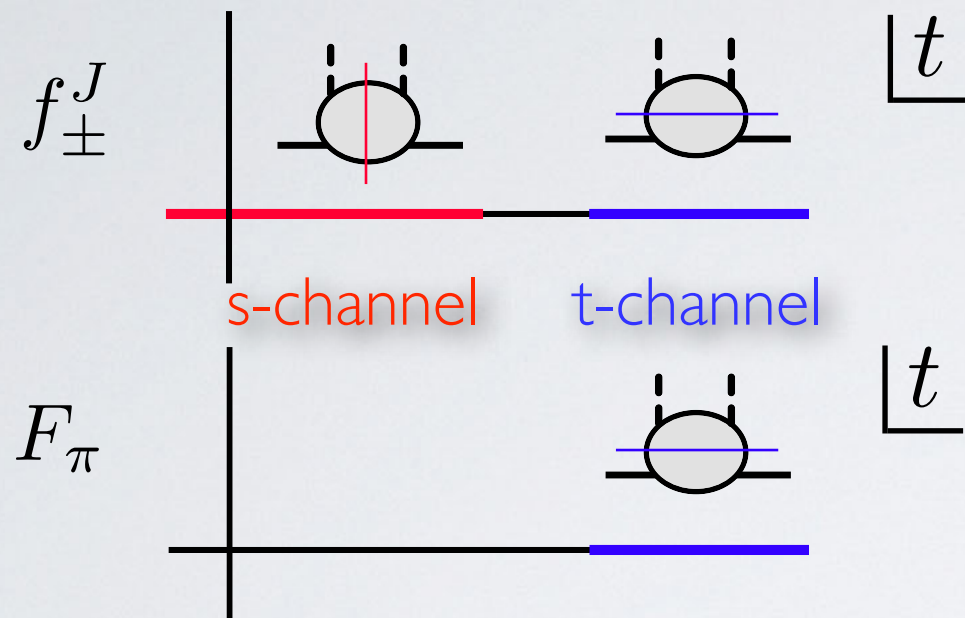


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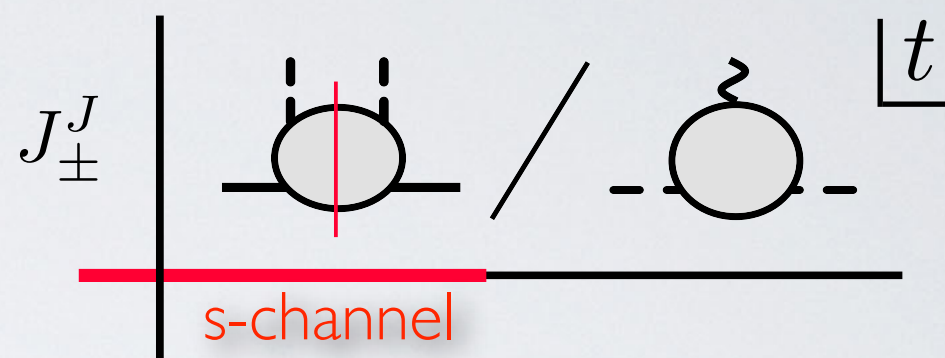
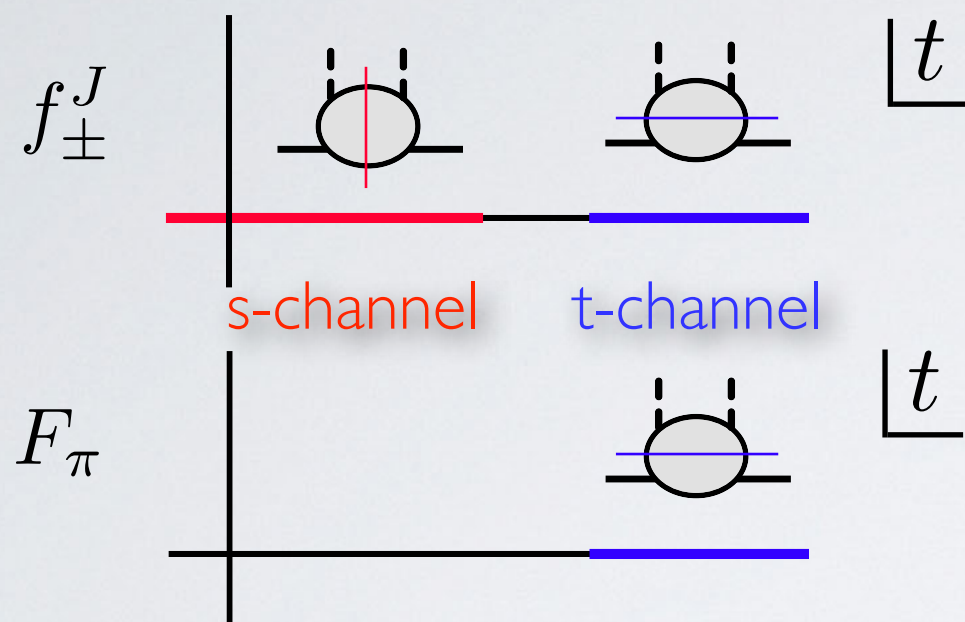
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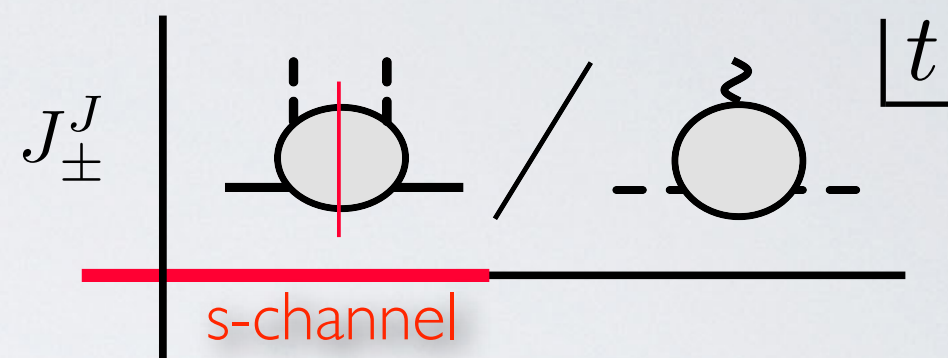
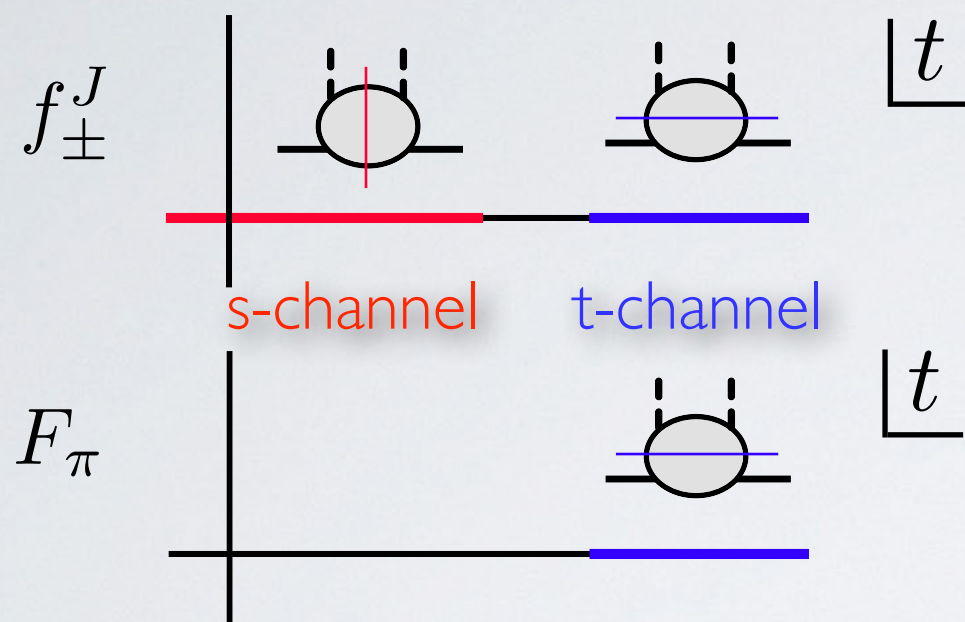
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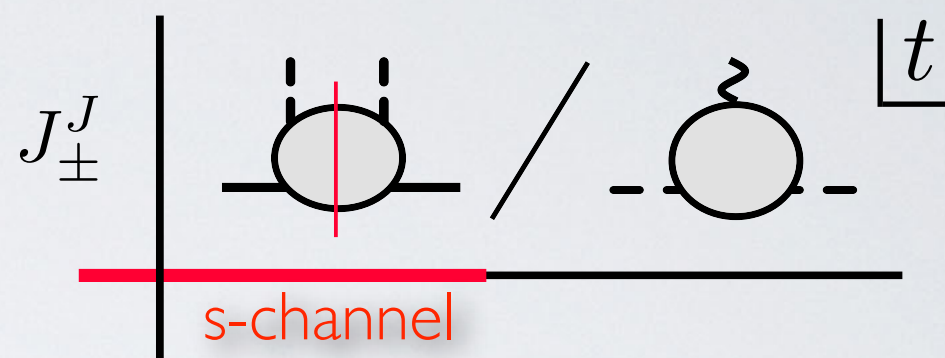
