Recent measurements of exclusive hadronic cross sections at *BABAR* and the implications for the muon *g-2* calculation

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Salamanca

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Outline

- The question of the muon *g*-2
- *BABAR* and the initial-state radiation (ISR) method

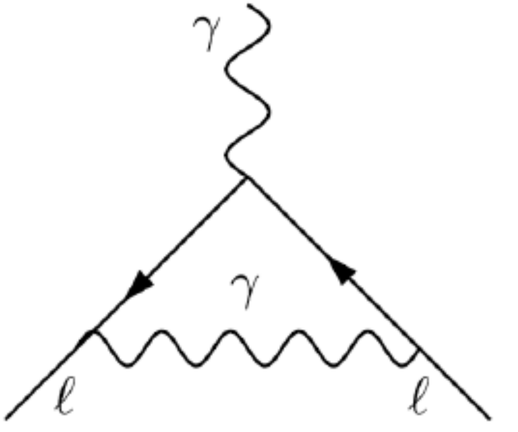
- Recent experimental results
  - $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ arXiv:1709.01171 (05-Sep-2017)
  - $e^+e^- \to \pi^+\pi^-\eta$ preliminary
  - $e^+e^- \to K_S K_L \pi^0$, $K_S K_L \pi^0\pi^0$, $K_S K_L \eta$ PRD 95, 052001 (2017)
  - $e^+e^- \to K_S K^+\pi^-\pi^0$, $K_S K^+\pi^-\eta$ PRD 95, 092005 (2017)
- Implications for the muon g-2

- Conclusions and perspectives

The anomalous magnetic moment of the lepton

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \qquad a = \frac{g-2}{2}$$

- (1928) pointlike Dirac particles: g=2, a=0
- (1948) anomaly discovered for the electron:
  - a_e^{exp}= (1.19 ± 0.05) 10^{-3} (Kusch-Foley)
- (1948) explained by O(α) QED corrections
  - $a_e^{th} = \alpha/2\pi = 1.16\ 10^{-3}$ (Schwinger)
- First triumph of QED!



1st order QED corrections

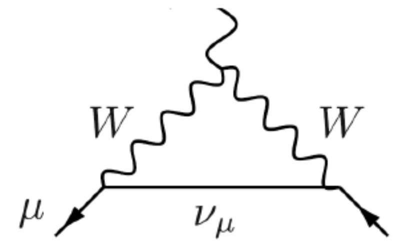
a_l sensitive to quantum fluctuations, not only from QED.
==> Must include all contributions for a precise calculation

More quantum fluctuations

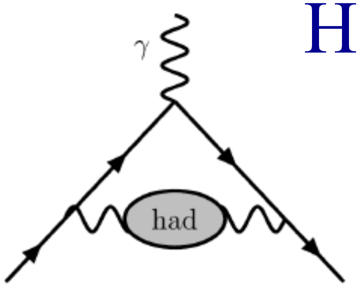
$$a^{Th} = a^{SM} + a^{NP}\ ? \qquad a^{SM} = a^{QED} + a^{had} + a^{EW}$$

QED

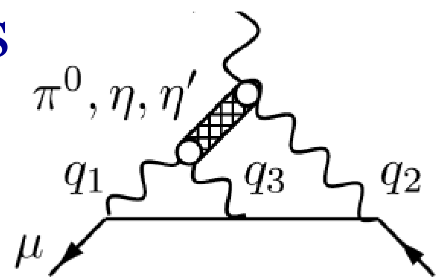
Electroweak



Hadrons

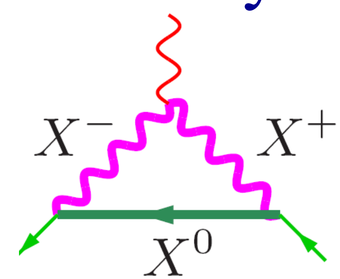


Vacuum Polarizations



Light-by-light scattering

New Physics



$$\delta a_\ell \propto \frac{m_\ell^2}{M^2}$$

a_μ much more sensitive than a_e to *NP*.
Typical gain of order $(m_\mu/m_e)^2 \approx 4 \times 10^4$

Status for a_μ before *BABAR* measurement

SM calculation *vs* experimental result (in units of 10^{-10})

QED	11 658 471.895	± 0.008
Weak	15.4	± 0.1
LO Hadronic Vacuum Polarization (HVP)	692.3	± 4.2
NLO HVP	-9.79	± 0.09
Hadronic Light by Light	10.5	± 2.6
a_μ^{SM}	11 659 180.2	± 4.9
a_μ^{exp} E821 @BNL: PRD73, 072003 (2006)	11 659208.9	± 6.3
$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM}$	**28.7**	**± 8.0**

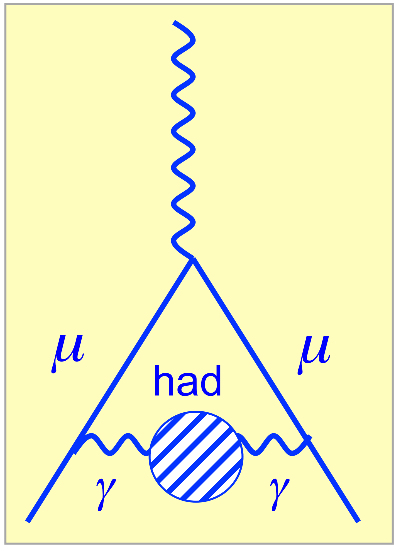
→ 3.6σ

DHMZ: Eur. Phys. J **C** 71, 1515 (2011)

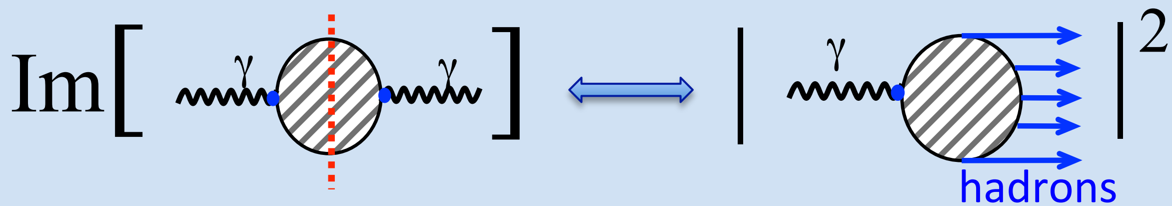
Main theoretical uncertainties from hadronic corrections;

- Light-by-Light: becoming important, improvements rely on lattice-QCD calculations
- Leading HVP : estimated using experimental data on e^+e^- annihilations

HVP calculations



- Quark loops not computable from QCD (low mass scale)
- Can use dispersion relations and optical theorem to relate the vertex corrections to the $e^+e^- \to hadrons$ cross section

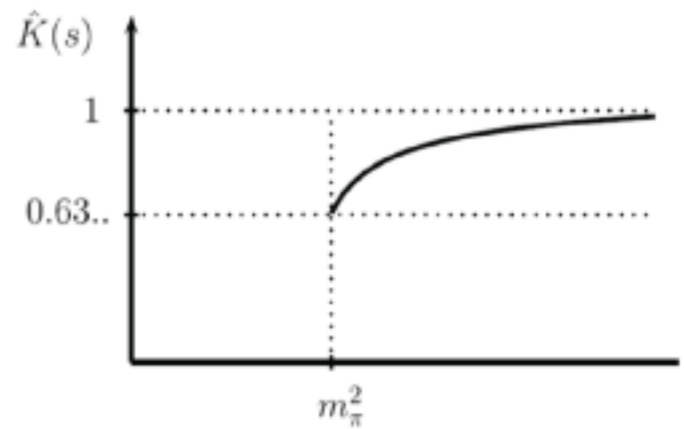


$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\,[e^+e^- \to \mathrm{hadrons}\,(\gamma)]}{\sigma_{pt}} \equiv R(s)\,; \quad \sigma^0(s) = \sigma(s)\left(\frac{\alpha}{\alpha(s)}\right)^2$$

Dispersion integral

$$a_\mu^{had,LO-HVP} = \frac{\alpha^2 m_\mu^2}{9\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, \frac{\hat{K}(s)}{s^2} R(s)$$

