Charm quark mass with calibrated uncertainty

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Work done in collaboration with
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Universitat Autònoma de Barcelona

Outline

- Motivation and Introduction
- Using Sum Rules to extract m_Q
  - overview
  - our proposal
- Conclusions and outlook

Motivation: why precise m_Q?

Techniques

Υ-spectroscopy $$m(\Upsilon(1S)) = 2M_b - \mathcal{C}\alpha^2 M_b + \cdots$$

lattice: HPQCD '14

$$\overline{m_c}(3\mathrm{GeV}) = 986(6)\mathrm{MeV}$$

$$\overline{m_b}(10\mathrm{GeV}) = 3617(25)\mathrm{MeV}$$

QCD Sum Rules $$\int \frac{\mathrm{d}s}{s^{n+1}} R_q(s) \sim \left(\frac{1}{m_q}\right)^{2n}$$

Motivation: why precise m_Q?

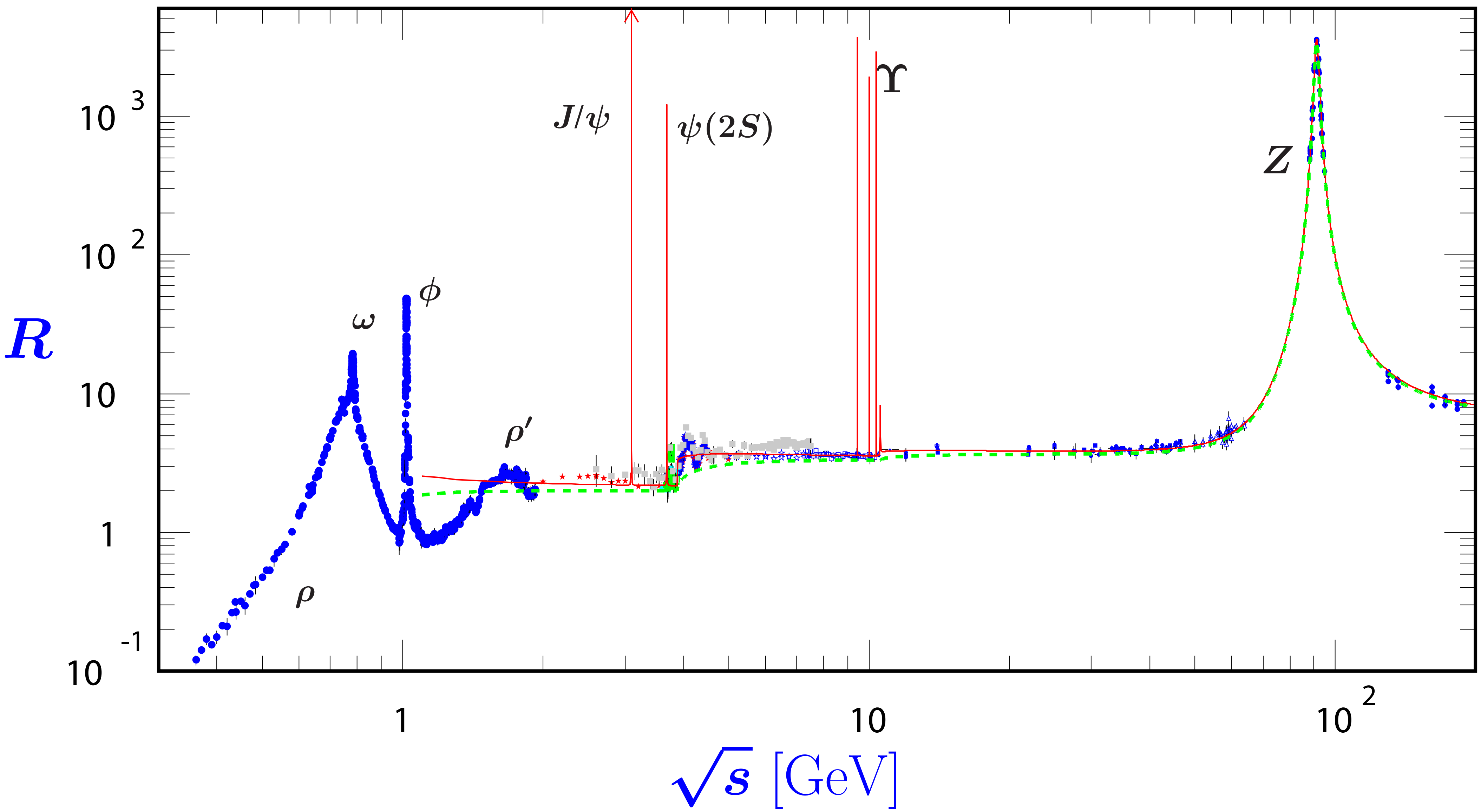
$\overline{m_c}(\overline{m_c})$MeV	method	reference
$1335 \pm 43^{+40}_{-11}$	HERA DIS	xFitter, 1605.01946
1246 ± 23	quarkonium 1S	Kiyo et al, 1510.07072
1288 ± 20	1st moment SR $J/\Psi, \Psi, R$	Dehnadi et al, 1504.07638
1271.5 ± 9.5	lattice ($N_f = 4$), PS current	HPQCD, 1408.4169
1348 ± 46	lattice (2+1+1), M_D	ETM, 1403.4504
1274 ± 36	lattice ($N_f = 2$), f_D	ALPHA, 1312.7693
1240 ± 50	$c\bar{c}$ X-section DIS	Alekhin et al, 1310.3059
1260 ± 65	$c\bar{c}$ X-section NLO fit	HI and ZEUS, 1211.1182
1262 ± 17	SR $J/\Psi, \Psi(2S - 6S)$	Narison, 1105.5070
1260 ± 36	lattice (2+1), f_D	PACS-CS, 1104.4600
1278 ± 9	SR $J/\Psi, \Psi, R$	Bodenstain et al, 1102.3835
1282 ± 24	1st moment SR $J/\Psi, \Psi, R$	Dehnadi et al, 1102.2264
1280 ± 70	lattice + pQCD in static potential	Laschka et al, 1102.0945
1279 ± 13	1st moment SR $J/\Psi, \Psi, R$	Chetyrkin et al, 1010.6157
1.28 ± 0.03GeV	PDG average	PDG 2016

QCD Sum Rules

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

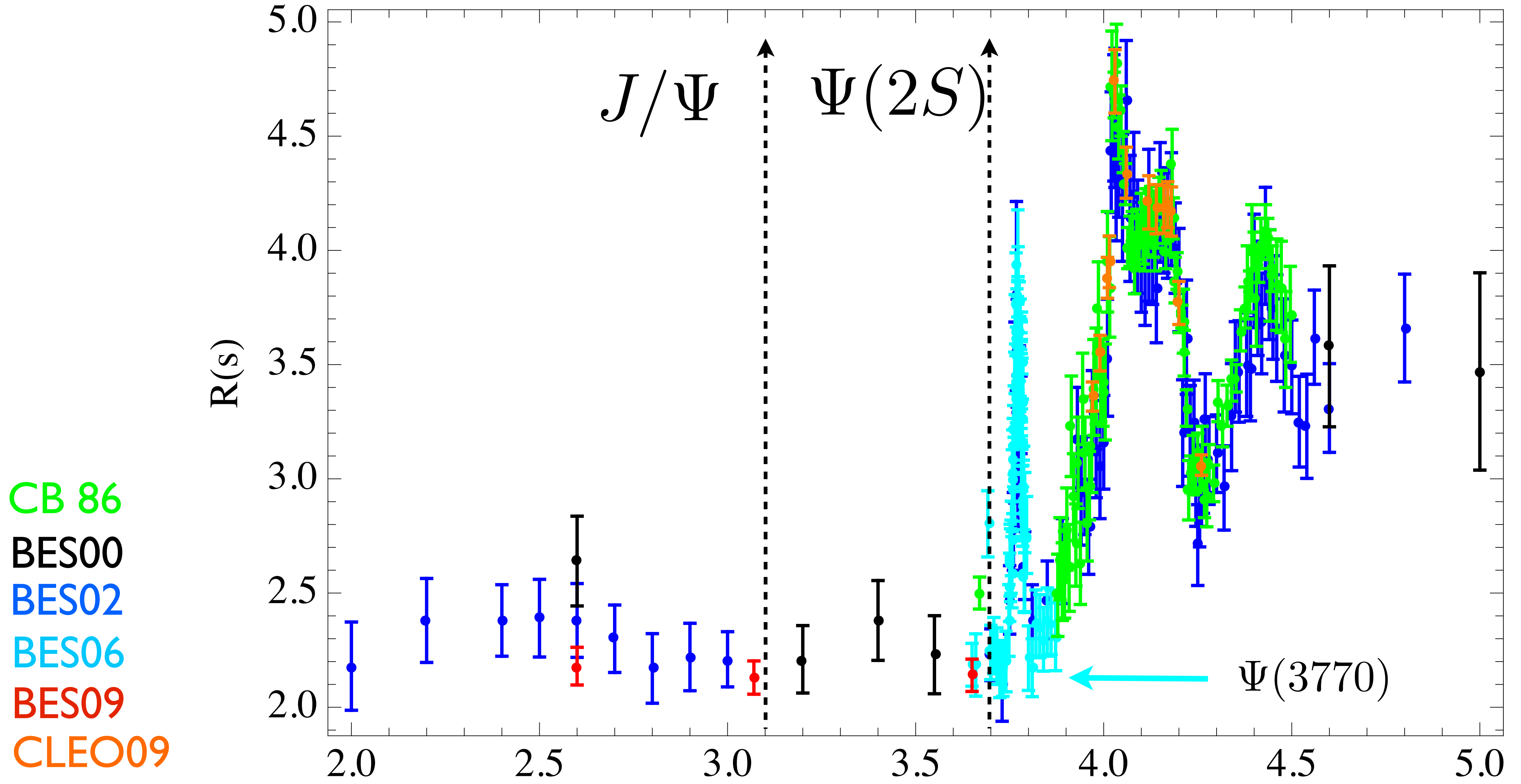
$$\sigma(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha_{\text{em}}(s)^2/3s$$

[PDG]



QCD Sum Rules

$$R(s) = R_{\mathrm{uds}}(s) + R_q(s) \qquad R_q(s) = R_q^{\mathrm{Res}}(s) + R_q^{\mathrm{th}}(s) + R_q^{\mathrm{cont}}(s)$$