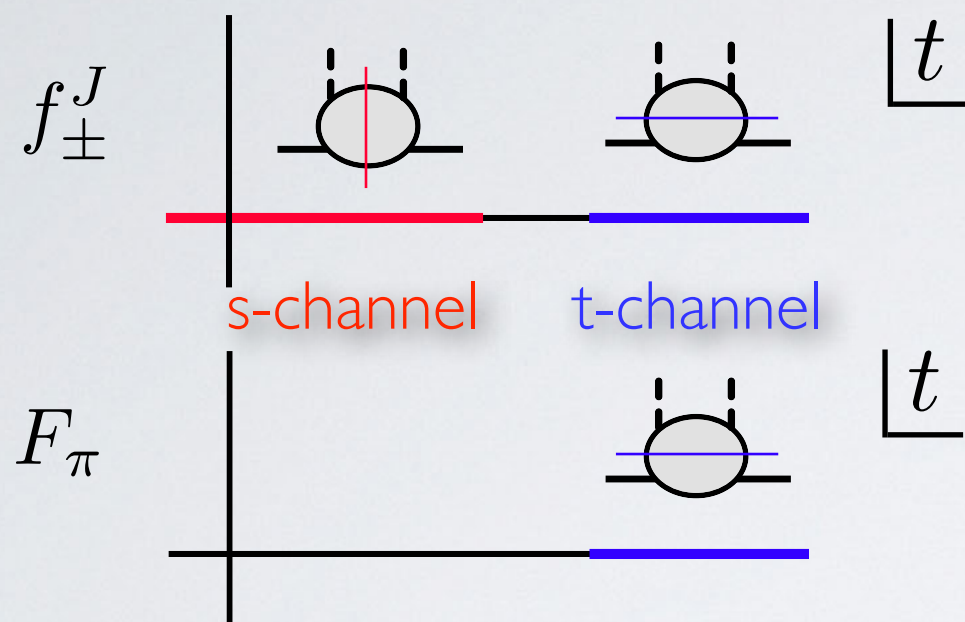
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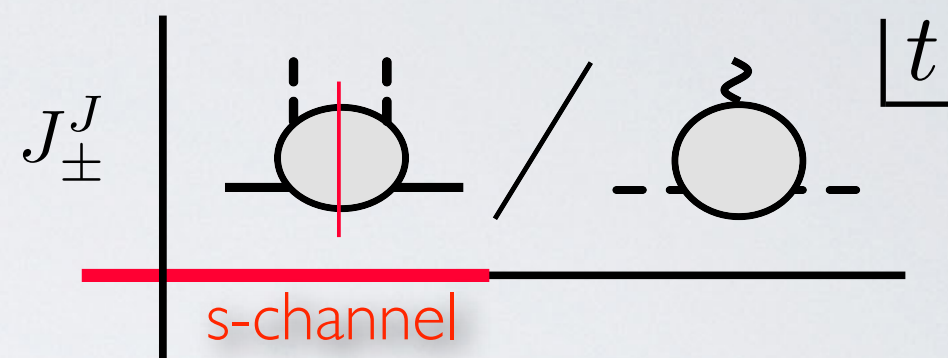
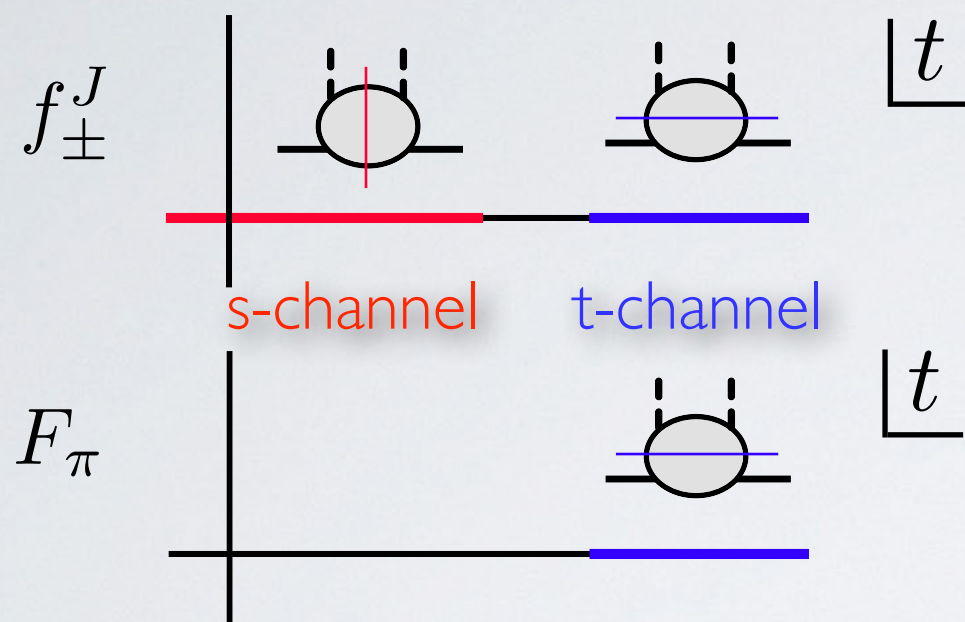
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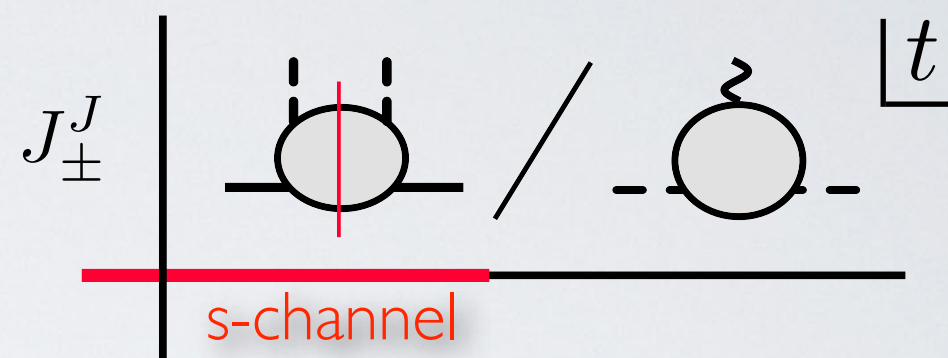
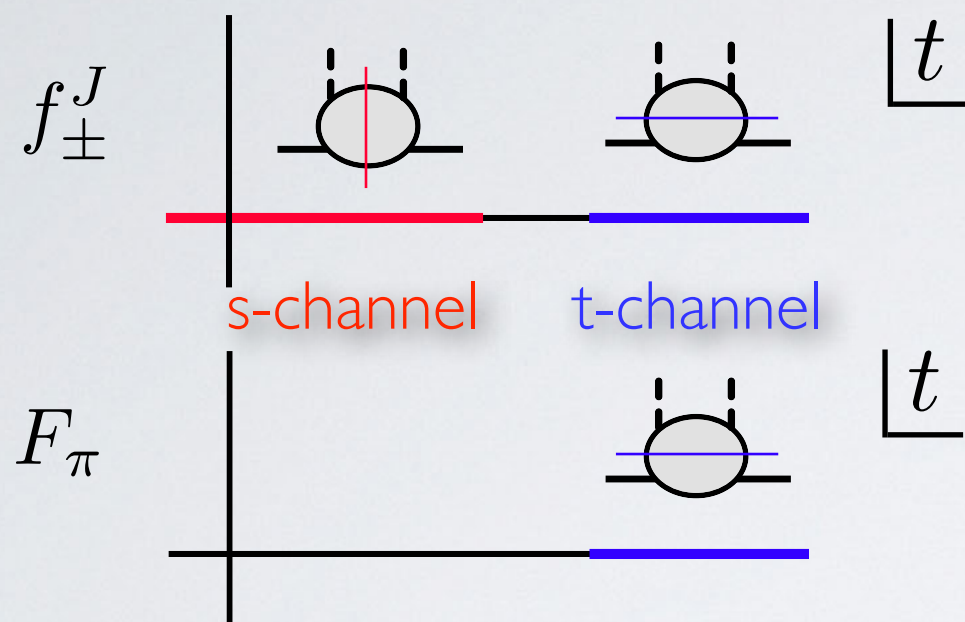
