Partonic quasi-distributions of the pion in chiral quark models

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Outline

- Parton distributions – basic properties of hadrons
- Soft matrix elements, accessible from low-energy models of QCD
- Chiral quark models of the pion
- Parton quasi-distributions, designed for Euclidean QCD lattices

- Results and predictions for quasi-distributions of the pion from chiral quark models
Introduction
Parton distribution

\[ Q^2 = -q^2, \quad x = \frac{Q^2}{2p \cdot q}, \quad Q^2 \to \infty \]

Factorization of soft and hard processes, Wilson’s OPE

\[ \langle J(q)J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle O_i(\mu) \rangle \]

Twist expansion \( \to \)

\[ F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \ldots \]
Parton distribution

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Twist expansion \( \to \) \( F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \cdots \)

Bj limit \( \to \) light-cone momentum is constrained: \( k^+ \equiv k^0 + k^3 = x P^+ \)

\( x \in [0, 1] \)
Distribution amplitude (DA) of the pion

Enters various measures of exclusive processes, e.g., pion-photon transition form factor
Field-theoretic definition
(here for quarks in the pion, leading twist)

Parton Distribution Function (DF):

\[
q(x) = \int \frac{dz^-}{4\pi} e^{ix P^+ z^-} \langle P | \bar{\psi}(0) \gamma^+ [0, z] \psi(z) | P \rangle \big|_{z^+ = 0, z^\perp = 0}
\]

Parton Distribution Amplitude (DA):

\[
\phi(x) = \frac{i}{F_{\pi}} \int \frac{dz^-}{2\pi} e^{i(x-1) P^+ z^-} \langle P | \bar{\psi}(0) \gamma^+ \gamma_5 [0, z] \psi(z) | \text{vac} \rangle \big|_{z^+ = 0, z^\perp = 0}
\]

(isospin suppressed)

**P** - pion momentum, \( v^\pm \equiv v^0 \pm v^3 \) - light-cone basis

\([z_1, z_2] = \exp \left( -ig_s \int_{z_1}^{z_2} d\xi \lambda^a A^+_a(\xi) \right)\) - Wilson’s gauge link

**x** - fraction of the light-cone mom. \( P^+ \) carried by the quark, \( x \in [0, 1] \)
Remarks

- Only *indirect* experimental information for the *pion* distributions:
  - DF from Drell-Yan in E615, DA from dijets in E791 and from exclusive processes involving pions
  - Impossibility to implement PDF or PDA on the euclidean lattices, only lowest moments can be obtained

- However, there exist (largely forgotten) simulations on *transverse* lattices – discussed later
Quasi-distributions
Parton quasi-distributions (quarks in the pion)

Parton Quasi-Distribution Function (QDF):

\[ V(y; P_3) = \int \frac{dz^3}{4\pi} e^{iyP_3 z^3} \langle P | \bar{\psi}(0) \gamma^3 [0, z] \psi(z) | P \rangle \Big|_{z^0 = 0, z^\perp = 0} \]

Parton Quasi-Distribution Amplitude (PDA):

\[ \tilde{\phi}(y; P_3) = \frac{i}{F_\pi} \int \frac{dz^3}{2\pi} e^{iy-1P_3 z^3} \langle P | \bar{\psi}(0) \gamma^+ \gamma_5 [0, z] \psi(z) | \text{vac} \rangle \Big|_{z^0 = 0, z^\perp = 0} \]

\( y \) - fraction of pion’s \( P_3 \) carried by the quark

Analogy to DF and DA, but \( y \) is not constrained

Basic property:

\[ \lim_{P_3 \to \infty} V(x; P_3) = q(x), \quad \lim_{P_3 \to \infty} \tilde{\phi}(x; P_3) = \phi(x) \]
QDF and QDA in the momentum representation

Constrained longitudinal momenta, but \( y \in (-\infty, \infty) \)
(partons can move “backwards”)
Chiral quark models
Chiral quark models

- $\chi_{SB}$ breaking $\rightarrow$ massive quarks
- Point-like interactions
- Soft matrix elements with pions (and photons, $W$, $Z$)
- One-quark loop, regularization:
  1) Pauli-Villars (PV)
  2) Spectral Quark Model (SQM) - implements VMD

Quantities evaluated at the quark model scale (where constituent quarks are the only degrees of freedom)
Chiral quark models

- $\chi_{SB}$ breaking $\rightarrow$ massive quarks
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Need for evolution
Gluon dressing, renorm-group improved
Scale and evolution