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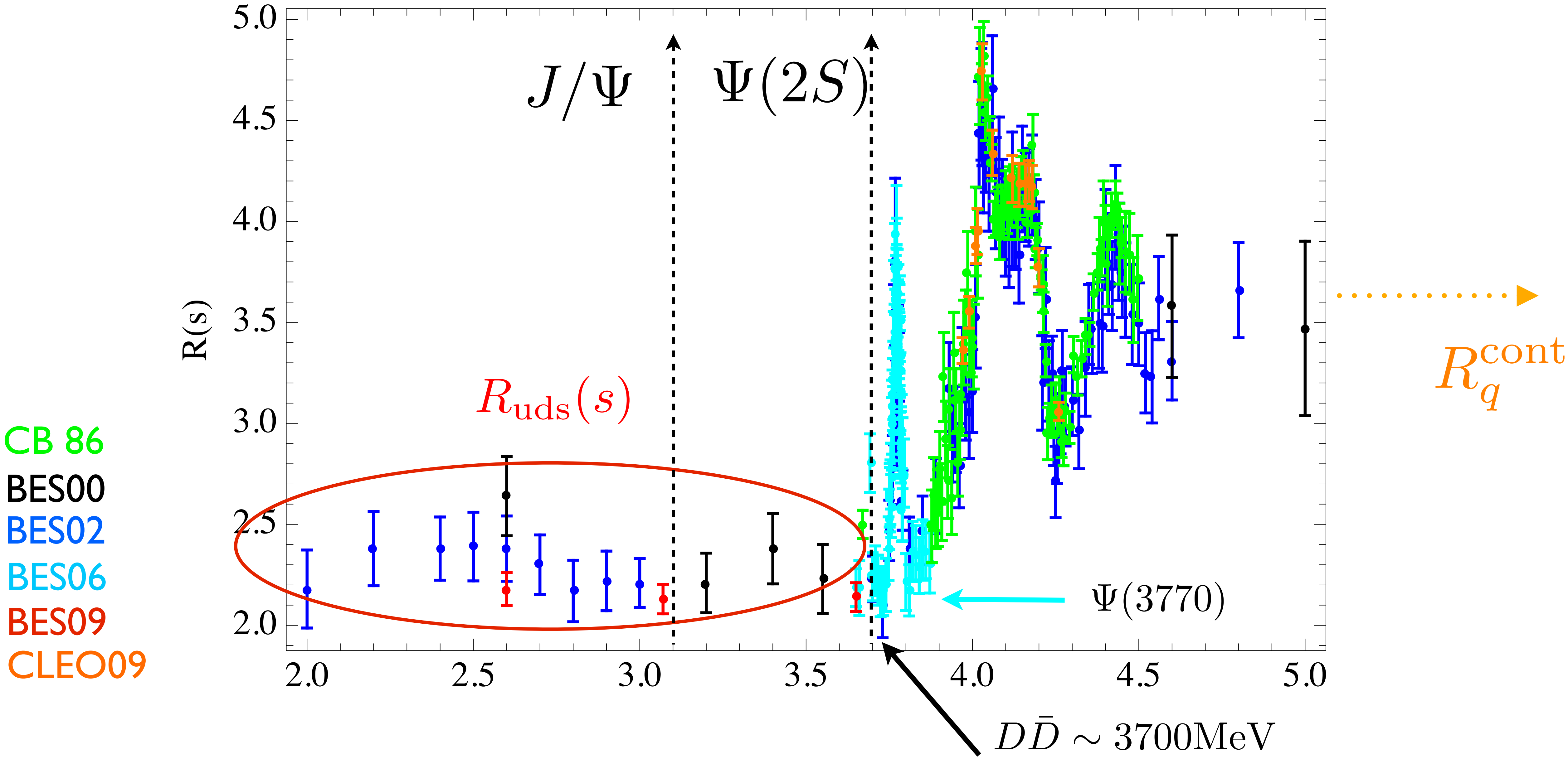

QCD Sum Rules

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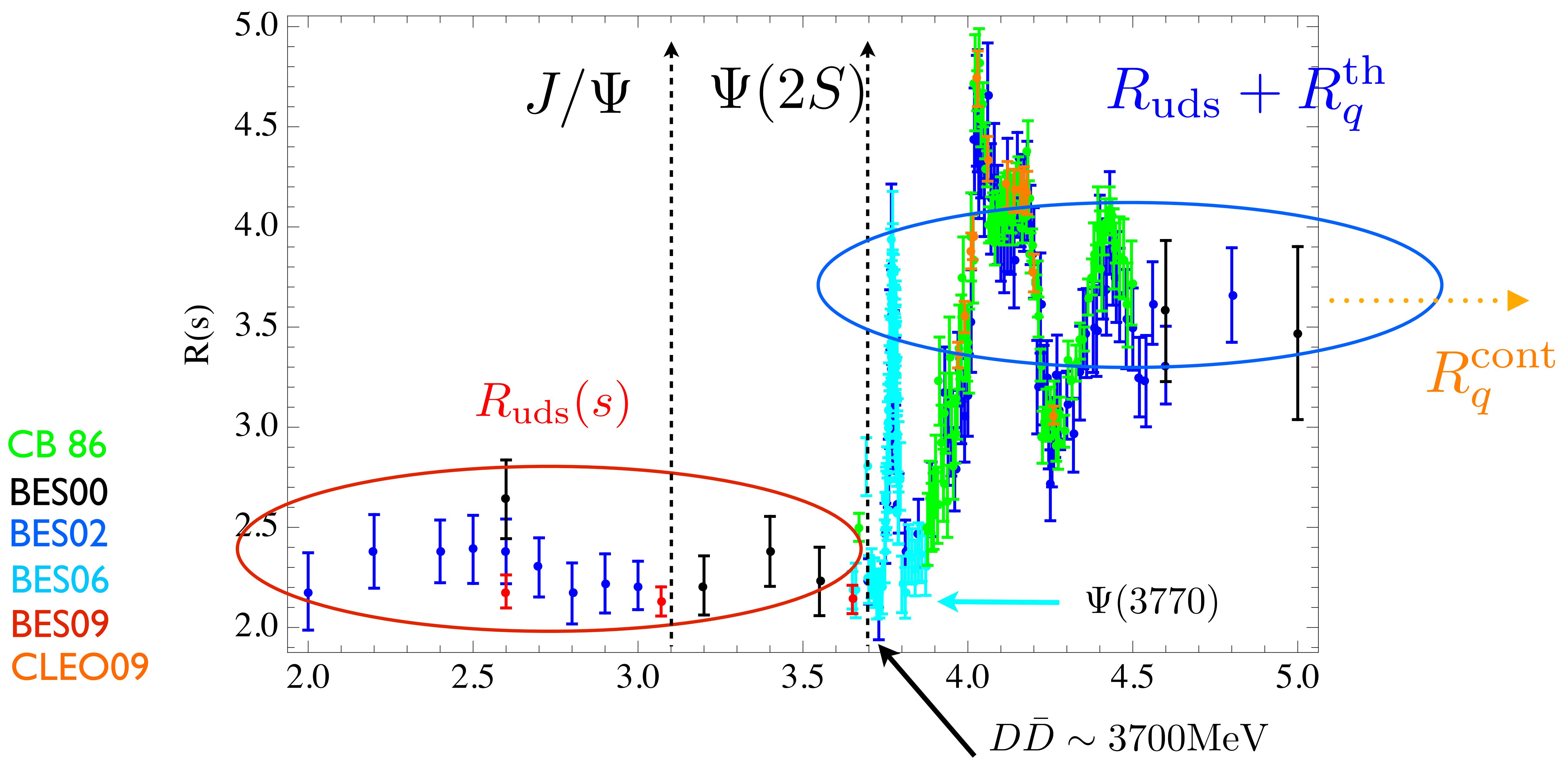

QCD Sum Rules

Using the optical theorem: [SVZ,'79]

$$R(s) = 12\pi \mathrm{Im}[\Pi(s + i\epsilon)]$$

$\Pi_q(s)$ is the correlator of two heavy-quark vector currents which can be calculated in pQCD order by order and satisfies a Dispersion Relation:

$\hat{\Pi}_q(s)$ in $\overline{MS}$

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

For t→0

$$\boxed{\mathcal{M}_n := \frac{12\pi^2}{n!} \frac{d^n}{dt^n} \hat{\Pi}_q(t)\Big|_{t=0} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R_q(s)}$$

QCD Sum Rules

$\hat{\Pi}_q(s)$ can be Taylor expanded:

$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2} \right)^n$$

QCD Sum Rules

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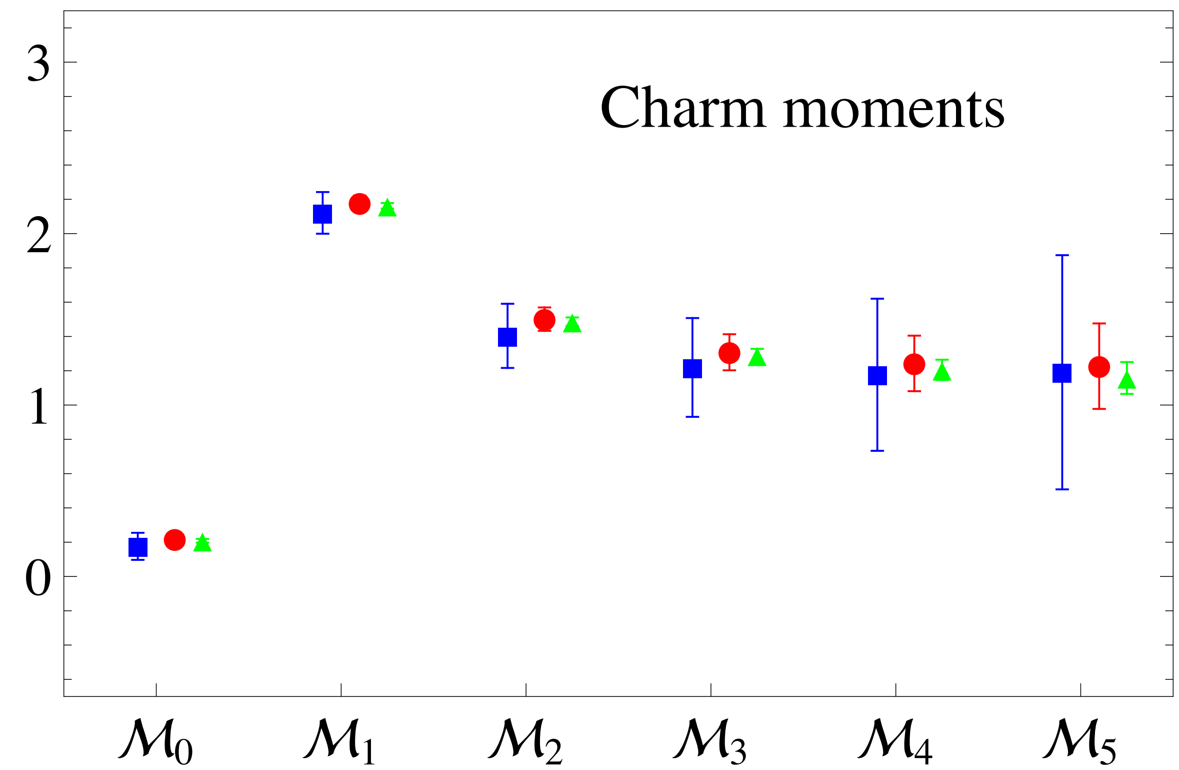


$$\mathcal{M}_n^{\mathrm{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n} \bar{C}_n$$

$$\hat{\alpha} = \hat{\alpha}(\overline{m_q})$$

$$\bar{C}_n = \bar{C}_n^{(0)} + \left(\frac{\hat{\alpha}}{\pi}\right) \bar{C}_n^{(1)} + \left(\frac{\hat{\alpha}}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\hat{\alpha}}{\pi}\right)^3 \bar{C}_n^{(3)} + \mathcal{O}\left(\frac{\hat{\alpha}}{\pi}\right)^4$$