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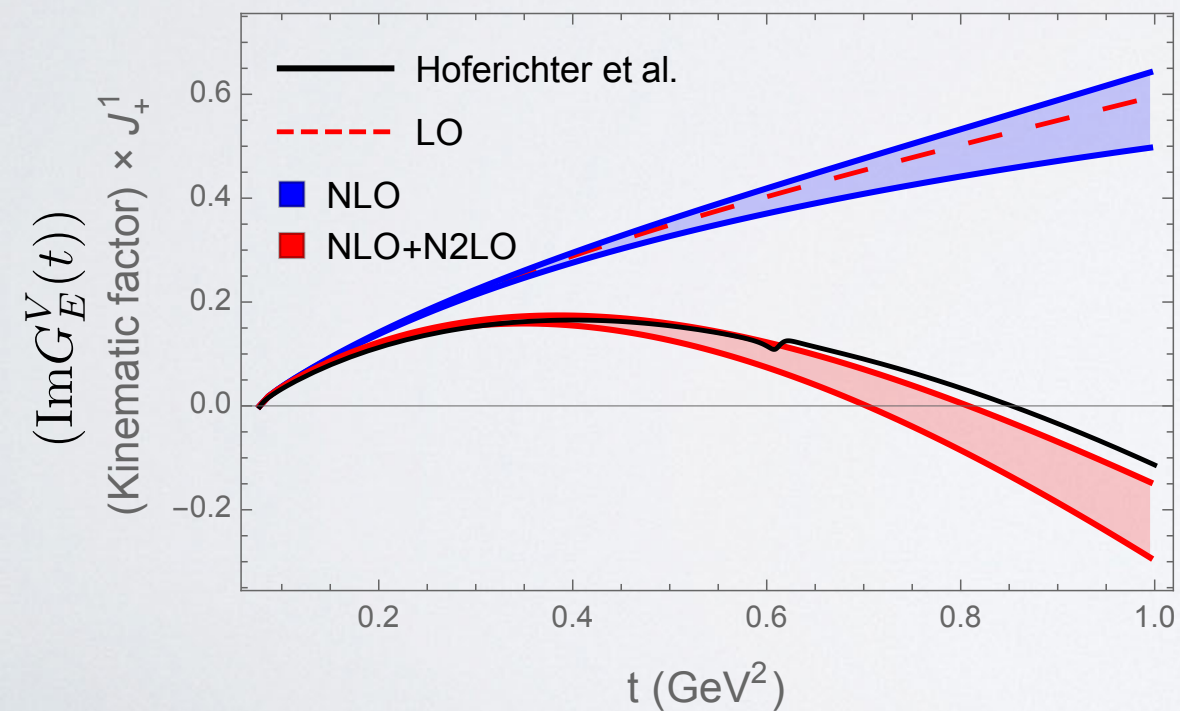
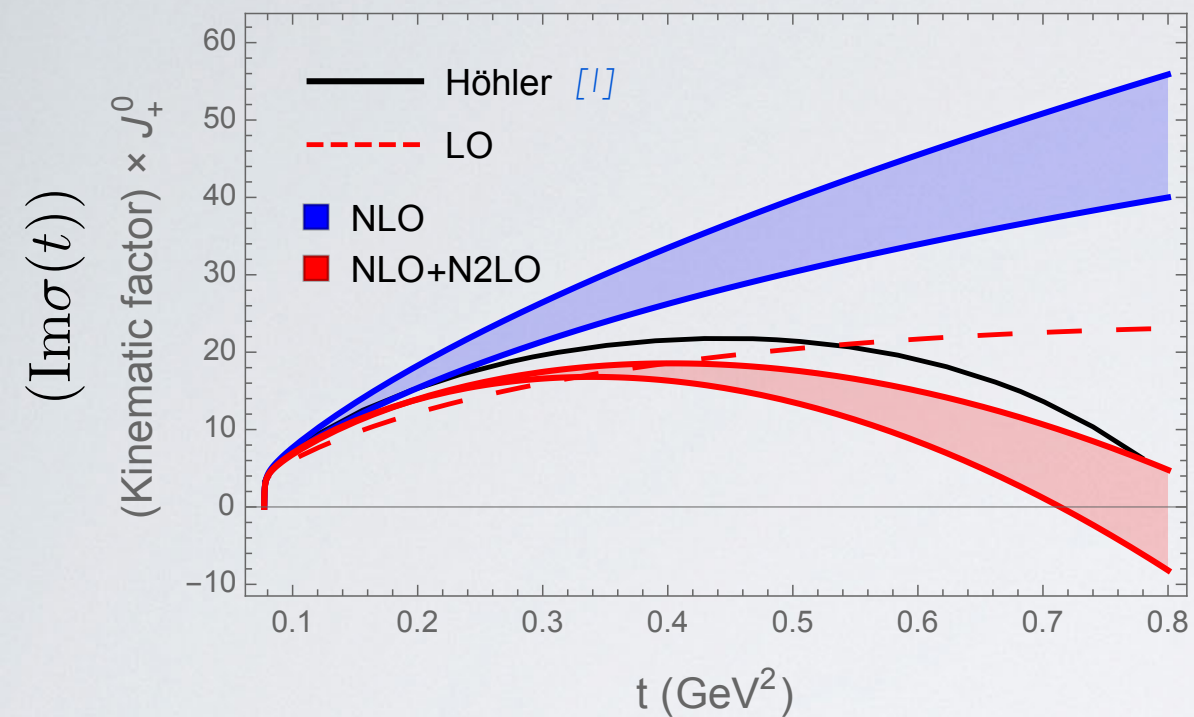
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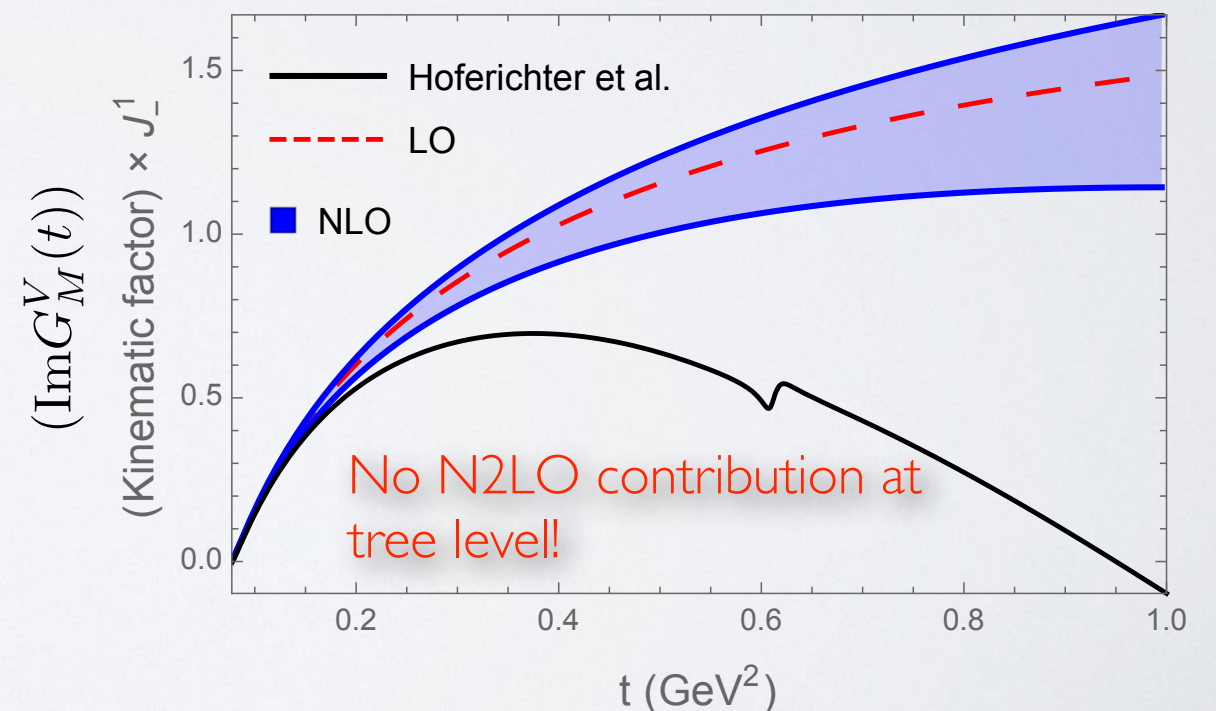
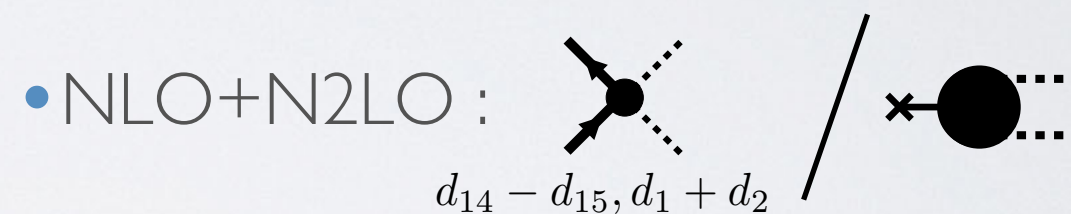
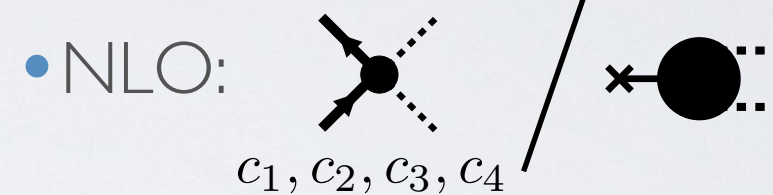
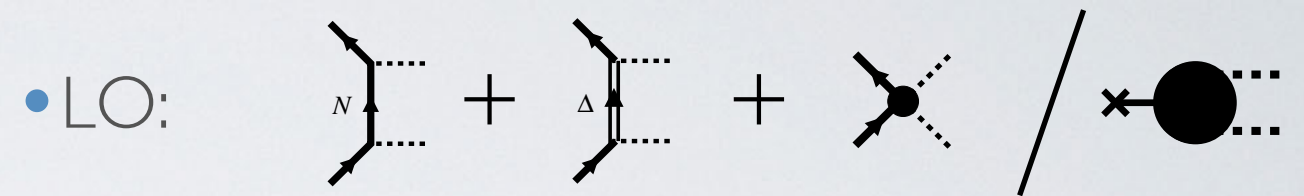
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$D\chi EFT$

