# MC top mass calibration

Vicent Mateu

VNiVERSiDAD D SALAMANCA

In collaboration with M. Buttenschön, B. Dehnadi
A. Hoang, M. Preisser, I. Stewart

HADRON – Salamanca 29-09-2017

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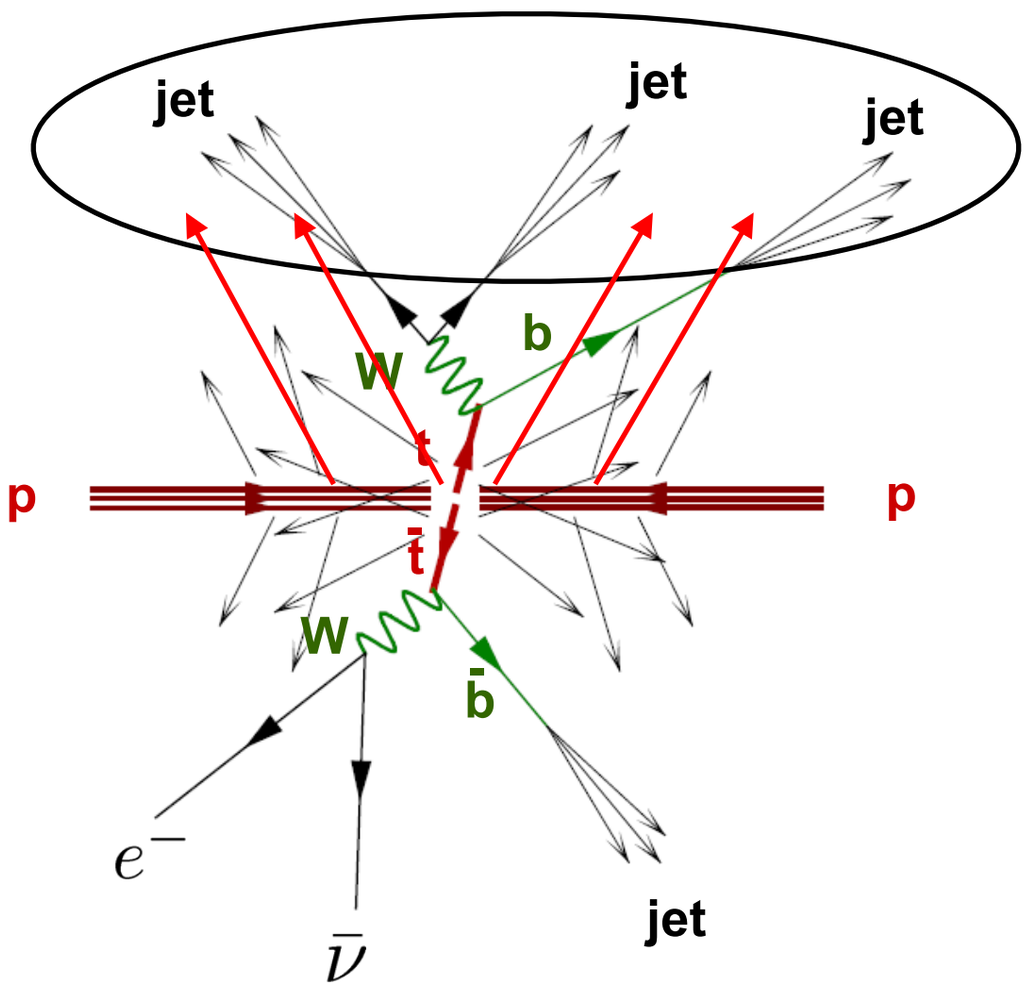
# Outline

- The top mass: determination and schemes
- MC generators and the top quark mass
- Theoretical setup
- Pythia fits for the top mass
- Conclusions and Outlook

# Top quark mass: measurements

# Top quark mass reconstruction

LCH case (Tevatron is similar)




basic idea: identify ALL top decay products

$$\Big(\sum_{i\sim\text{decay}} p_i^\mu\Big)^2 = m_t^2 + \Delta m_t^2$$

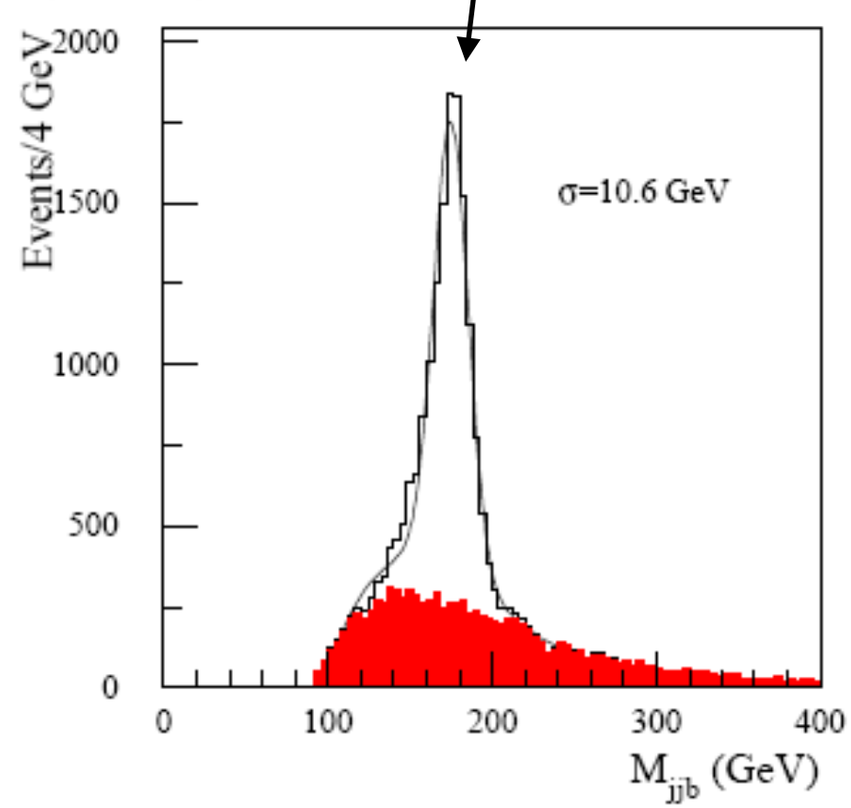
pole mass + "contamination"

invariant mass distribution

$$m_t + \Delta m_t^{\text{hadronic}}$$

Jet algorithm
hadronization
Soft physics
Underlying event

peak position



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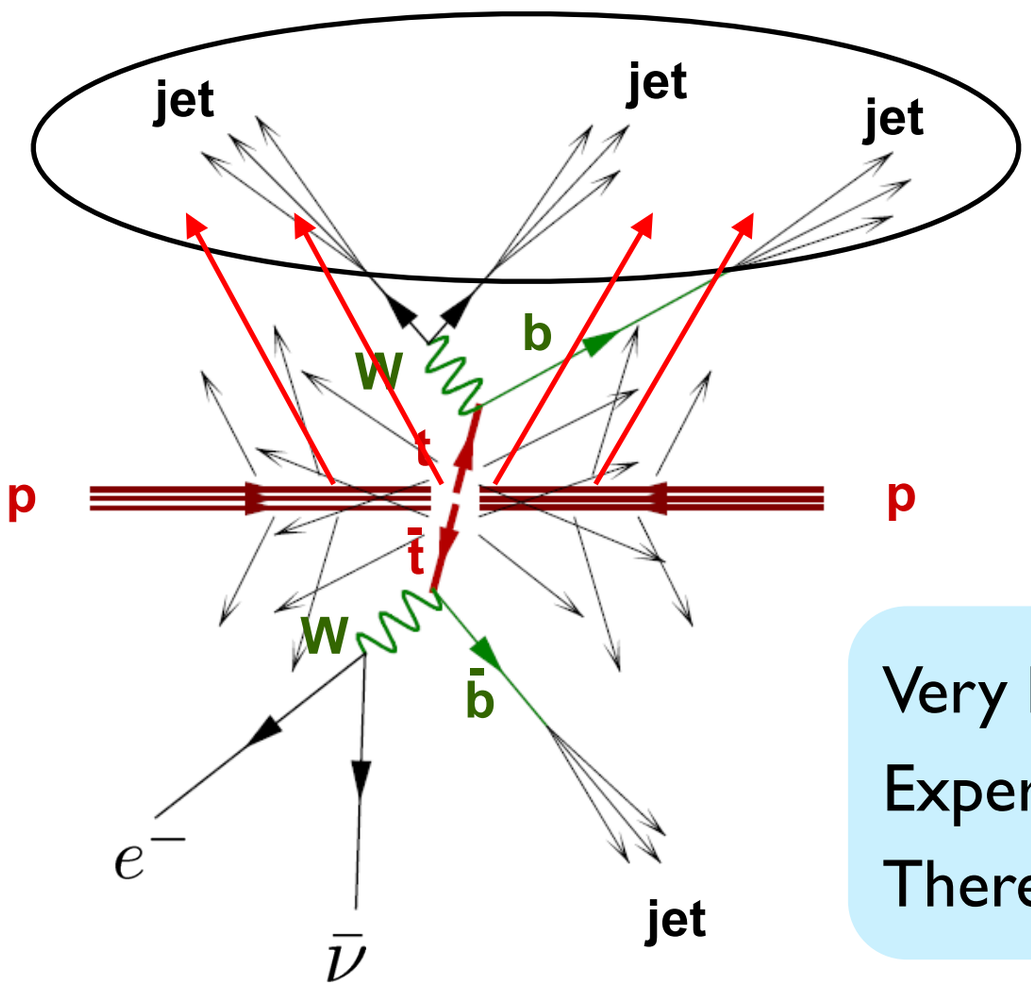



invariant mass distribution

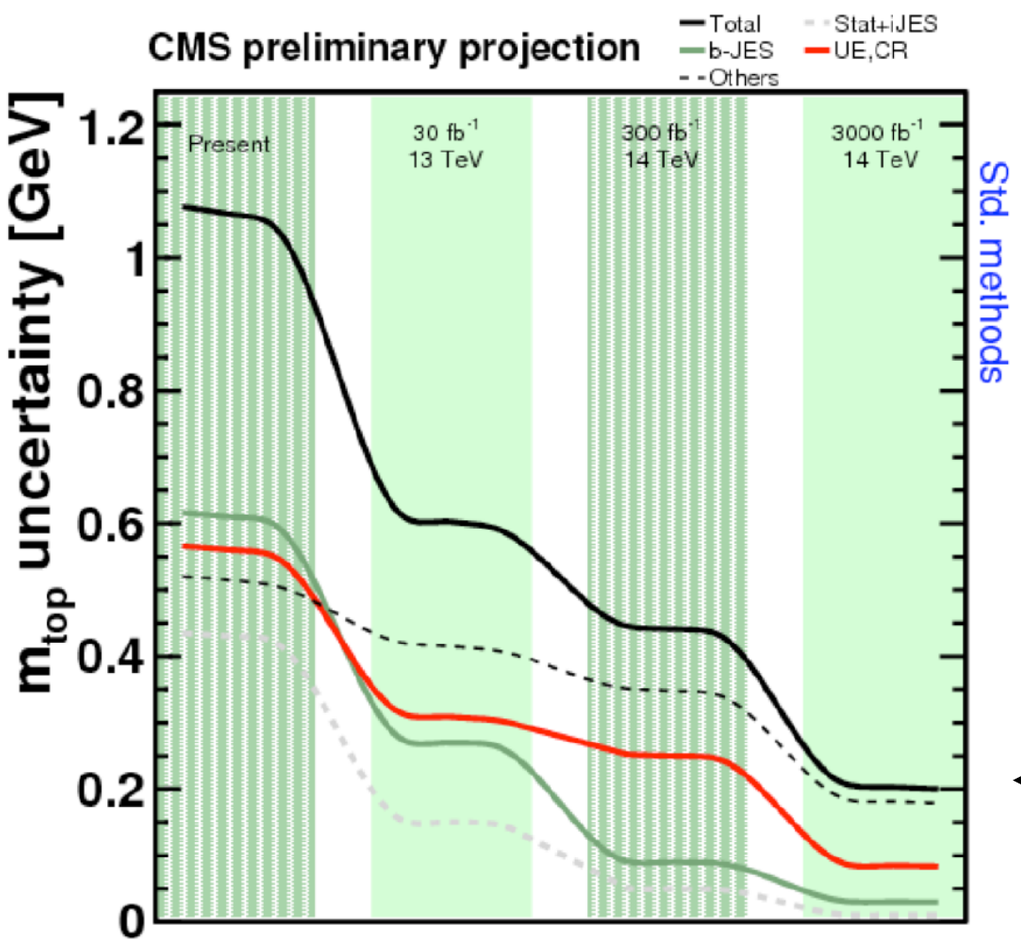
$$m_t^{\text{MC}} + \Delta m_t^{\text{hadronic, MC}}$$

Very hard to compute these effects from first principles
Experimentalists use Parton shower MC to estimate them
Therefore the parameter determined is in fact $m_t^{\text{MC}}$

**<u>MC top mass</u>:** mass of top propagator prior to top decay… scheme?

historically all-order identification with the pole mass

# Small history of top quark measurements




(Note some discrepancies between LHC and Tevatron)

Direct reconstruction has the best sensitivity to the top mass

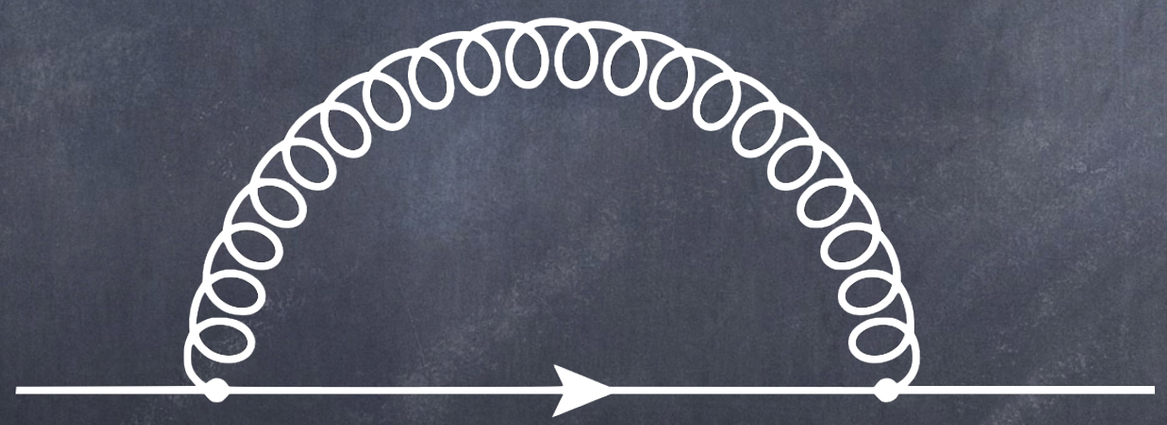
(projection)

$\longleftarrow \quad \Delta m_t \sim 200\,\mathrm{MeV}$

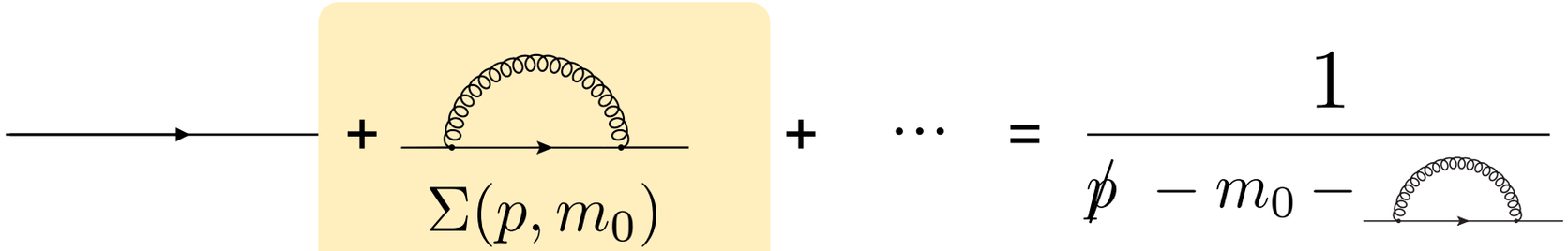
With this precision the following questions become pressing:

How precise is the MC ?
What is the precise meaning of $m_t^{\mathrm{MC}}$ ?
Could it be related to a short-distance mass ?

