MC top mass calibration

Vicent Mateu

In collaboration with M. Buttenschön, B. Dehnadi A. Hoang, M. Preisser, I. Stewart

HADRON — Salamanca 29-09-2017
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¿HADRON? - Salamanca 29-09-2017
Outline

- The top mass: determination and schemes
- MC generators and the top quark mass
- Theoretical setup
- Pythia fits for the top mass
- Conclusions and Outlook
Top quark mass: measurements
Top quark mass reconstruction

LCH case (Tevatron is similar)

basic idea: identify ALL top decay products

\[
(\sum_{i\sim\text{decay}} p_i^\mu)^2 = m_t^2 + \Delta m_t^2
\]

core idea: invariant mass distribution

\[
m_t + \Delta m_t^{\text{hadronic}}
\]

Jet algorithm
hadronization
Soft physics
Underlying event

Graph showing invariant mass distribution with peak position at approximately 10.6 GeV.
Top quark mass reconstruction

LCH case (Tevatron is similar)

basic idea: identify ALL top decay products

\[( \sum_{i \sim \text{decay}} p_i^\mu )^2 = m_t^2 + \Delta m_t^2 \]

pole mass + “contamination”

invariant mass distribution

\[ m_t^{\text{MC}} + \Delta m_t^{\text{hadronic,MC}} \]

Very hard to compute these effects from first principles
Experimentalists use Parton shower MC to estimate them
Therefore the parameter determined is in fact \( m_t^{\text{MC}} \)

**MC top mass**: mass of top propagator prior to top decay… scheme?
historically all-order identification with the pole mass
Small history of top quark measurements

Direct reconstruction has the best sensitivity to the top mass

(projection) \[ \Delta m_t \sim 200 \text{ MeV} \]

With this precision the following questions become pressing:

How precise is the MC?
What is the precise meaning of \( m_t^{\text{MC}} \)?
Could it be related to a short-distance mass?
Top quark mass: schemes
Quark masses and schemes in QFT

\[ \mu \text{- independent has divergences} \]

\[ \Sigma(p, m_0) \]

\[ \frac{1}{p^2 - m_0^2} \]

\( m_0 \) = bare mass

quark mass defined in context of perturbation theory

Pole scheme: propagator has a pole for \( p^2 \to m_p^2 \)

\[ m_p = m_0 + \Sigma(m_p, m_0) \]

pole mass is \( \mu \) - independent

The whole diagram is absorbed into the mass definition !!!!

Absorbs into mass parameter UV fluctuations from scales \( > 0 \)

Linear sensitivity to infrared momenta leads to factorially growing coefficients in perturbation theory

asymptotic behavior, but impacts lower orders

Similar behavior in other diagrams for a given observable

\[ \Sigma(m, m) \sim m \sum_n \alpha_s^{n+1}(2\beta_0)^n n! \]
Quark masses and schemes in QFT

\[ m_0 = \text{bare mass} \]

\[ m_{\text{MS}}(\mu) = m_0 + \Sigma(m_p, m_0)_{\text{finite}} \quad \text{MS mass is } \mu \text{- dependent} \]

\[ m_p - m_{\text{MS}}(\mu) = \Sigma(m_p, m_0)_{\text{finite}} \equiv \delta m_{\text{MS}}(\mu) \quad \text{Relation to the pole mass is used to define any other short-distance scheme} \]

\[ \delta m_{\text{MS}}(\bar{m}) = \bar{m} \sum_{n=1}^{\infty} \left[ \frac{\alpha_s(\bar{m})}{4\pi} \right]^n a_{n0} \quad \text{Renormalon is mass independent} \]
Quark masses and schemes in QFT

\[ \mu \text{- independent has divergences} \]

\[ \sum(p, m_0) \]

\[ \mu \text{- independent has divergences} \]

\[ m_p - m_{\text{MSR}}(R) \equiv \delta m_{\text{MSR}}(R) = R \sum_{n=1} \left[ \frac{\alpha_s(R)}{4\pi} \right]^n a_{n0} \]

\[ m_{\text{MSR}}(R = 0) = m_{\text{pole}} \]

\[ m_{\text{MSR}}[R = \overline{m}(\overline{m})] = \overline{m}(\overline{m}) \]

MSR interpolates between pole and MS

\[ m_{\text{MSR}}[R \sim \Lambda_{\text{QCD}}] \]

similar to pole mass or kinetic mass but without renormalon problem!