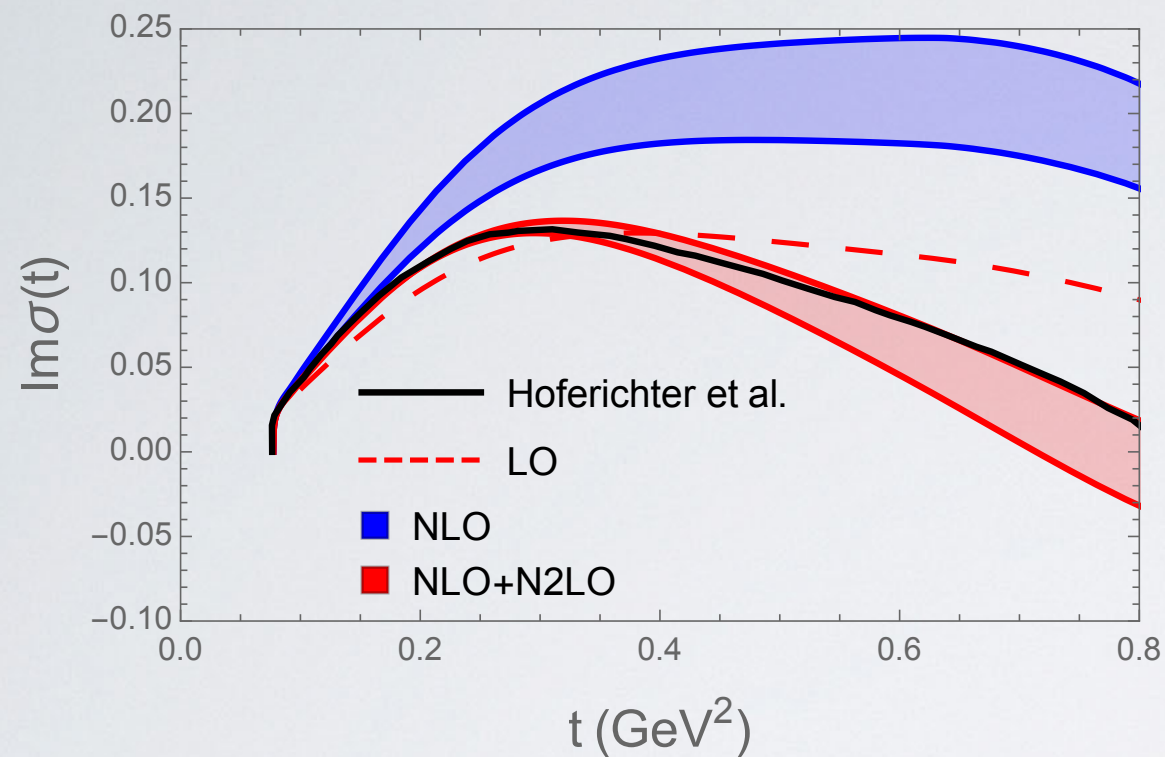


- $J_\pm^J \equiv f_\pm^J / F_\pi$ smooth → suited for ChEFT.



[1] Höhler, in Landolt-Börnstein, 9b2, ed. H. Schopper (Springer, Berlin, 1983)

$D|\chi EFT$



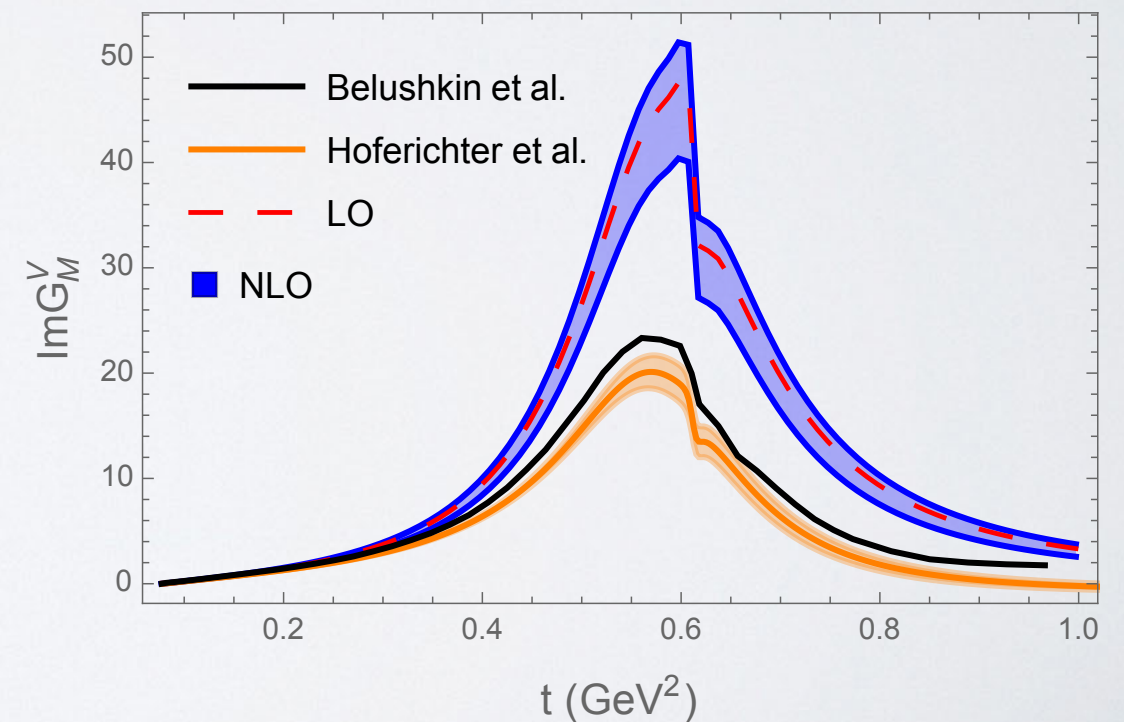
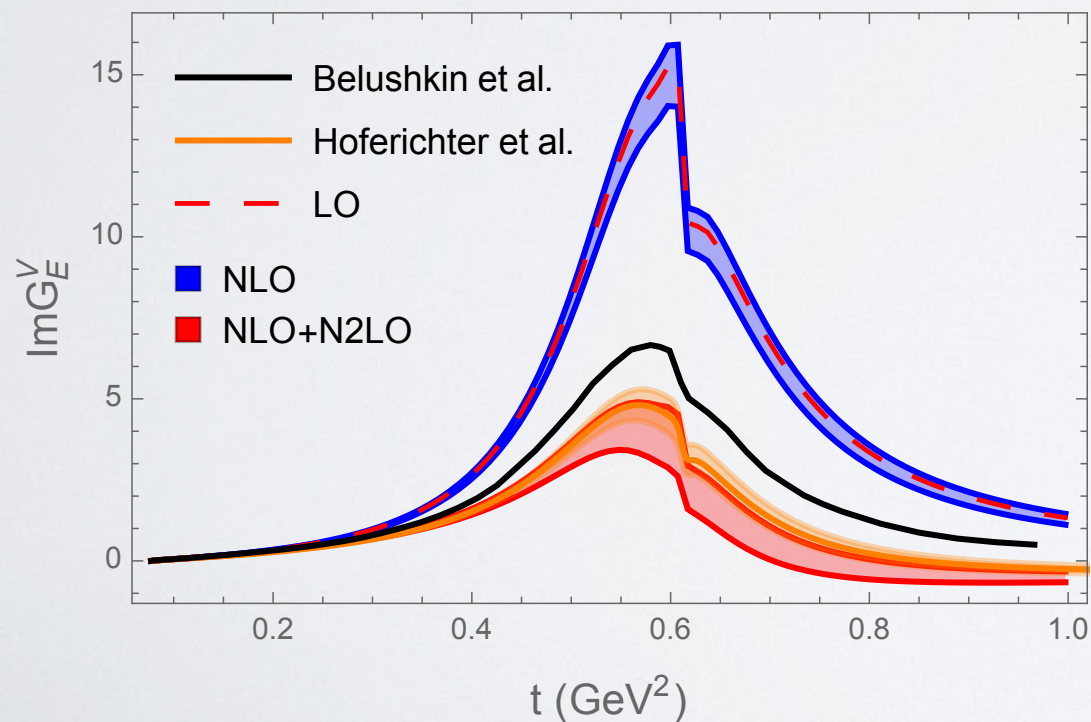
$$\text{Im}\sigma(t) = \frac{3k_{cm}}{4\sqrt{t}(m_N^2 - t/4)} |\Gamma_\pi(t)|^2 J_+^0(t)$$