[Maier et al, '08]
[Chetyrkin, Steinhauser'06]
[Melnikov, Ritberger'03]

[Kiyo et al '09]
[Hoang et al '09]
[Greynat et al '09]

QCD Sum Rules

Sum Rules:

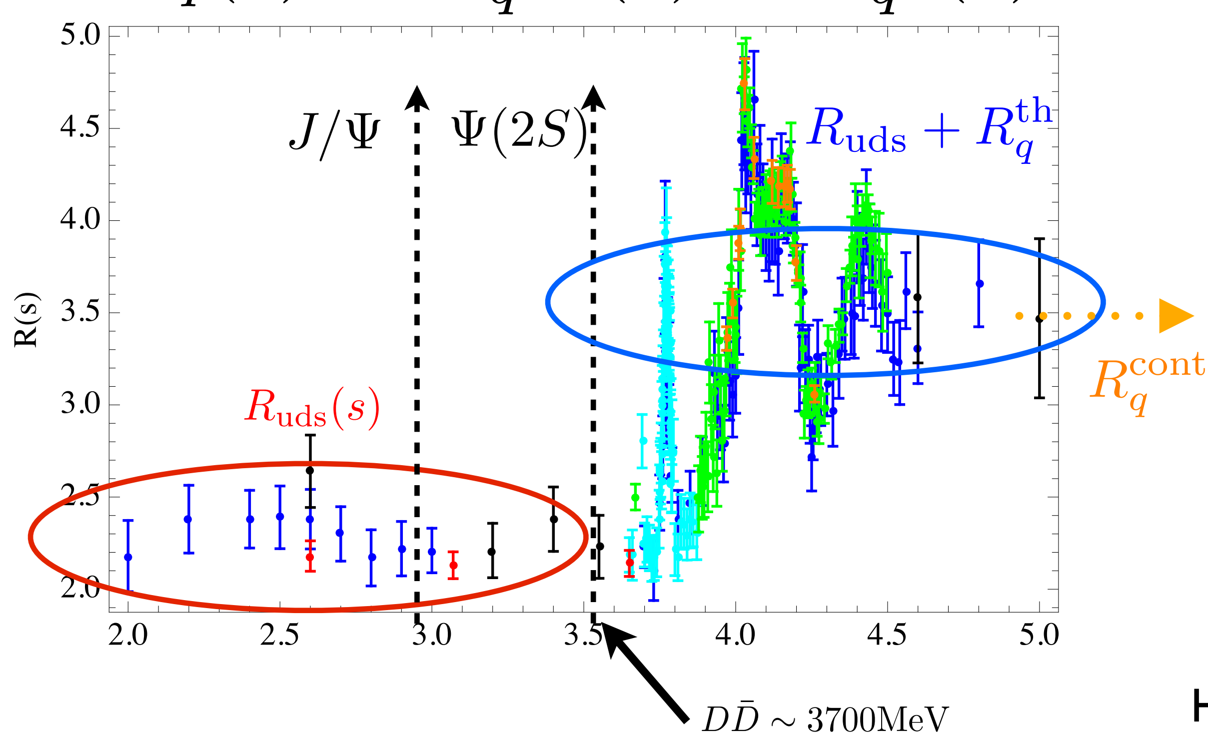
$$\boxed{\mathcal{M}_n = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} R_q(s)}$$

L.h.s. from theory

$$\mathcal{M}_n^{\mathrm{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n$$

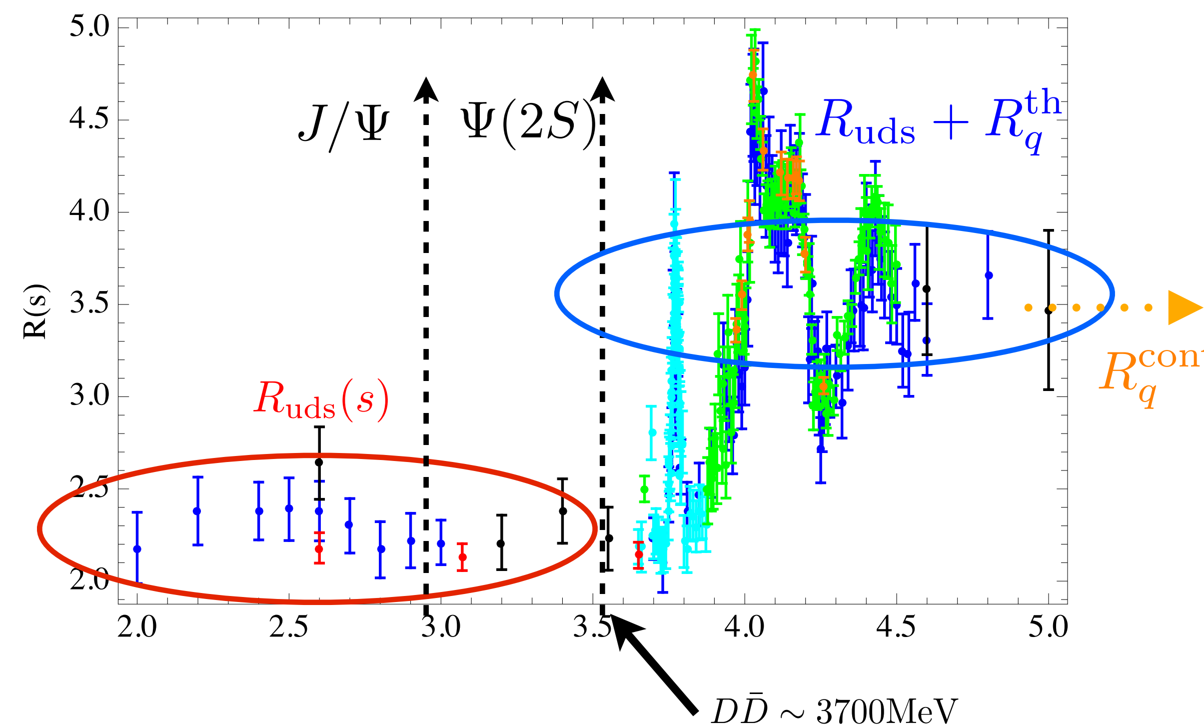
R.h.s. from experiment

$$R_q(s) = R_q^{\mathrm{Res}}(s) + R_q^{\mathrm{th}}(s) + R_q^{\mathrm{cont}}(s)$$



QCD Sum Rules

$$R_q(s) = R_q^{\rm Res}(s) + R_q^{\rm th}(s) + R_q^{\rm cont}(s)$$



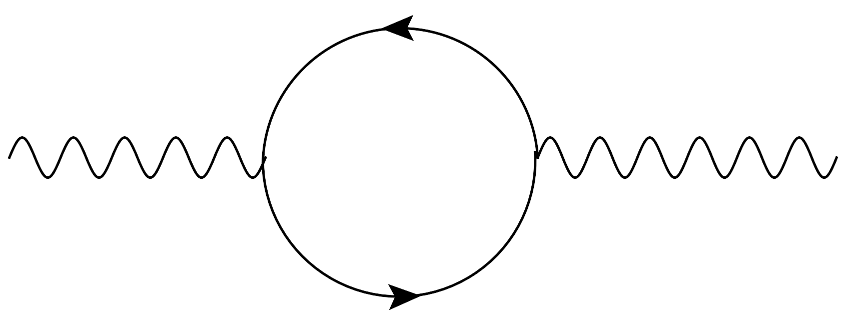
$$R_q^{\rm Res}(s) = \frac{9\pi M_R \Gamma_R^e}{\alpha_{\rm em}^2(M_R)}\delta(s - M_R^2)$$

$$R_q^{\rm th}(s) = R_q(s) - R_{\rm background}$$

$$(2M_D \leq \sqrt{s} \leq 4.8{\rm GeV})$$

$R_q^{\rm cont}(s)$ calculated using pQCD

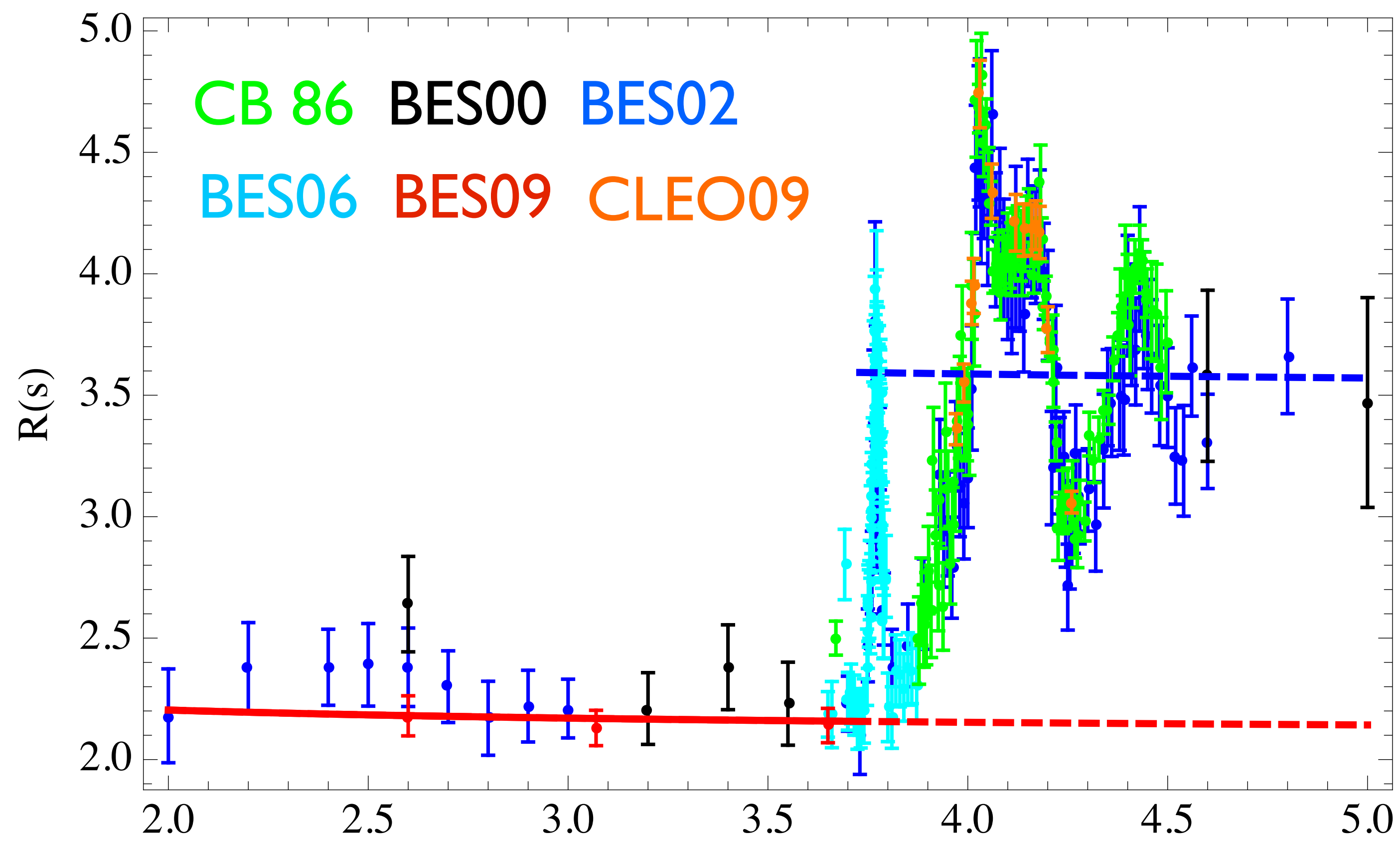
$$(\sqrt{s} \geq 4.8{\rm GeV})$$



Background

$$R_{\rm background} = R_{\rm uds} + R_{\rm uds(cb)} + R_{\rm sing} + R_{\rm QED}$$