$K(s)/s^2 \sim 1/s^2$ emphasizes the role of the processes at low energies

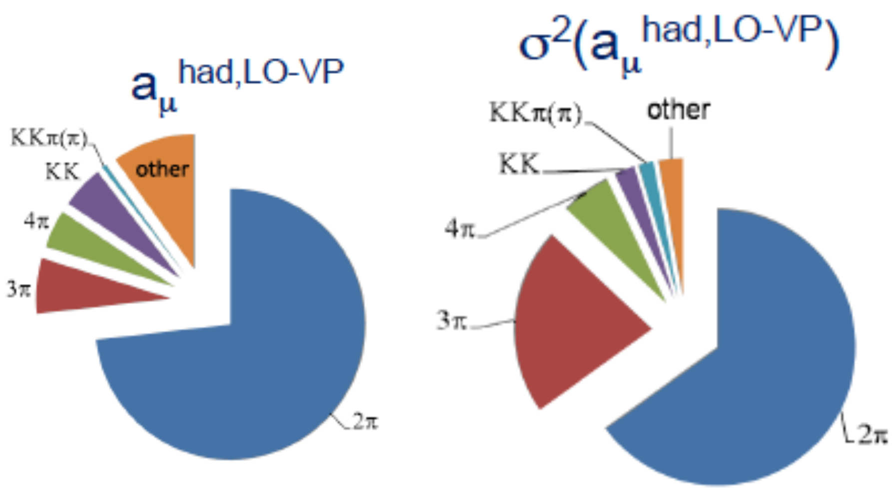


The hadronic cross section

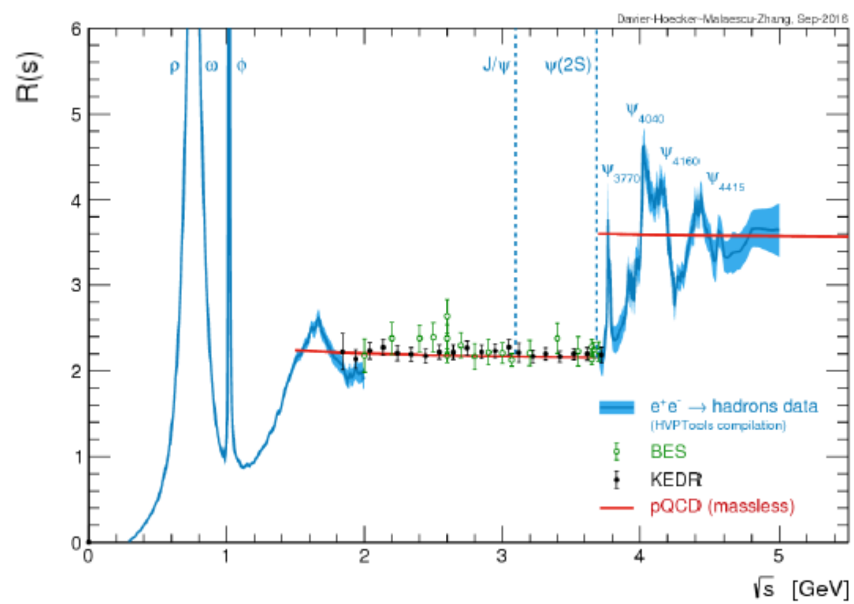
In order to calculate the total cross section:

- Sum up the main contributions from experimentally measured exclusive cross sections (ππ, 3π, 4π, KK, KKπ, KKππ,…)
- Estimate the missing channels (*e.g.* from isospin relations)
- Above a certain energy (typically 1.8 GeV), use pQCD and cross-check with inclusive $R(s)$ measurements

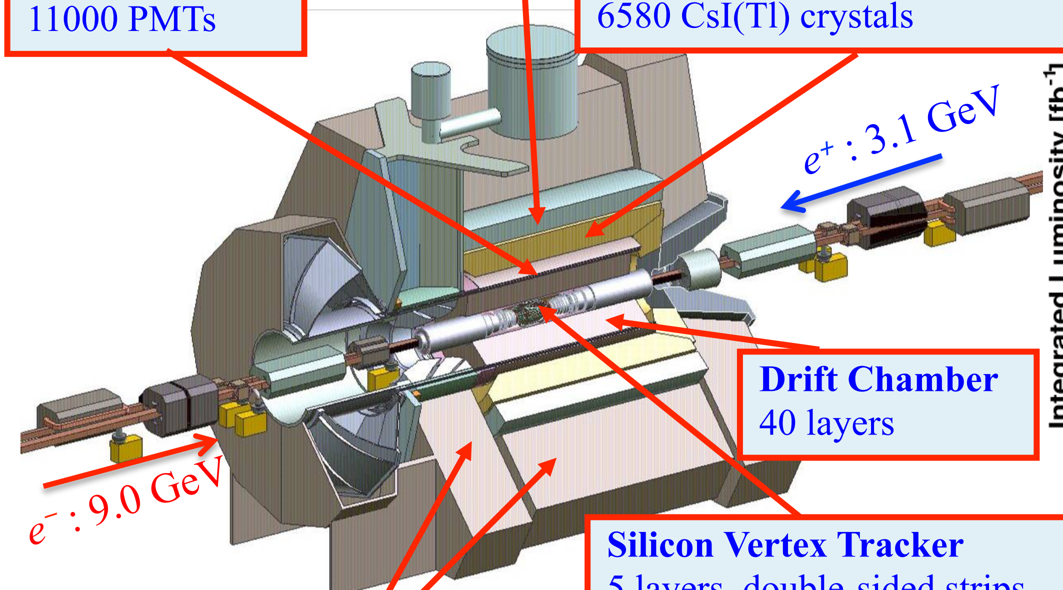
Main contributions to a_μ and to its error
(new *BABAR* measurements included)



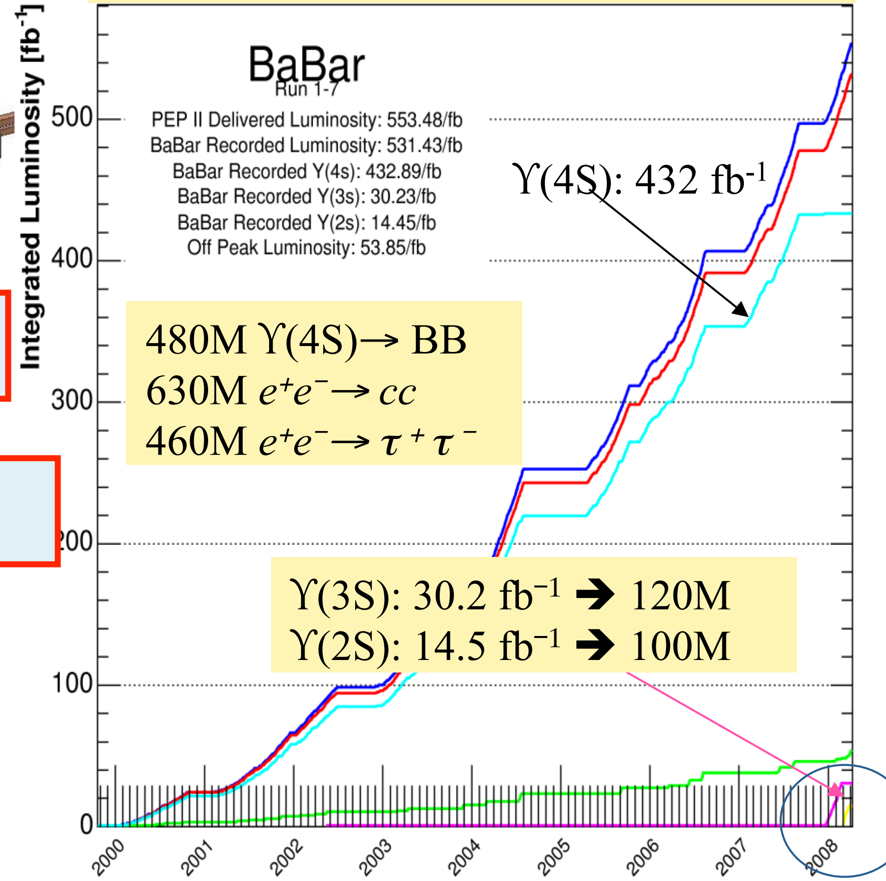
DHMZ , TAU 2016, arXiv:1612.02743



BABAR detector and collected data sample



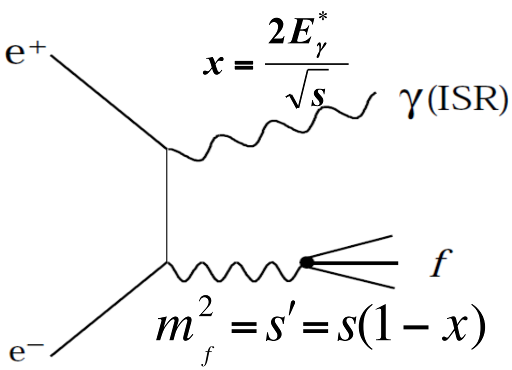
BABAR recorded luminosity and data set:



Detector details and performances in:
- NIM **A479**,1 (2002),
- NIM **A729**, 615 (2013)

Ten years of operation from 1999 to 2008
- Integrated luminosity: **~530 fb^{-1}**

Initial State Radiation at *B*-factories



$$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f)}{dm_f\, d\cos\theta^*_\gamma} = \frac{2m_f}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}(m_f)$$

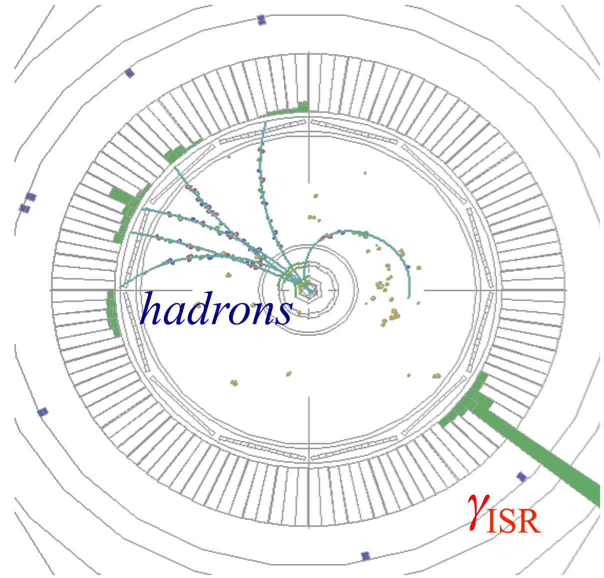
- The hadronic cross section $e^+e^-\to f$ can be extracted from the ISR cross section $e^+e^-\to\gamma f$.
- The radiator function *W(s,x)* is calculated in QED with accuracy better than 1% level

ISR studies at the Υ(4S) yield the same observables as low energy e^+e^- experiments!