[Hoang, Jain, Scimemi, Stewart, ‘08]  [Hoang, Jain, Lepenik, VM, Preisser, Scimemi, Stewart, ‘17]

MSR mass absorbs into mass parameter UV fluctuations from scales > R

\[ m_{\text{pole}} - m_{\text{MSR}}(R) = \sum_{n=1} \left[ \frac{\alpha_s(R)}{4\pi} \right]^n a_{n0} \]

[See tuesday talk by P.G. Ortega for an application to bottomonium]
MC generators and the top quark mass
\[ m_t^{MC} = (173.34 \pm 0.27_{\text{stat}} \pm 0.71_{\text{syst}}) \text{GeV} \]

LHC-Tevatron combination

Let us assume that, to some extent, MC perform ab initio QCD computations
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**Important fact:** MC’s do not include quark self-energy corrections. Therefore one can consider these are absorbed into the mass parameter… … but only for energy scales above the shower cutoff $\Lambda_{\text{shower}} \sim \Lambda_{\text{QCD}}$

$$m_t^{\text{MC}} = (173.34 \pm 0.27_{\text{stat}} \pm 0.71_{\text{syst}}) \text{ GeV}$$  \text{LHC-Tevatron combination}

$$m_t^{\text{MC}} = m^{\text{MSR}}(R = 1 \text{ GeV})$$
Let us assume that, to some extent, MC perform ab initio QCD computations

**Important fact:** MC’s do not include quark self-energy corrections. Therefore one can consider these are absorbed into the mass parameter... ...but only for energy scales above the shower cutoff $\Lambda_{\text{shower}} \sim \Lambda_{\text{QCD}}$

**Parton shower and hadronization model** modify the shape of the distribution and further modify the peak location.

$$m_t^{MC} = m^{MSR}(R = 1 \text{ GeV}) + \Delta_{t,MC}(R = 1 \text{ GeV}) \sim 1 \text{ GeV}$$
Calibrating the top quark mass

Outline

• Introduction
• Monte Carlo generators and the top quark mass
• Calibration of the Monte Carlo top mass parameter
• Preliminary detailed results of first serious systematic analysis
• Summary, future plans

In collaboration with:
M. Butenschön
B. Dehnadi,
V. Mateu,
M. Preisser
I. Stewart

CALIPER
Calibrating the MC mass

Buttenschön, Dehnadi, Hoang, VM, Preisser, Stewart, 2016

Strategy: “measure” the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.
Calibrating the MC mass

Strategy: “measure” the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.

Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !) \( e^+ e^- \rightarrow t \bar{t} + X \) [Buttenschön, Dehnadi, Hoang, VM, Preisser, Stewart, 2016]
Calibrating the MC mass

Strategy: “measure” the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.

Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)

\( e^+ e^- \rightarrow t \bar{t} + X \)

We will use massive event shapes (jets) as our observable: thrust, HJM, etc…
Calibrating the MC mass

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Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !) $e^+ e^- \rightarrow t \bar{t} + X$

We will use massive event shapes (jets) as our observable: thrust, HJM, etc…

We will use SCET as our first principle QCD calculator:

- Full mass scheme control: \( \overline{\text{MS}} \) or MSR
- Full hadronization control: shape function
- Full resummation perturbation theory + control: \( N^2\text{LL} + \text{NLO} \)
Calibrating the MC mass

Strategy: “measure” the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.

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Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !)

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We will use SCET as our first principle QCD calculator:

- Full mass scheme control: \( \overline{\text{MS}} \) or MSR
- Full hadronization control: shape function
- Full resummation perturbation theory + control: N\(_2\)LL + NLO

We will use Pythia as our parton-shower MC… to start with
Theoretical setup
Massive event shape

\[ \tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \approx \frac{M_1^2 + M_2^2}{Q^2} \]

Simplification: consider boosted tops

Shifts the whole distribution by \( \sim \frac{M_1^2 + M_2^2}{Q^2} \)

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

Peak very sensitive to mass

\[ \tau_{\text{peak}}^{\text{tree-level}} = 1 - \sqrt{1 - \frac{4 m_t^2}{Q^2}} \approx \frac{2 m_t^2}{Q^2} \]

Higher mass sensitivity for lower \( Q \)

Invariant mass distribution in the resonance region of wide hemisphere jets!!
Factorization theorem for massless event shapes

\[
\frac{1}{\sigma_0} \frac{d\sigma}{d\epsilon} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\text{QCD}}}{Q}\right)
\]

- **Universal Wilson Coefficient**
- **Jet function**
- **Soft function**
- **Nonsingular terms, power corrections**

Calculable in perturbation theory

Perturbative and nonperturbative components

[Bauer, Lee, Fleming, Sterman]
[Berger, Kuks, Sterman]
The factorization theorem needs to be modified to include massive particles