QM provide non-perturbative result at a low scale $Q_0$

$$F(x, Q_0)_{\text{model}} = F(x, Q_0)_{\text{QCD}}, \quad Q_0 - \text{the matching scale}$$

Determination of $Q_0$ via momentum fraction: quarks carry 100% of momentum at $Q_0$. One adjusts $Q_0$ in such a way that when evolved to $Q = 2$ GeV, the quarks carry the experimental value of 47%

LO DGLAP evolution:

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

[Davidson, Arriola 1995]:

$$q(x; Q_0) = 1$$
Older results from chiral quark models w/ evolution
LO DGLAP evolution to the scale $Q^2 = (4 \text{ GeV})^2$:

points: Fermilab E615, Drell-Yan

line: QM evolved to $Q = 4$ GeV
Pion quark DF, QM vs. transverse lattice

points: transverse lattice [Dalley, van de Sande 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS param. at 0.5 GeV
points: E791 data from dijet production in $\pi + A$

solid line: QM at $Q = 2$ GeV

dashed line: asymptotic form $6x(1-x)$ at $Q \to \infty$
Pion DA, QM vs. transverse lattice

points: transverse lattice data [Dalley, van de Sande 2003]
line: QM at $Q = 0.5$ GeV+

NEW: Quasi-distributions from QM
Analytic formulas (in the chiral limit)

SQM (at the QM scale):

\[
\tilde{\phi}(y, P_z) = V(y, P_z) = \frac{1}{\pi} \left[ \frac{2m_\rho P_z y}{m_\rho^2 + 4P_z^2 y^2} + \arctg \left( \frac{2P_z y}{m_\rho} \right) \right] + (y \to 1 - y)
\]

(similar simplicity for PV NJL)

Satisfy the proper normalization

\[
\int_{-\infty}^{\infty} dy \tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dy V(y, P_z) = 1, \quad \int_{-\infty}^{\infty} dy 2y V(y, P_z) = 1
\]

and the limit

\[
\lim_{P_z \to \infty} \tilde{\phi}(y, P_z) = \lim_{P_z \to \infty} V(y, P_z) = \theta[y(1 - y)] = \phi(x) = q(x), \quad (y = x)
\]
(a) Quark QDA of the pion in NJL ($m_\pi = 0$) at various $P_z$, plotted vs. $y$

(b) The same, but for the quark QDF multiplied with $2y$
Quark QDA of the pion in NJL (a) and SQM (b) ($m_\pi = 310$ MeV, $P_z = 0.9$ and 1.3 GeV), plotted vs. $y$ and compared to the lattice at $Q = 2$ GeV (LaMET [Zhang et al. 2017])
Evolution of QDF
Relation $k_T$-unintegrated quantities (TMA, TMD)

Radyushkin’s formula [2016] – follows just from Lorentz invariance

\[ \tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_0^1 dx P_z TMA(x, k_1^2 + (x - y)^2 P_z^2). \]

QDA can be obtained from TMA via a double integration!

Analogously

\[ V(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_0^1 dx P_z TMD(x, k_1^2 + (x - y)^2 P_z^2). \]
Evolution of unintegrated DF
UDF or TMD

Kwieciński’s method [2003], based on one-loop CCFM
DGLAP-like evolution, diagonal in $b$-space conjugate to $k_T$
For the non-singlet case:

\[
Q^2 \frac{\partial f(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz P_{qq}(z) \times \left[ \Theta(z - x) J_0[(1 - z)Qb] f\left(\frac{x}{z}, b, Q\right) - f(x, b, Q) \right]
\]
Results of evolution of pion QDF in $Q$ at fixed $P_z$

Strength moved to lower $y$ as $Q$ increases
Changing $P_z$ at fixed $Q$

$Q^2 = 4\text{GeV}^2$

$P_z \rightarrow \infty$ limit achieved fastest at $y \sim 0.6 - 0.9$
Conclusions
Conclusions

- Model evaluation of quasi-distributions of the pion (at the QM scale)
- Very simple analytic results, all consistency conditions met, illustration and check of definitions and methods
- Results at finite $P_z$ are interesting per se (Radyushkin’s relation, relation to Ioffe-time distributions, ET wave functions), can be (favorably) compared to QDA from Euclidean lattice QCD
- For QDF of the pion predictions made for various $Q$ (Kwieciński’s evolution) and $P_z$
  - $P_z \sim 1$ GeV, accessible presently on the lattice, may not be sufficiently close to $P_z \to \infty$ limit
  - Convergence fastest for intermediate $y$, suggesting the domain where lattice may work best

- Recent activity also on related objects: pseudo-distributions, Ioffe-time distributions... Lattice efforts for both $N$ and $\pi$