- Quantum numbers at production vertex $J^{PC}=1^{--}$
- Continuous ISR spectrum:
  - Access a large energy range from threshold up to √s ~ 8 GeV
- α_{em} suppression compensated by the huge luminosity
- Comparable or better sensitivity than previous measurements based on energy scan

Common analysis strategy

- Events selection for an ISR tagged analysis:
  - ISR photon is the γ with highest E_γ^* & with E_γ^*>3 GeV
  - All particles detected inside a fiducial volume
  - Back-to-back topology between ISR γ and the rest of the event
  - $\pi/K/p$ discrimination using dE/dx and Cherenkov angle
  - Kinematic fit requiring $\vec{p}$ and E conservation
    - mass constraint for intermediate narrow states
  - Fit χ^2 used for signal selection and background subtraction



- Monte Carlo simulations and data control samples are used for detector acceptances, selection efficiencies and estimates of different background sources

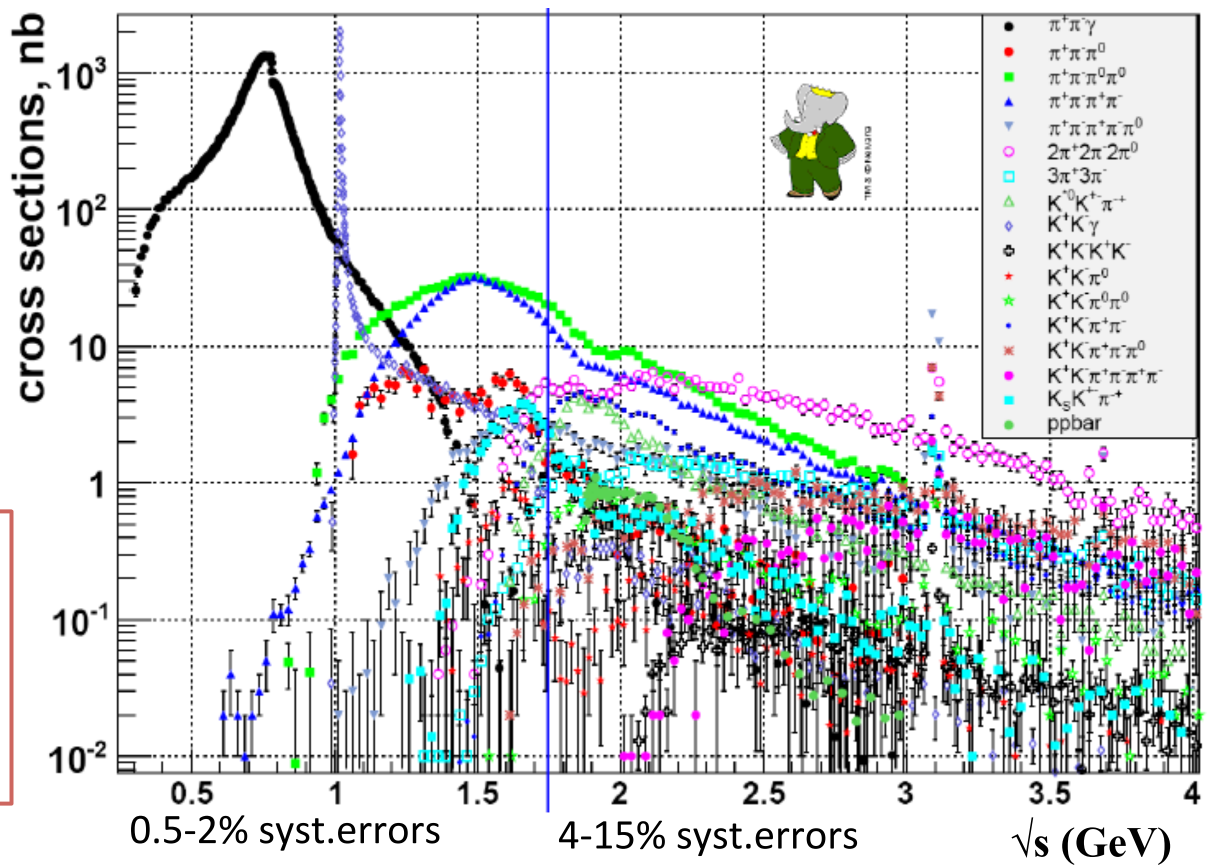
The ISR program for Light Quark Mesons in *BABAR*

Summary of cross sections measured by BABAR

- long list of published papers
- >20 final states studied

- Four new analyses, with 7 final states discussed in this talk

✓$\pi^+\pi^-\pi^0\pi^0$
✓$\pi^+\pi^-\eta$
✓$K_SK_L\pi^0$, $K_SK_L\eta$, $K_SK_L\pi^0\pi^0$
✓$K_SK^+\pi^-\pi^0$, $K_SK^+\pi^-\eta$

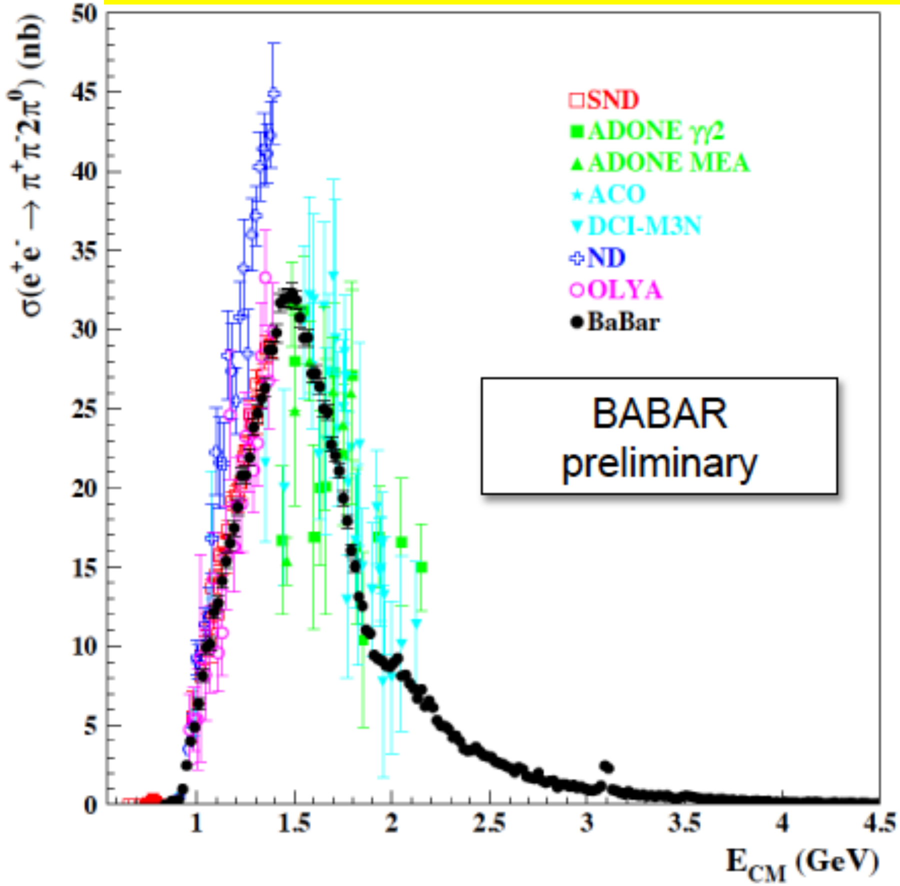


Courtesy Fedor V. Ignatov, via J. Chauveau

Before *BABAR* measurement (for most channels):
- limited precision
- big differences among experiments
- small energy ranges covered

$e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$

arXiv:1709.01171 (submitted to Phys.Rev.D)



E_{CM}(GeV)	Syst. unc.
1.2 – 2.7	3.1%
2.7 – 3.2	6.7%
> 3.2	7.1%

- Dominant ISR-background $\pi^+\pi^-3\pi^0$ removed by using data
- Most precise measurement to date
- Widest energy range
  - $0.85 < E_{CM} < 4.5$ GeV