# Top quark mass: schemes



# Quark masses and schemes in QFT




$m_0$ = bare mass

$\mu$- independent

has divergences

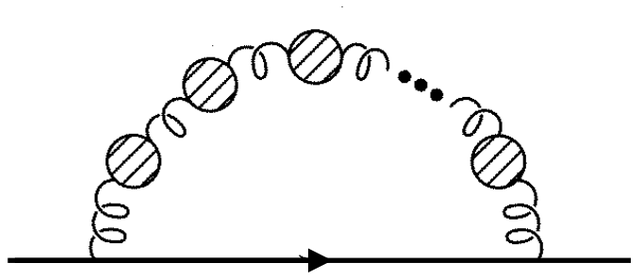
quark mass defined in context of perturbation theory

**Pole scheme**: propagator has a pole for $\not{p} \to m_p$

$m_p = m_0 + \Sigma(m_p, m_0)$ pole mass is $\mu$ - independent

The whole diagram is absorbed into the mass definition !!!!

Absorbs into mass parameter UV fluctuations from scales > 0



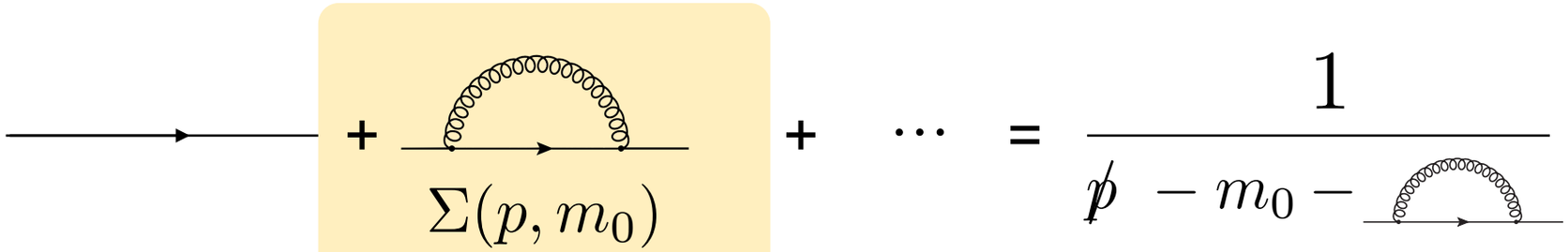
$$\Sigma(m, m) \sim m \sum_n \alpha_s^{n+1} (2\beta_0)^n \, n!$$

Linear sensitivity to infrared momenta leads to factorially growing coefficients in perturbation theory

asymptotic behavior, but impacts lower orders

Similar behavior in other diagrams for a given observable

# Quark masses and schemes in QFT




$m_0$ = bare mass

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Absorbs into mass parameter UV fluctuations from scales > $\overline{m}(\overline{m})$

**$\overline{\text{MS}}$ scheme**: propagator is finite, subtract only $\dfrac{1}{\epsilon}$ in dimensional regularization

$$\overline{m}(\mu) = m_0 + \Sigma(m_p, m_0)|_{\frac{1}{\epsilon}}$$

MS mass is $\mu$ - dependent

no renormalon problem

$$m_p - \overline{m}(\mu) = \Sigma(m_p, m_0)|_{\text{finite}} \equiv \delta m_{\overline{\text{MS}}}(\mu)$$

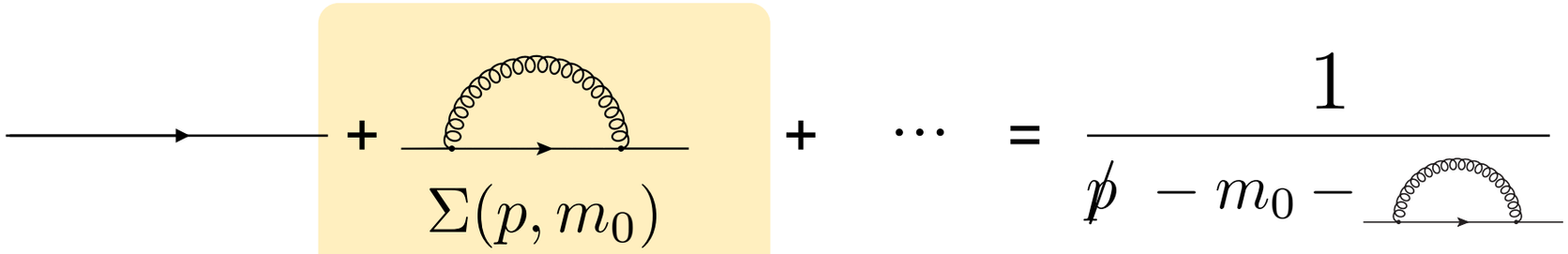
$\mu$ - dependent

Relation to the pole mass is used to define any other short-distance scheme

$$\delta m_{\overline{\text{MS}}}(\overline{m}) = \overline{m} \sum_{n=1} \left[ \frac{\alpha_s(\overline{m})}{4\pi} \right]^n a_{n0}$$

Renormalon is mass independent !!!

# Quark masses and schemes in QFT




$m_0$ = bare mass

$\mu$- independent

has divergences

quark mass defined in context of perturbation theory

*[Hoang, Jain, Scimemi, Stewart, '08]* *[Hoang, Jain, Lepenik, VM, Preisser, Scimemi, Stewart, '17]*

MSR mass absorbs into mass parameter UV fluctuations from scales > R

$$m_p - m^{\rm MSR}(R) \equiv \delta m_{\rm MSR}(R) = R \sum_{n=1} \left[\frac{\alpha_s(R)}{4\pi}\right]^n a_{n0}$$

[See tuesday talk by P. G. Ortega for an application to bottomonium]

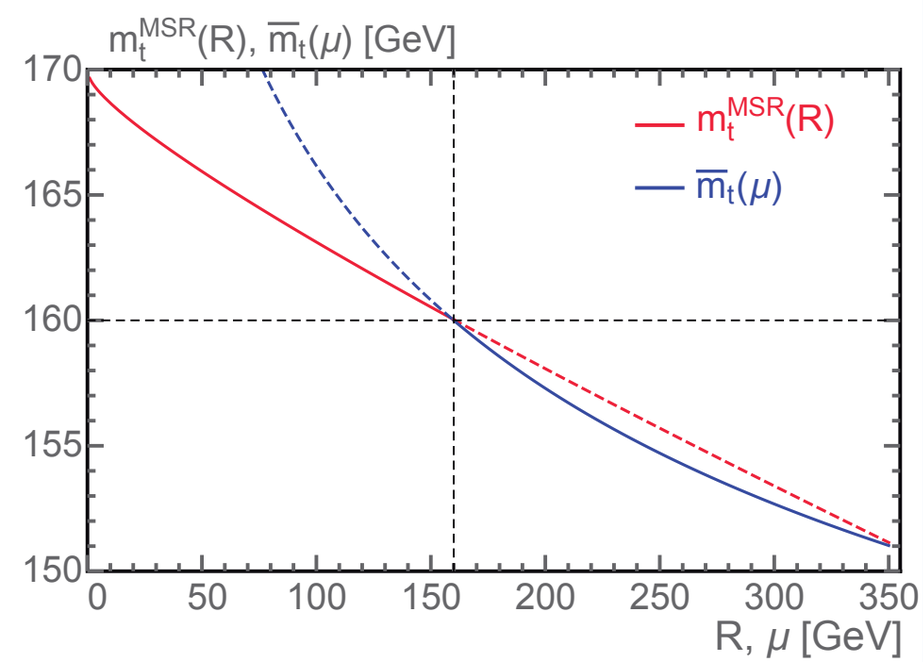
$$m^{\rm MSR}(R=0) = m_{\rm pole}$$

$$m^{\rm MSR}[R = \overline{m}(\overline{m})] = \overline{m}(\overline{m})$$

MSR interpolates between pole and $\overline{\rm MS}$

$m^{\rm MSR}[R \sim \Lambda_{\rm QCD}]$

similar to pole mass or kinetic mass but without renormalon problem!



# MC generators and the top quark mass

# MC mass

*based on [Hoang '14]*
*[Hoang, Stewart '08]*

$m_t^{\rm MC} = (173.34 \pm 0.27_{\rm stat} \pm 0.71_{\rm syst})\,{\rm GeV}$ LHC-Tevatron combination

Let us assume that, to some extent, MC perform ab initio QCD computations

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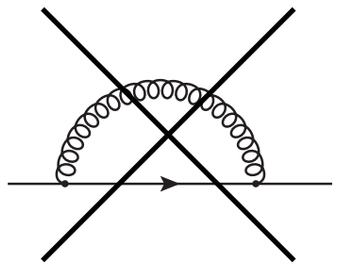
Important fact: MC's do not include quark self-energy corrections

Therefore one can consider these are absorbed into the mass parameter…



… but only for energy scales above the shower cutoff $\Lambda_{\rm shower} \sim \Lambda_{\rm QCD}$

$$m_t^{\rm MC} = m^{\rm MSR}(R = 1\,{\rm GeV})$$

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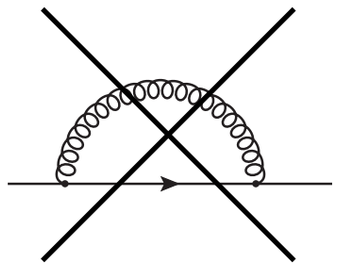

… but only for energy scales above the shower cutoff $\Lambda_{\rm shower} \sim \Lambda_{\rm QCD}$

**Parton shower and hadronization model** modify the shape of the distribution and further modify the peak location.

$$m_t^{\rm MC} = m^{\rm MSR}(R = 1\,{\rm GeV}) + \Delta_{t,{\rm MC}}(R = 1\,{\rm GeV}) \sim 1\,{\rm GeV}$$

# Calibrating the top quark mass

# Calibrating the MC mass

*[Buttenschön, Dehnadi, Hoang, VM, Preisser, Stewart, 2016]*

Strategy: "measure" the MC mass using a completely independent hadron level QCD prediction of a strongly mass-dependent observable.

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Strongly mass-sensitive hadron level observable (as closely as possible related to reconstructed invariant mass distribution !) $e^+ e^- \to t\,\bar{t} + X$

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**We will use SCET as our first principle QCD calculator:**

Full mass scheme control: $\overline{\text{MS}}$ or MSR
Full hadronization control: shape function
Full resummation perturbation theory + control: $\text{N}^2\text{LL}$ + NLO

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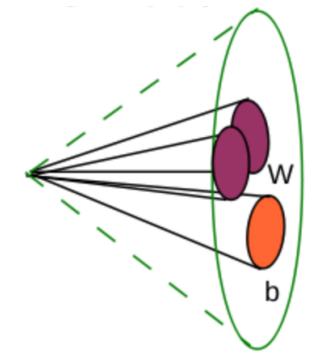
We will use Pythia as our parton-shower MC… to start with

# Theoretical setup

# Massive event shape

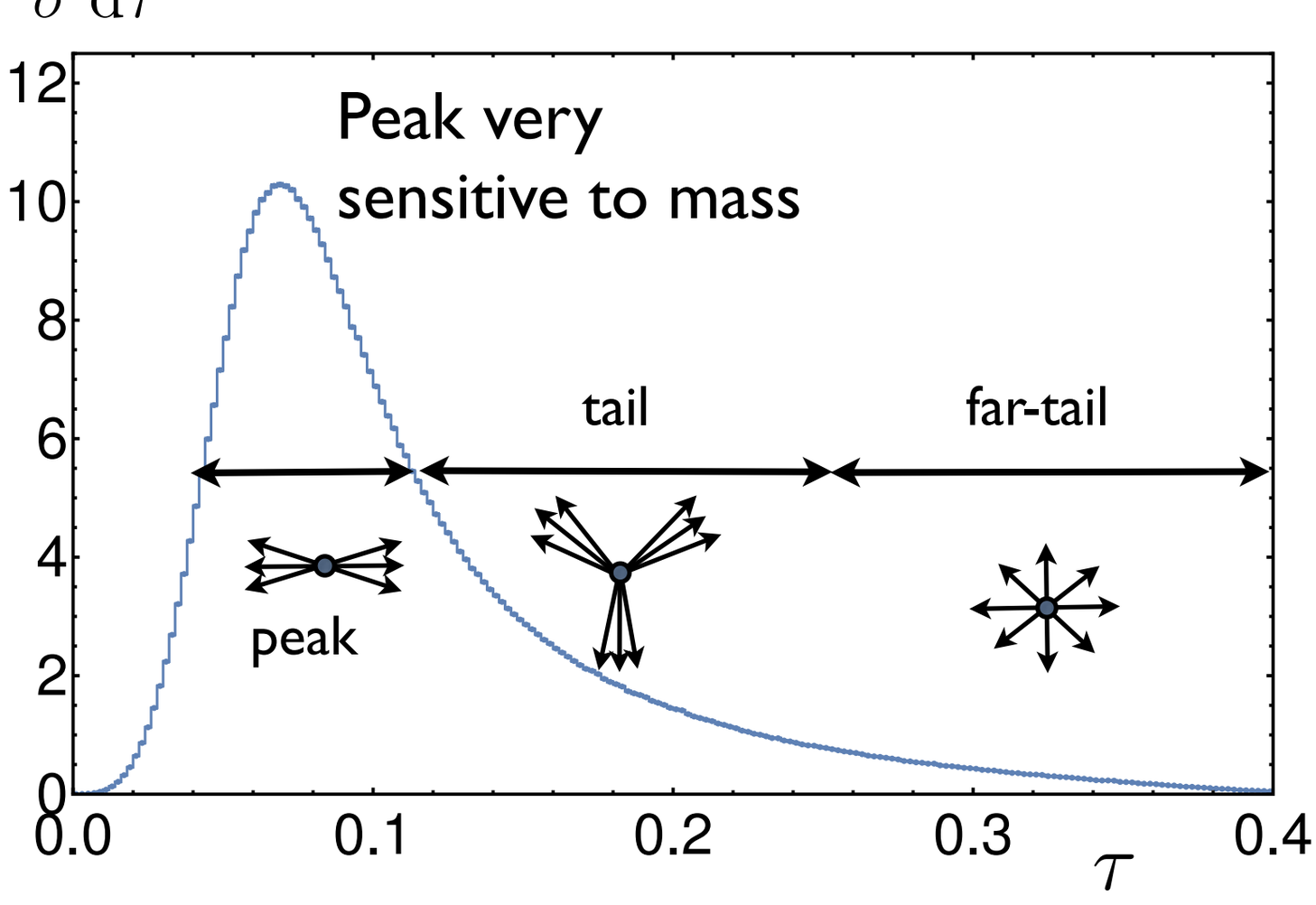
$$\tau_J = 1 - \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{Q} \underset{Q\,\tau_J \ll Q}{\sim} \frac{M_1^2 + M_2^2}{Q^2}$$

Simplification: consider boosted tops




Shifts the whole distribution by ~ $\tau_{\rm peak}^{\rm tree-level} = 1 - \sqrt{1 - 4\,\frac{m_t^2}{Q^2}} \approx 2\,\frac{m_t^2}{Q^2}$

Higher mass sensitivity for lower Q




Invariant mass distribution in the resonance region of wide hemisphere jets !!