- Explicit mass dependence in matrix elements
- Account for primary and secondary heavy quark radiation
- Running with different number of flavors according to thresholds
- Matching to a new EFT in the peak region
Renormalization group evolution

- Hard scale: $\mu_H \sim Q$
- Jet scale: $\mu_J \sim Q \sqrt{\tau}$
- Soft scale: $\mu_S \sim Q \tau$
- $\Lambda_{QCD}$

Local running: $\log^n \left( \frac{Q}{\mu_J} \right)$
Non-local running: $\log^n \left( \frac{\mu_J}{\mu_S} \right)$
$\log^n \left( \frac{\Lambda_{QCD}}{\mu_S} \right)$
A massive quark pair has an invariant mass that is bounded. The distribution is closely related to the production of massive secondary heavy quark production in the thrust distribution for a fixed c.m. energy. The conceptual setup to define these quark mass corrections is partially guided by identifying the hard coe− cient and the jet and soft functions. The nonperturbative component which can be parametrized through a convolution with a soft model function that can be determined through fits to experimental data in a framework to carry out this task systematically.

Accounting for quark masses in this context adds an− other problem related to the fact that dispersions are related to specific divergences of collinear and soft scales, which themselves depend on the value of the quark masses. The framework of SCET – properly ex− tended to carry out this task systematically – demonstrated in Ref. [15] that also the conceptual is− sues in Ref. [14] where it has been shown that the problem of secondary heavy quark production in the thrust distribution is related to the fact that...
Primary Heavy quark production

New kinematical regime for

\[ Q^2 \tau - 2m^2 \sim 2m\Gamma \ll 2m^2 \]

New class of logs, match to a new EFT

boosted HQET (or bHQET)

In practice: new matching coefficient, new jet function, new renormalization scale

Takes place in Peak region

In this theory one can treat finite width effects from first principles (essential for top, irrelevant for bottom)

[M. Butenschön, B. Dehnadi, A. Hoang, VM, M. Preisser, I. Stewart, 2016]
boosted HQET Factorization Theorem

\( \frac{1}{\sigma_0} \frac{d\sigma}{d\tau} \sim H^{(n_l+1)} \times H_m^{(n_f)} \times J_B^{(n_l)} \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m_t^2}{Q^2}, \frac{\Gamma_t}{m_t}, \tau\right) \)

QCD to SCET matching
SCET to bHQET matching
bHQET jet function
SCET soft function

corrections to fact. theorem (most of them accounted for!)

in SCET regime \( \overline{MS} \) mass has correct behavior
in bHQET regime MSR mass has correct behavior

\[ \begin{align*}
\mu_i \text{ [GeV]} \\
\mu_B \\
\mu_J \\
\mu_S \\
\overline{m}(\overline{m})
\end{align*} \]

\[ \begin{align*}
m_t^{MSR}(R), \overline{m}_t(\mu) \text{ [GeV]} \\
- \text{ red: } m_t^{MSR}(R) \\
- \text{ blue: } \overline{m}_t(\mu)
\end{align*} \]
Results
Convergence and Stability: MSR vs pole

500 profiles ( = 500 fits) tune 7
\[ \alpha_s(m_Z) = 0.1181 \quad \Gamma_t = 1.4 \text{ GeV} \]
\[ Q = 700, 1000, 1400 \text{ GeV} \quad \text{peak (60/80)} \%
\]
input: \( m_t^{MC} = 173 \text{ GeV} \)  \hspace{1cm}  fit to find \( m_t^{MSR} \) or \( m_t^{pole} \)

Good convergence and stability for \( m_t^{MSR}(1 \text{ GeV}) \)
\( m_t^{MSR}(1 \text{ GeV}) \) theoretically the closest to the MC top mass
\( m_t^{MSR}(1 \text{ GeV}) \) numerically close to \( m_t^{MC} \)

\[ m_t^{MC} = 173 \text{ GeV} \quad \left( \tau_2^{e^+e^-} \right) \]

<table>
<thead>
<tr>
<th>Mass</th>
<th>Order</th>
<th>Central</th>
<th>Perturb.</th>
<th>Incompatibility</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_t^{MSR} )</td>
<td>NLL</td>
<td>172.80</td>
<td>0.26</td>
<td>0.14</td>
<td>0.29</td>
</tr>
<tr>
<td>( m_t^{MSR} )</td>
<td>NNLL</td>
<td>172.82</td>
<td>0.19</td>
<td>0.11</td>
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Spread of results from 21 datasets
Convergence and Stability: MSR vs pole