$$\text{Im}G_{\{E,M\}}^V(t) = \frac{k_{cm}^3}{\{m_N, \sqrt{2}\}\sqrt{t}} |F_\pi(t)|^2 J_\pm^1(t)$$

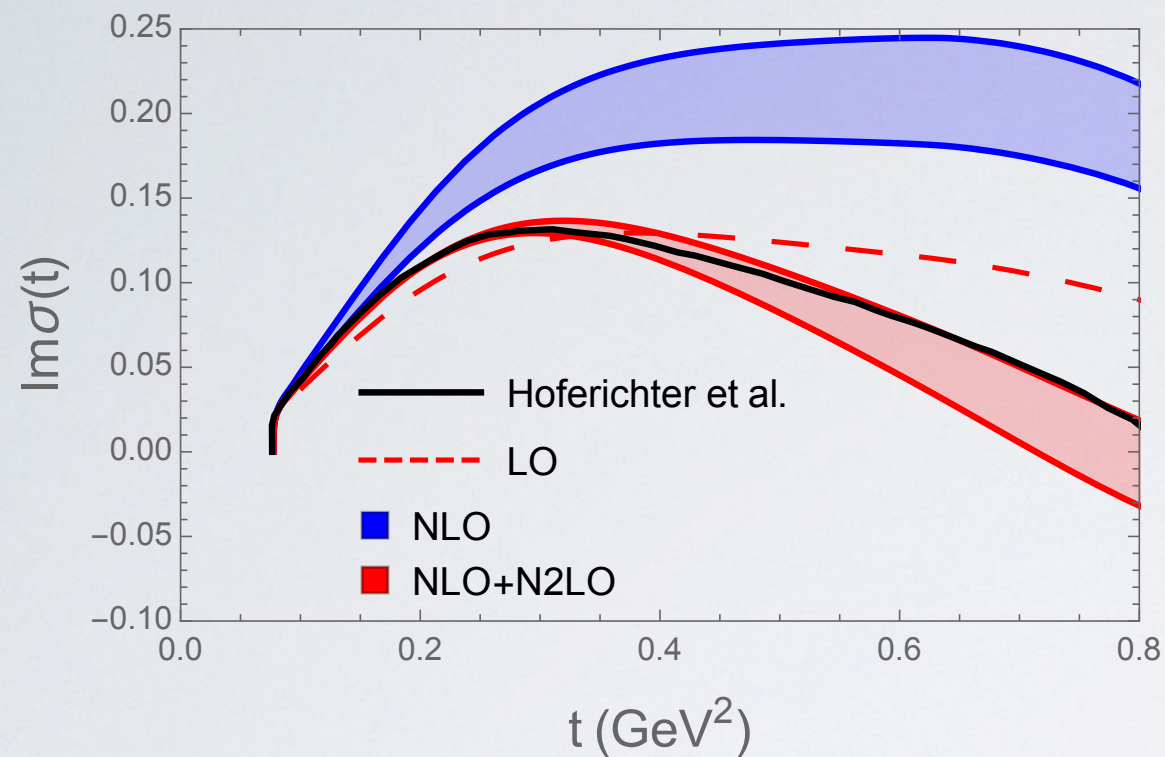
[1] Hoferichter, Ditsche, Kubis, Meißner, *JHEP* 063 (2012)

[2] Belushkin, Hammer and Meißner, *PRC* 75 (2007)

[3] Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meißner *EPJA* 52 (2016)



$D|\chi EFT$



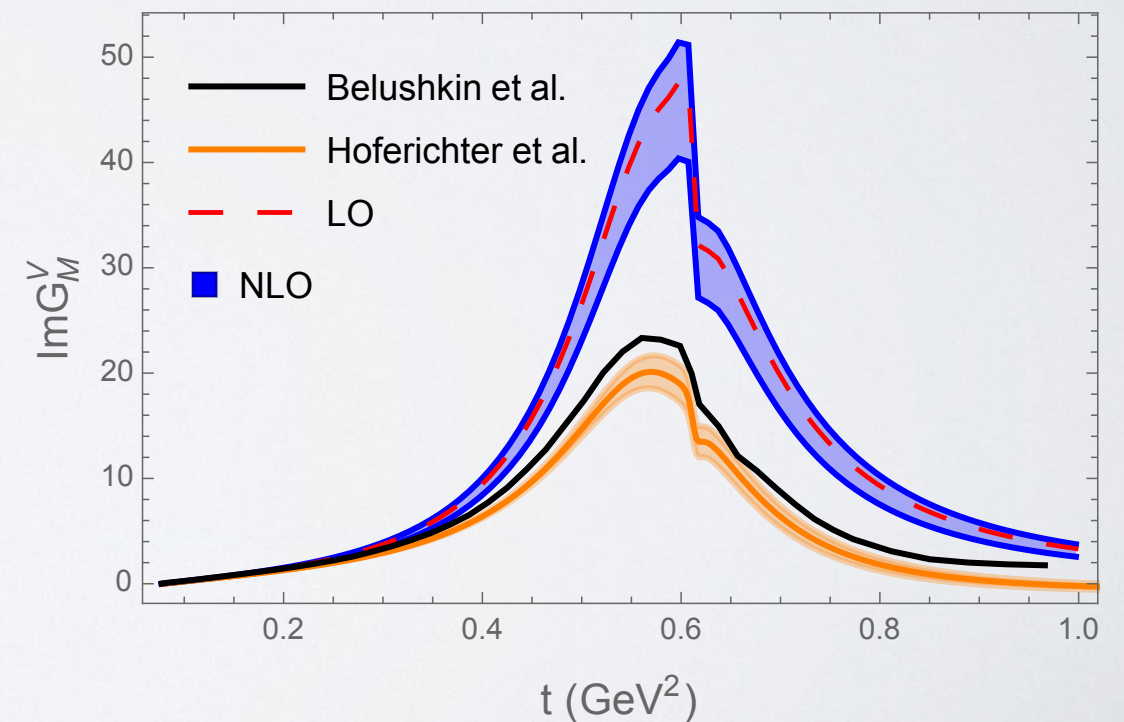
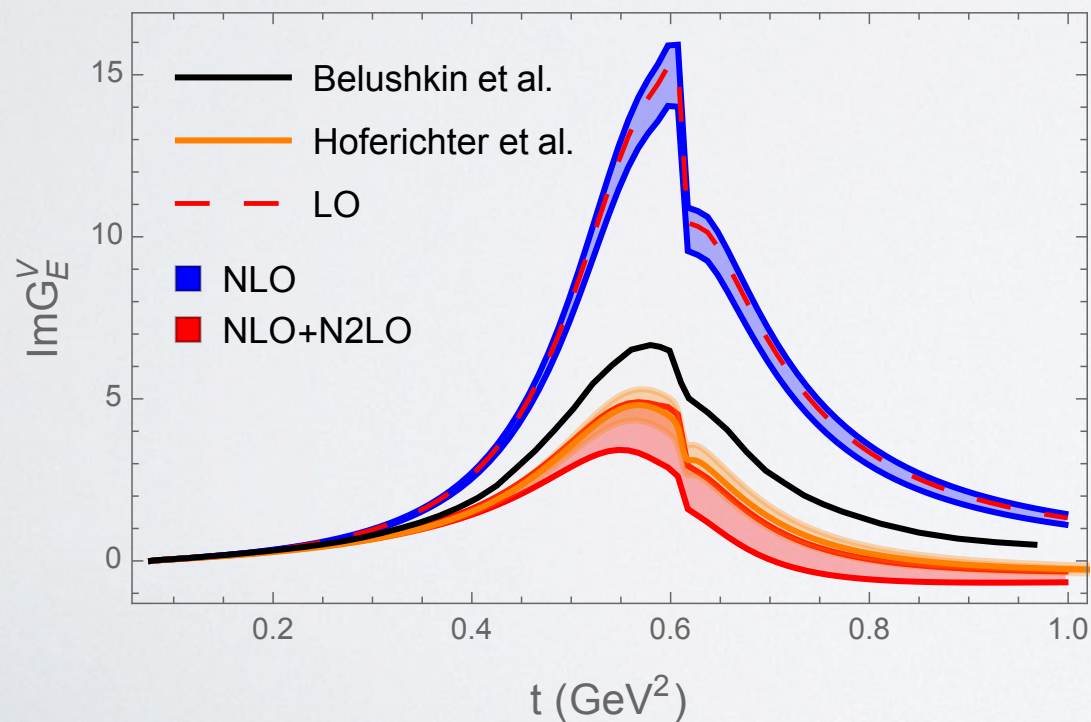
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[1] Hoferichter, Ditsche, Kubis, Meißner, JHEP 063 (2012)