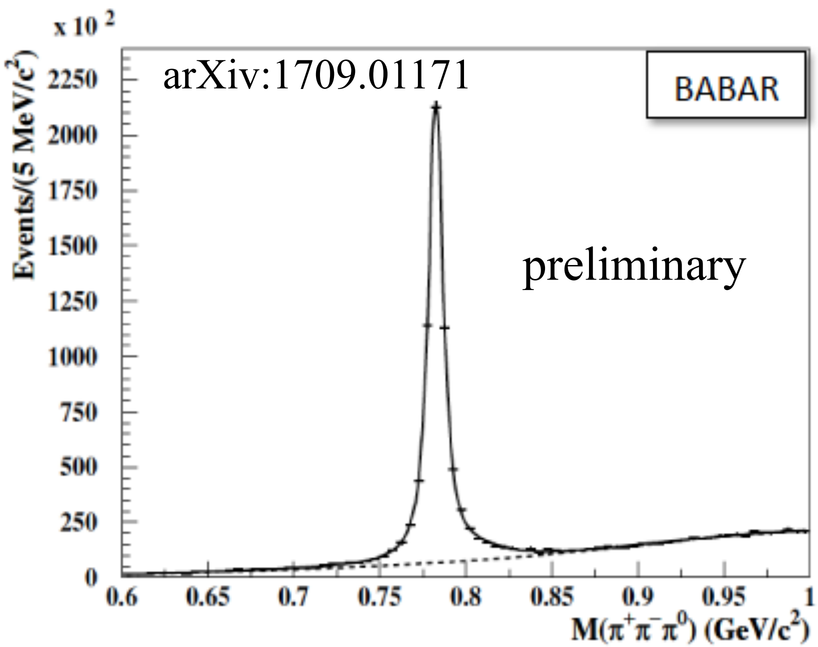
Contribution to $g_\mu - 2$

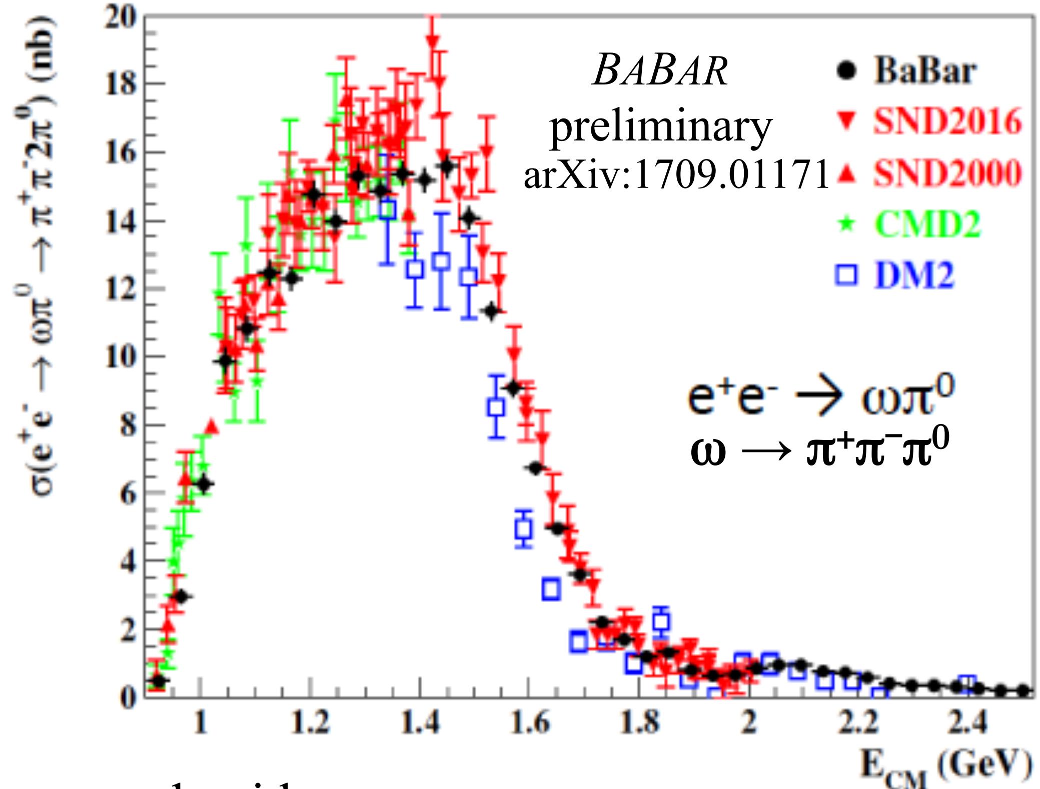
$$a_\mu(0.85 < \sqrt{s} < 1.8\,\text{GeV}) = (17.9 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-10}$$

- BABAR data reduce uncertainty in $a_\mu^{\pi+\pi-\pi0\pi0}$ by a factor of 2.5

$e^+e^- \rightarrow \omega\pi^0 \rightarrow (\pi^+\pi^-\pi^0)\pi^0$

Large contribution from $e^+e^- \rightarrow \omega\pi^0$ with $\omega \rightarrow \pi^+\pi^-\pi^0$

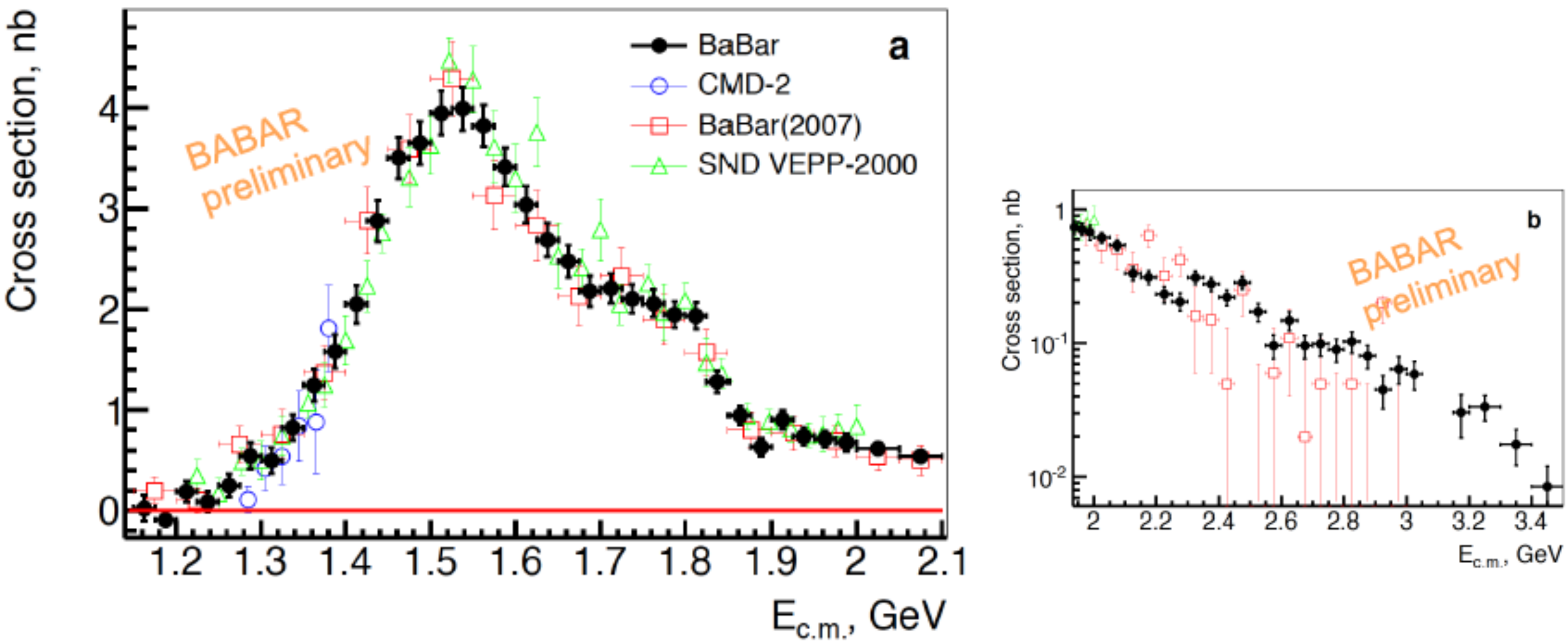




- BABAR data more precise and on much wider range than previous experiments.
- Some discrepancies resolved.

$e^+e^- \to \pi^+\pi^-\eta,\ \eta\to\gamma\gamma$

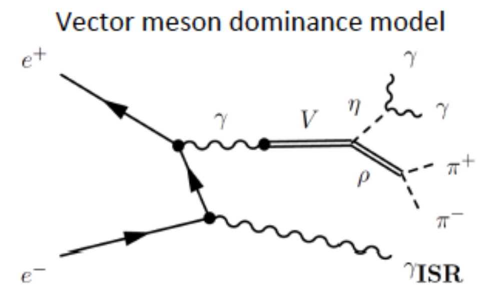
- About 8000 signal events
- Complements and improves the precision of the *BABAR* result from 2007 [PRD 76, 092005], based on 232 fb^{-1} and the $\eta\to\pi^+\pi^-\pi^0$ decay mode



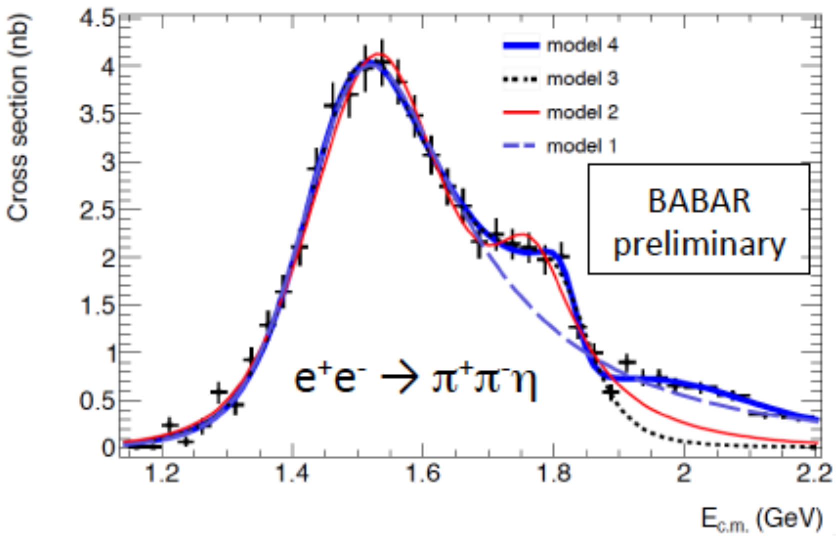
- Data in agreement with previous data
- Similar precision below 1.6 GeV, significantly better accuracy above
- The total systematic uncertainty near the cross section maximum, 1.35-1.80 GeV, is 4.5%

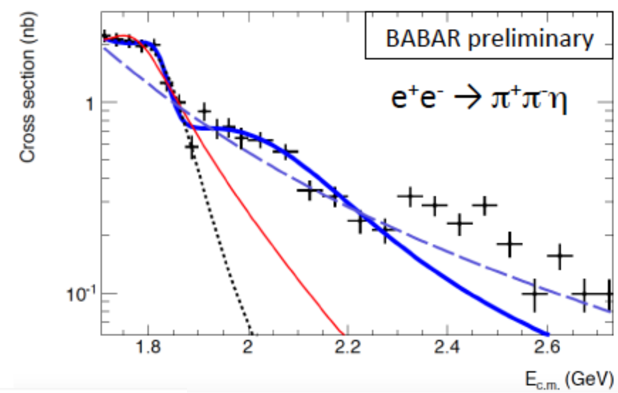
$e^+e^- \rightarrow \pi^+\pi^-\eta,\ \eta\rightarrow\gamma\gamma$

- Process dominated by the $\rho(770)\eta$ intermediate state, but it has a complex E_{CM} structure



V = ρ(770), ρ(1450), ρ(1700), higher ρ excitations





model	Resonance model	Good fit for
0	ρ(770) + ρ(1450)	Doesn't fit
1	ρ(770) − ρ(1450)	E_{cm} < 1.7 GeV
2	ρ(770) − ρ(1450) − ρ(1700)	E_{cm} < 1.9 GeV
3	ρ(770) − ρ(1450) + ρ(1700)	E_{cm} < 1.9 GeV
4	ρ(770) − ρ(1450) + ρ(1700) + ρ(2150)	E_{cm} < 2.2 GeV