# Factorization theorem for massless event shapes

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

*[Bauer, Lee, Fleming, Sterman]*

*[Berger, Kuks, Sterman]*

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

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Jet function

Soft function

Nonsingular terms, power corrections

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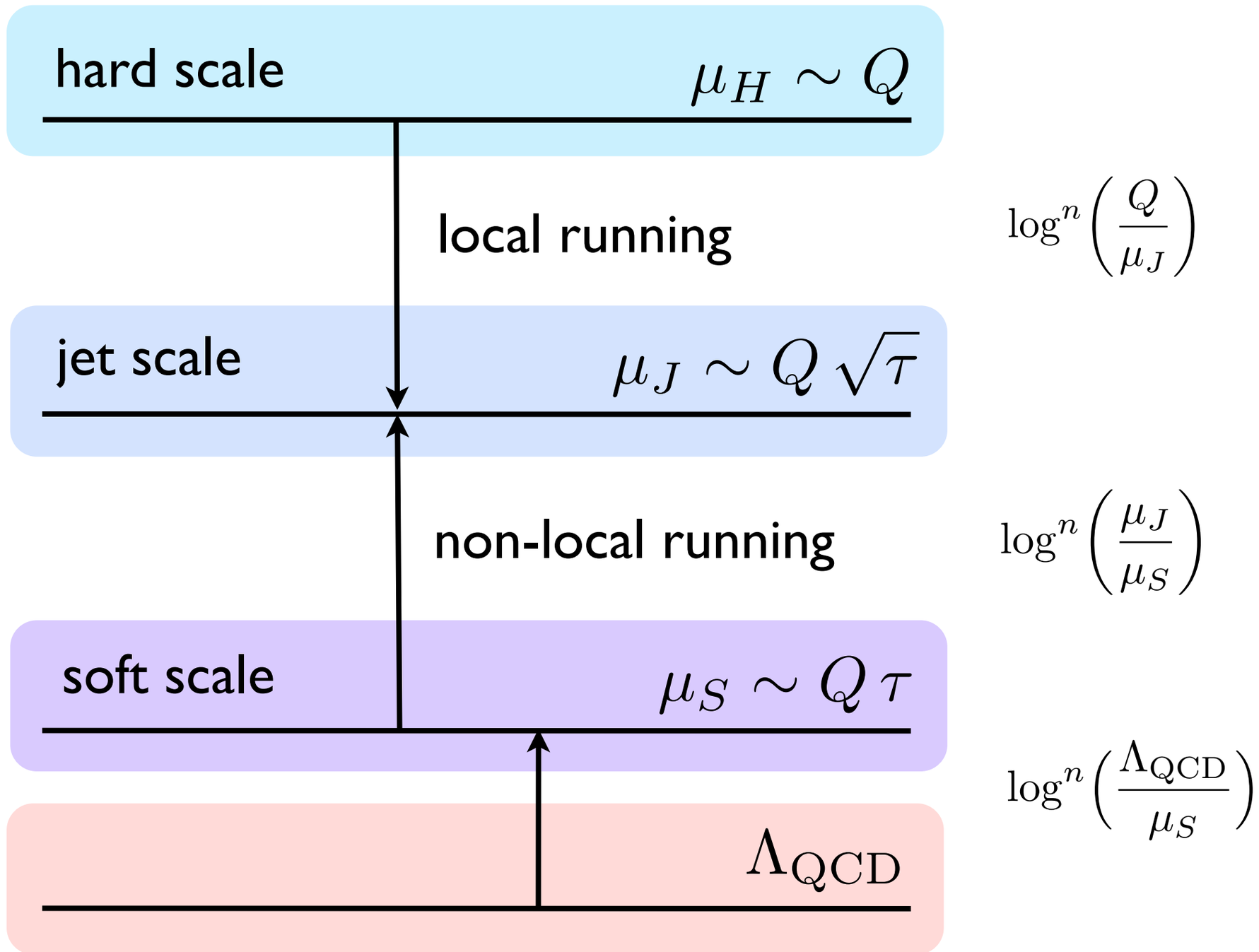
Perturbative and nonperturbative components

The factorization theorem needs to be modified to include massive particles

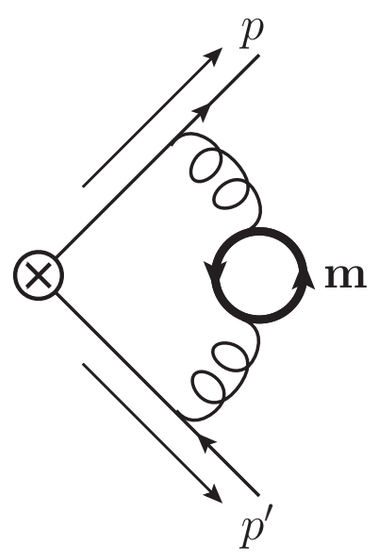
- Explicit mass dependence in matrix elements
- Account for primary and secondary heavy quark radiation
- Running with different number of flavors according to thresholds
- Matching to a new EFT in the peak region

# Renormalization group evolution



# Renormalization group evolution

**Secondary mass effects**




hard scale $\mu_H \sim Q$

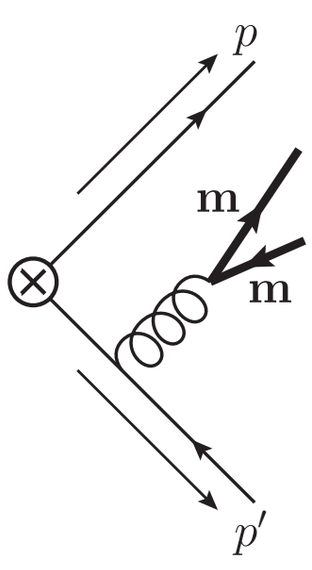
$n_l + 1$ massless evolution

jet scale $\mu_J \sim Q\sqrt{\tau}$

m

matching coefficient

$n_l$ massless evolution




soft scale $\mu_S \sim Q\,\tau$

Peak region

relevant for us, mas sensitivity

When including massive quarks, a new scale enters the game!

# Primary Heavy quark production

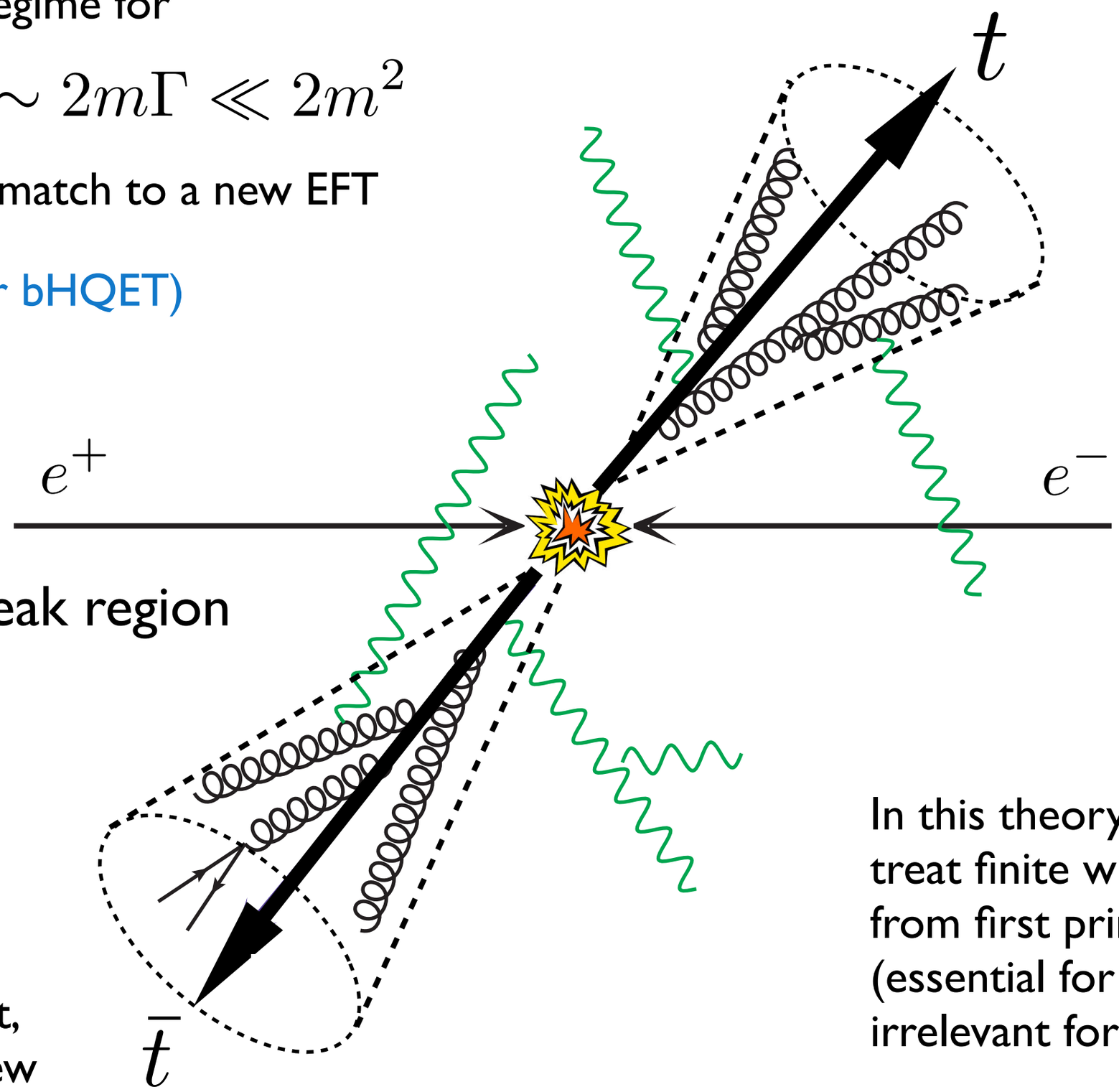
*[Fleming, Hoang, Mantry, Stewart]*

New kinematical regime for

$$Q^2\tau - 2m^2 \sim 2m\Gamma \ll 2m^2$$

New class of logs, match to a new EFT

boosted HQET (or bHQET)




Takes place in Peak region

In practice: new matching coefficient, new jet function, new renormalization scale

In this theory one can treat finite width effects from first principles (essential for top, irrelevant for bottom)

*[M. Butenschön, B. Dehnadi, A. Hoang, VM, M. Preisser, I. Stewart, 2016]*

# boosted HQET Factorization Theorem

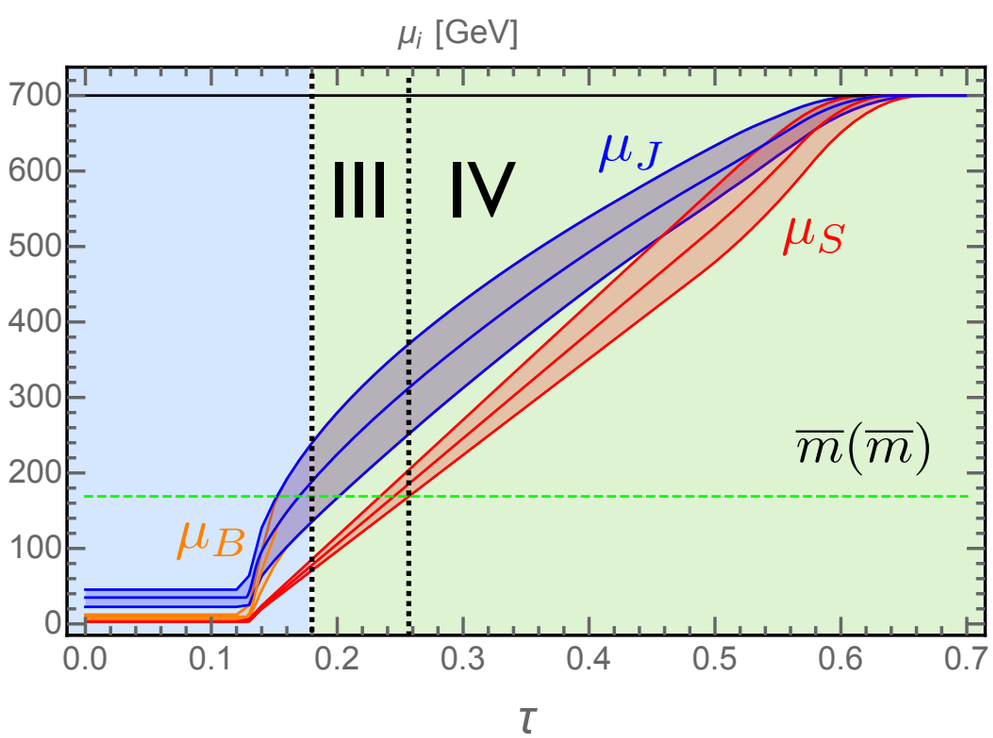
*[Fleming, Hoang, Mantry, Stewart]*

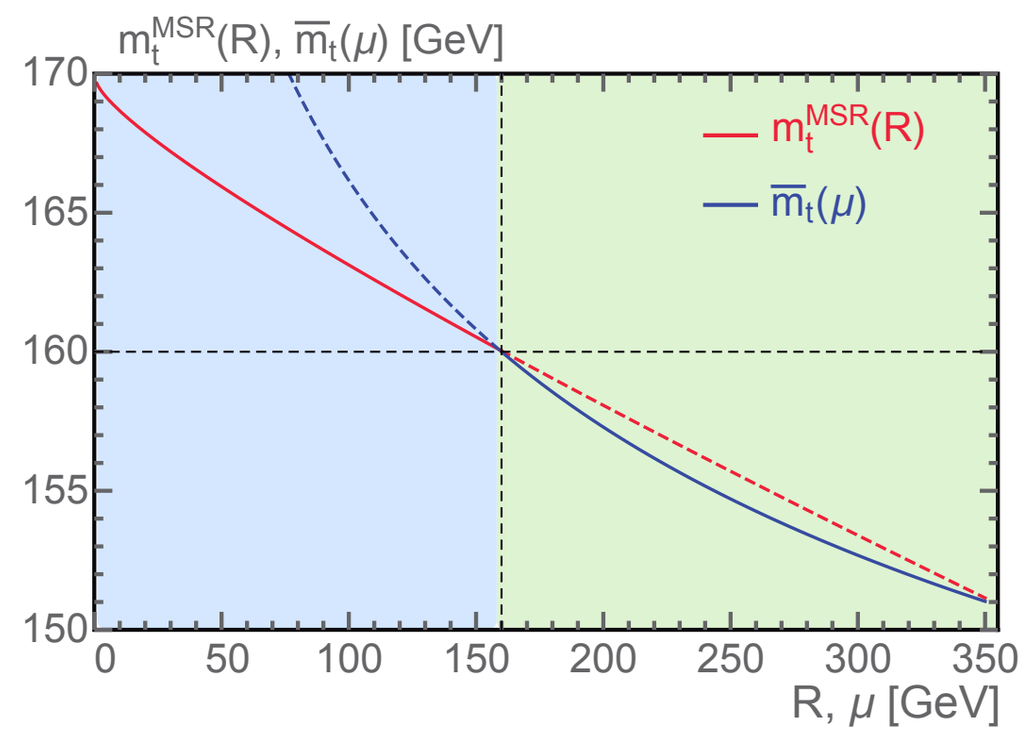
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} \sim H^{(n_l+1)} \times H_m^{(n_f)} \times J_B^{(n_l)} \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m_t^2}{Q^2}, \frac{\Gamma_t}{m_t}, \tau\right)$$

| | | | | |
|---|---|---|---|---|
| QCD to SCET matching | SCET to bHQET matching | bHQET jet function | SCET soft function | corrections to fact. theorem (most of them accounted for!) |

in SCET regime $\overline{\mathrm{MS}}$ mass has correct behavior

in bHQET regime MSR mass has correct behavior



# Results

# Convergence and Stability: MSR vs pole

500 profiles ( = 500 fits) tune 7

$\alpha_s(m_Z) = 0.1181 \qquad \Gamma_t = 1.4\,\mathrm{GeV}$

$Q = 700, 1000, 1400\,\mathrm{GeV}$ peak (60/80)%

input : $m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$ fit to find $m_t^{\mathrm{MSR}}$ or $m_t^{\mathrm{pole}}$