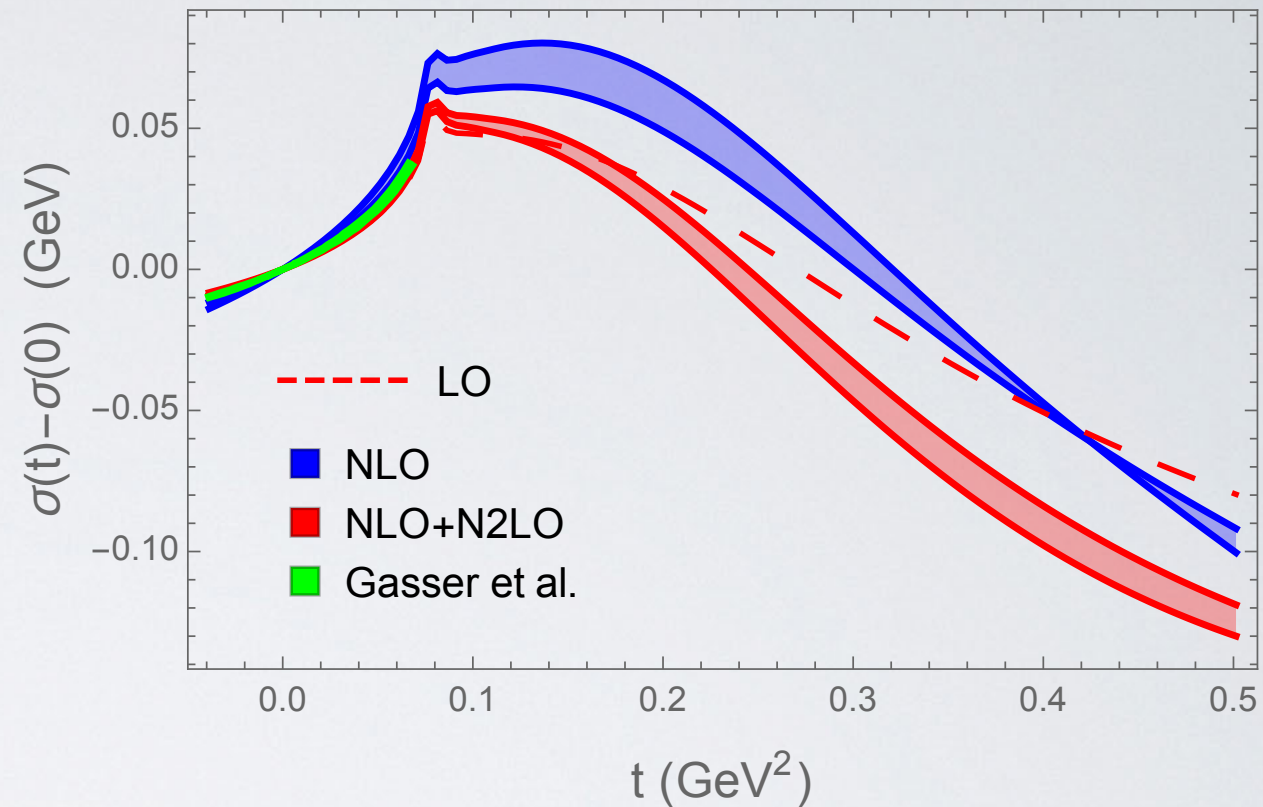
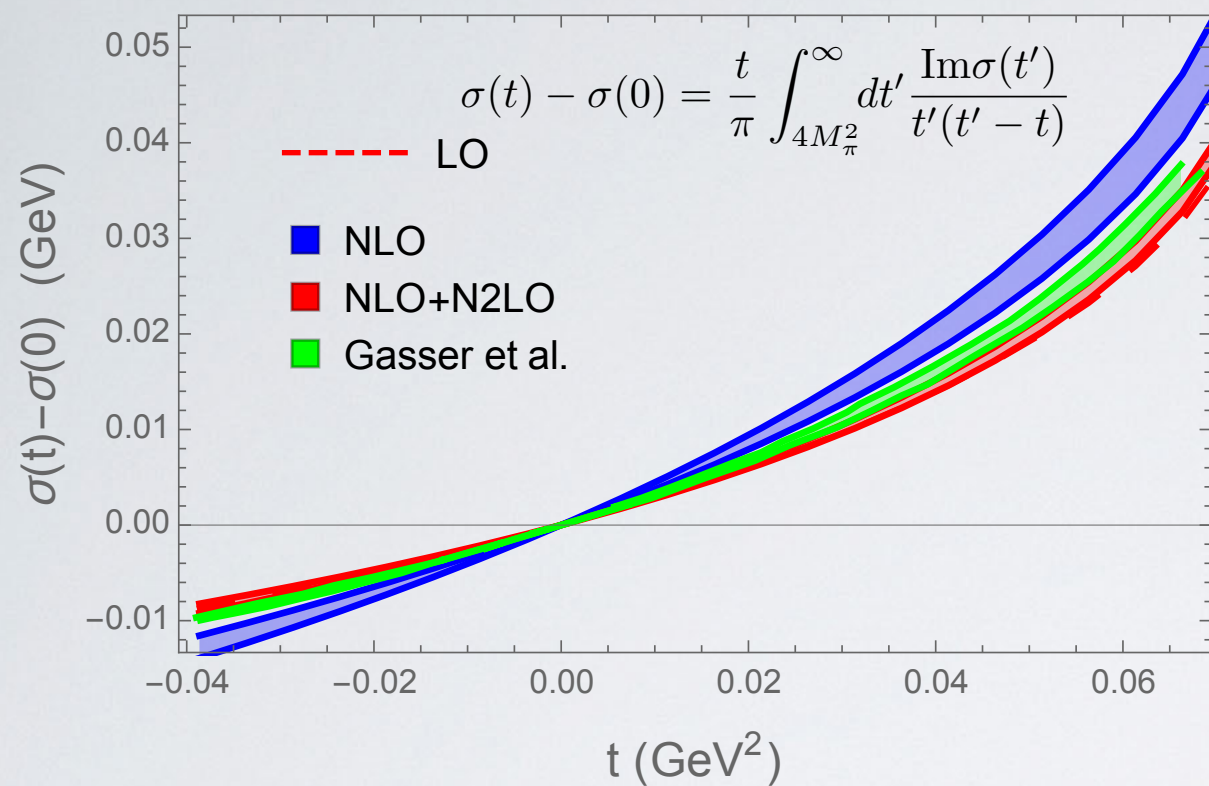
[2] Belushkin, Hammer and Meißner, PRC 75 (2007)