Light flavor contribution in charm region

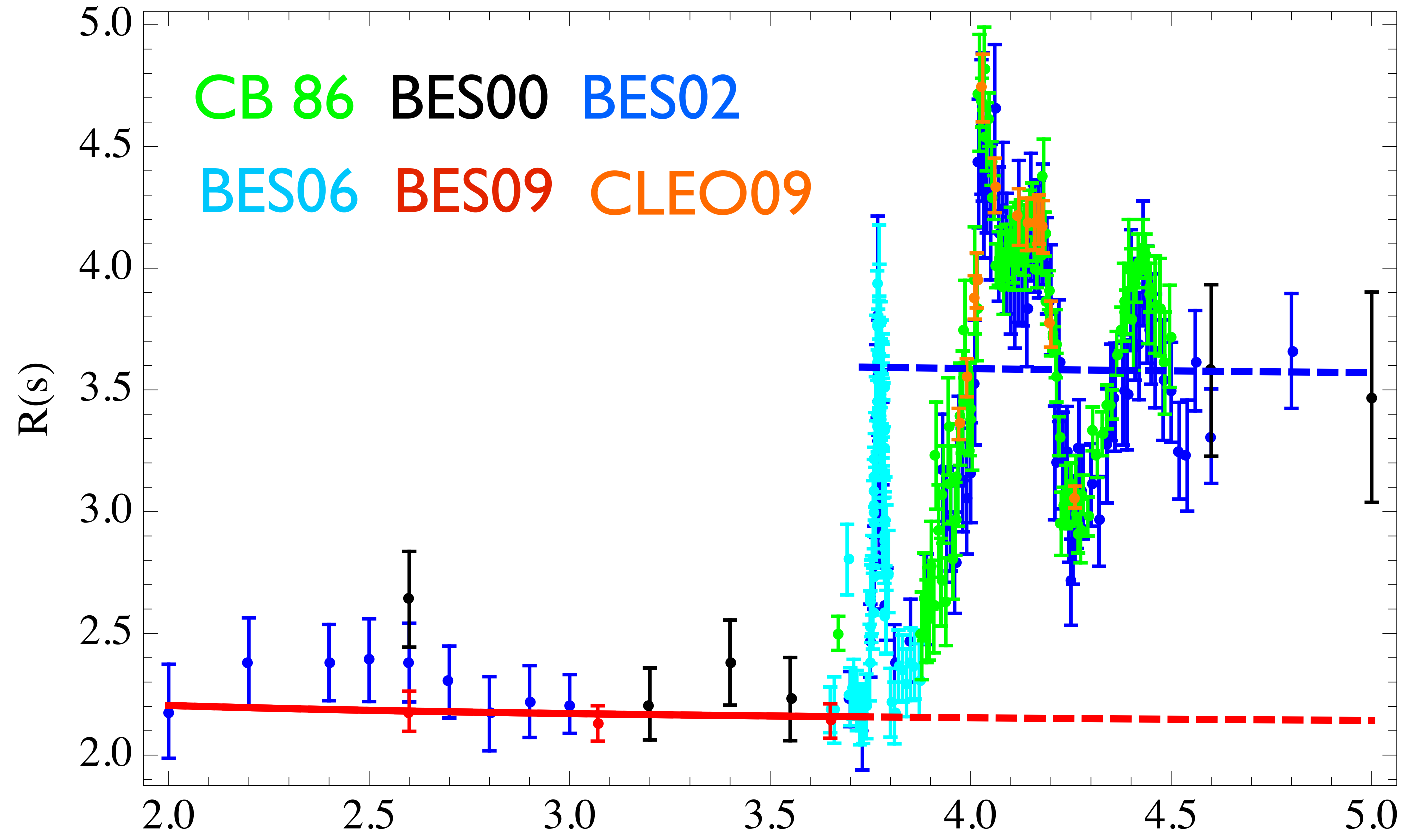


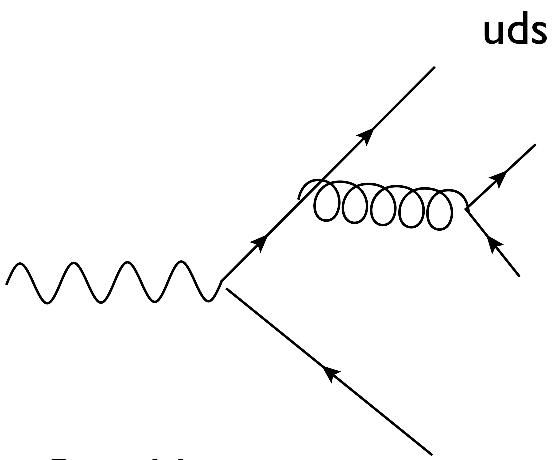
Using pQCD below threshold, calculate R, and extrapolate

Background

$$R_{\rm background} = R_{\rm uds} + R_{\rm uds(cb)} + R_{\rm sing} + R_{\rm QED}$$

Light flavor contribution in charm region
+
secondary production



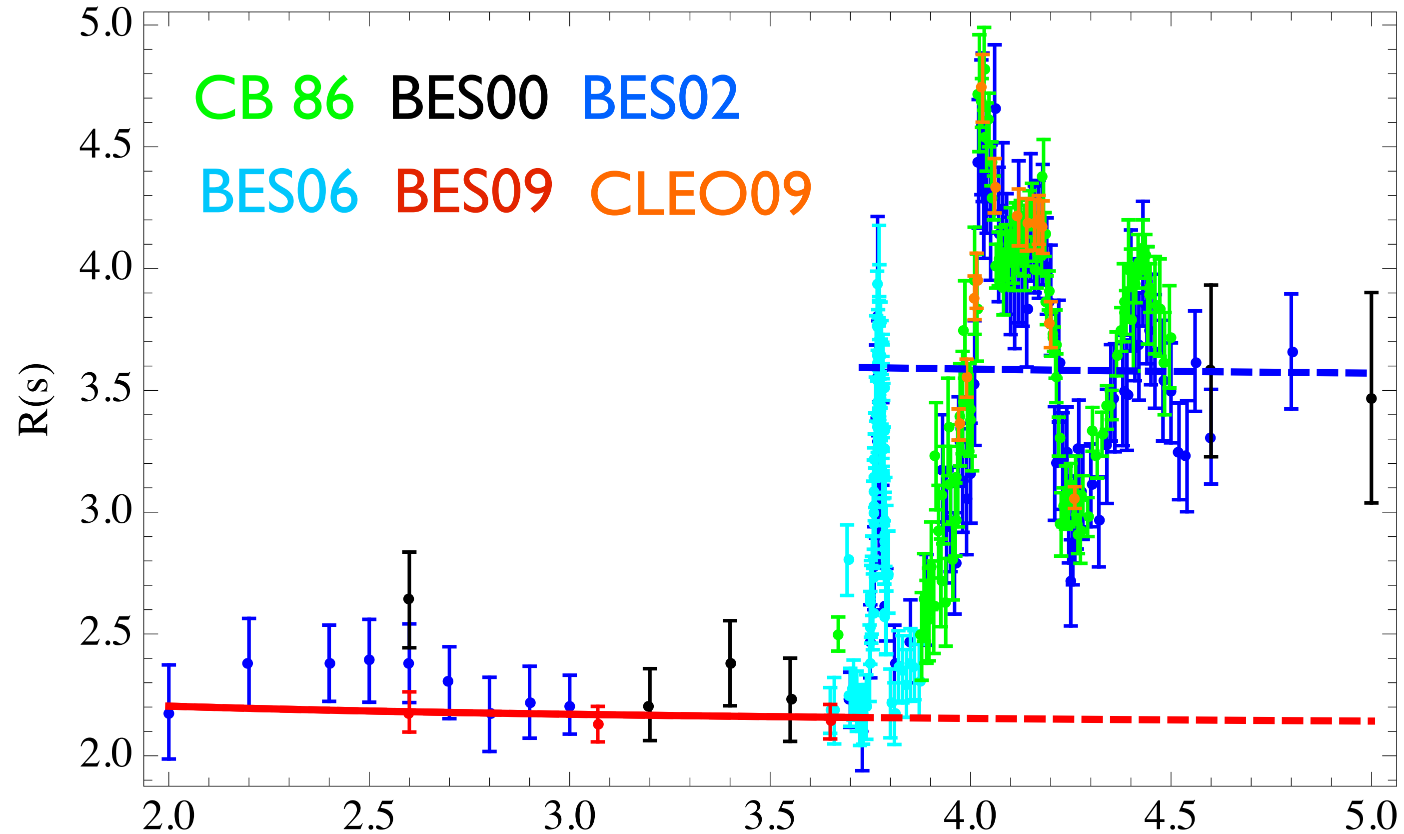


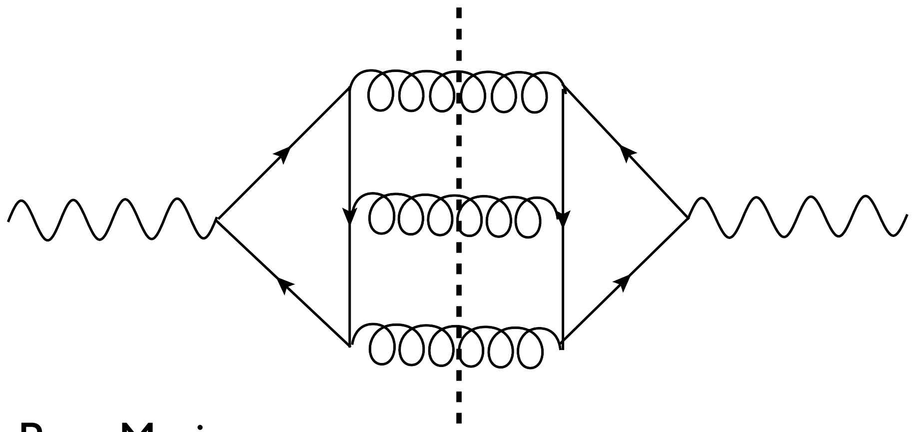
$\sim \alpha^2$

Background

$$R_{\rm background} = R_{\rm uds} + R_{\rm uds(cb)} + R_{\rm sing} + R_{\rm QED}$$

Light flavor contribution in charm region
+
secondary production
+
singlet contribution



