Isospin – breaking effects in decay constants of heavy mesons from QCD sum rules

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We propose a new method for calculating the dependences of the decay constants of heavy-light mesons on the light-quark mass \( m_q \) based on QCD sum rules at infinitely large Borel mass parameter (local-duality limit). For a specific choice of the correlation functions, all condensate contributions vanish and the \( m_q \)-dependence of the decay constants is shown to be mainly determined by the known analytic \( m \)-dependence of the diagrams of perturbative QCD. The results for strong IB in the decay constants of heavy pseudoscalar and vector mesons are reported.

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QCD sum rule for 2 – point function

A typical Borel QCD sum rule for the decay constant $f_H$ of a heavy (pseudoscalar or vector) $\bar{Q}q$ meson $H$ of mass $M_H$, consisting of a heavy quark $Q$ with mass $m_Q$ and a light quark $q$ with mass $m_q$, has the form

$$f_H^2(M_H^2)^N \exp(-M_H^2 \tau) = \int \frac{ds}{(m_Q+m_q)^2} \exp(-s \tau) s^N \rho_{\text{pert}}(s, m_Q, m_q, \alpha_s) + \Pi_{\text{power}}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \ldots).$$

Nonperturbative effects appear at two places:

(i) as power corrections given in terms of vacuum condensates and

(ii) in the effective threshold $s_{\text{eff}}^{(N)}(\tau, m_Q, m_q, \alpha_s)$.

Depending on $N$, nonperturbative effects are distributed in a different way between power corrections and the effective threshold.

The usual procedure: work in a “windows” of (nonzero) $\tau$, try to fix $s_{\text{eff}}$ by one or another criterion then obtain the decay constant. This procedure works more or less successfully and allows one to obtain predictions for the decay constants (LMS, Phys. Lett. B765, 365, 2017).

QCD input parameters:

$\alpha_s$, quark masses, perturbative spectral densities, condensates + in some cases hadron masses.
Another perspective: one is interested in particular dependence of the hadron observable on $m_q$ and can make use of some “external” inputs. A different approach is promising and allows one to improve strongly the accuracy of the predictions.

**Borel QCD sum rule in LD limit $\tau \to 0$**

LD limit in Borel sum rules was introduced by Radyushkin in the context of relations between 2-point and 3-point sum rules and obtaining predictions for hadron form factor in a broad range of momentum transfers.

We propose other application of this limit:

(i) Consider Borelized correlation function of dimension-2 in LD limit. *LD limit is well-defined.*

(ii) In dimension-2 correlator ALL power corrections vanish in LD

\[
\frac{f_q^2}{s_{eff}^{(q)}} = \int_{(m_Q+m_q)^2} d\rho_{\text{pert}}(s, m_Q, m_q, \alpha_s | m_{\text{sea}})
\]
\[ f_q^2 = \int_{(m_Q+m_q)^2}^{(q)_{\text{eff}}} ds \rho_{\text{pert}}(s, m_Q, m_q, \alpha_s|m_{\text{sea}}) \]

Here \( m_q = m_u, m_d, \) or \( m_s, \) and \( m_{\text{sea}} \) denotes the set of values \( m_u, m_d, \) and \( m_s. \)

The IB is related to

\[
\delta \Pi_{\text{dual}} = \Pi_{\text{dual}}(s^d_{\text{eff}}, m_d|m_{\text{sea}}) - \Pi_{\text{dual}}(s^u_{\text{eff}}, m_u|m_{\text{sea}}).\]

- **What is known in QCD?** \( \rho_{\text{pert}} \) is calculated as expansion in powers of \( a \equiv \alpha_s/\pi: \)

\[
\rho_{\text{pert}}(s, m_Q, m_q, \alpha_s) = \rho^{(0)}(s, m_Q, m_q) + a\rho^{(1)}(s, m_Q, m_q) + a^2\rho^{(2)}(s, m_Q, m_q|m_{\text{sea}}) + \ldots
\]

Order \( a^2 \) is the first order where the “sea-quark” masses appear. The second-order spectral density is known approximately, i.e. for massless quarks only, \( \rho^{(2)}(s, m_Q, m_q = 0|m_{\text{sea}} = 0). \)

Does this approximate OPE allow us to obtain IB and with what accuracy?
• For decay constants of heavy mesons, the OPE error is $O(a^2 m_s) \sim O(\text{a few MeV})$, $a \leq 0.1$