Good convergence and stability for $m_t^{\mathrm{MSR}}(1\,\mathrm{GeV})$

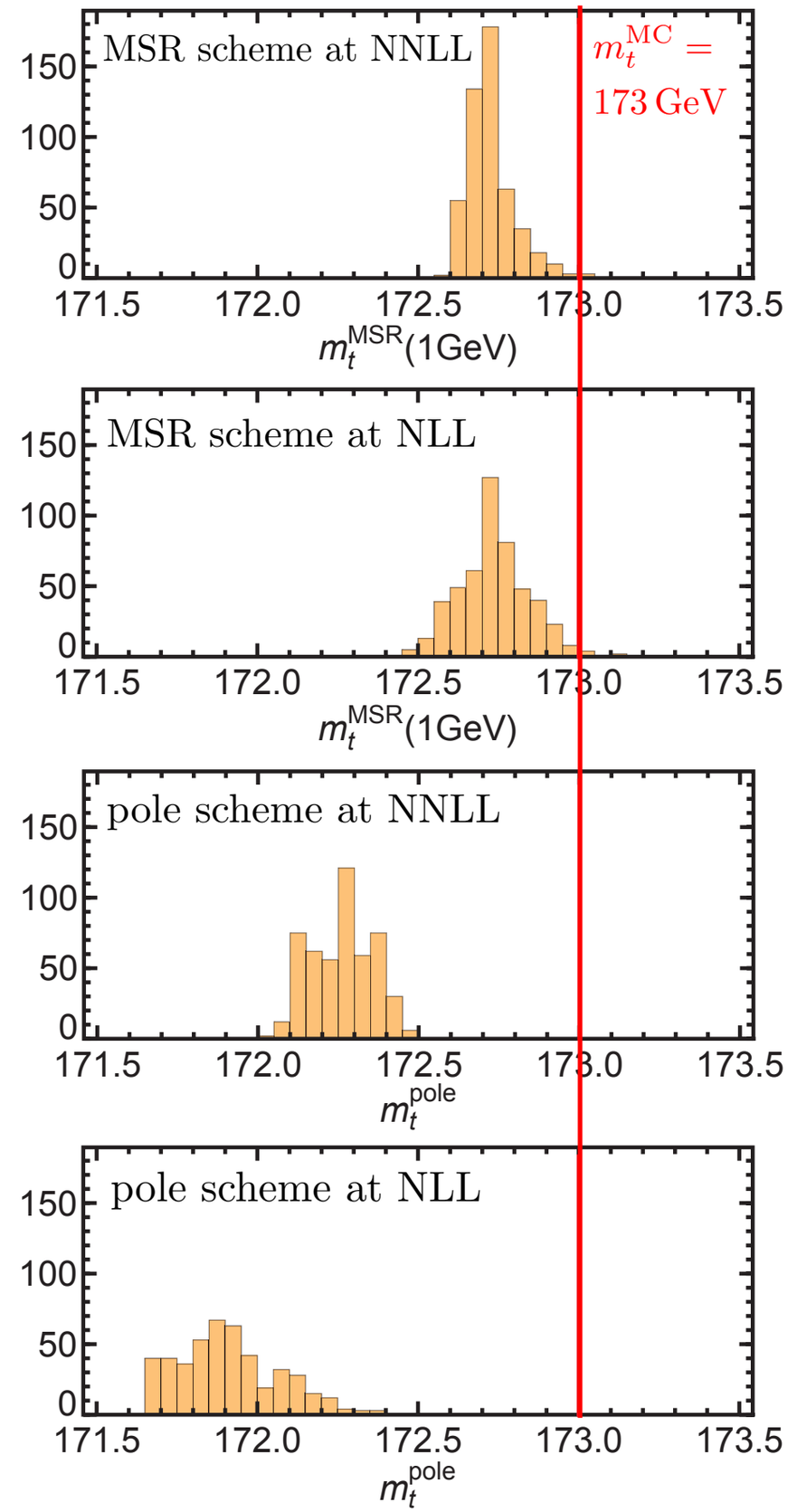
$m_t^{\mathrm{MSR}}(1\,\mathrm{GeV})$ theoretically the closets to the MC top mass

$m_t^{\mathrm{MSR}}(1\,\mathrm{GeV})$ numerically close to $m_t^{\mathrm{MC}}$

$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV} \quad \left(\tau_2^{e^+e^-}\right)$

| Mass | Order | Central | Perturb. | Incompatibility | Total |
|---|---|---|---|---|---|
| $m_{t,1\,\mathrm{GeV}}^{\mathrm{MSR}}$ | NLL | 172.80 | 0.26 | 0.14 | 0.29 |
| $m_{t,1\,\mathrm{GeV}}^{\mathrm{MSR}}$ | NNLL | 172.82 | 0.19 | 0.11 | 0.22 |

Spread of results from 21 datasets



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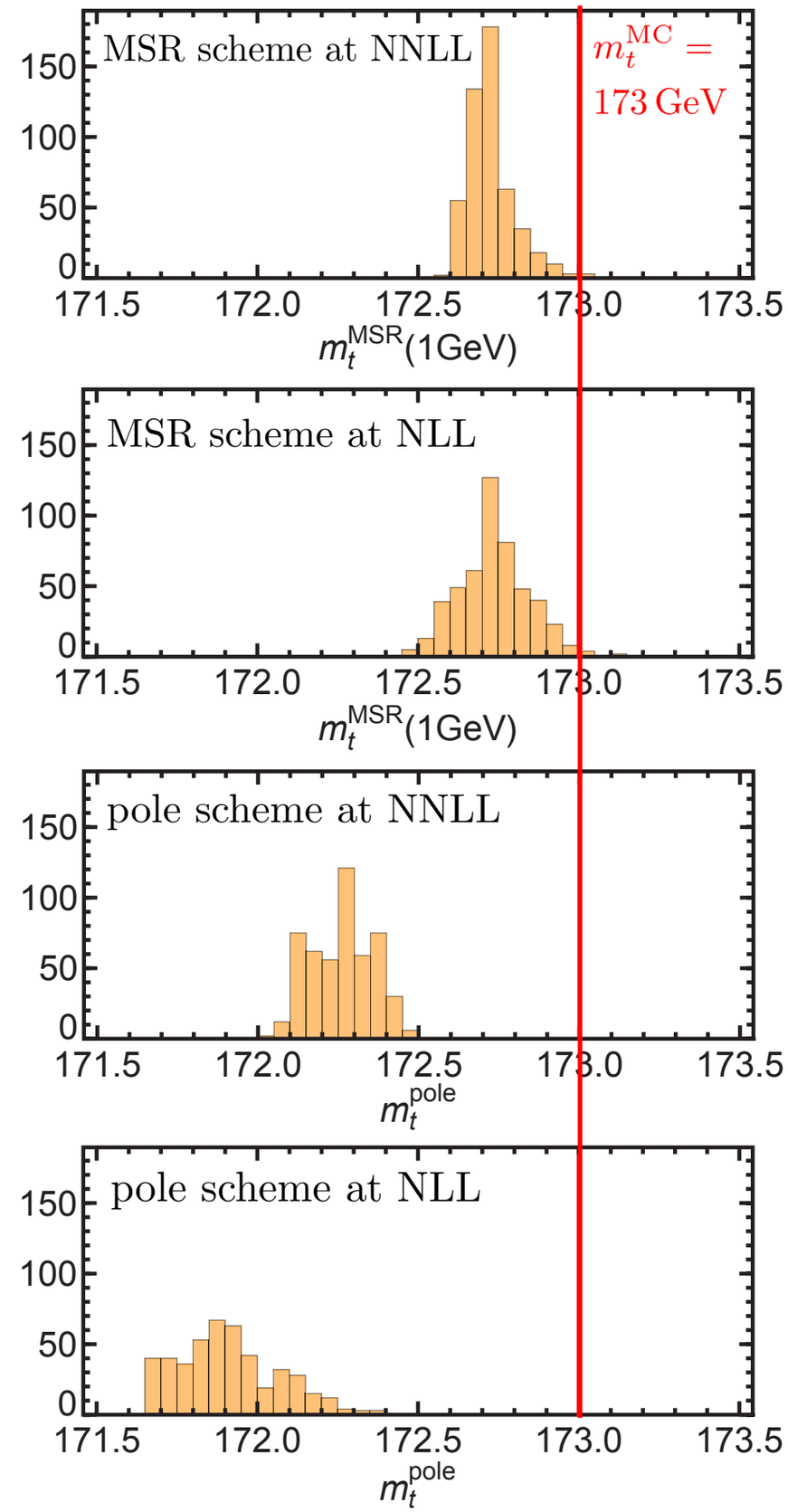
Pole mass shows worse convergence

$m_t^{\mathrm{pole}}$ not at all close to $m_t^{\mathrm{MC}}$

900/600 MeV difference at NLL/NNLL !!!

$m_t^{\mathrm{pole}} \neq m_t^{\mathrm{PYTHIA}}$

Similar findings from the other 20 data sets



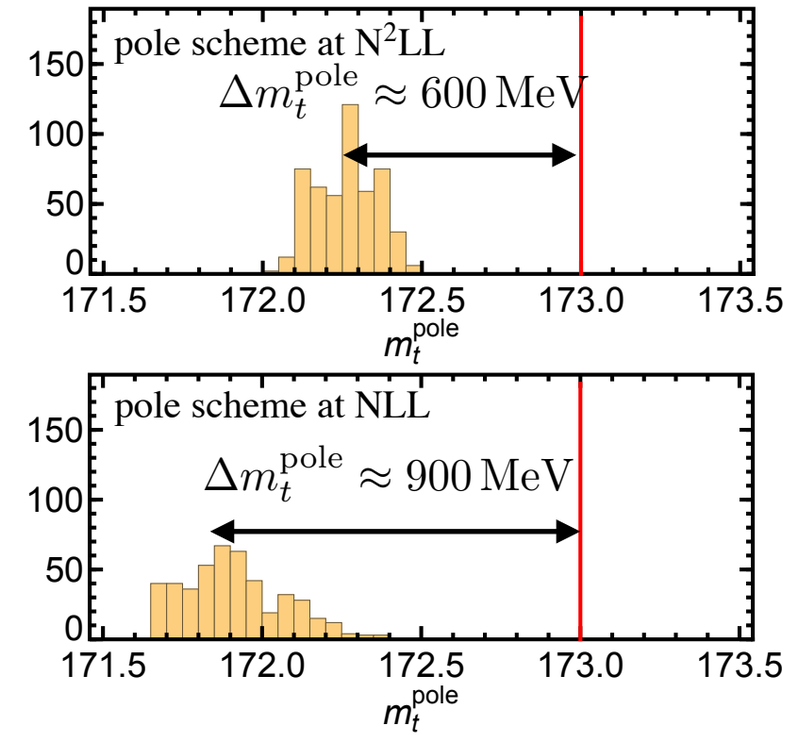
# Results for the pole mass

## 1) Pole mass implemented in code (pole fits)

$$m_t^{\rm MC} = 173\,{\rm GeV} \quad \left(\tau_2^{e^+e^-}\right)$$

| Mass | Order | Central | Perturb. | Incompatibility | Total |
|---|---|---|---|---|---|
| $m_t^{\rm pole}$ | NLL | 172.10 | 0.34 | 0.16 | 0.38 |
| $m_t^{\rm pole}$ | NNLL | 172.43 | 0.18 | 0.22 | 0.28 |

$$(m_t^{\rm pole})_{\rm NLL/N^2LL} < m_t^{\rm MSR}(1\,{\rm GeV}) < m_t^{\rm MC}$$




## 2) Pole mass determined from the MSR mass $\alpha_s(m_Z) = 0.1181 \qquad n_f = 5$

$$\begin{array}{cccc} \mathcal{O}(\alpha_s) & \mathcal{O}(\alpha_s^2) & \mathcal{O}(\alpha_s^3) & \mathcal{O}(\alpha_s^4) \end{array}$$

$$m_t^{\rm pole} - m_t^{\rm MSR}(1\,{\rm GeV}) = 0.173 + 0.139 + 0.160 + 0.237\ {\rm GeV}$$

$$m_t^{\rm pole} > m_t^{\rm MSR}(1\,{\rm GeV})$$

all coefficients positive (for any R)