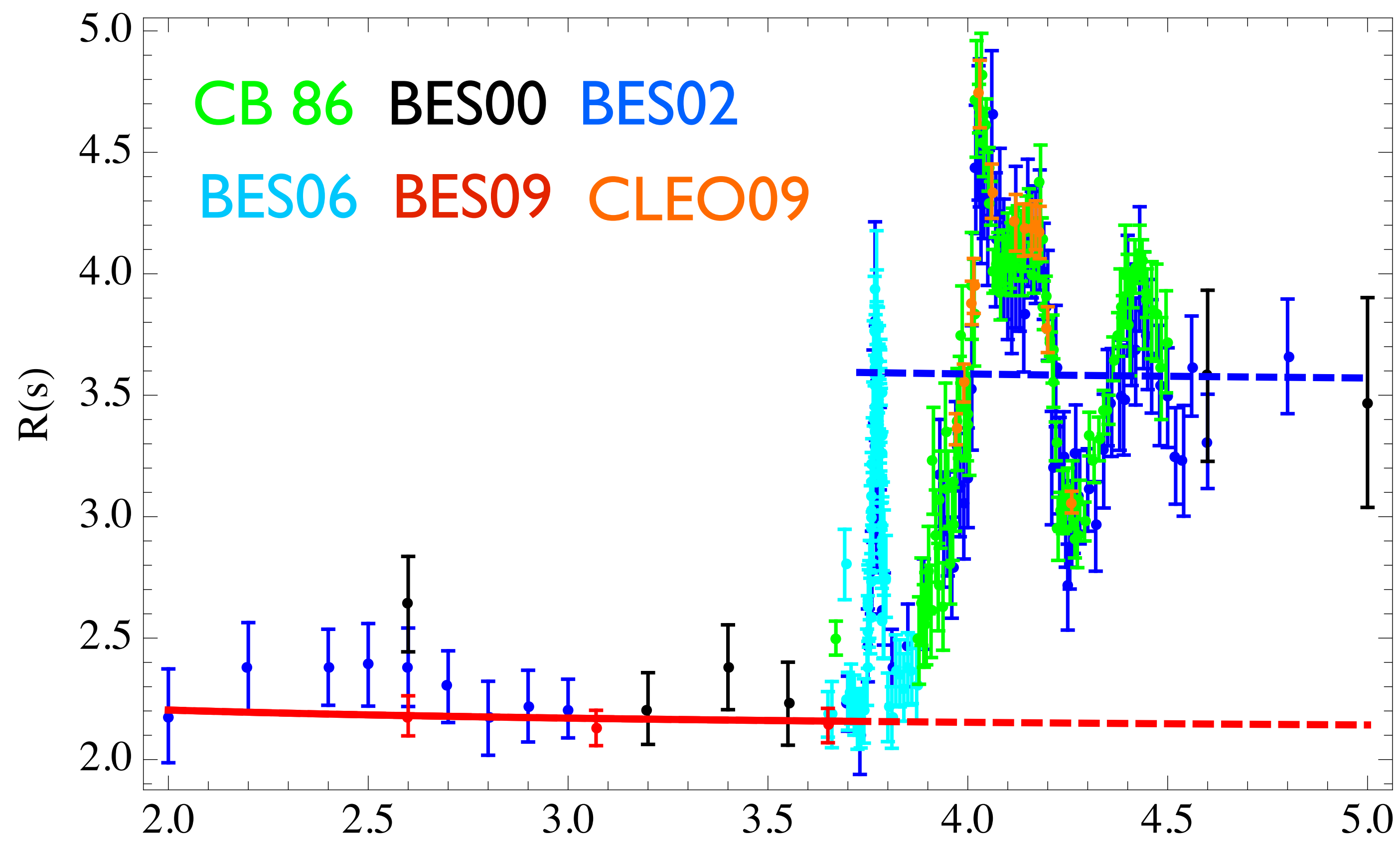


$\sim \alpha^3$

Background

$$R_{\rm background} = R_{\rm uds} + R_{\rm uds(cb)} + R_{\rm sing} + R_{\rm QED}$$

Light flavor contribution in charm region
+
secondary production
+
singlet contribution
+
2loop QED



Non-perturbative effects

Non-perturbative effects due to gluon condensates to the moments are:

[Chetyrkin et al '12]

$$\mathcal{M}_n^{\rm nonp}(\mu^2) = \frac{12\pi^2 Q_q^2}{(4\hat{m}_q^2)^{n+2}} \mathrm{Cond}\, a_n \left(1 + \frac{\alpha_s(\hat{m}_q^2)}{\pi} b_n\right)$$

$a_n\,,\, b_n$ are numbers, and $\mathrm{Cond} = \langle \frac{\alpha_s}{\pi} G^2 \rangle = (5 \pm 5) \cdot 10^{-3} \mathrm{GeV}^4$ [Dominguez et al '14]

from fits to tau data

$$\frac{\mathcal{M}_n^{\rm nonp}(\hat{m}_c)}{\mathcal{M}_n^{\rm th}} \sim 0.5\% - 2\% \longrightarrow \Delta\hat{m}_c(\hat{m}_c) \sim 2\mathrm{MeV} - 8\mathrm{MeV}$$

QCD Sum Rules

Our approach

- Consider global duality
- Do not use experimental data on threshold region, only resonances
  - Exp data in threshold only for error estimation
- Use two different moments to extract the mass

QCD Sum Rules

Our approach

For a global duality:

$\hat{\Pi}_q(s)$ in $\overline{MS}$

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

$t \to \infty \quad \text{define the } \mathcal{M}_0$

[Erler, Luo '03]

QCD Sum Rules

Our approach

For a global duality:

$\hat{\Pi}_q(s)$ in $\overline{MS}$

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

$t \to \infty$ $\quad$ define the $\mathcal{M}_0$ $\quad$ (but has a divergent part) $\quad$ [Erler, Luo '03]

$$\lim_{t\to\infty} \hat{\Pi}_q(-t) \sim \log(t) \quad \longleftrightarrow \quad \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s}{s} R_q(s) \sim \log(\infty)$$

Fortunately, divergence given by the zero-mass limit of R(s)

QCD Sum Rules

Our approach

[Chetyrkin, Harlander, Kühn,'00]

$$\lambda_1^q(s) = 1 + \frac{\alpha_s(s)}{\pi} + \left[\frac{\alpha_s(s)}{\pi}\right]^2 \left[\frac{365}{24} - 11\zeta(3) + n_q\left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)\right]$$
$$+ \left[\frac{\alpha_s(s)}{\pi}\right]^3 \left[\frac{87029}{288} - \frac{121}{8}\zeta(2) - \frac{1103}{4}\zeta(3) + \frac{275}{6}\zeta(5)\right.$$
$$+n_q\left(-\frac{7847}{216} + \frac{11}{6}\zeta(2) + \frac{262}{9}\zeta(3) - \frac{25}{9}\zeta(5)\right)$$
$$\left.+n_q^2\left(\frac{151}{162} - \frac{1}{18}\zeta(2) - \frac{19}{27}\zeta(3)\right)\right]$$

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s} \lambda_1^q(s)$$

$$= -\frac{5}{3} + \frac{\hat{\alpha}_s}{\pi}\left[4\zeta(3) - \frac{7}{2}\right] \qquad \hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

$$+ \left(\frac{\hat{\alpha}_s}{\pi}\right)^2 \left[\frac{2429}{48}\zeta(3) - \frac{25}{3}\zeta(5) - \frac{2543}{48} + n_q\left(\frac{677}{216} - \frac{19}{9}\zeta(3)\right)\right]$$

$$+ \left(\frac{\hat{\alpha}_s}{\pi}\right)^3 \left[-9.86 + 0.40\, n_q - 0.01\, n_q^2\right]$$

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\boxed{\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)}} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s} \lambda_1^q(s)$$

$$\hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

[PDG]

R	M_R [GeV]	Γ_R^e [keV]
J/Ψ	3.096916	5.55(14)
$\Psi(2S)$	3.686109	2.36(4)

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule:

$$\sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{3Q_q^2 M_R \hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q^{\text{cont}}}{3Q_q^2} - \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s} \lambda_1^q(s)$$

$$\hat{\alpha}_s = \alpha_s(\hat{m}_q^2)$$

$$\hat{\alpha}_{em}(0) \sim 0.98\hat{\alpha}_{em}(M_{J/\Psi})$$

$$\Delta\hat{\alpha}_{em} \to \Delta m_c \sim 12\text{MeV}$$

n_q active flavors

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

Two parameters to determine: $m_q\ ,\ \lambda_3^q$

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

Two parameters to determine: $m_q\ ,\ \lambda_3^q$

We need two equations: zeroth moment + nth moment

$$\frac{9}{4}Q_q^2\left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n}\bar{C}_n = \sum_{\rm resonances}\frac{9\pi\Gamma_R^e}{M_R^{2n+1}\hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty}\frac{{\rm d}s}{s^{n+1}}R_q(s)$$

$$n \geq 1$$

QCD Sum Rules

Our approach

Zeroth Sum Rule: [Erler, Luo '03]

$$R_q^{\rm cont}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

Simpler version of analytic reconstruction [Greynat, PM, Peris'12] $s' = s + 4(\hat{m}_q^2(2M) - M^2)$

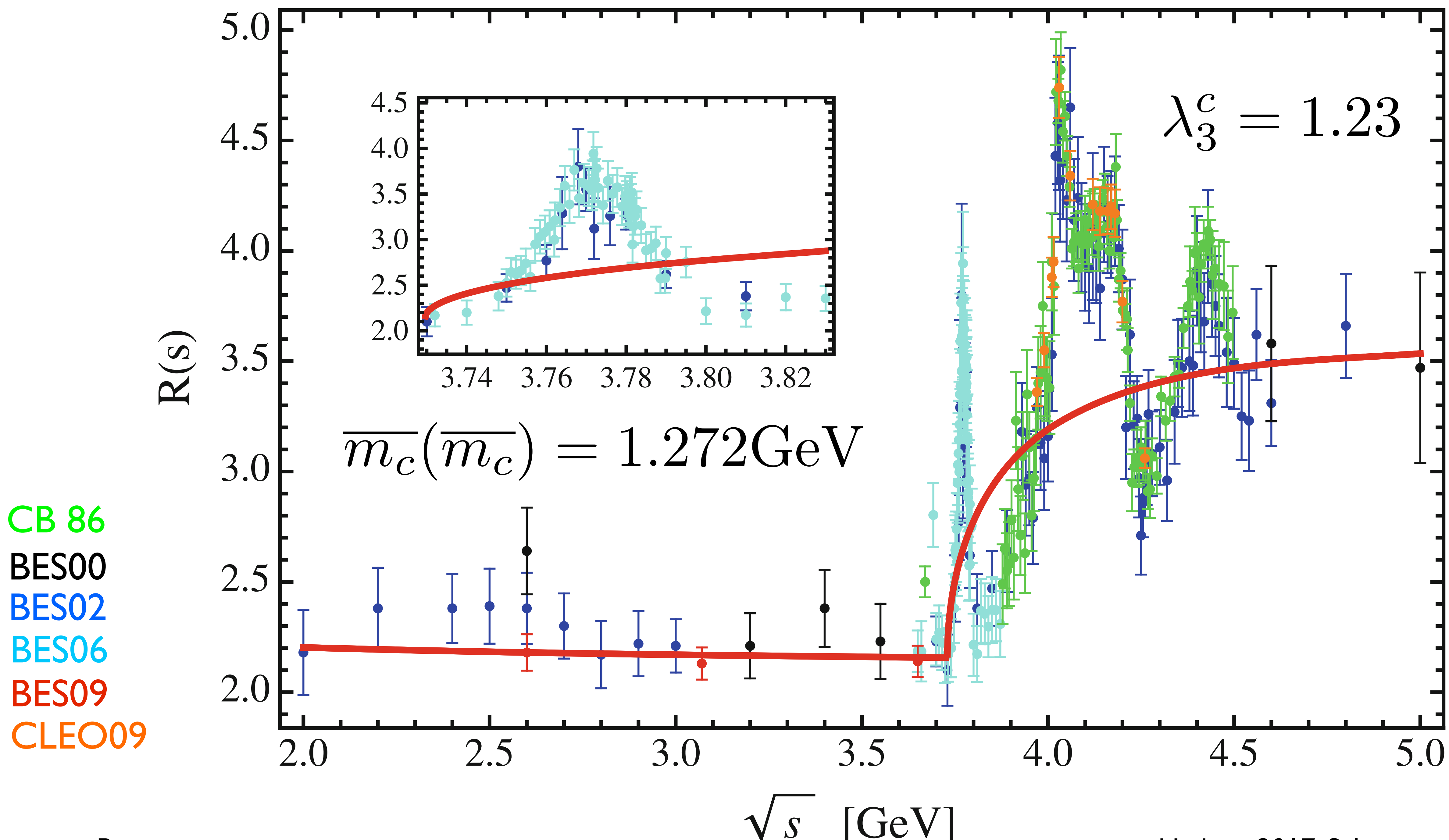
Two parameters to determine: $m_q\ ,\ \lambda_3^q$