[3] Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meißner EPJA 52 (2016)



Scalar Form Factor

DI χ EFT



		LO	NLO	NLO+N2LO	GLS [1]	HKMS[2]
$\langle r^2 \rangle_S$ (fm ²)	$(\sigma(0) = 59 \text{ MeV})$	1.06	1.40–1.67	1.03–1.13	–	1.07(4)
	$(\sigma(0) = 45 \text{ MeV})$	1.38	1.83–2.19	1.34–1.49	1.6	–

	LO	NLO	NLO+N2LO	GLS [3]	HDKM [4]	ChPT $\mathcal{O}(p^3)$	ChPT $\mathcal{O}(p^4)$
Δ_σ (MeV)	13.3	17.4 - 20.6	13.3 - 14.5	15.2(4)	13.9(3)	4.6	$14.0 + 4M_\pi^4 \bar{e}_2$

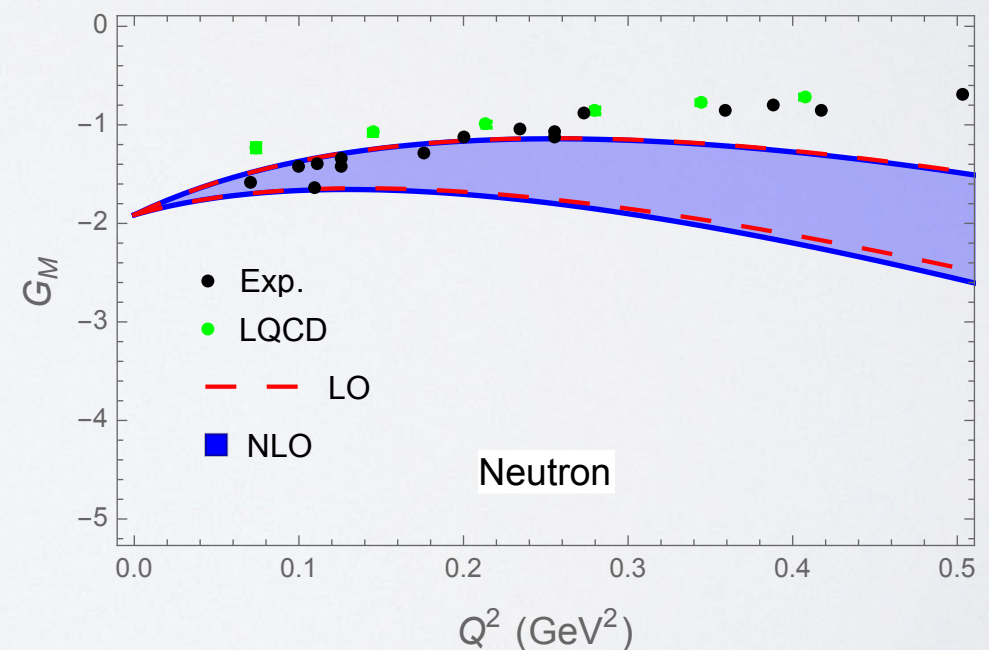
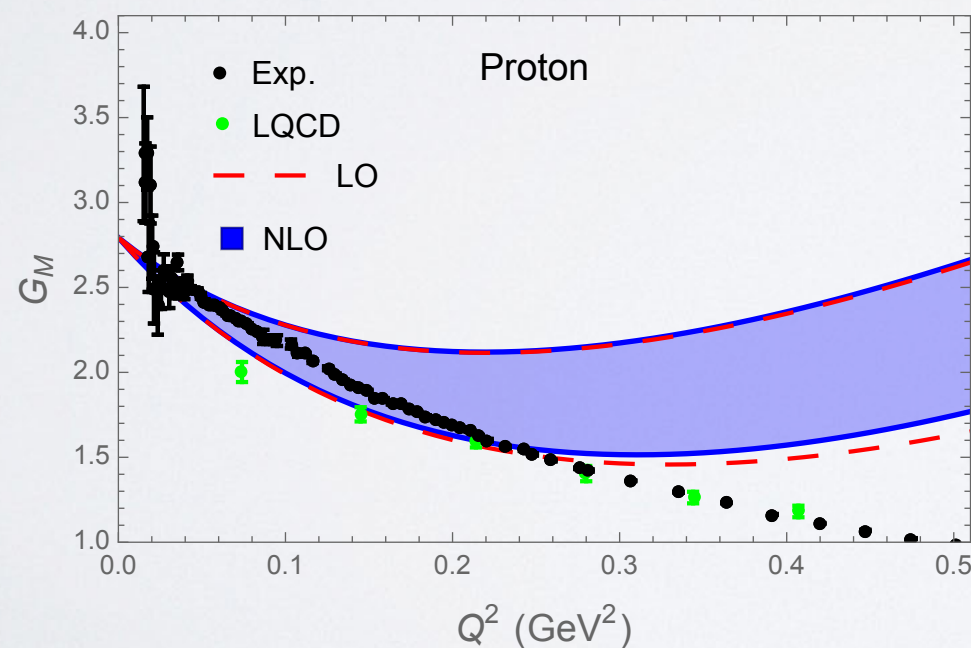
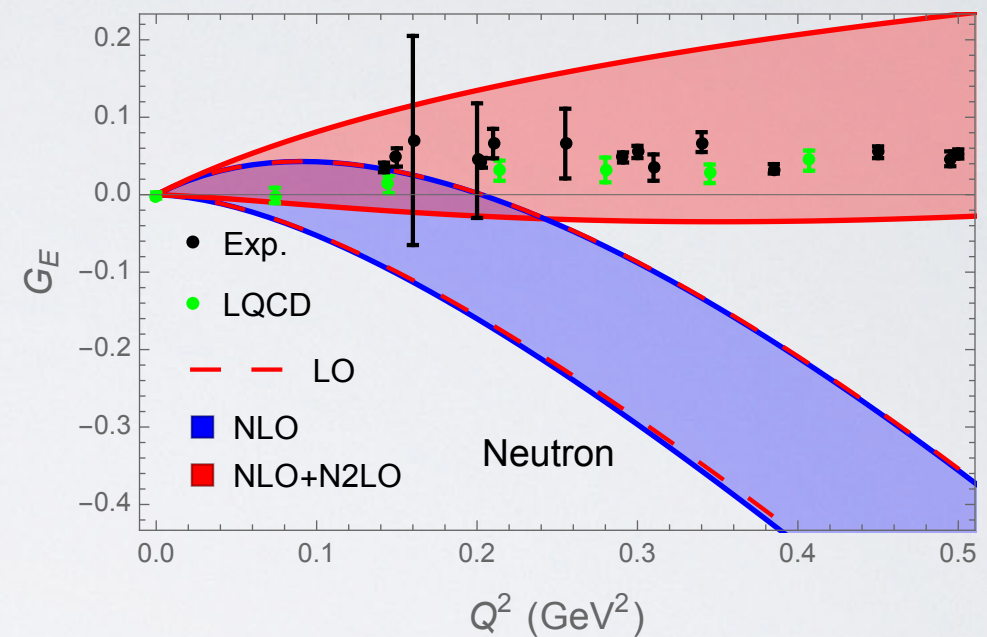
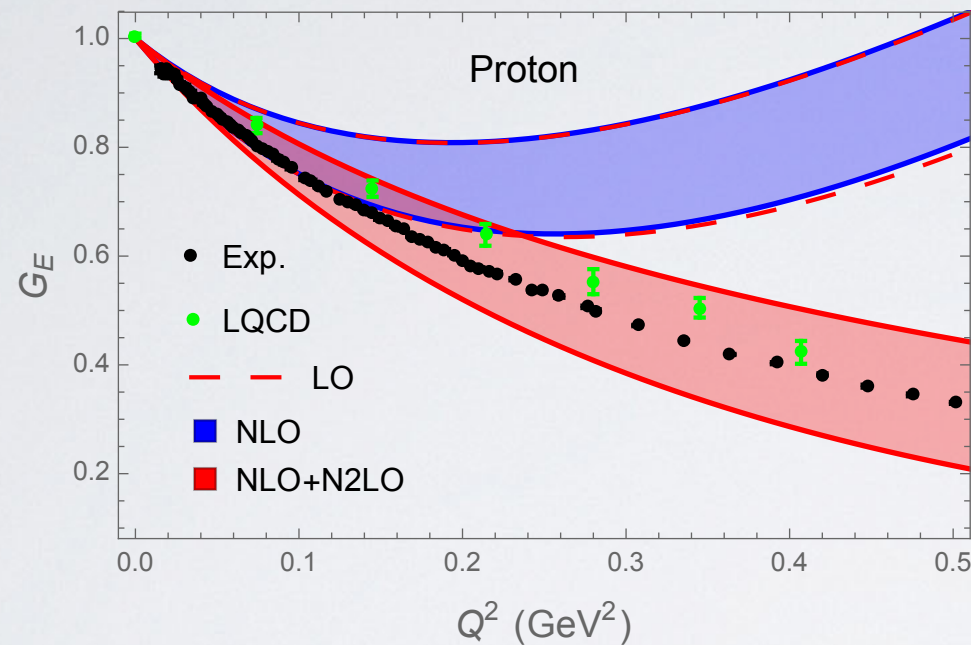
[1] Gasser, Leutwyler, Sainio, PLB 253 260-264, [2] Hoferichter, Klos, Menéndez, Schwenk PRD 94 (2016)