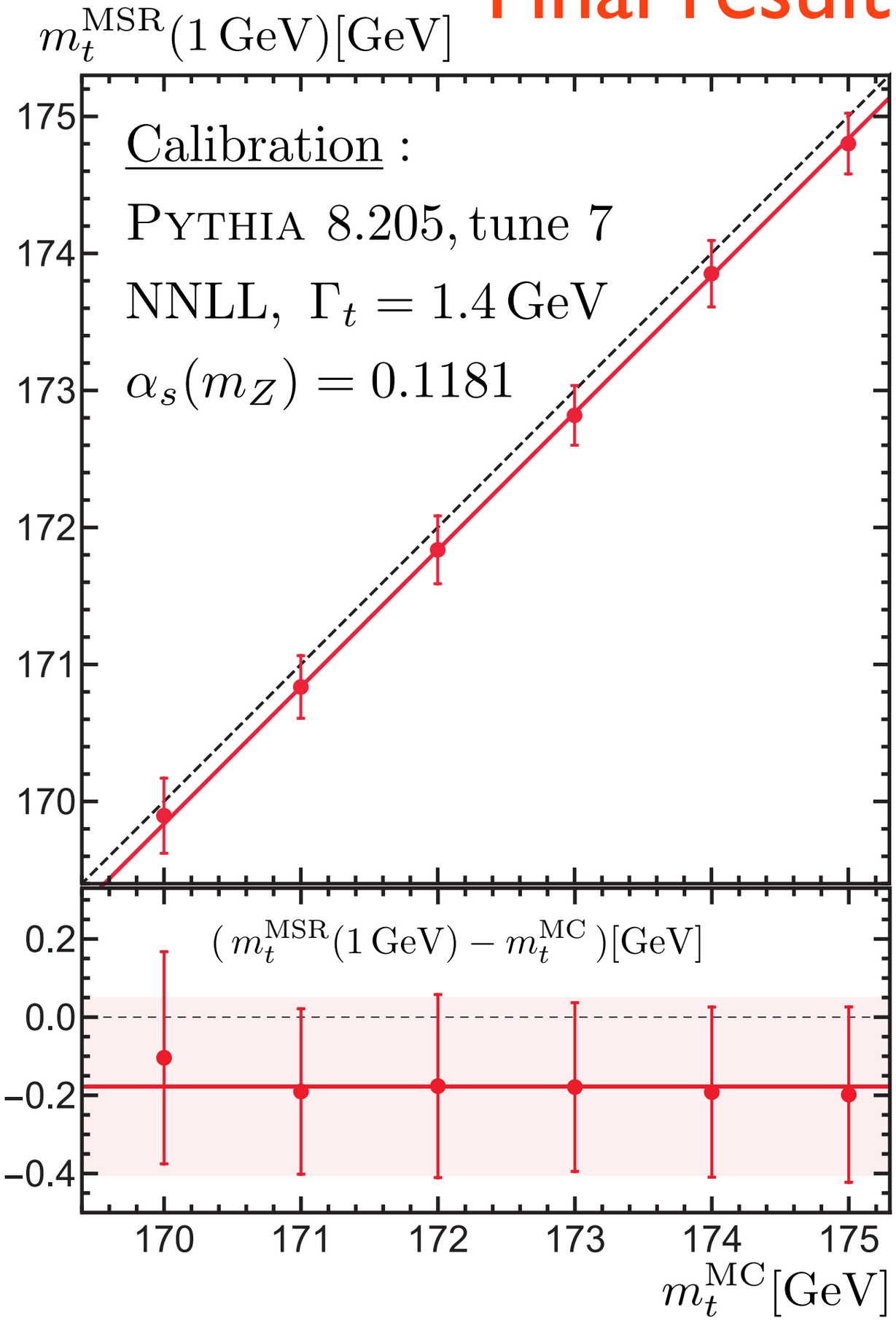
$$m_t^{\rm pole} = (172.99\,{\rm GeV})_{\rm N^2LL},\ (172.80\,{\rm GeV})_{\rm NLL}$$

Calibration in terms of the pole mass involves large higher-order perturbative corrections

Additional uncertainty on pole mass

$$(m_t^{\rm pole})_{\rm N^2LL} = 172.72 \pm 0.40\,{\rm GeV}$$
$$(m_t^{\rm pole})_{\rm NLL} = 172.45 \pm 0.52\,{\rm GeV}$$

# Final result for $m_t^{\mathrm{MSR}}(1\,\mathrm{GeV})$




All investigated MC top mass values show a consistent picture

The MC top quark mass parameter is indeed closely related to the MSR mass at a low scale

Linear dependence between

$m_t^{\mathrm{MSR}}(1\,\mathrm{GeV})$ and $m_t^{\mathrm{MC}}$

Within uncertainties $m_t^{\mathrm{MSR}}(1\,\mathrm{GeV}) \approx m_t^{\mathrm{MC}}$

$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$ $(\tau_2^{e^+e^-})$

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Spread of results from 21 datasets

$$\Omega_1^{\mathrm{PY}} = 0.41 \pm 0.07 \pm 0.02\,\mathrm{GeV}(\mathrm{NLL})$$

$$\Omega_1^{\mathrm{PY}} = 0.42 \pm 0.07 \pm 0.03\,\mathrm{GeV}(\mathrm{N^2LL})$$

# Conclusions

# Conclusions

- Top behaves almost as a real particle... but not quite.
- MC top mass can be calibrated by comparison to a hadron-level ab initio "QCD calculator" (SCET).
- Electron-positron collision, simplest possible setup.
- MC mass close to MSR mass but NOT to pole mass.
- Consistency: $N^3LL + N^2LO$ (CALIPER, fortran 2008 code).
- Further checks (on the beam pipe): C-Parameter, Heavy Jet Mass, etc...

# Backup slides

# Details of the calibration

renormalization scales (estimate perturbative uncertainties)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = f(m_t^{\mathrm{MSR}}, \alpha_s(m_Z), \Omega_i, \mu_j, \Gamma_t)$$

top width (check MC accuracy)

any scheme (calibrate!)

non-perturbative (fit together with mass)

**PYTHIA (8.205) samples:**

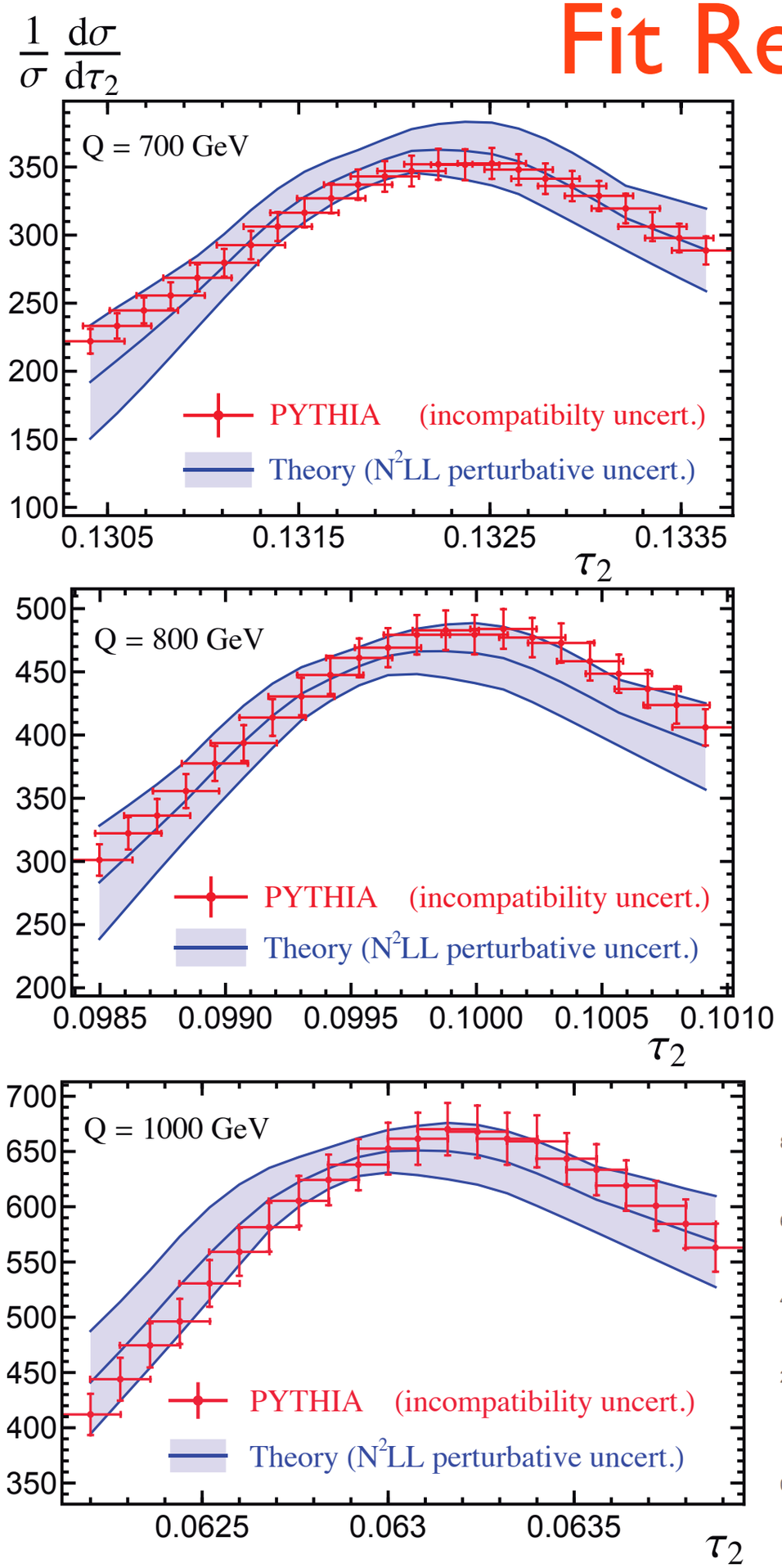
**Q values**: 600, 700, 800, …, 1400 GeV
Various **top widths**: dynamic (mass dependent), hard-wired 1.4 GeV, 2.0 GeV
Various **tunes**: 1(very old), 3 (LEP), 7 ("Monash")
**Statistics**: $10^7$ events.

**Fitting strategy:**

**Fit parameters**: $m_t^{\mathrm{MSR}}, \Omega_i$ . No sensitivity to $\alpha_s$ , hence use world average
**Standard fit** based on $\chi^2$ minimization (using MC statistical errors).
Analysis with 500 sets of profiles (renormalization scales).
Different **Q-sets**: 7 sets with energies between 600 and 1400 GeV.
Different **n-sets**: 3 choices of fit-ranges - (x/y)% of the maximum peak hight.

21 datasets

# Fit Results: Pythia vs Theory

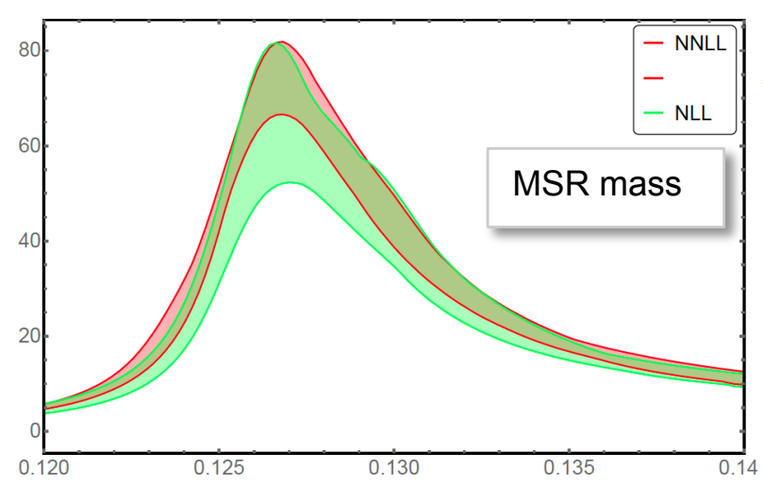



We only compare peak data, since otherwise Pythia is not reliable. Also peak gives higher mass sensitivity.

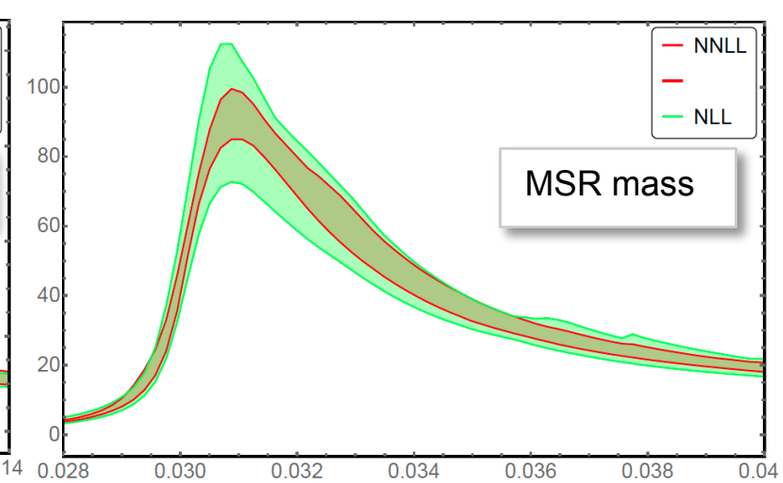
- Good description of Pythia 8.2 default output with default scale setting NNLL + NLO QCD.
- Increasing discrepancies in distribution tail and for higher energies due to off shell effects in NS

Theoretical accuracy at NNLL / NLL order

Q = 700 GeV




Q = 1400 GeV



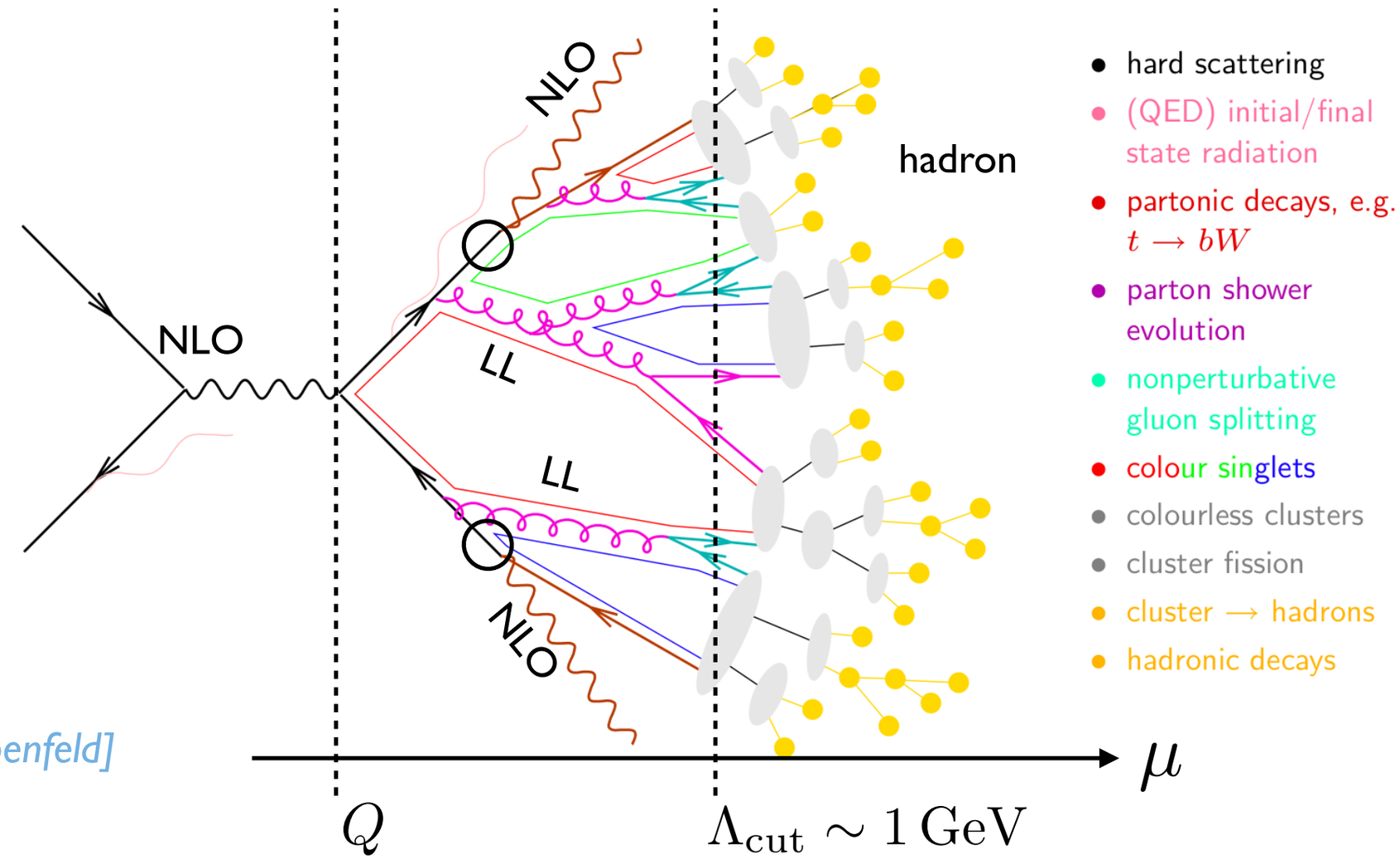
# Overview of parton shower MC

Pythia

Sherpa

Herwig

Geneva




*[Original figure by D. Zeppenfeld]*

**<u>Hard matrix element</u>**: annihilates initial particles into tops + other hard partons

**<u>Parton shower</u>**: QCD resummation at LL of large Sudakov logs

~ partial NLO matrix elements

**<u>Top mass</u>:** mass of top propagator prior to top decay… scheme?

historically all-order identification with the pole mass

# Tune dependence

500 profiles ( = 500 fits)