We use Zeroth + 2nd moments
(no experimental data on R(s) so far)

n	Resonances	Continuum	Total	Theory
0	1.231 (24)	−3.229(+28)(43)(1)	−1.999(56)	**Input** (11)
1	1.184 (24)	0.966(+11)(17)(4)	2.150(33)	2.169(16)
2	1.161 (25)	0.336(+5)(8)(9)	1.497(28)	**Input** (25)
3	1.157 (26)	0.165(+3)(4)(16)	1.322(31)	1.301(39)
4	1.167 (27)	0.103(+2)(2)(26)	1.270(38)	1.220(60)
5	1.188 (28)	0.080(+1)(1)(38)	1.268(47)	1.175(95)

QCD Sum Rules

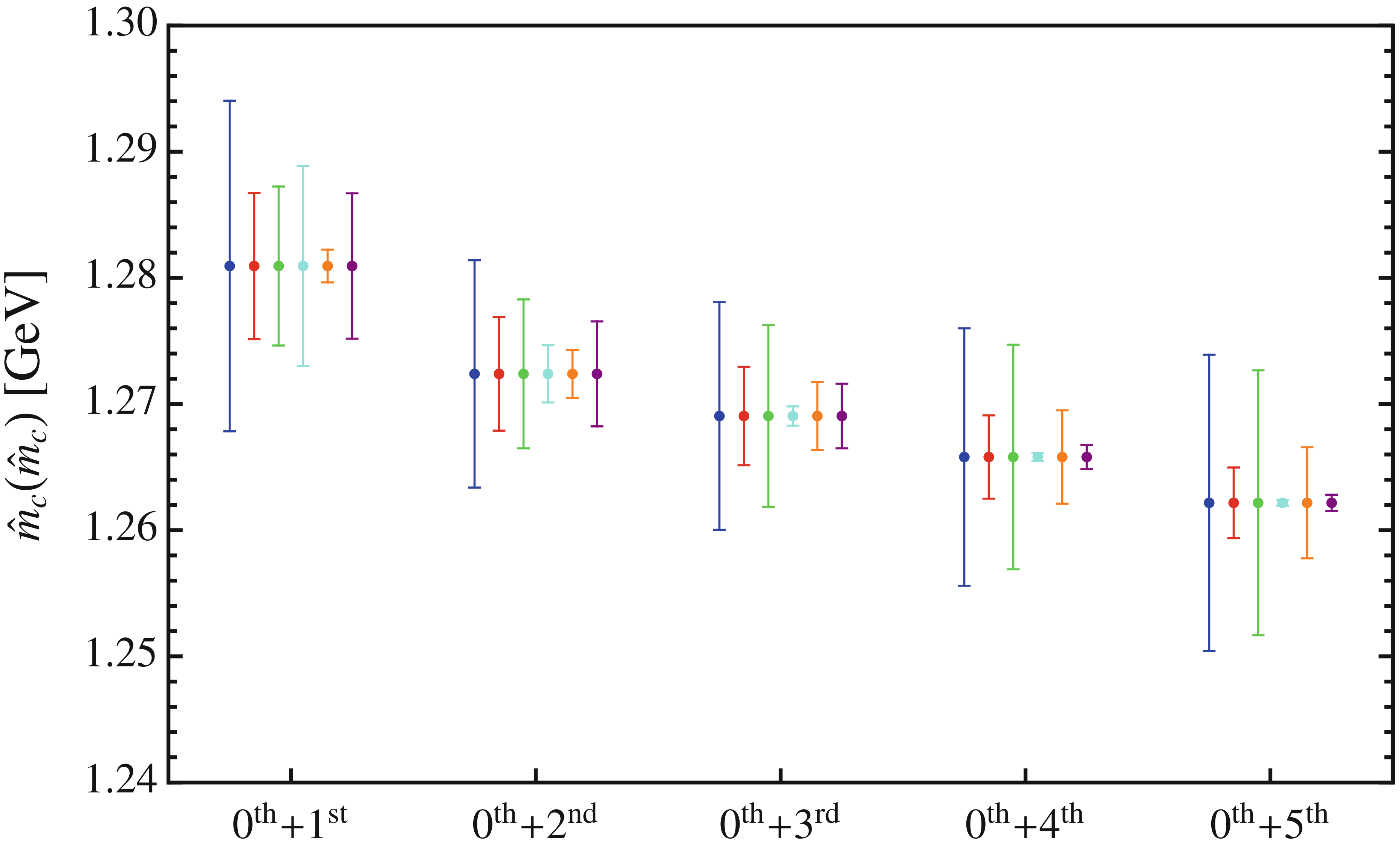
Our approach



QCD Sum Rules

Our approach

Repeat for each pair Zeroth+nth moment



Total
Resonances
Truncation error
Comparison with R^{Exp} threshold data
Condensates
$\Delta\alpha_s(M_z)$

Good consistency between different sum rules

QCD Sum Rules

Our approach: error budget

Resonances:

$$\frac{9}{4}Q_q^2\left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n}\bar{C}_n = \sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{M_R^{2n+1}\hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty}\frac{\mathrm{d}s}{s^{n+1}}R_q(s)$$

R	M_R [GeV]	Γ_R^e [keV]
J/Ψ	3.096916	5.55(14)
$\Psi(2S)$	3.686109	2.36(4)

from 6 MeV to 3 MeV

(0th+1st) (0th+5th)

(completely dominated by J/Ψ)

QCD Sum Rules

Our approach: error budget

Truncation Error (theory error):

$$\mathcal{M}_n^{\mathrm{pQCD}} = \frac{9}{4} Q_q^2 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n} \bar{C}_n$$

$$\bar{C}_n = \bar{C}_n^{(0)} + \left(\frac{\hat{\alpha}}{\pi}\right) \bar{C}_n^{(1)} + \left(\frac{\hat{\alpha}}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\hat{\alpha}}{\pi}\right)^3 \bar{C}_n^{(3)} + \mathcal{O}\left(\frac{\hat{\alpha}}{\pi}\right)^4$$

[Erler, Luo '03]

$$\boxed{\Delta\mathcal{M}_n^{(4)} = \pm N_C C_F C_A^3 \, Q_q^2 \left[\frac{\hat{\alpha}_s(\hat{m}_q)}{\pi} \right]^4 \left(\frac{1}{2\hat{m}_q(\hat{m}_q)} \right)^{2n}}$$

from 5 MeV to 10 MeV

(0th+1st) (0th+5th)

Example known orders

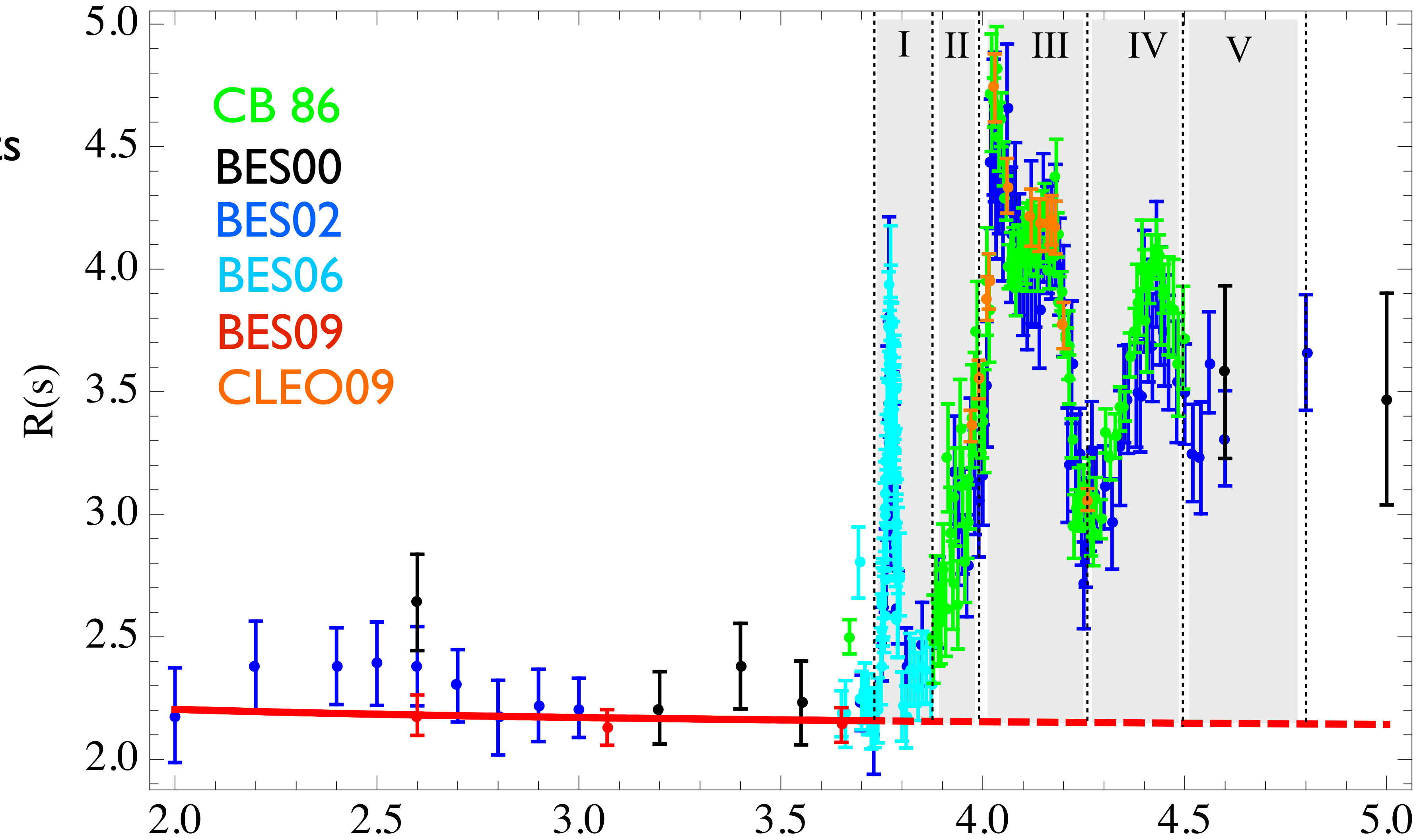
n	$\frac{\Delta\mathcal{M}_n^{(2)}}{\lvert\mathcal{M}_n^{(2)}\rvert}$	$\frac{\Delta\mathcal{M}_n^{(3)}}{\lvert\mathcal{M}_n^{(3)}\rvert}$
0	1.88	3.03
1	2.14	2.84
2	1.92	4.58
3	3.25	5.63
4	6.70	4.30
5	19.18	3.62

QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data: $(2M_D \leq \sqrt{s} \leq 4.8\text{GeV})$

Calculate Exp moments



QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data:

Collab.	n	$[2M_{D^0}, 3.872]$	$[3.872, 3.97]$	$[3.97, 4.26]$	$[4.26, 4.496]$	$[4.496, 4.8]$
CB86	0	–	0.0339(22)(24)	0.2456(25)(172)	0.1543(27)(108)	–
	1	–	0.0220(14)(15)	0.1459(16)(102)	0.0801(14)(56)	–
	2	–	0.0143(9)(10)	0.0868 (9)(61)	0.0416(7)(29)	–
BES02	0	0.0334(24)(17)	0.0362(29)(18)	0.2362(41)(118)	0.1399(38)(70)	0.1705(63)(85)
	1	0.0232(17)(12)	0.0235(19)(12)	0.1401(24)(70)	0.0726(20)(36)	0.0788(30)(39)
	2	0.0161(12)(8)	0.0152(13)(8)	0.0832(15)(42)	0.0378(10)(19)	0.0365(14)(18)
BES06	0	0.0311(16)(15)	–	–	–	–
	1	0.0217(11)(11)	–	–	–	–
	2	0.0151(8)(7)	–	–	–	–
CLEO09	0	–	–	0.2591(22)(52)	–	–
	1	–	–	0.1539(13)(31)	–	–
	2	–	–	0.0915(8)(18)	–	–
Total	0	0.0319(14)(11)	0.0350(18)(15)	0.2545(18)(46)	0.1448(27)(59)	0.1705(63)(85)
	1	0.0222(9)(8)	0.0227(12)(10)	0.1511(11)(27)	0.0752(14)(31)	0.0788(30)(39)
	2	0.0155(6)(6)	0.0147(8)(6)	0.0899(6)(16)	0.0391(7)(16)	0.0365(14)(18)

QCD Sum Rules

Our approach: error budget

Comparison with $R^{\rm Exp}$ threshold data:

$$\int_{(2M_{D^0})^2}^{(4.8\,\mathrm{GeV})^2} \frac{\mathrm{d}s}{s} R_c^{\rm cont}(s)\Big|_{\hat{m}_c=1.272\,\mathrm{GeV}} = \mathcal{M}_0^{\rm Data} = 0.6367(195) \longrightarrow \lambda_3^{\rm c,exp} = 1.34(17)$$

$$(2M_D \leq \sqrt{s} \leq 4.8\mathrm{GeV})$$

Error induced to Quark mass:

I) $\lambda_3^c = 1.23 \to \lambda_3^{\rm c,exp} = 1.34$

from $+6.4$ MeV to $+0.2$ MeV

II) $\Delta\lambda_3^{\rm c,exp} = 0.17$

from 4.7 MeV to 0.1 MeV

n	Data	$\lambda_3^c = 1.34(17)$	$\lambda_3^c = 1.23$
0	0.6367(195)	0.6367(195)	0.6239
1	0.3500(102)	0.3509(111)	0.3436
2	0.1957(54)	0.1970(65)	0.1928
3	0.1111(29)	0.1127(38)	0.1102
4	0.0641(16)	0.0657(23)	0.0642
5	0.0375(9)	0.0389(14)	0.0380

QCD Sum Rules

Our approach: error budget

Comparison with R^{Exp} threshold data:

$$\int_{(2M_{D^0})^2}^{(4.8\,\mathrm{GeV})^2} \frac{\mathrm{d}s}{s} R_c^{\mathrm{cont}}(s)\Big|_{\hat{m}_c=1.272\,\mathrm{GeV}} = \mathcal{M}_0^{\mathrm{Data}} = 0.6367(195) \longrightarrow \lambda_3^{c,\mathrm{exp}} = 1.34(17)$$

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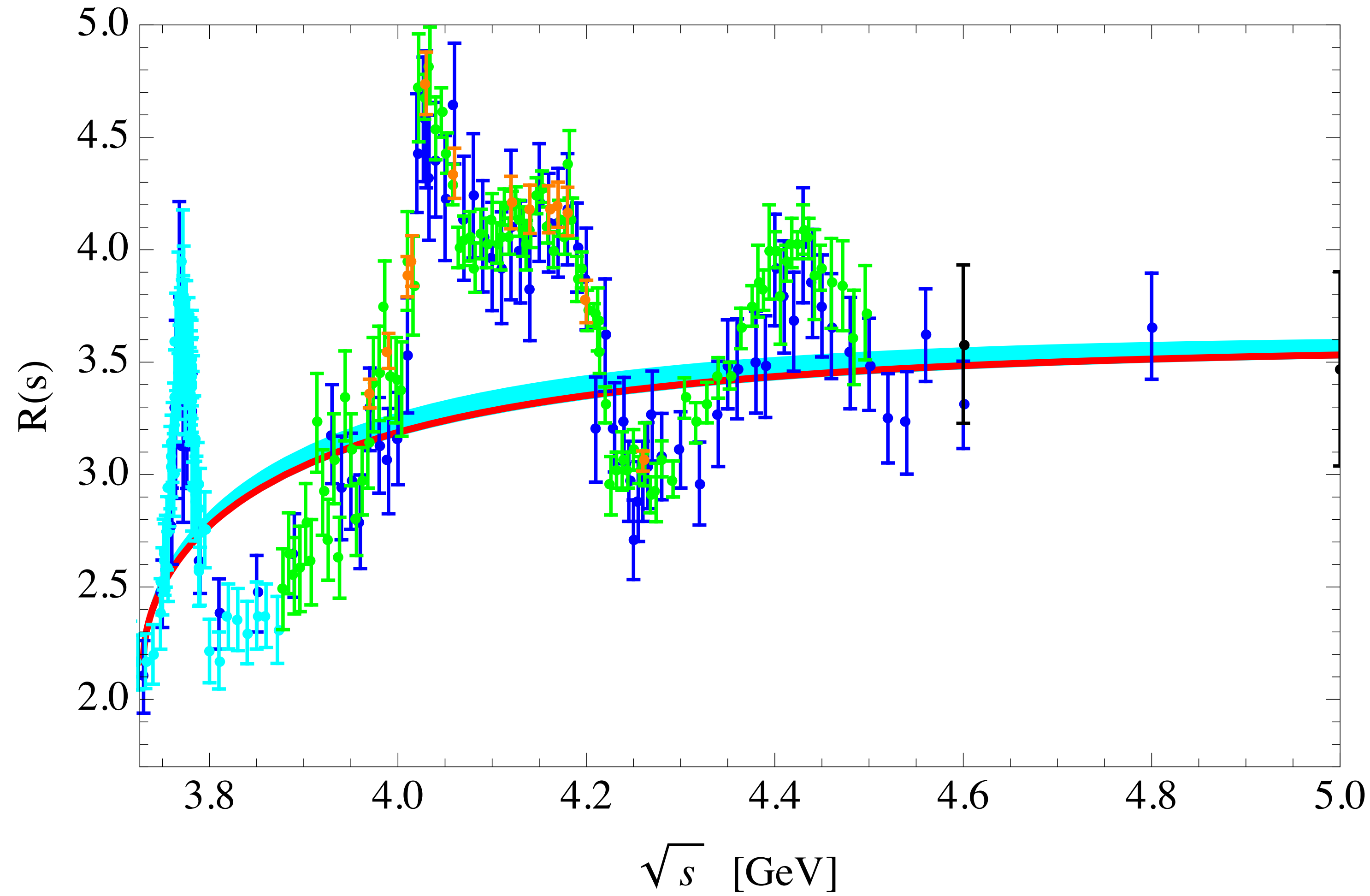


QCD Sum Rules

Our approach: error budget

Condensates:

Non-perturbative effects due to gluon condensates to the moments are: [Chetyrkin et al '12]

$$\mathcal{M}_n^{\rm nonp}(\mu^2) = \frac{12\pi^2 Q_q^2}{(4\hat{m}_q^2)^{n+2}} \,{\rm Cond}\, a_n \left(1 + \frac{\alpha_s(\hat{m}_q^2)}{\pi} b_n\right)$$

$a_n \,, b_n$ are numbers, and ${\rm Cond} = \langle \frac{\alpha_s}{\pi} G^2 \rangle = (5 \pm 5) \cdot 10^{-3} {\rm GeV}^4$ [Dominguez et al '14]

$$\Delta \langle \frac{\alpha_s}{\pi} G^2 \rangle = 5 \cdot 10^{-3} {\rm GeV}^4 \longrightarrow$$

from 1 MeV to 4 MeV
(0th+1st) (0th+5th)

(but this is only the first condensate)

Parametric error:

$$\Delta\overline{m_c}(\overline{m_c})[{\rm MeV}] = -0.5 \cdot 10^3 \frac{\rm MeV}{{\rm GeV}^4} \Delta \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

QCD Sum Rules

Our approach: error budget

$\Delta\alpha_s(M_z)$ $\qquad$ $\alpha_s(M_z) = 0.1182(16)$ $\qquad$ from PDG

$$\Delta\alpha_s(M_z) = 0.0016 \longrightarrow \text{from 6 MeV to 1 MeV}$$

Parametric error:

(0th+1st) $\quad \Delta\overline{m_c}(\overline{m_c})[\mathrm{MeV}] = 3.6 \cdot 10^3 \Delta\alpha_s(M_z)$