• However, for IB effects, the strange sea-quark contributions cancel in the difference and the OPE accuracy increases strongly:

$$\delta \Pi_{\text{dual}} = \Pi_{\text{dual}}(\tau, s^d_{\text{eff}}, m_d|m_u, m_d, m_s) - \Pi_{\text{dual}}(\tau, s^u_{\text{eff}}, m_u|m_u, m_d, m_s)$$

$$= \Pi_{\text{dual}}(\tau, s^d_{\text{eff}}, m_d|m_{\text{sea}} = 0) - \Pi_{\text{dual}}(\tau, s^u_{\text{eff}}, m_u|m_{\text{sea}} = 0) + O(a^2 \delta m).$$

This relation suggests the following algorithm for the calculation of the IB effects:

(i) Obtain $f(m_q)$ corresponding to the correlation function in which the light-quark mass in the LO and the NLO spectral densities is equal to $m_q$ which we choose in the range $m_{ud} < m_q < m_s$, whereas in the NNLO spectral density the light $u$, $d$ and $s$ quarks are taken massless.

(ii) Calculates $f(m_d) - f(m_u)$; the OPE error in this quantity compared to the IB in “real” QCD (i.e. corresponding to the physical sea-quark masses) is of order $O(a^2 \delta m) \approx \delta m/100$. It is important to emphasize that the known OPE allows one to address properly the IB effects.

What to do with the $m_q$-dependence of $s_{\text{eff}}$?

Parametrize $s_{\text{eff}} = s^{(0)}_{\text{eff}} + m_q s^{(1)}_{\text{eff}} + \cdots$ and determine $s^{(0)}_{\text{eff}}$ and $s^{(1)}_{\text{eff}}$ using a few “external” lattice QCD results for strange mesons and isosymmetric mesons.
Parametrize \( s_{\text{eff}} = s_{\text{eff}}^{(0)} + m_q s_{\text{eff}}^{(1)} + \cdots \) and determine \( s_{\text{eff}}^{(0)} \) and \( s_{\text{eff}}^{(1)} \) using a few “external” lattice QCD results for strange mesons and isosymmetric mesons.

*Interesting subtleties:*

- HQ limit in pole mass, our LD sum rule reproduces correctly the full QCD result for \( f_V/f_P \) at LO, NLO, NNLO in \( \alpha_s \). A convincing evidence that LD sum rules are consistent.

- HQ limit in pole mass vs HQ limit in running mass. Subtleties about the corresponding effective thresholds

- Chiral logs, which may influence IB effects. Where do these chiral logs come from in our approach?

*The answer:* the only source of nonperturbative effects now is \( s_{\text{eff}} \). ALL phenomena originating from long-distances are now encoded in \( s_{\text{eff}} \). In particular, chiral logs, which are known in the HQ limit. We have taken these effects into account.
\[ R(x_q) = 1 + R_L x_q \log(x_q) + R_1 x_q + \ldots \]
Summary and conclusions

• We present the first application of QCD sum rules in LD limit to IB in the decay constants of heavy mesons and show that it is possible to obtain accurate predictions for $\delta f/f$. This was not obvious since the typical accuracy of the SR predictions for $f_{B,B^*,D,D^*}$ is about 10-15 MeV.

• The known OPE (full $m_q$-dependence at LO and NLO, massless light quarks in NNLO) allows one to access IB with $O(a^2 m_u, a^2 m_d)$ accuracy, whereas the accuracy of the individual $f$ is $O(a^2 m_s)$.

• Knowing the explicit dependence of the OPE on $m_q$ and obtaining the decay constants as a function $f(m_q)$ opens the possibility to access the IB effects.

• Making use of lattice QCD results for strange and isosymmetric heavy mesons, we report the following IB effects:

\[
\begin{align*}
 f_{D^+} - f_{D^0} &= (0.96 \pm 0.09) \text{ MeV}, \\
 f_{B^0} - f_{B^+} &= (1.01 \pm 0.10) \text{ MeV} \\
 f_{D^{*+}} - f_{D^{*0}} &= (1.18 \pm 0.35) \text{ MeV}, \\
 f_{B^{*0}} - f_{B^{*+}} &= (0.89 \pm 0.30) \text{ MeV}
\end{align*}
\]

(i) The main IB (70-80%) is due to the $m_q$ dependence of the spectral densities
(ii) The accuracy is limited mainly by the uncertainties of the available lattice results for $f_H$. 