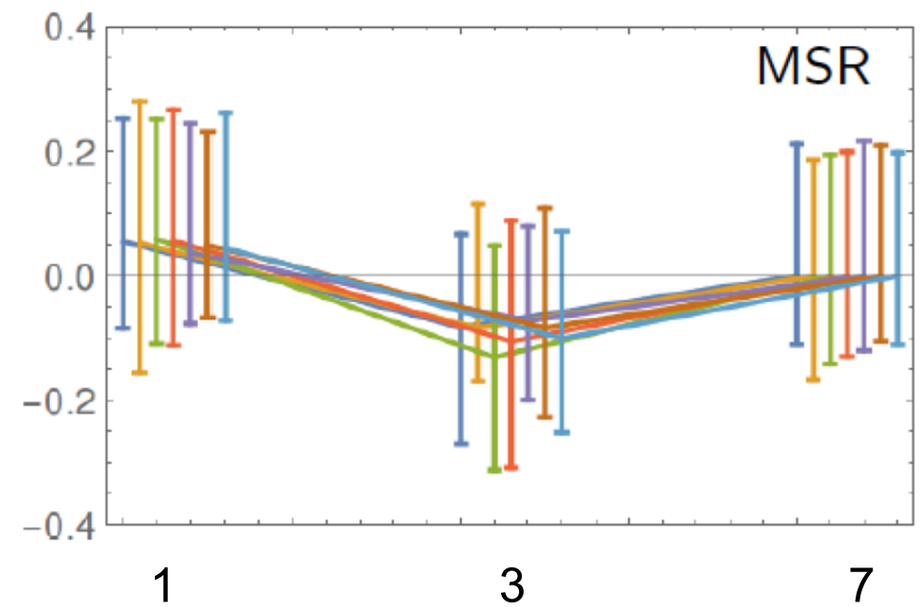
$\alpha_s(m_Z) = 0.1181 \qquad \Gamma_t = 1.4\,\mathrm{GeV}$

Different Q-sets peak (60/80)%

input : $m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$

tune dependence:

$[m_t^{\mathrm{MSR}}]^{\mathrm{tune}} - [m_t^{\mathrm{MSR}}]^{\mathrm{tune=7}}$




Clear sensitivity to the tune

$m_t^{\mathrm{MC}}$ will depend on tune

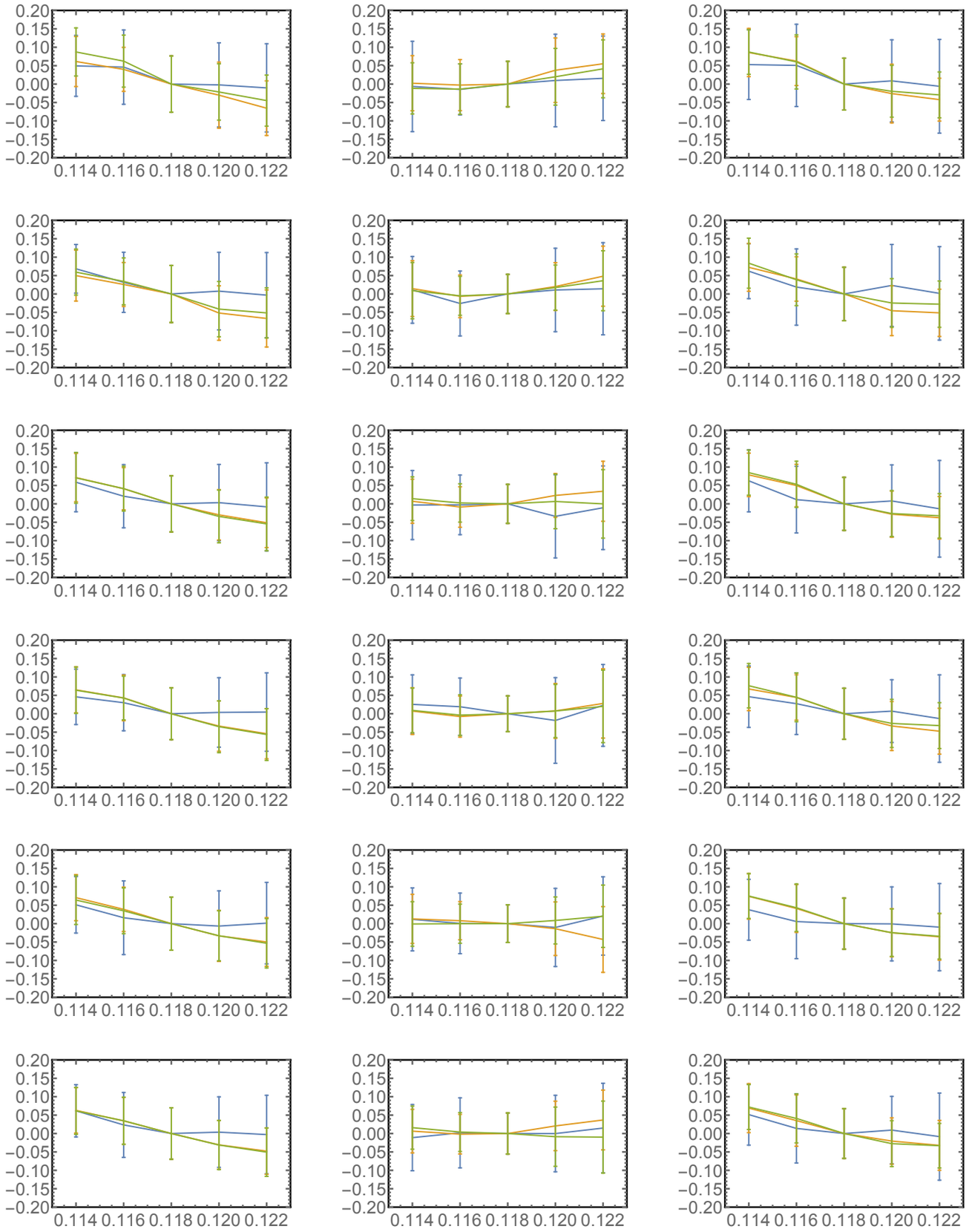
Tune dependence is **not** a calibration uncertainty

(different tune $\Rightarrow$ different MC $\Rightarrow$ different calibration)

# Fits to Pythia data

very preliminary

tune 1 | tune 3 | tune 7




$m^{\mathrm{MC}} = 170\,\mathrm{GeV}$

$m^{\mathrm{MC}} = 175\,\mathrm{GeV}$

$m^{\mathrm{MSR}}(1\,\mathrm{GeV})$ vs $\alpha_s(m_Z)$

error bar: fit + theory uncertainty

various lines for various widths

# Top width dependence

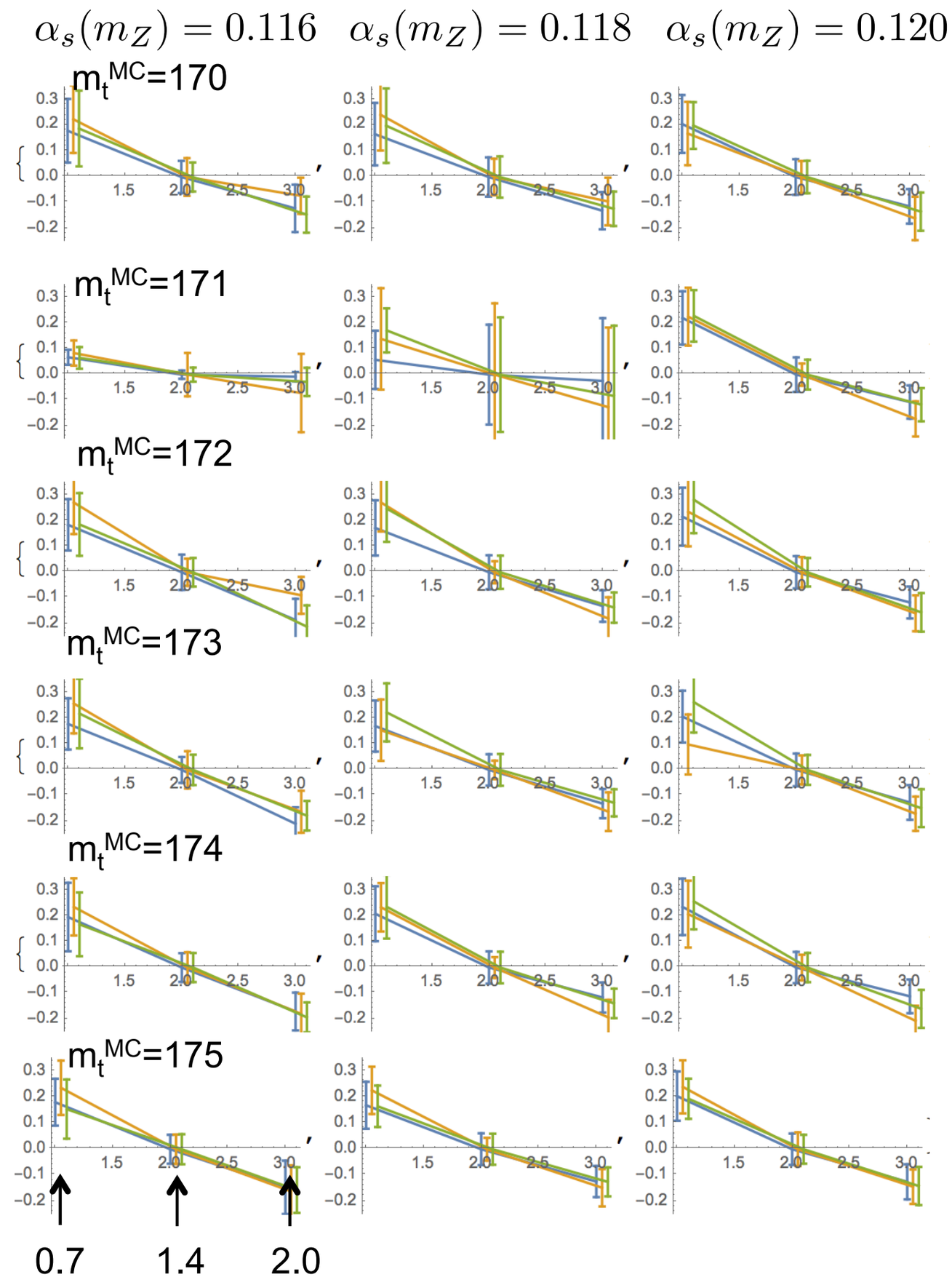
500 profiles ( = 500 fits)

$$\alpha_s(m_Z) = 0.1181$$

Different Q-sets peak (60/80)%

Three tunes (different colors)

$$\Gamma_t = (0.7,\ 1.4,\ 2.0)\ \mathrm{GeV}$$

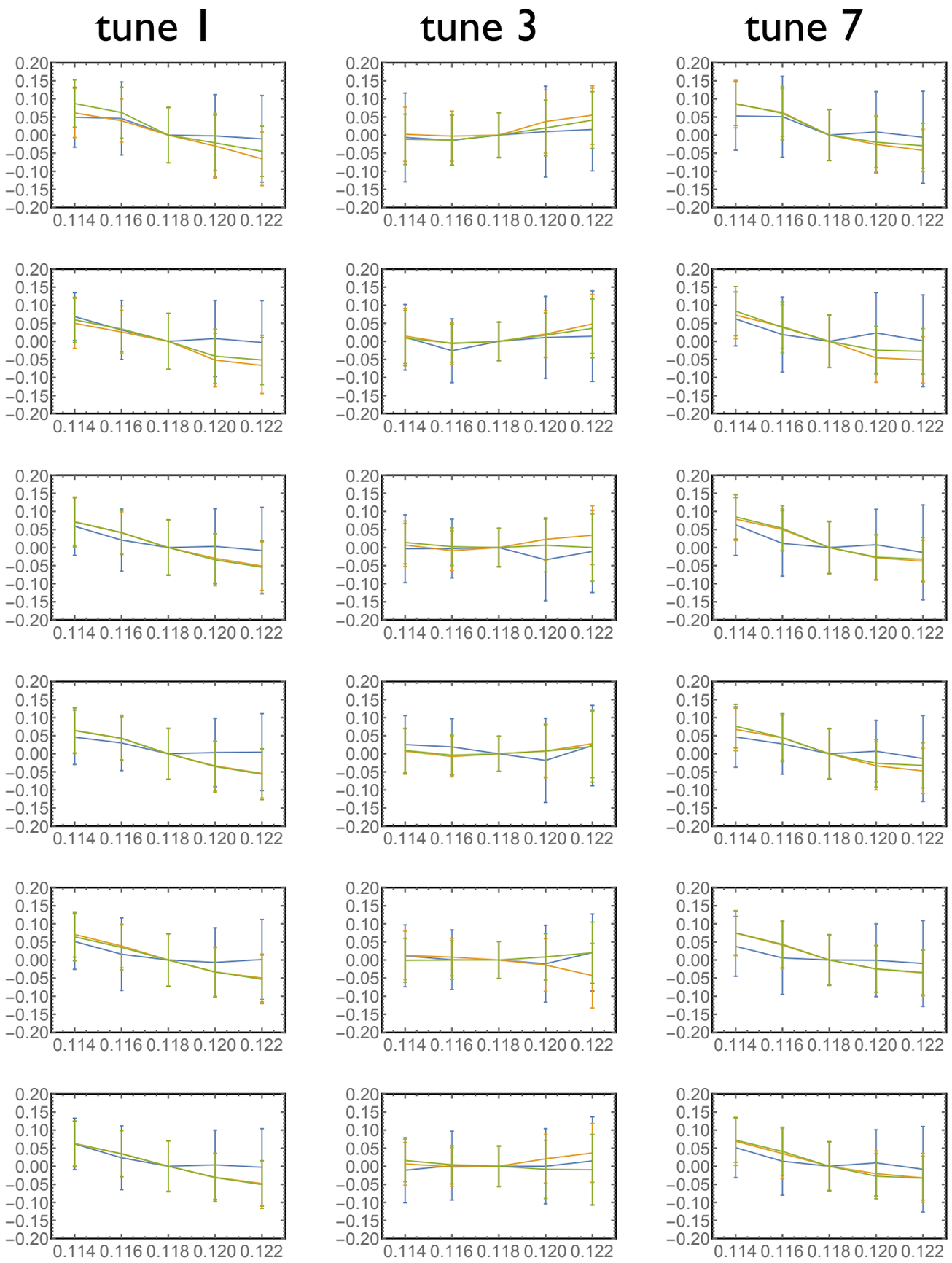



Clear sensitivity to top width value

Can be interpreted as observable dependence or MC modeling dependence.