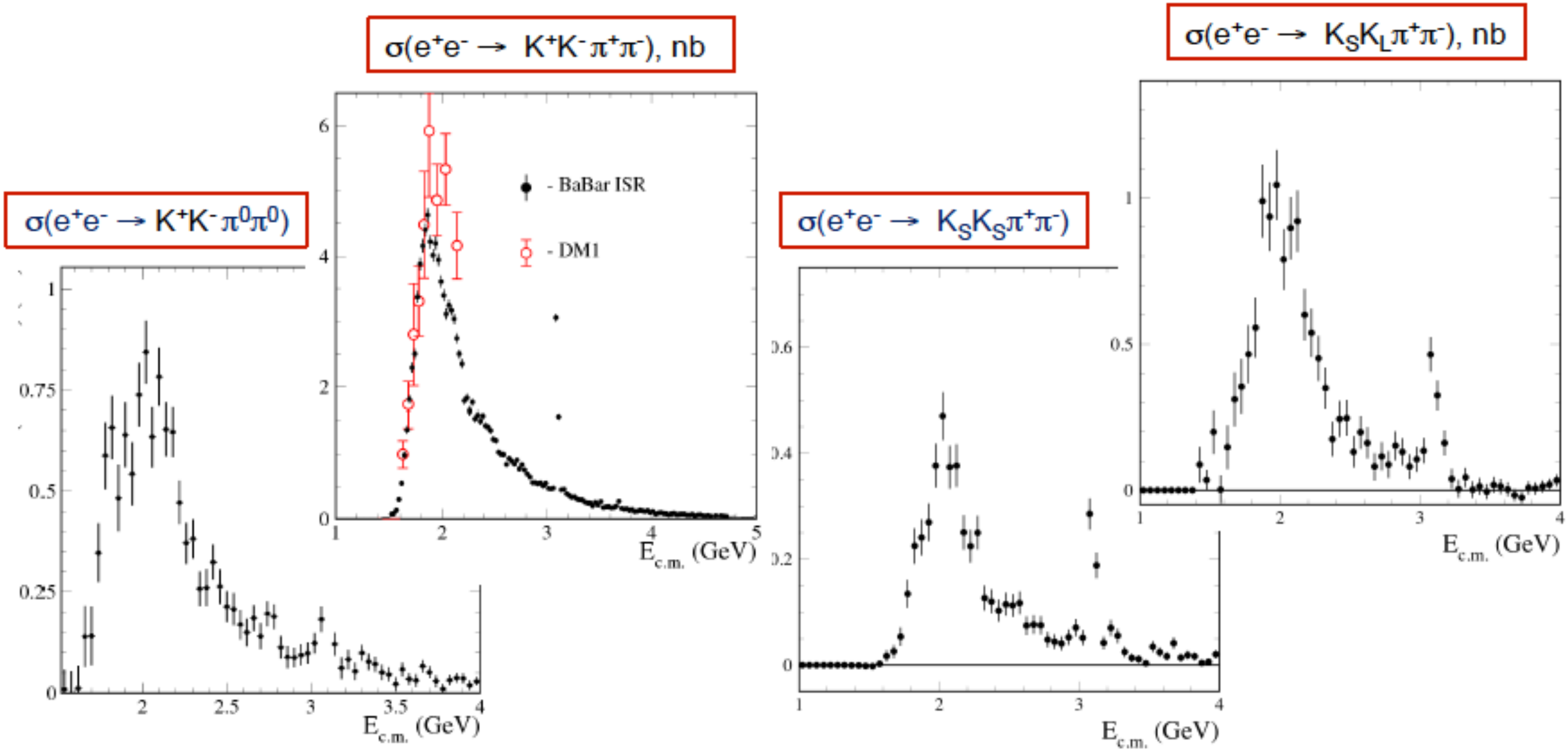
- Coupling constants governing the decays ~real: phase differences are 0 or π only
- Need an additional resonance to describe data above E_{cm}= 2.3 GeV

$e^+e^- \rightarrow K\bar{K}\pi\pi$

- Six different combinations in the $e^+e^- \rightarrow KK\pi\pi$ processes. Four were previously measured by *BABAR*:

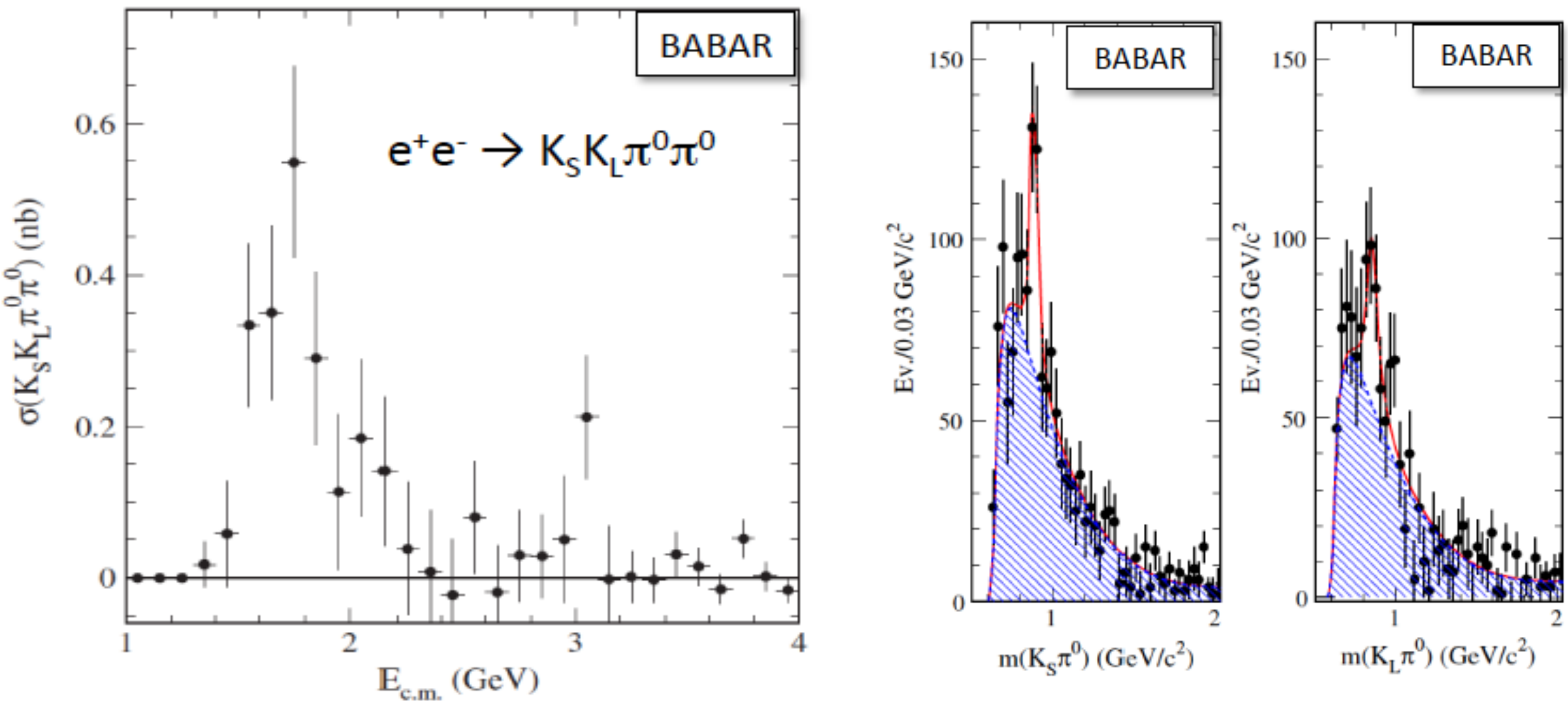
Phys. Rev. D 86, 012008 (2012)

Phys. Rev. D 89, 092002 (2014)



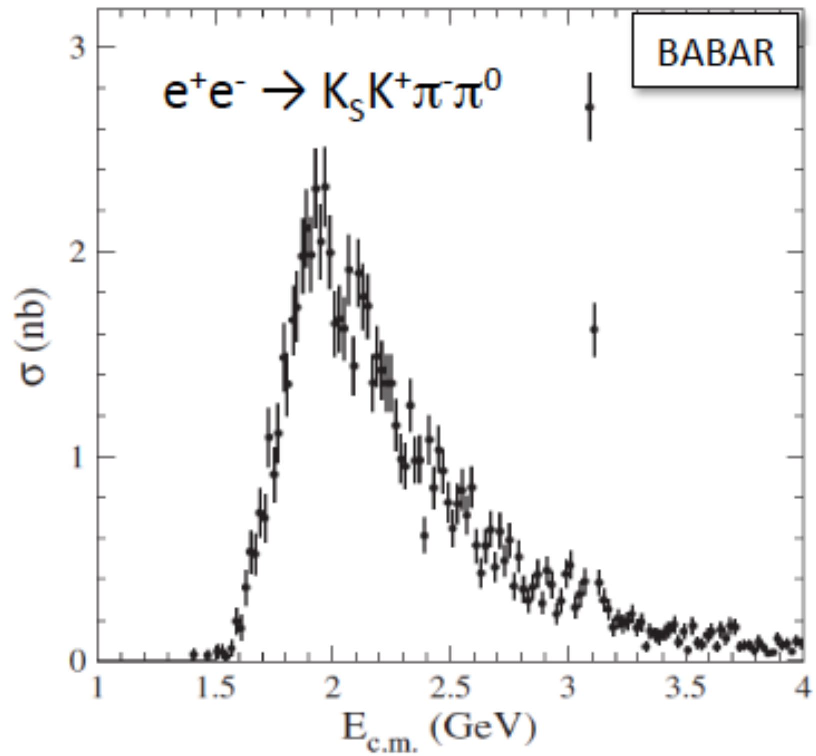
$e^+e^- \rightarrow K_S K_L \pi^0 \pi^0$

Phys. Rev. D95, 052001 (2017)