500 profiles ( = 500 fits)    tune 7
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\[ m_t^{MSR}(1 \text{ GeV}) \text{ theoretically the closest to the MC top mass} \]
\[ m_t^{MSR}(1 \text{ GeV}) \text{ numerically close to } m_t^{MC} \]

Pole mass shows worse convergence
\[ m_t^{pole} \text{ not at all close to } m_t^{MC} \]

900/600 MeV difference at NLL/NNLL !!!
\[ m_t^{pole} \neq m_t^{PYTHIA} \]

Similar findings from the other 20 data sets
Results for the pole mass

1) Pole mass implemented in code (pole fits)

\[ m_t^{\text{MC}} = 173 \text{ GeV} \ (\tau^+_L e^-) \]

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<tr>
<td>( m_t^{\text{pole}} )</td>
<td>NLL</td>
<td>172.10</td>
<td>0.34</td>
<td>0.16</td>
<td>0.38</td>
</tr>
<tr>
<td>( m_t^{\text{pole}} )</td>
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<td>172.43</td>
<td>0.18</td>
<td>0.22</td>
<td>0.28</td>
</tr>
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\[ (m_t^{\text{pole}})_{\text{NLL/N}^2\text{LL}} < m_t^{\text{MSR}} (1 \text{ GeV}) < m_t^{\text{MC}} \]

2) Pole mass determined from the MSR mass \( \alpha_s (m_Z) = 0.1181 \quad n_f = 5 \)

\[ m_t^{\text{pole}} - m_t^{\text{MSR}} (1 \text{ GeV}) = 0.173 + 0.139 + 0.160 + 0.237 \text{ GeV} \]

\[ m_t^{\text{pole}} = (172.99 \text{ GeV})_{N^2\text{LL}}, (172.80 \text{ GeV})_{\text{NLL}} \]

Calibration in terms of the pole mass involves large higher-order perturbative corrections

Additional uncertainty on pole mass

\[ (m_t^{\text{pole}})_{N^2\text{LL}} = 172.72 \pm 0.40 \text{ GeV} \]

\[ (m_t^{\text{pole}})_{\text{NLL}} = 172.45 \pm 0.52 \text{ GeV} \]
Final result for $m_t^{\text{MSR}}(1 \text{ GeV})$

All investigated MC top mass values show a consistent picture

The MC top quark mass parameter is indeed closely related to the MSR mass at a low scale

Linear dependence between $m_t^{\text{MSR}}(1 \text{ GeV})$ and $m_t^{\text{MC}}$

Within uncertainties $m_t^{\text{MSR}}(1 \text{ GeV}) \approx m_t^{\text{MC}}$

Spread of results from 21 datasets

$\Omega_1^{\text{PY}} = 0.41 \pm 0.07 \pm 0.02 \text{ GeV (NLL)}$

$\Omega_1^{\text{PY}} = 0.42 \pm 0.07 \pm 0.03 \text{ GeV (N}^2\text{LL)}$

TABLE I. Results of the calibration for $m_t^{\text{MSR}}(1 \text{ GeV})$

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Calibration:

PYTHIA 8.205, tune 7
NNLL, $\Gamma_t = 1.4 \text{ GeV}$
$\alpha_s(m_Z) = 0.1181$
Conclusions
Conclusions

- Top behaves almost as a real particle... but not quite.
- MC top mass can be calibrated by comparison to a hadron-level ab initio “QCD calculator” (SCET).
- Electron-positron collision, simplest possible setup.
- MC mass close to MSR mass but NOT to pole mass.
- Consistency: \( N^3LL + N^2LO \) (CALIPER, fortran 2008 code).
- Further checks (on the beam pipe): C-Parameter, Heavy Jet Mass, etc...
Backup slides
Details of the calibration

\[
\frac{d\sigma}{d\tau} = f(m_t^{\text{MSR}}, \alpha_s(m_Z), \Omega_i, \mu_j, \Gamma_t)
\]

any scheme (calibrate!)

renormalization scales (estimate perturbative uncertainties)

top width (check MC accuracy)

non-perturbative (fit together with mass)

PYTHIA (8.205) samples:

Q values: 600, 700, 800, ..., 1400 GeV
Various top widths: dynamic (mass dependent), hard-wired 1.4 GeV, 2.0 GeV
Various tunes: 1 (very old), 3 (LEP), 7 (“Monash”)
Statistics: $10^7$ events.

Fitting strategy:

Fit parameters: $m_t^{\text{MSR}}, \Omega_i$. No sensitivity to $\alpha_s$, hence use world average

Standard fit based on $\chi^2$ minimization (using MC statistical errors).