Our conclusion: Pythia has an error in describing the correct top width dependence.

# $\alpha_s(m_Z)$ dependence




$m^{\mathrm{MC}} = 170\,\mathrm{GeV}$

$m^{\mathrm{MC}} = 175\,\mathrm{GeV}$

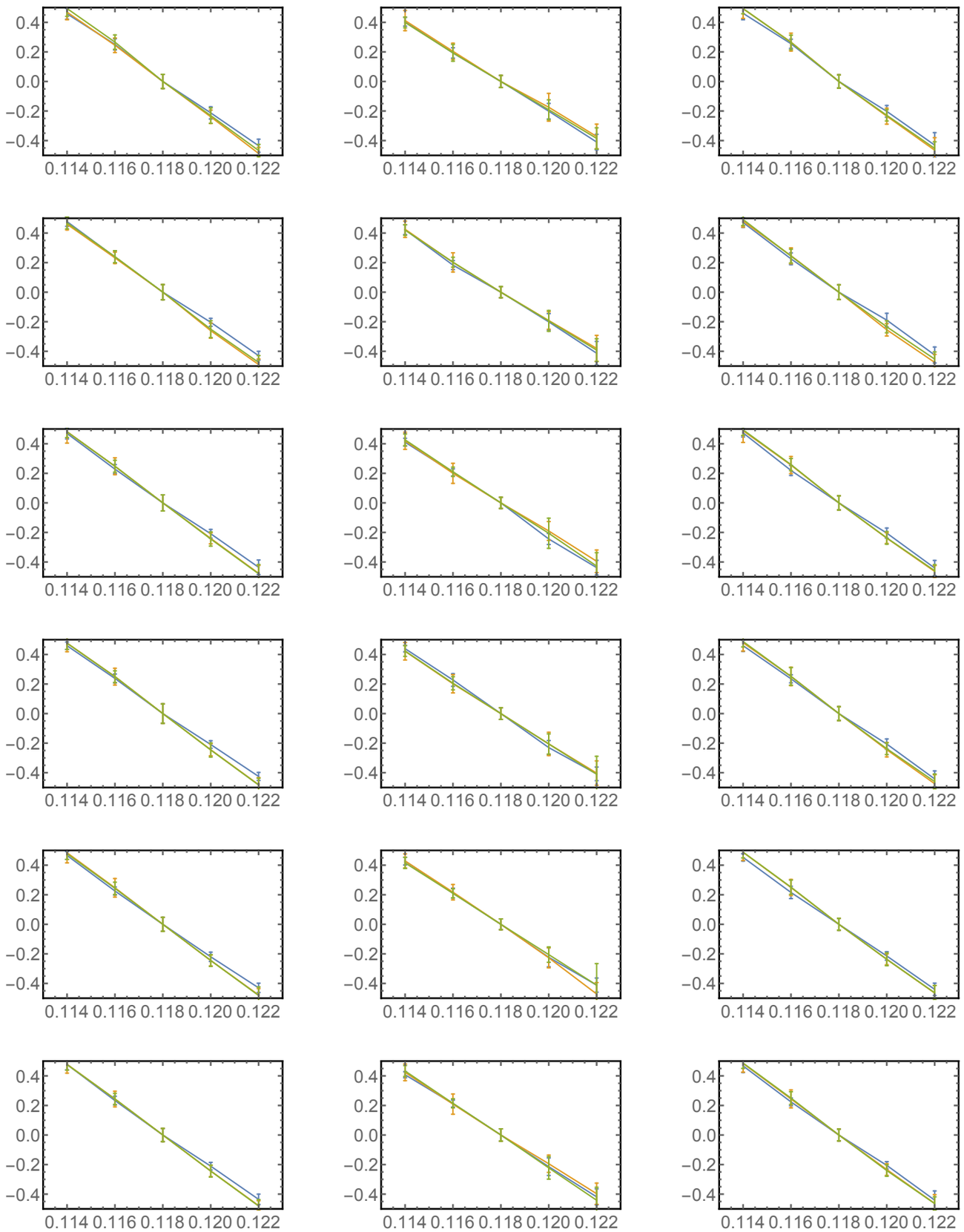
$m^{\mathrm{MSR}}(1\,\mathrm{GeV})$ vs $\alpha_s(m_Z)$

error bar: fit + theory uncertainty

various lines for various widths

# $\alpha_s(m_Z)$ dependence

tune 1 tune 3 tune 7




$m^{\mathrm{MC}} = 170\,\mathrm{GeV}$

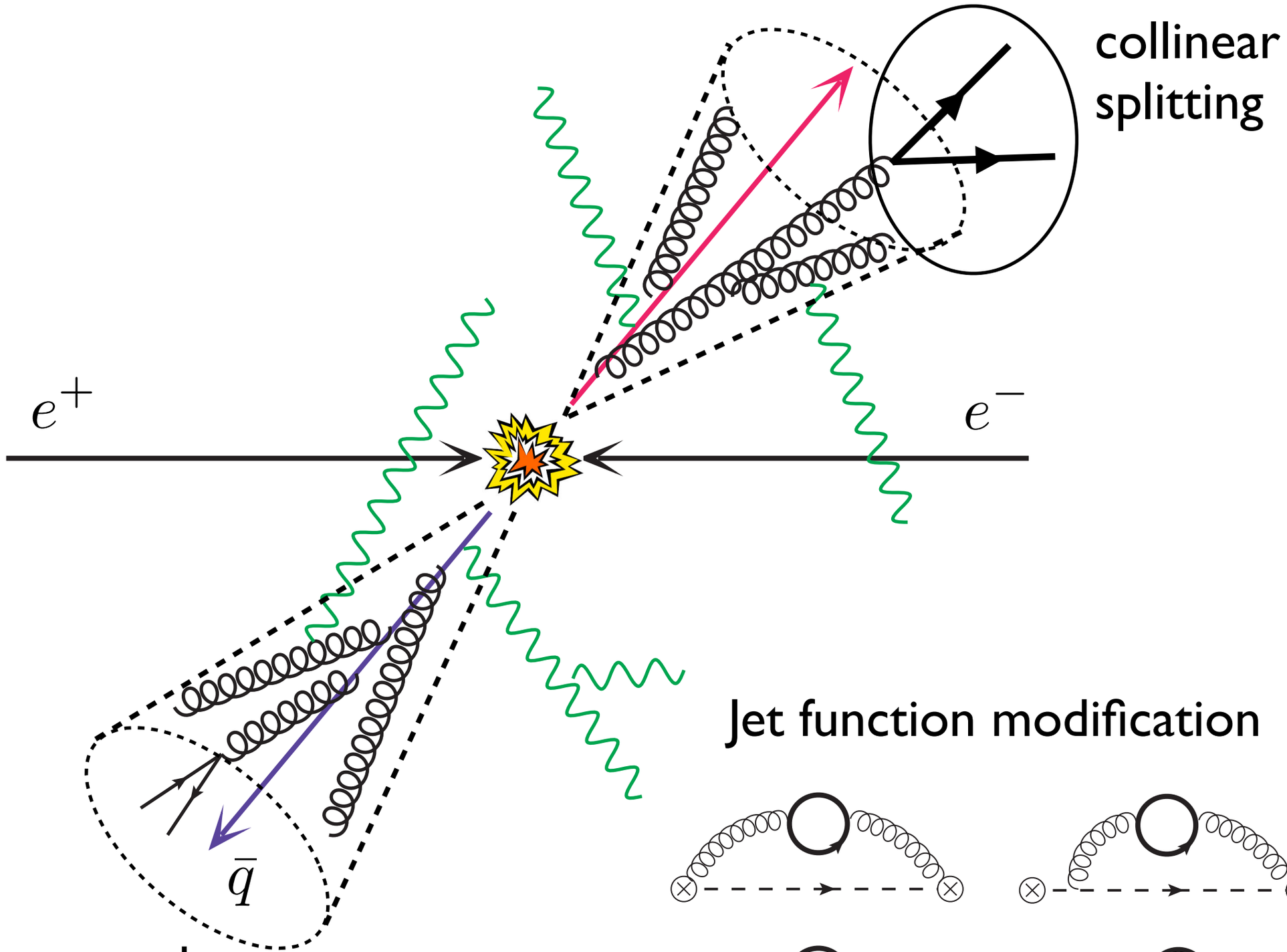
$m^{\mathrm{MC}} = 175\,\mathrm{GeV}$

$\overline{m}(\overline{m})$ vs $\alpha_s(m_Z)$

error bar: fit + theory uncertainty

various lines for various widths

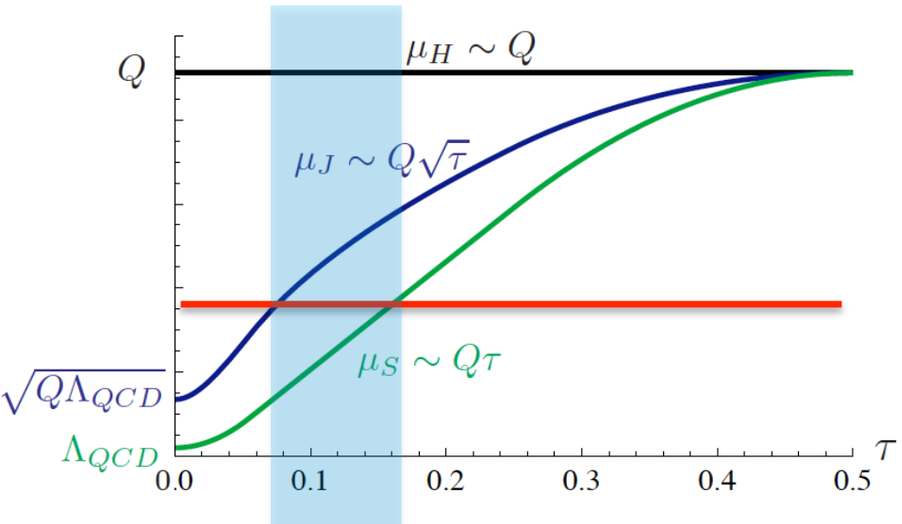
# Heavy quark production through gluon splitting

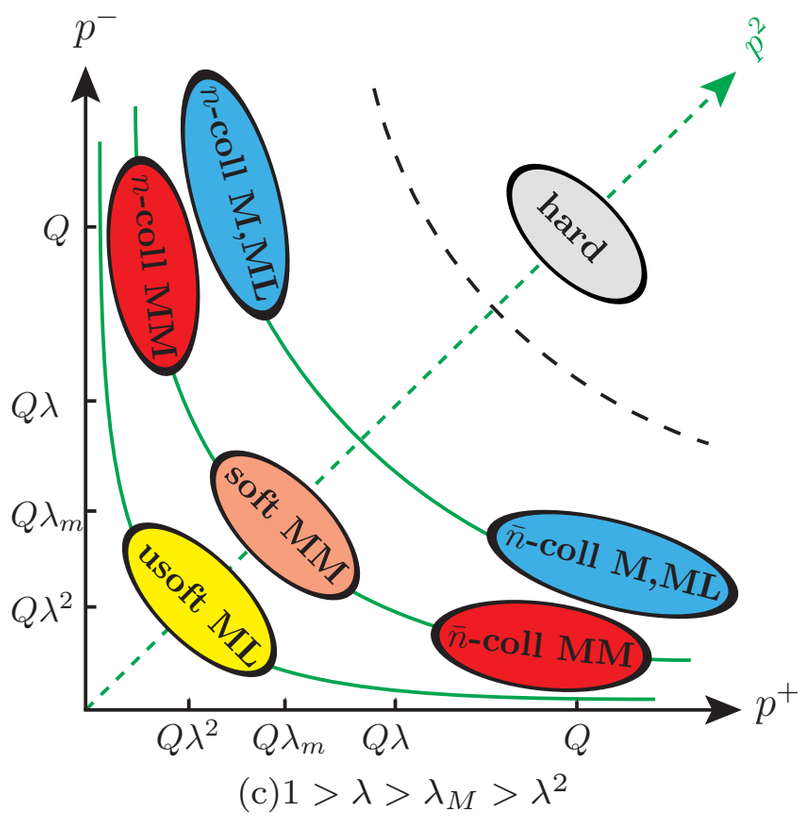



Jet function modification

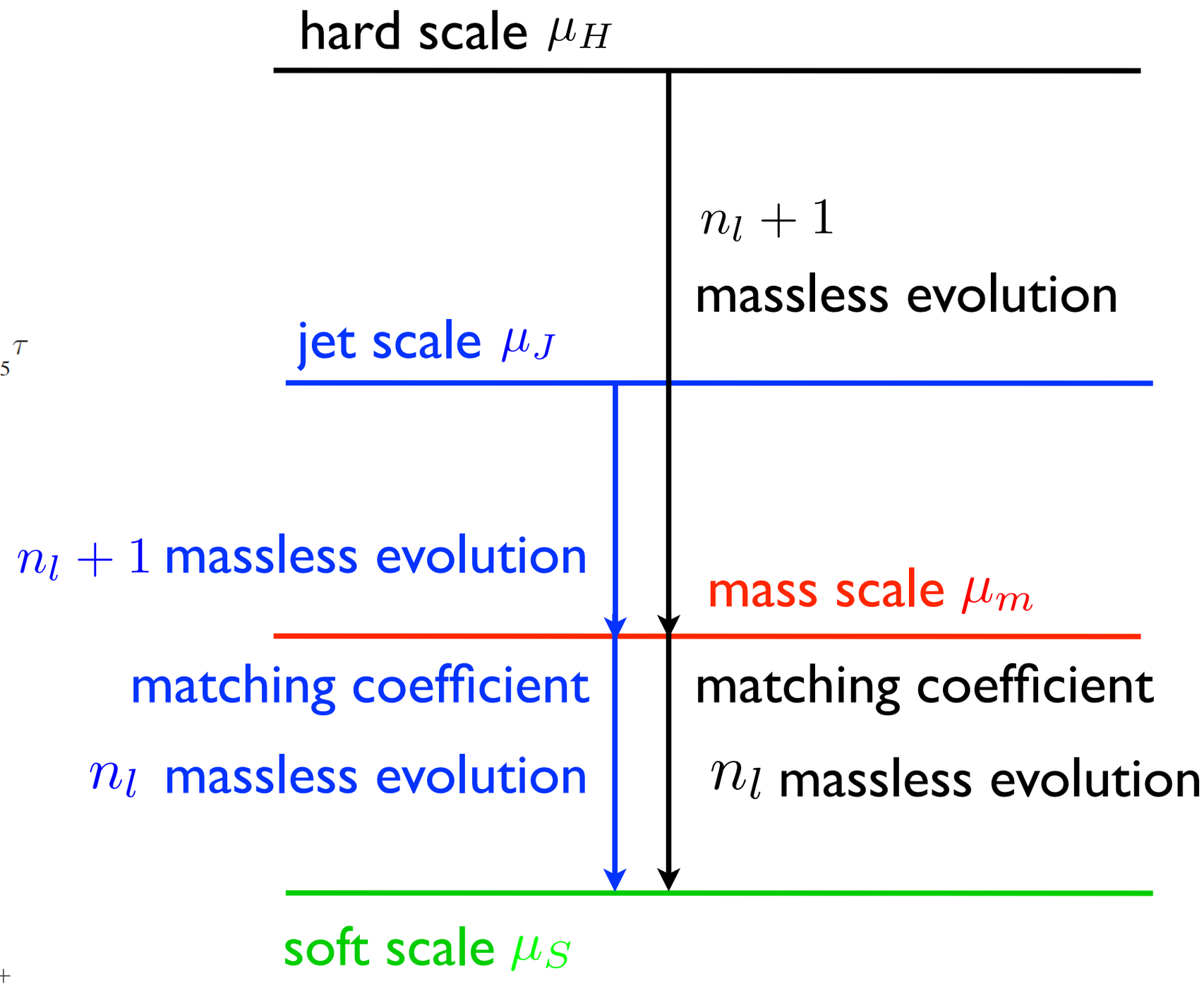
if enough energy to produce
pair of heavy quarks