- First measurement of this channel
- Systematic uncertainties are 25% at the peak, grows to 60% at 2 GeV
- Dominant contribution from K*(892)Kπ intermediate state

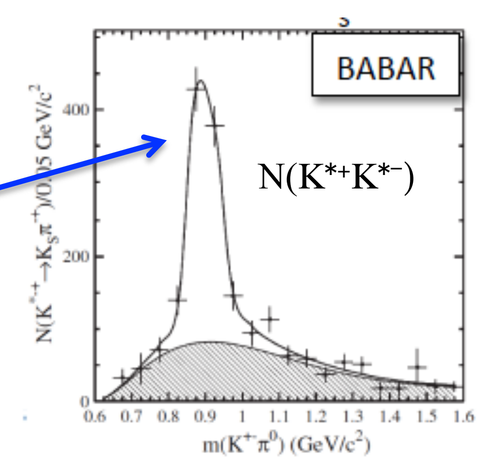
$e^+e^- \to K_S K^+ \pi^- \pi^0$



Phys. Rev. D95, 092005 (2017)

Intermediate state
$K^{*0} K^0_S \pi^0$
$K^{*0} K^{\pm} \pi^{\mp}$
$K_2^*(1430)^0 K^0_S \pi^0$
$K_2^*(1430)^0 K^{\pm} \pi^{\mp}$
$K^*(892)^{\pm} K^0_S \pi^{\mp}$
$K^*(892)^{\pm} K^{\mp} \pi^0$
$K_2^*(1430)^{\pm} K^0_S \pi^{\mp}$
$K_2^*(1430)^{\pm} K^{\mp} \pi^0$
$K^{*0} \overline{K}^{*0}$
$K^*(892)^+ K^*(892)^-$
$K^0_S K^{\pm} \rho(770)^{\mp}$

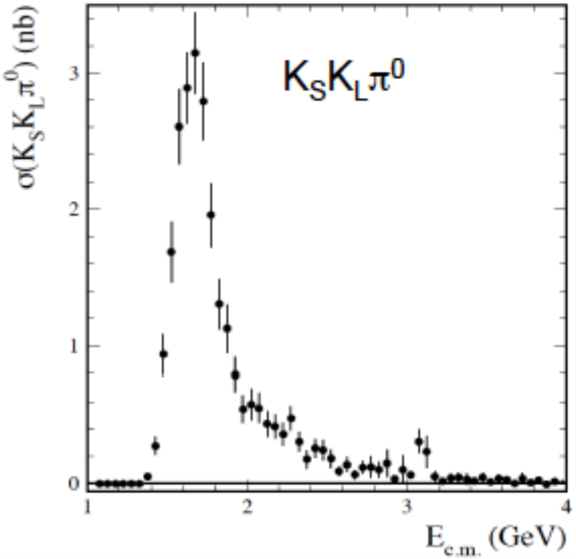
- First measurement of this channel (6400 signal events)
- Systematic uncertainties are 6-7% below 2 GeV
- Very reach intermediate state composition
  - Dominant contribution from $K^*(892)K\pi$
  - Large $K_S K^+ \rho(770)$ contribution; $K^*(892)^+ K^*(892)^-$ is ~15%
- Large $J/\psi \to K_S K^+ \pi^- \pi^0$ peak (first observation of this J/ψ decay)

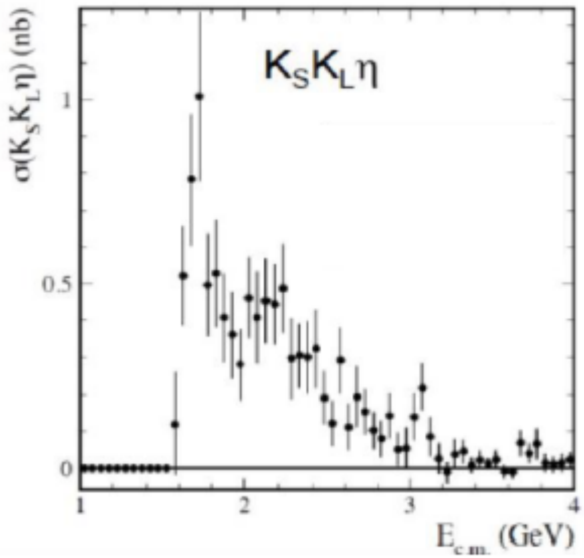


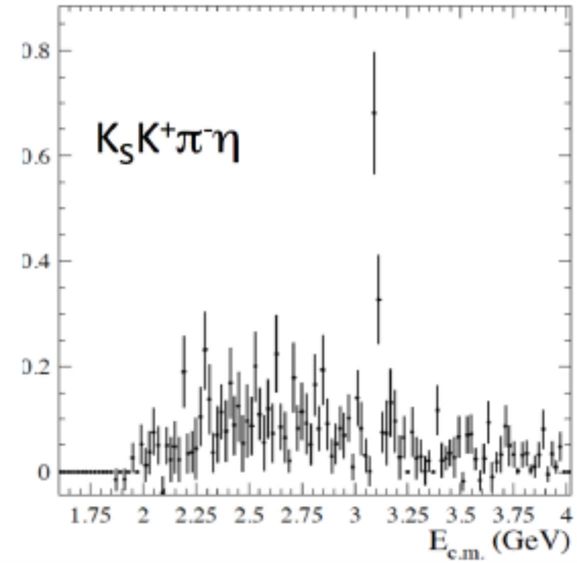
$e^+e^- \to K_S K_L \pi^0$, $K_S K_L \eta$, $K_S K^+ \pi^- \eta$

Phys. Rev. D95, 052001 (2017)

Phys. Rev. D95, 092005 (2017)





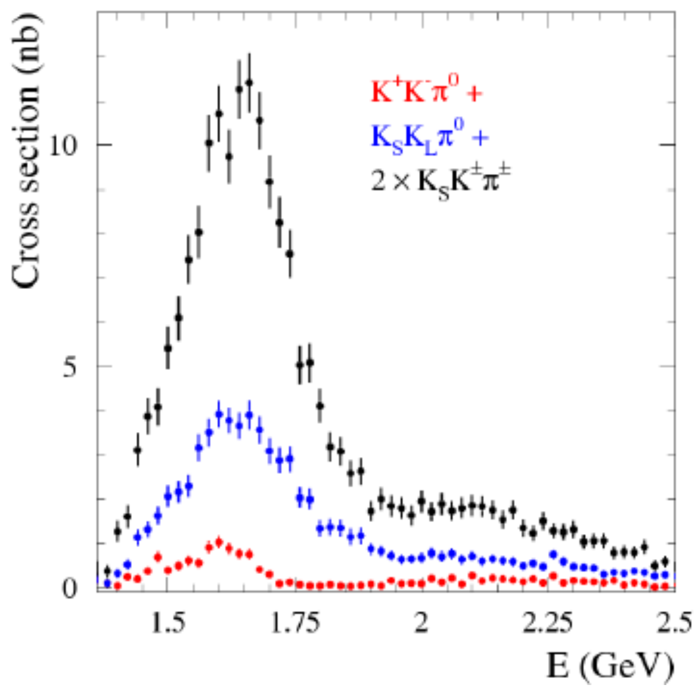


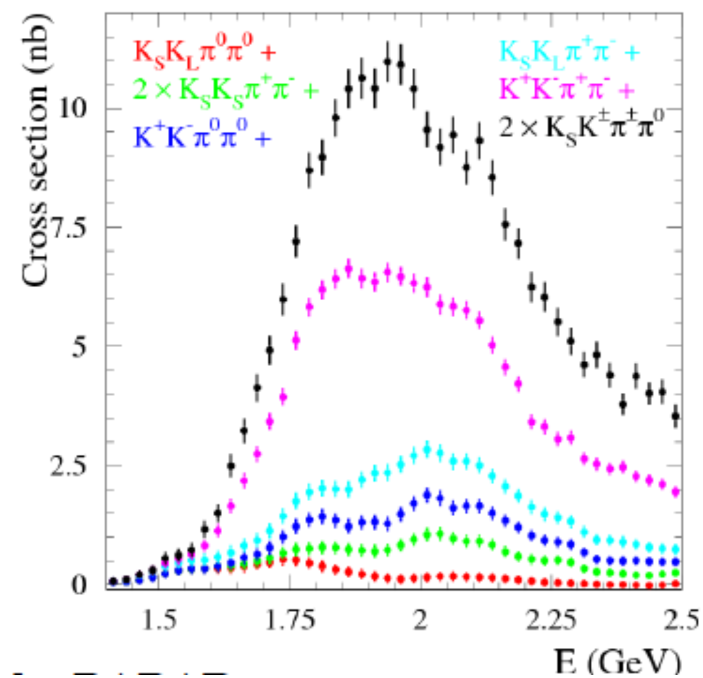
- First measurement
- Systematic uncertainty is 10% near the peak, grows to 30% at 3.0 GeV
- Dominant $K^*(892)\bar{K}$ intermediate state

- First measurement
- Systematic uncertainty is 25% at the peak, grows to 60% at 2 GeV

- First measurement
- Systematic uncertainty is 12-19% below 3 GeV
- Dominant $K^*(892)\bar{K}\eta$ intermediate state.