Analysis with 500 sets of profiles (renormalization scales).

Different Q-sets: 7 sets with energies between 600 and 1400 GeV.

Different n-sets: 3 choices of fit-ranges - (x/y)% of the maximum peak height.

\{ 21 datasets \}
**Fit Results: Pythia vs Theory**

- Good description of Pythia 8.2 default output with default scale setting NNLL + NLO QCD.
- Increasing discrepancies in distribution tail and for higher energies due to off shell effects in NS.

**Theoretical accuracy at NNLL / NLL order**

- We only compare peak data, since otherwise Pythia is not reliable. Also peak gives higher mass sensitivity.

- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution
- Higher mass sensitivity for lower Q ($p_T$)
- Finite lifetime effects included
- Dependence on non-perturbative parameters

**Top Quark WG Meeting, CERN, May 17-18, 2016**
Overview of parton shower MC

Monte-Carlo Event Generators
• Full simulation of all processes.
• QCD-inspired: partly first principles QCD partly model
• All experimental aspects can be implemented.
• Description power of data better than intrinsic accuracy.
• Essential for full simulation of particle collisions and experimental analyses.

Pythia
Sherpa
Herwig
Geneva

Original figure by D. Zeppenfeld

Hard matrix element: annihilates initial particles into tops + other hard partons

Parton shower: QCD resummation at LL of large Sudakov logs
~ partial NLO matrix elements

Top mass: mass of top propagator prior to top decay… scheme?
historically all-order identification with the pole mass
500 profiles ( = 500 fits)

\[ \alpha_s(m_Z) = 0.1181 \quad \Gamma_t = 1.4 \text{ GeV} \]

Different Q-sets \quad peak (60/80)\%

input : \[ m_t^{MC} = 173 \text{ GeV} \]

tune dependence:

\[ [m_t^{MSR}]_{\text{tune}} - [m_t^{MSR}]_{\text{tune}=7} \]

Clear sensitivity to the tune

\[ m_t^{MC} \] will depend on tune

Tune dependence is **not** a calibration uncertainty

(different tune \[\Rightarrow\] different MC \[\Rightarrow\] different calibration)
Fits to Pythia data

very preliminary

\[ m_{MC} = 170 \text{ GeV} \]

\[ m_{MSR}^{1 \text{ GeV}} \text{ vs } \alpha_s(m_Z) \]

error bar: fit + theory uncertainty

various lines for various widths

\[ m_{MC} = 175 \text{ GeV} \]
Top width dependence

500 profiles ( = 500 fits)
\[ \alpha_s(m_Z) = 0.1181 \]
Different Q-sets peak (60/80)%
Three tunes (different colors)
\[ \Gamma_t = (0.7, 1.4, 2.0) \text{ GeV} \]

Clear sensitivity to top width value

Can be interpreted as observable dependence or MC modeling dependence.

Our conclusion: Pythia has an error in describing the correct top width dependence.
\( \alpha_s(m_Z) \) dependence

\[
m^{MC} = 170 \text{ GeV}
\]

\[
m^{\text{MSR}}(1 \text{ GeV}) \text{ vs } \alpha_s(m_Z)
\]

error bar: fit + theory uncertainty

various lines for various widths

\[
m^{MC} = 175 \text{ GeV}
\]
\( \alpha_s(m_Z) \) dependence

\[ m^{MC} = 170 \text{ GeV} \]

\( \overline{m}(\overline{m}) \) vs. \( \alpha_s(m_Z) \)

error bar: fit + theory uncertainty

various lines for various widths

\[ m^{MC} = 175 \text{ GeV} \]
Heavy quark production through gluon splitting

Jet function modification

if enough energy to produce pair of heavy quarks
Scenario III

\[ \begin{align*}
&\text{hard scale } \mu_H \\
&\text{jet scale } \mu_J \\
&\text{mass scale } \mu_m \\
&\text{soft scale } \mu_S
\end{align*} \]

\[ n_l + 1 \text{ massless evolution} \]

\[ n_l \text{ massless evolution} \]

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\[ n_l \text{ massless evolution} \]
Heavy quark production through gluon splitting

![Diagram showing heavy quark production](image)

**Soft function modification**

if enough energy to produce pair of heavy quarks
Scenario IV

\[ \text{Scenario IV} \]

**hard scale** \( \mu_H \)

\[ n_l + 1 \]

massless evolution

**jet scale** \( \mu_J \)

\[ n_l + 1 \]

massless evolution

**soft scale** \( \mu_S \)

mass scale \( \mu_m \)

no matching at the mass scale!