# Scenario III

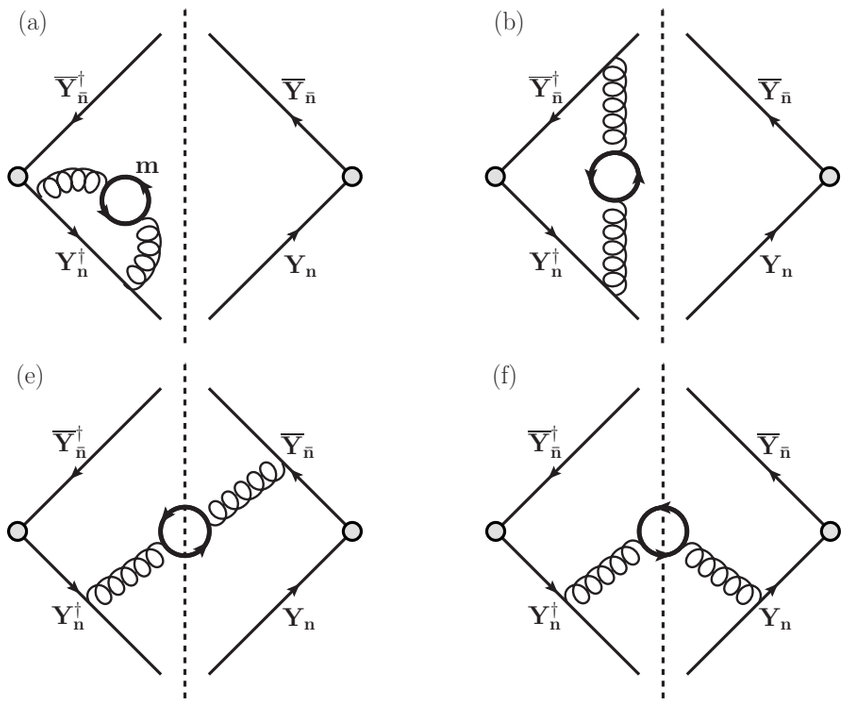


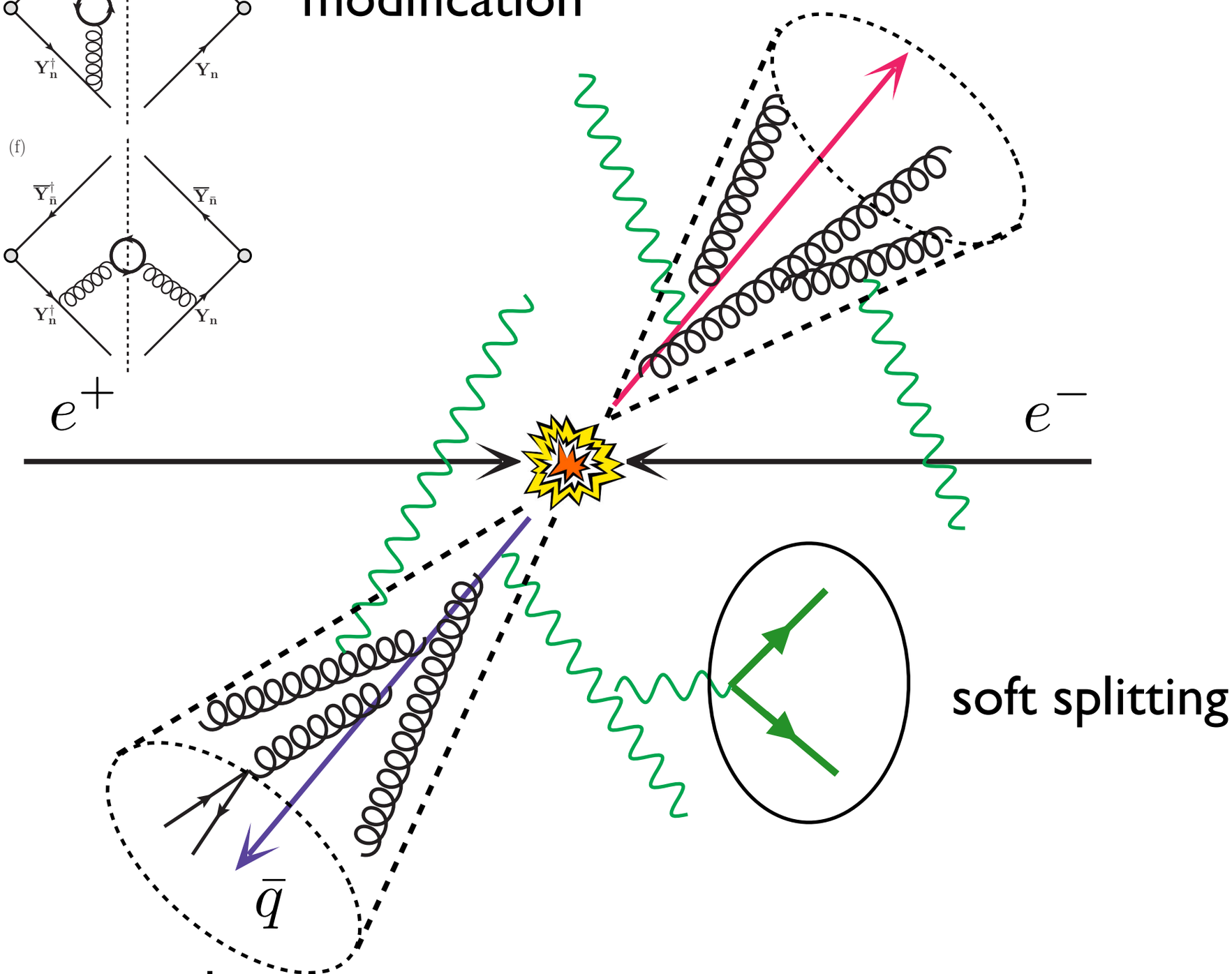

(c)$1 > \lambda > \lambda_M > \lambda^2$



# Heavy quark production through gluon splitting

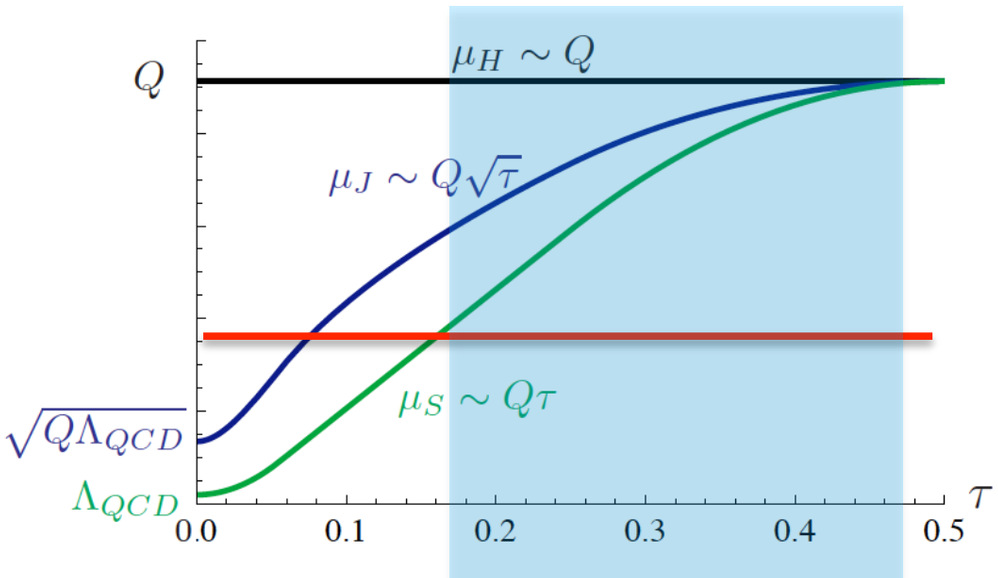


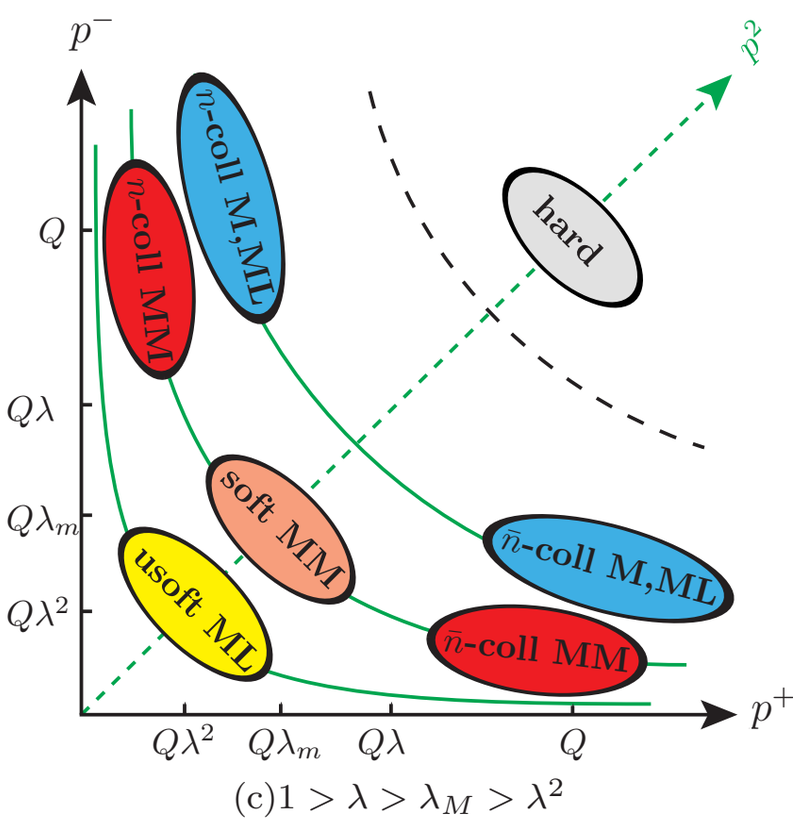

Soft function modification

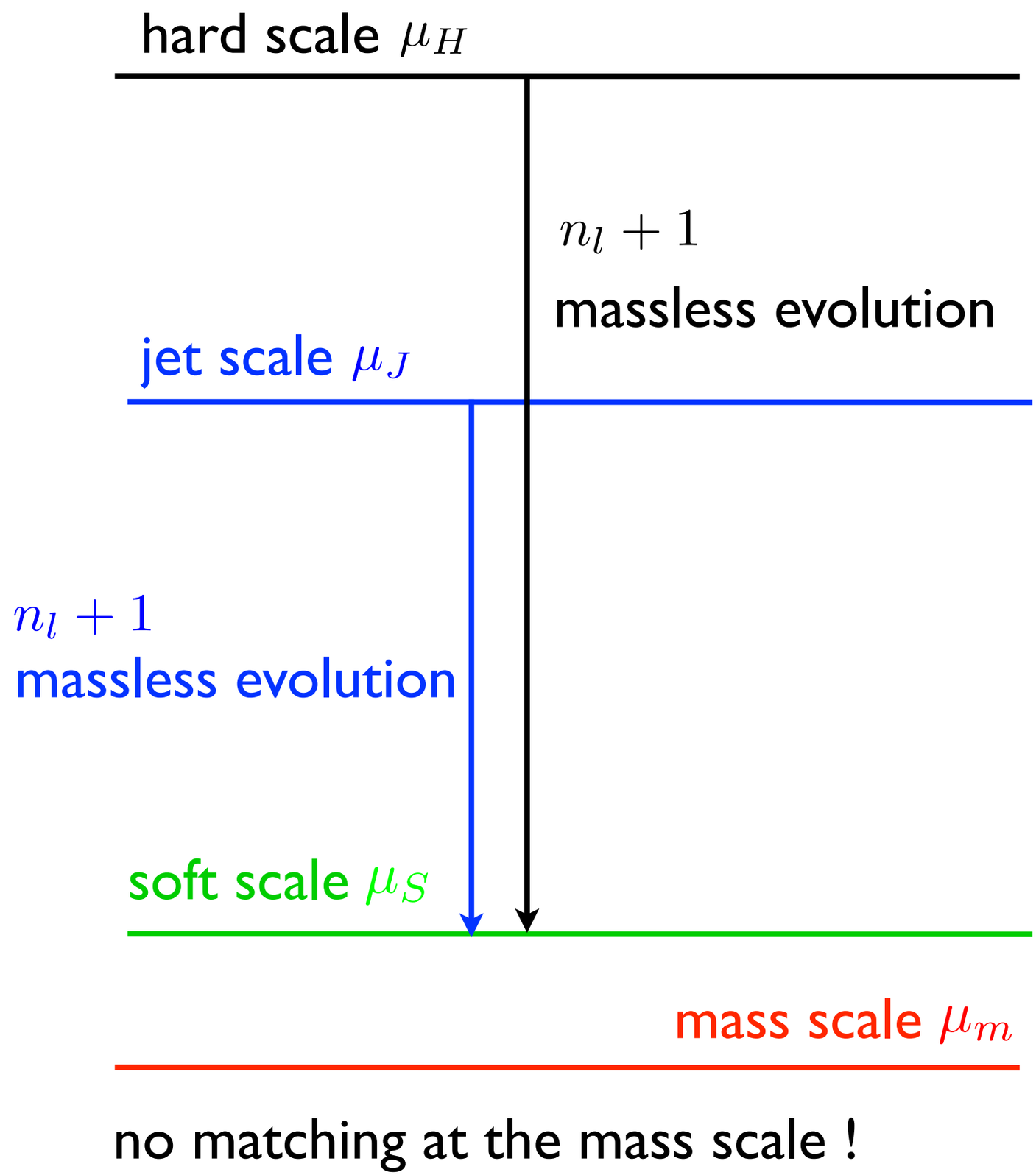



if enough energy to produce pair of heavy quarks

# Scenario IV


$Q$
$\mu_H \sim Q$
$\mu_J \sim Q\sqrt{\tau}$
$\mu_S \sim Q\tau$
$\sqrt{Q\Lambda_{QCD}}$
$\Lambda_{QCD}$
0.0
0.1
0.2
0.3
0.4
0.5
$\tau$


$p^-$
$p^+$
$p^2$
$Q$
$Q\lambda$
$Q\lambda_m$
$Q\lambda^2$
$\bar{n}$-coll M,ML
$\bar{n}$-coll MM
hard
soft MM
usoft ML
$\bar{n}$-coll M,ML
$\bar{n}$-coll MM
(c)$1 > \lambda > \lambda_M > \lambda^2$


hard scale $\mu_H$
$n_l + 1$
massless evolution
jet scale $\mu_J$
$n_l + 1$
massless evolution
soft scale $\mu_S$
mass scale $\mu_m$
no matching at the mass scale !