[3] Gasser, Leutwyler, Sainio, PLB 253 252-259, [4] Hoferichter, Ditsche, Kubis, Meißner, JHEP 1206 (2012)

Electromagnetic Form Factors

$DI\chi EFT$

$$G_{E,M}^{p,n}(Q^2) = G_{E,M}^{p,n}(0) \mp \frac{\langle r_{E,M} \rangle^V}{6} Q^2 \pm \frac{Q^4}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^V(t')}{t'^2(t' + Q^2)} - \frac{Q^2}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^S(t')}{t'(t' + Q^2)}$$



● *AI Collaboration* ● *ETM Collaboration, 1706.00469 (2017)*

- Radii

	LO	NLO	NLO+N2LO	Exp.
$\langle r_E^2 \rangle^p$ (fm ²)	(1.11, 1.49)	(1.05, 1.52)	(0.46, 0.94)	(0.71 , 0.77)
$\langle r_M^2 \rangle^p$ (fm ²)	(1.19, 1.46)	(1.04, 1.54)	–	(0.60 , 0.76)
$\langle r_E^2 \rangle^n$ (fm ²)	(-0.84, -0.47)	(-0.88, -0.40)	(-0.29, 0.18)	-0.12
$\langle r_M^2 \rangle^n$ (fm ²)	(1.29 , 1.64)	(1.08, 1.81)	–	(0.77, 0.79)

- Radii

	LO	NLO	NLO+N2LO	Exp.
$\langle r_E^2 \rangle^p$ (fm ²)	(1.11, 1.49)	(1.05, 1.52)	(0.46, 0.94)	(0.71 , 0.77)
$\langle r_M^2 \rangle^p$ (fm ²)	(1.19, 1.46)	(1.04, 1.54)	–	(0.60 , 0.76)
$\langle r_E^2 \rangle^n$ (fm ²)	(-0.84, -0.47)	(-0.88, -0.40)	(-0.29, 0.18)	-0.12
$\langle r_M^2 \rangle^n$ (fm ²)	(1.29 , 1.64)	(1.08, 1.81)	–	(0.77, 0.79)

- Higher moments

$$G_E(Q^2) = 1 - \frac{\langle r_E^2 \rangle}{3!} Q^2 + \frac{\langle r_E^4 \rangle}{5!} Q^4 - \frac{\langle r_E^6 \rangle}{7!} Q^6 + \frac{\langle r_E^8 \rangle}{9!} Q^8 + \dots$$

$$\frac{G_M(Q^2)}{\mu_N} = 1 - \frac{\langle r_M^2 \rangle}{3!} Q^2 + \frac{\langle r_M^4 \rangle}{5!} Q^4 - \frac{\langle r_M^6 \rangle}{7!} Q^6 + \frac{\langle r_M^8 \rangle}{9!} Q^8 + \dots$$

- Radii

	LO	NLO	NLO+N2LO	Exp.
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G_E^p	LO	NLO	NLO+N2LO
$\langle r^4 \rangle$ (fm ⁴)	(2.09, 2.48)	(2.00, 2.53)	(1.16, 1.70)
$\langle r^6 \rangle$ (fm ⁶)	(10.77, 11.70)	(10.46, 11.86)	(7.59, 9.00)
$\langle r^8 \rangle$ (fm ⁸)	(144.35, 148.22)	(142.04, 149.46)	(121.32, 128.74)

G_E^n	LO	NLO	NLO+N2LO
$\langle r^4 \rangle$ (fm ⁴)	(-1.53, -1.13)	(-1.58, -1.04)	(-0.74, -0.20)
$\langle r^6 \rangle$ (fm ⁶)	(-8.94, -8.02)	(-9.11, -7.71)	(-6.24, -4.84)
$\langle r^8 \rangle$ (fm ⁸)	(-135.09, -131.21)	(-136.32, -128.91)	(-115.60, -108.18)

G_M^p	LO	NLO
$\langle r^4 \rangle$ (fm ⁴)	(2.38, 2.68)	(2.14, 2.81)
$\langle r^6 \rangle$ (fm ⁶)	(13.91, 14.61)	(12.86, 15.17)
$\langle r^8 \rangle$ (fm ⁸)	(204.60, 207.57)	(193.78, 213.38)

G_M^n	LO	NLO
$\langle r^4 \rangle$ (fm ⁴)	(3.30, 2.87)	(3.49, 2.51)
$\langle r^6 \rangle$ (fm ⁶)	(19.62, 18.60)	(20.44, 17.07)
$\langle r^8 \rangle$ (fm ⁸)	(295.06, 290.72)	(303.54, 274.93)

Summary and Conclusions

Summary and Conclusions

- Chiral EFT can be combined with dispersion theory improve calculation of Form Factors.
- Studying the analytic structure of the matrix element allows us to separate the perturbative vs the non-perturbative part:
 - t-channel \rightarrow non-perturbative $\rightarrow |F_\pi|^2$ (data, lattice, dispersion theory)
 - s-channel \rightarrow perturbative \rightarrow ChEFT \rightarrow Prediction from πN scattering.
- $DI\chi EFT$ achieves good **predictions** for the spectral functions up to $t \sim 0.3 \text{ GeV}^2$ and potentially up to 1 GeV^2 .
- Direct application to $G = +1$ operators \rightarrow Scalar and EM FFs
 - EFT of DM detection (scalar FF).
 - Proton Radius Puzzle (higher order derivatives).
- Promising new approach to unveil the structure of the nucleon from first principles.

FIN

Spares

DI χ EFT

- We estimate the size of the N2LO corrections by considering only the tree level contributions.

- Born Terms are accounted for though $g_A \rightarrow g_A - 2d_{18}M_\pi^2$

- Contact terms depend on d_i

- Scalar

$$A^+ = -\frac{4\nu^2 m_N}{f_\pi^2} (d_{14} - d_{15})$$

$$B^+ = \frac{4\nu m_N}{f_\pi^2} (d_{14} - d_{15})$$

- Vector

$$A^- = \frac{2\nu}{f_\pi^2} [2(d_1 + d_2 + 2d_5)M_\pi^2 - (d_1 + d_2)t + 2d_3\nu^2] \quad B^- = 0$$

Dominant

- Estimate the value of $d_1 + d_2$ and $d_{14} - d_{15}$ by imposing the charge sum rules

$$\sigma(0) = \frac{1}{\pi} \int_{4M_\pi^2}^{\Lambda} dt' \frac{\text{Im}\sigma(t')}{t'} \quad | \text{ GeV}^2$$

$$G_{E,M}(0) = \frac{1}{\pi} \int_{4M_\pi^2}^{\Lambda} dt' \frac{\text{Im}G_{E,M}(t')}{t'} \quad | \text{ GeV}^2$$

- To reconstruct the EM form factors, we need the isoscalar component as well.
- One cannot apply the same approach as in the isovector case.
- In the isospin limit, only odd number of pions contribute ($G = -1$)
- The isoscalar component is dominated by the ω and ϕ exchanges.
- We model the isoscalar spectral functions through the exchange of these VM in the narrow width approximation.

$$\text{Im}F_i^S = \pi \sum_{V=\omega,\phi} a_i^V \delta(t - M_V^2) \quad (i = 1, 2)$$

a_1^ω	a_1^ϕ	a_2^ω	a_2^ϕ
(0.58, 0.85)	(-0.49, 0.26)	(-0.13, 0.38)	(-0.23, 0.28)

- We use SU(3) symmetry, some assumptions about the F/D ratio and empirical $g_{\omega NN}$ couplings from [\[Machleidt PRC 63 \(2001\)\]](#) [\[Belushkin et al., PRC 75 \(2007\)\]](#)