Total $e^+e^- \to KK\pi$ and $e^+e^- \to KK\pi\pi$ cross sections





From V.P. Druzhinin, EPJ Web of Conferences 142, 01013 (2017)

- All modes have now been measured by *BABAR*!
  - $KK\pi$ covers ~12% of the total cross section for E_{CM}~1.6 GeV
  - $KK\pi\pi$ covers ~25% of the total cross section for E_{CM}~2 GeV
- Precision on $(g-2)_\mu$ significantly improved (no reliance on isospin)

$a_\mu^{KK\pi}$ = (2.45 ± 0.15) x10^{-10}
previously (2.39 ± 0.16) x10^{-10}

$a_\mu^{KK\pi\pi}$ = (0.85 ± 0.05) x10^{-10}
previously (1.35 ± 0.39) x10^{-10}

Updated a_μ status

SM calculation *vs* experimental result with latest *BABAR* results included

(in units of 10^{-10})

QED	11 658 471.895	± 0.008	
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a_μ^{SM}	11 659 181.7	± 4.2	(2011) ± 4.9
a_μ^{exp}	11 659209.1	± 6.3	
$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM}$	**27.4**	**± 7.6**	(2011) ± 8.0

M. Davier, arXiv:1612.0274 (2017)

still 3.6σ

Conclusions and perspectives

- *BABAR* pioneered the use of the ISR method to precisely measure low-energy exclusive hadronic cross sections

- New *BABAR* results reduce the uncertainty in $a_\mu^{had,LO}$:
  - $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$: from ~7% to ~3%
  - $e^+e^- \to KK\pi\pi$ (all channel are now measured!): from ~30% to ~6%

- New measurement expected soon from *BABAR* and other experiments (BES, CMD-3, SND)
  - in particular a new measurement $e^+e^- \to \pi^+\pi^-(\gamma)$ from *BABAR* is in progress

- Expectation from direct measurement
  - g-2 @ Fermilab target: improve precision from 0.54 to 0.14 ppb
  - E34 @ Jparc target: 0.1 ppb

➢ Potentially a SM breaking at ~7σ could be observed with the combined progresses expected for e^+e^- data and direct *g-2* measurements, should the present experiment-theory discrepancy be confirmed

Conclusions and perspectives

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  - g-2 @ Fermilab [illegible]get: improve precision from 0.54 to 0.14 ppb
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THANK YOU!

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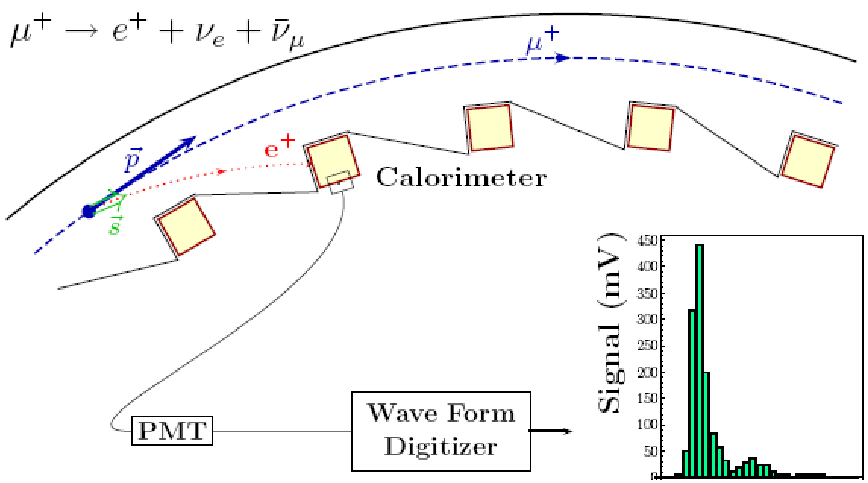
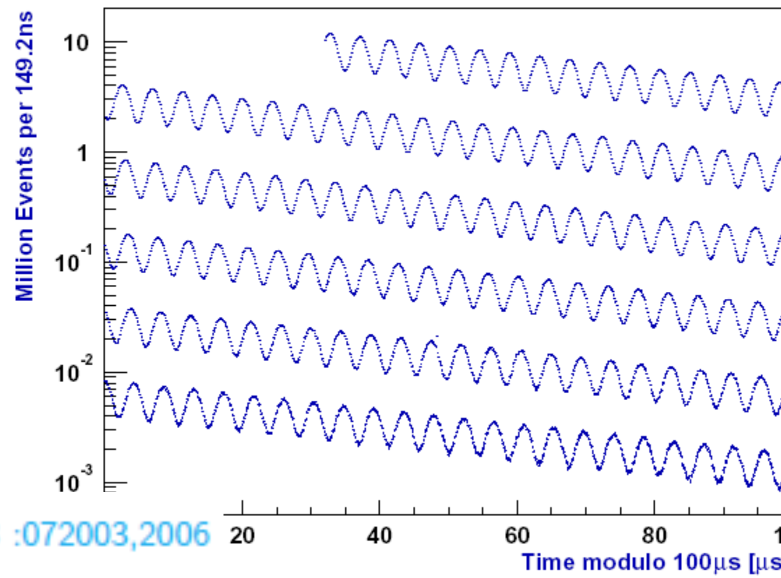
Backup Slides

The E-821 direct a_μ measurement at BNL



Bennett Phys.Rev.D73 :072003,2006



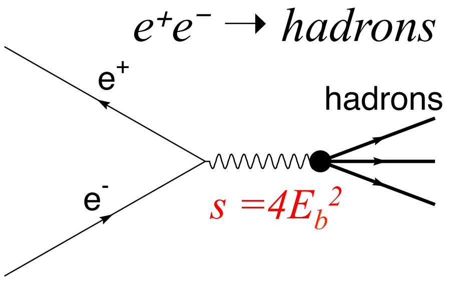
- $\pi^+ \rightarrow \mu^+\nu$ violates P ==> μ^+ polarized
- μ^+ stored in a cyclotron: constant $\vec{B}$ field
  - μ rotating with frequency ω_c; spin precessing with freq. ω_s
  - $\omega_a = \omega_s - \omega_c = a_\mu \, eB/m_\mu$
- $\mu \rightarrow e\nu\bar{\nu}$ violates P ==> e direction (energy in LAB) remembers μ polarization
  - fraction of detected e with $E>E_{threshold}$ modulated with freq. ω_a

$a_\mu^{exp} = (11\,659\,208.0 \pm 5.4 \pm 3.3)\, 10^{-10}$ E821 PRD73.072003.2006
(± 6.3) (0.54 ppm)

How to measure σ_{had}

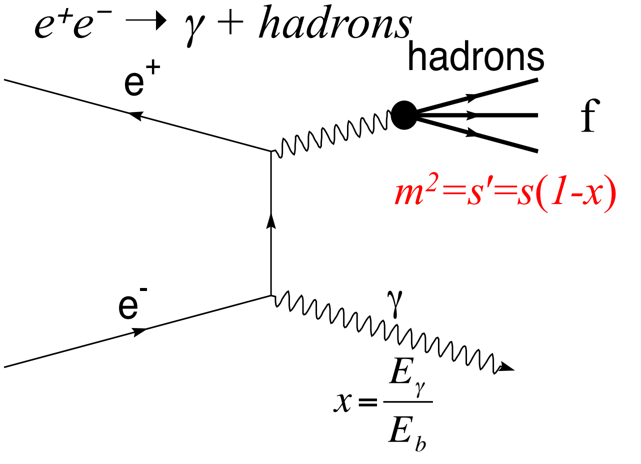
"Conventional" method: Energy Scan

- σ_{had} measured varying the beam energies within the accessible range ==> $\sqrt{s} = 2E_b$
- Effect of ISR mitigated requiring $(E,\vec{p})$ balance, between initial and final state



Alternative method: Radiative Return

- Developed by KLOE and BABAR
- The emission of an ISR photon lower the effective c.m. energy of the e^+e^- collision : $s \to s' = s(1\text{-}x)$
  - Study e^+e^- annihilations for a continous and wide spectrum of energies $\sqrt{s'}$ below the nominal $\sqrt{s}$
  - No change of operating conditions of the collider
  - Optimal use of the available luminosity



ISR studies at the $\Upsilon(4S)$ can yield the same observables as the low energy e^+e^- experiments!

ISR method in a nutshell

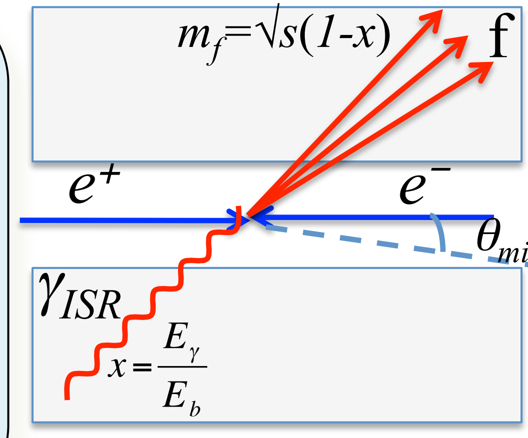
Born approximation $$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f,\theta^*_\gamma)}{dm\, d\cos\theta^*_\gamma} = \frac{2m}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}(m_f)$$

$x = \frac{E_\gamma}{E_b}$ $\quad m^2 = s' = s(1-x)$

θ_γ^*: ISR photon polar angle in the e^+e^- c.m.