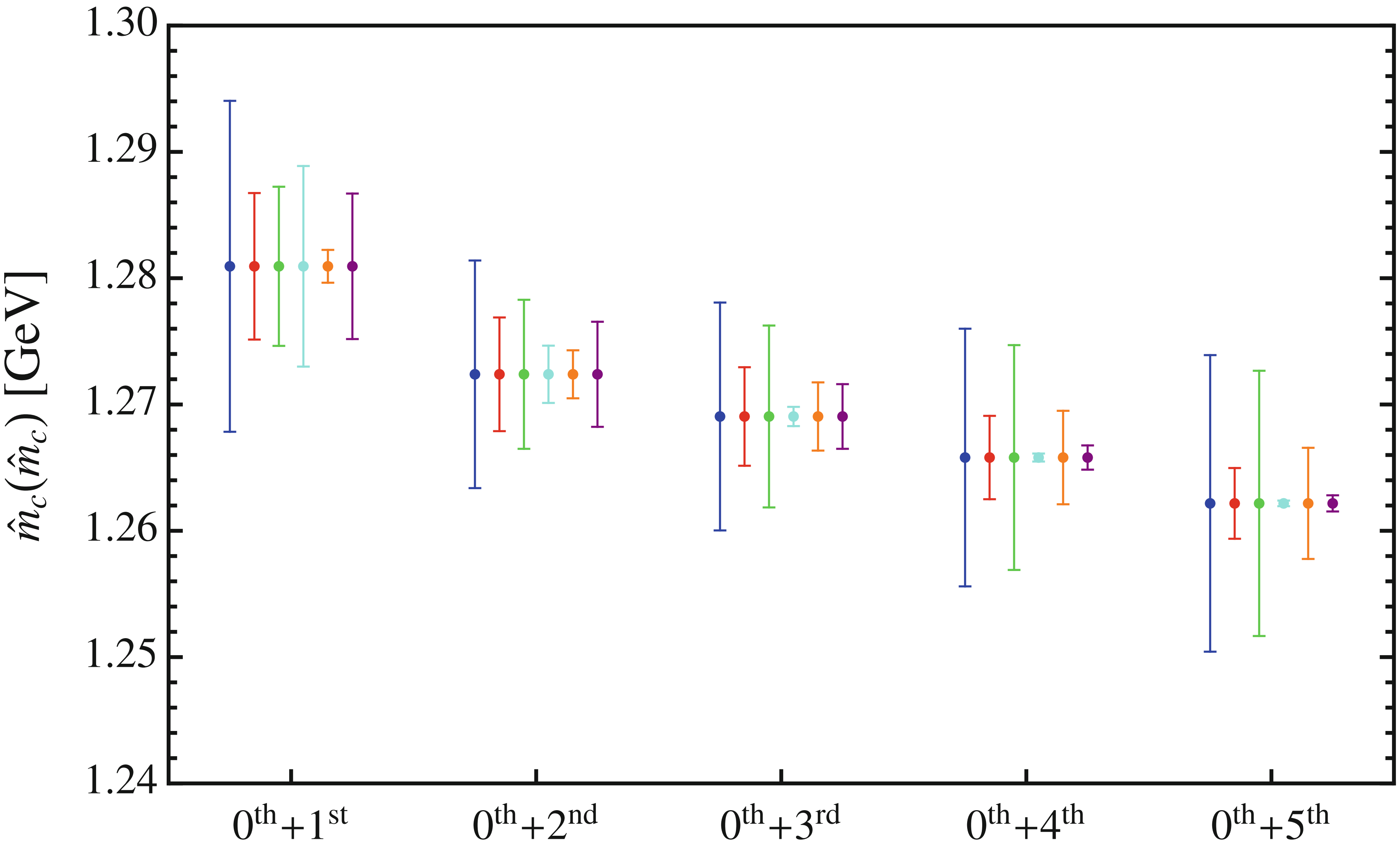
(0th+5th) $\quad \Delta\overline{m_c}(\overline{m_c})[\mathrm{MeV}] = -0.4 \cdot 10^3 \Delta\alpha_s(M_z)$

QCD Sum Rules

Our approach

preliminary results

What pair/result to choose?



Resonances

Truncation error

Comparison with R^{Exp} threshold data

Condensates

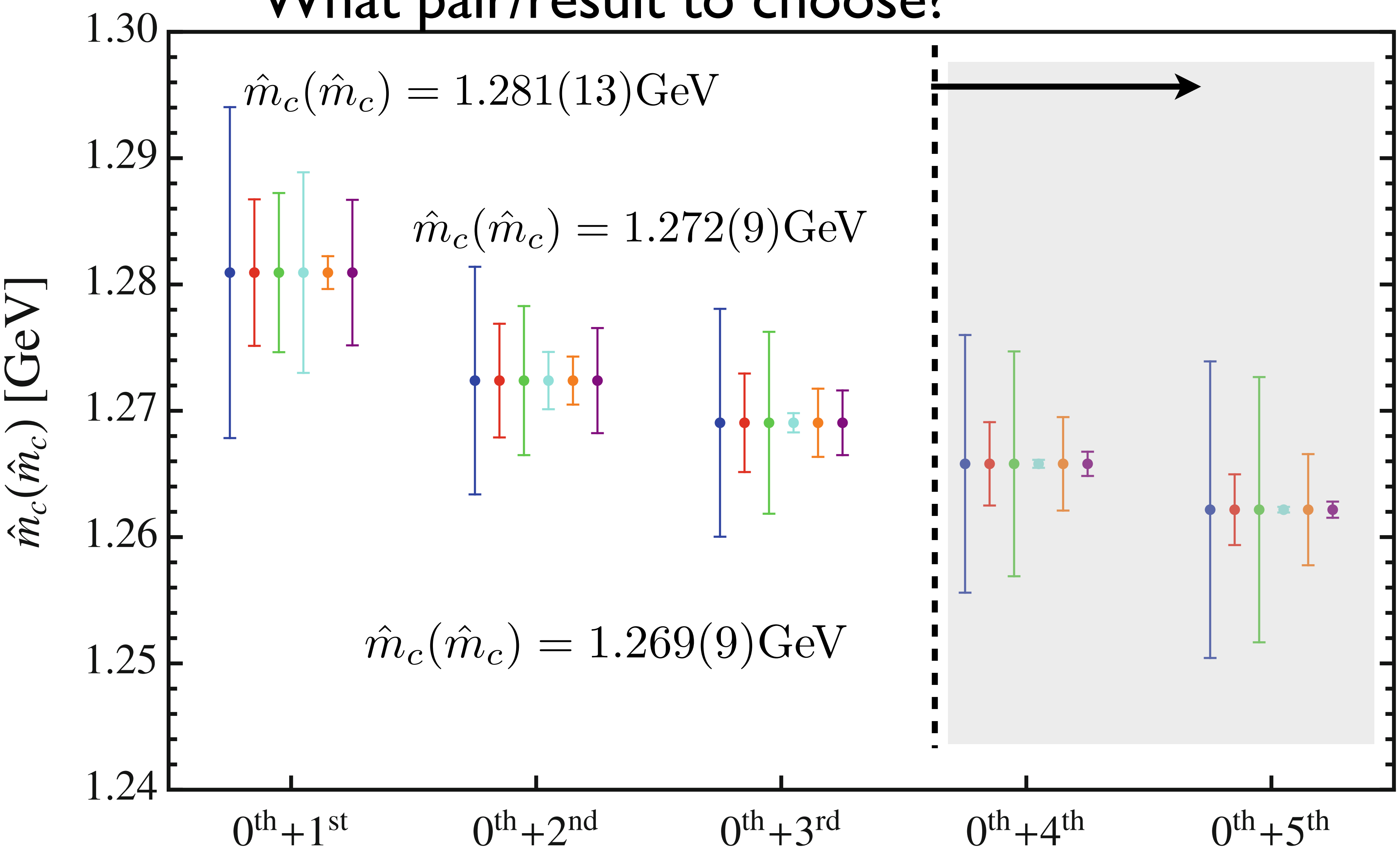
$\Delta\alpha_s(M_z)$

QCD Sum Rules

Our approach

preliminary results

What pair/result to choose?



Resonances

Truncation error

Comparison with R^{Exp} threshold data

Condensates

$\Delta\alpha_s(M_z)$

Large condensate effects
+
new condensates will matter

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

$$\chi^2 = \frac{1}{2}\sum_{n,m} \left(\mathcal{M}_n - \mathcal{M}_n^{\mathrm{pQCD}}\right) \left(\mathcal{C}^{-1}\right)^{nm} \left(\mathcal{M}_m - \mathcal{M}_m^{\mathrm{pQCD}}\right) + \chi_c^2$$

$$\mathcal{C} = \frac{1}{2}\sum_{n,m} \rho^{\mathrm{Abs}(n-m)} \Delta\mathcal{M}_n^{(4)} \Delta\mathcal{M}_m^{(4)}$$

ρ a correlation parameter

$$\chi_c^2 = \left(\frac{\Gamma^e_{J/\Psi(1S)} - \Gamma^{e,\mathrm{exp}}_{J/\Psi(1S)}}{\Delta\Gamma^e_{J/\Psi(1S)}}\right)^2 + \left(\frac{\Gamma^e_{\Psi(2S)} - \Gamma^{e,\mathrm{exp}}_{\Psi(2S)}}{\Delta\Gamma^e_{\Psi(2S)}}\right)^2 + \left(\frac{\hat{\alpha}_s(M_z) - \hat{\alpha}_s(M_z)^{\mathrm{exp}}}{\Delta\hat{\alpha}_s(M_z)}\right)^2 + \left(\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle - \langle\frac{\alpha_s}{\pi}G^2\rangle^{\mathrm{exp}}}{\Delta\langle\frac{\alpha_s}{\pi}G^2\rangle}\right)^2$$

QCD Sum Rules

Our approach: more than two moments?

Define a χ^2 function:

ρ	Constraints	$(\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2)_\rho$ -0.06	$\mathcal{M}_0,\ (\mathcal{M}_1, \mathcal{M}_2)_\rho$ -0.05	$\mathcal{M}_0,\ (\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)_\rho$ 0.32
$\hat{m}_c(\hat{m}_c)$ [GeV]		1.275(8)	1.275(8)	1.271(7)
λ_3^c		1.19(8)	1.19(8)	1.19(7)
$\Gamma^e_{J/\Psi}$ [keV]	5.55(14)	5.57(14)	5.57(14)	5.59(14)
$\Gamma^e_{\Psi(2S)}$ [keV]	2.36(4)	2.36(4)	2.36(4)	2.36(4)
C_G [GeV^4]	0.005(5)	0.005(5)	0.005(5)	0.004(5)
$\hat{\alpha}_s(M_z)$	0.1182(16)	0.1178(15)	0.1178(15)	0.1173(15)

QCD Sum Rules

Our approach: more than two moments?

Preferred scenario:

	$0\text{th} + (1\text{st} + 2\text{nd})_\rho$ $\Delta\hat{m}_c(\hat{m}_c)$ [MeV]	$(0\text{th} + 2\text{nd})$ $\Delta\hat{m}_c(\hat{m}_c)$ [MeV]
Central value	1274.5	1272.4
$\Delta\Gamma^e_{J/\Psi}$	5.9	4.5
$\Delta\Gamma^e_{\Psi(2S)}$	1.4	0.4
Truncation	–	5.9
$\Delta\lambda^c_3$	3.0	2.3
Condensates	1.1	1.9
$\Delta\hat{\alpha}_s(M_Z)$	5.4	4.2
Total	8.7	9.0

Conclusions and Outlook

• Heavy quark masses are interesting: for being fundamental parameters as well as for their implications on many phenomenological scenarios.

• From the different strategies, one of the most precise is the use of SR. Quark mass determinations at the % or sub-% level.

• Using SR + global fit using different moments (χ^2), we extract $\hat{m}_c(\hat{m}_c)$

$$\hat{m}_c(\hat{m}_c) = 1.272(9)\mathrm{GeV}$$

Good agreement with other determinations based on SRs and lattice!

• Error sources are understood: seems a clear roadmap for improvements

• Next step: the bottom case

Thanks!