Radiator function (at lowest order):

$$w_0(s,x,\theta^*) = \frac{\alpha}{\pi x}\left[\frac{(2-2x+x^2)\sin^2\theta^* - \frac{x^2}{2}\sin^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)^2} - \frac{4m_e^2}{s}\frac{(1-2x)\sin^2\theta^* - x^2\cos^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)}\right]$$



cross section $\sigma_0(e^+e^- \to f)$

$$\sigma_0(m_i) = \frac{\Delta N(m_i)}{\Delta m}\frac{1}{\varepsilon(s,m_i)(1+\delta_{rad})\, d\mathcal{L}(m_i)/dm}$$

- $\varepsilon(s,m_i)$: reconstruction efficiency
- $(1+\delta_{rad})$: radiative corrections
- $d\mathcal{L}(m_i)/dm$: ISR luminosity

ISR differential luminosity $$\frac{d\mathcal{L}}{dm} = \frac{2m}{s}\frac{\alpha}{\pi\cdot x}\cdot\left((2-2x+x^2)\log\frac{1+C}{1-C} - x^2 C\right) L_{ee}$$

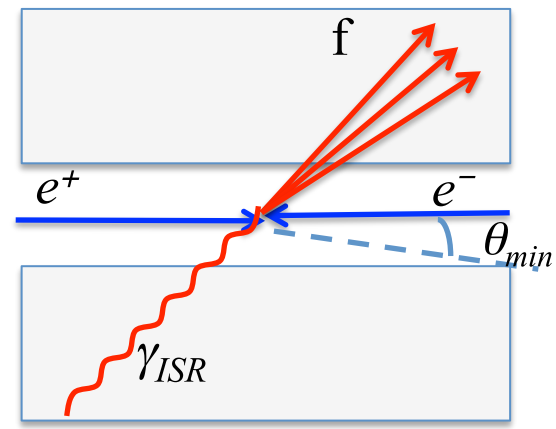
- L_{ee}: Luminosity integrated by the collider
- $C = \cos\theta^*_{\min}$: Detector angular acceptance

- obtained from integration of the radiator function over θ_γ^*
- $20^o < \theta_\gamma^* < 160^o$ ==> **acceptance for ISR photon ~15% in *BABAR***
- known at <1% level

To tag or not to tag

Tagged approach:
☺ fully reconstructed events ➞ great background reduction
☹ ~90% signal loss

Untagged approach:
☺ typically higher efficiency
☹ higher background reduced by requiring the missing mass consistent with zero

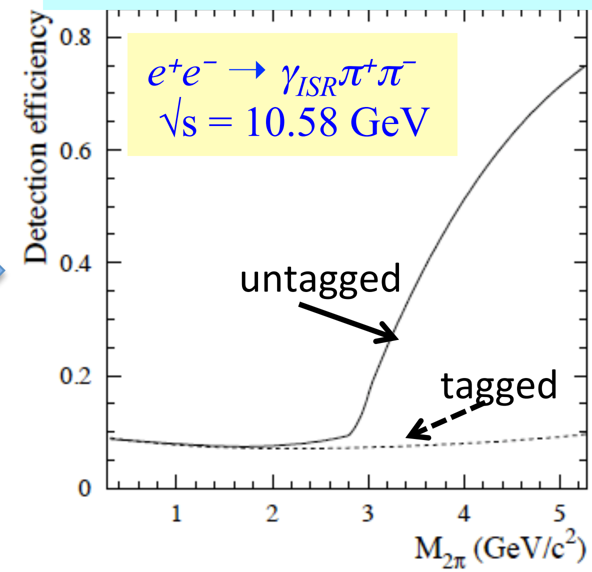


- So, what is the more convenient approach?

  ==> It depends on experimental situation

- At √s=10.58 GeV and for low m_f, (i.e. large x) the hadronic system has a large boost opposite to the photon direction ==> the efficiency is almost insensitive to tagging

Druzhinin et *al*, arXiv:1105.4975



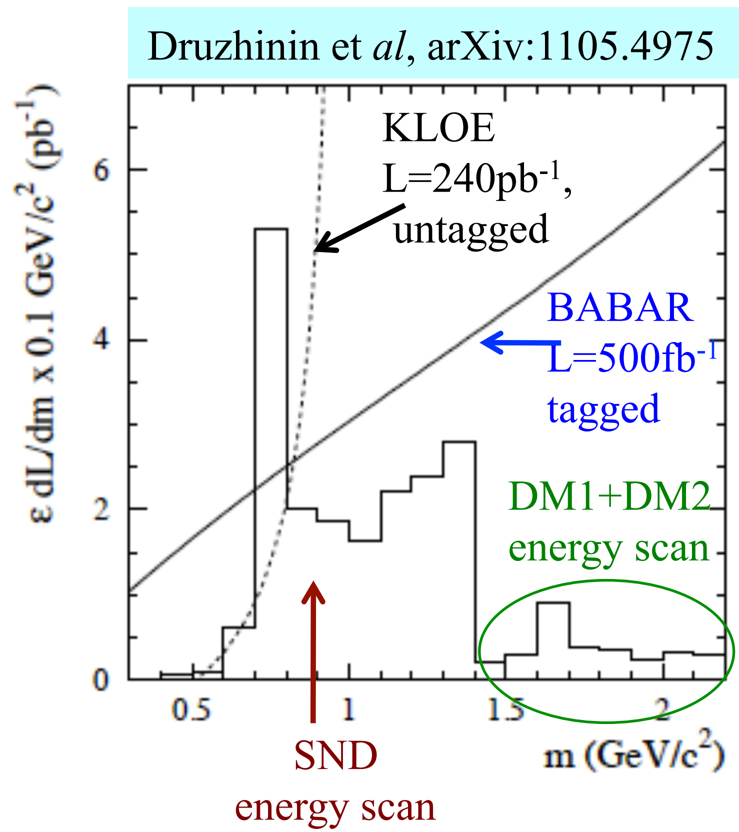
This is why at *BABAR*:

- Light Quarks final states ⇔ Tagged analyses
- Heavy Quarks final states ⇔ Untagged analyses

ISR vs Energy scan

- Uniform data quality all-over the energy range
  no systematics from point-to-point normalization
- Statistically very competitive samples

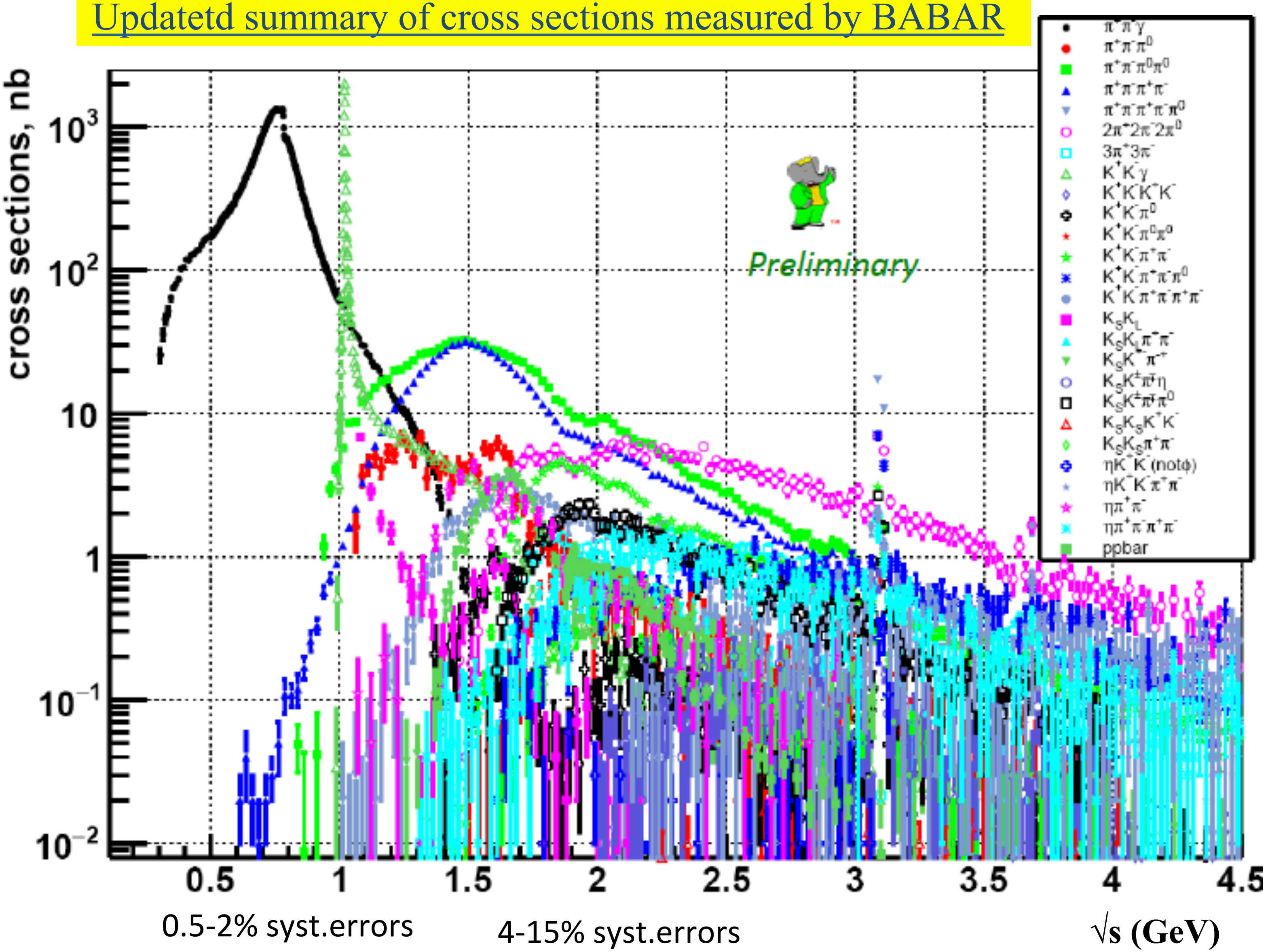
  α_{em} suppression compensated by the huge luminosity integrated by *B*- or Φ-factories
- Boost of the hadronic system:
  - $\varepsilon \neq 0$ at threshold
- If hard photon detected:
  - loose hadron selection
  - hadronic system at wide angle (in LAB ref)
    - large geometric acceptance
    - ε weakly dependent on angular distribution in the c.m. of the hadrons system



- Higher (and different) background sources
  - main backgrounds from different ISR processes and $e^+e^- \to qq$ production
- Limited mass resolution (~ few MeV)

The ISR program for Light Quark Mesons in *BABAR*

Updatetd summary of cross sections measured by BABAR



Background to $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$

Background subtracted using simulation normalized to data or using the data sideband

Largest ISR background: $\pi^+\pi^- 3\pi^0 \gamma_{ISR}$

- Cross section not well measured; only previous measurement is from 1979
- Perform new measurement using similar techniques to that used for the $\pi^+\pi^-\pi^0\pi^0$ cross section
- Obtain reliable background estimate, adjusting the shape and normalization of $e^+e^- \rightarrow \pi^+\pi^- 3\pi^0$ in the simulation