Pion wavefunction with dynamical spin effects

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Overview

1. Light-front wavefunction
2. Pion's special case
3. Dynamical spin wavefunction
4. meson LF Wavefunction with dynamical spin effects.
5. Predictions for radius and form factors

LFWF

$$H_{\rm QCD}^{\rm LF}|\Psi(P)\rangle = M^2|\Psi(P)\rangle$$

where $H_{\rm QCD}^{\rm LF} = P^+P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,h_i} \int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp i}] \frac{1}{\sqrt{x_i}} \Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)|n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[\mathrm{d}x_i] \equiv \prod_i^n \mathrm{d}x_i \delta(1 - \sum_{j=1}^n x_j) \qquad [\mathrm{d}^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \frac{\mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta^2(\sum_{j=1}^n \mathbf{k}_{\perp i}) \; .$$

Holographic Schrödinger equation

The valence meson LFWF can then be written in a factorized form:

$$\Psi(\zeta, x, \phi) = e^{iL\phi}\mathcal{X}(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

It can then be shown that reduces to a 1-dimensional Schrödinger-like wave equation for the transverse mode of LFWF of the valence ($n = 2$ for mesons) state, namely:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

the potential is given by

$$U(z_5, J) = \frac{1}{2}\varphi''(z_5) + \frac{1}{4}\varphi'(z_5)^2 + \left(\frac{2J-3}{4z_5}\right)\varphi'(z_5)$$

where $\varphi(z_5)$ is the dilaton field which breaks conformal invariance in AdS space. A quadratic dilaton, $\varphi(z_5) = \kappa^2 z_5^2$, profile results in a light-front harmonic oscillator potential in physical spacetime:

$$U(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

Solutions to holographic Schrödinger equation

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M^2 = 4\kappa^2 \left(n + L + \frac{S}{2} \right)$$

which, as expected, predicts a massless pion. The corresponding normalized eigenfunctions are given by

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left(\frac{-\kappa^2 \zeta^2}{2} \right) L_n^L(x^2 \zeta^2) \ .$$

To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $\mathcal{X}(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space. Either matching consistently results in $\mathcal{X}(x) = \sqrt{x(1-x)}$

Meson holographic LFWF

Brodsky, de Teramond (PRL, 09)
Brodsky, de Teramond, Dosch, Erlich (Phys. Rep. 15)
The meson holographic LFWFs for massless quarks can thus be written in closed form:

$$\Psi_{nL}(\zeta, x, \phi) = e^{iL\phi}\sqrt{x(1-x)}(2\pi)^{-1/2}\kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^L \exp\left(\frac{-\kappa^2\zeta^2}{2}\right)L_n^L(x^2\zeta^2)$$

$$\Psi^\pi(x,\zeta^2) = \mathcal{N}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

where $\mathcal{N}$ is a normalization constant fixed by requiring that

$$\int \mathrm{d}^2\mathbf{b}\mathrm{d}x|\Psi^\pi(x,\zeta^2)|^2 = P_{q\bar{q}}$$

where $P_{q\bar{q}}$ is the probability that the meson consists of the leading quark-antiquark Fock state.

Universal fundamental AdS/QCD scale κ

AdS/QCD scale κ can be chosen to fit the experimentally measured Regge slopes

- $\kappa = 590$ MeV for pseudoscalar mesons and $\kappa = 540$ MeV for vector mesons.
- A fit to the HERA data on diffractive ρ electroproduction, with $m_{u/d} = 140$ MeV, gives $\kappa = 560$ MeV.
- $\kappa = 550$ MeV (with $m_{u/d}[m_s] = 46[140]$ MeV) leads to a good simultaneous description of the HERA data on diffractive ρ and ϕ electroproduction.

Pion's LFWF

In earlier applications of LFH with massless quarks, much lower values of κ were required to fit the pion data:

- $\kappa = 375$ MeV in order to fit the pion EM form factor data
  Brodsky, de Teramond, PRD 77, 056007 (2008)
- $\kappa = 432$ MeV (with $P_{q\bar{q}} = 0.5$) to fit the photon-to-pion transition form factor data simultaneously at large Q^2 and $Q^2 = 0$
  Brodsky, Cao, de Teramond, PRD 84, 075012 (2011)

Higher value of $\kappa = 787$ MeV is also used with $m_{u/d} = 330$ MeV leading to the prediction $P_{q\bar{q}} = 0.279$ from fit to data, implying an important contribution of higher Fock states in the pion.
Vega, Schmidt, Branz, Gutsche, Lyubovitskij, PRD 80, 055014 (2009)
When a universal $\kappa = 550$ MeV is used, together with a constituent quark mass $m_{u/d} = 420$ MeV, $P_{q\bar{q}} = 0.6$ is fixed for the pion only: for the kaon, $P_{q\bar{q}} = 0.8$ and for all other mesons, $P_{q\bar{q}} = 1$
Branz, Gutsche, Lyubovitskij, Schmidt,Vega, PRD 82, 074022 (2010)

Pion's special treatment

More recently with $m_{u/d} = 330$ MeV, a universal $\kappa = 550$ MeV for all meson but fix the wavefunction normalization for the pion so as to fit the decay constant. Consequently, this implies that $P_{q\bar{q}} = 0.61$ only for the pion.

Swarnkar, Chakrabarti, PRD 92, 074023 (2015)

Observation:

- All these previous studies seem to indicate that a special treatment is required at least for the pion either by using a distinct AdS/QCD scale κ or/and relaxing the normalization condition on the holographic wavefunction, i.e. invoking higher Fock states contributions.
- Pion observables are predicted using the holographic wavefunction with the helicity dependence is always assumed to decouple from the dynamics, i.e. the helicity wavefunction is taken to be momentum-independent.

Dynamical spin wavefunction

It is possible to achieve a better description of the pion observables by using a universal AdS/QCD scale κ and without the need to invoke higher Fock state contributions

To restore the helicity dependence of the holographic wavefunction, we assume that

$$\Psi(x,\mathbf{k}) \to \Psi_{h\bar{h}}(x,\mathbf{k}) = S_{h\bar{h}}(x,\mathbf{k})\Psi(x,\mathbf{k})$$

where $S_{h\bar{h}}(x,\mathbf{k})$ corresponds to the helicity wavefunction for a point-like meson-$q\bar{q}$ coupling and, in most general form, can be written as

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \frac{\bar{v}_{\bar{h}}((1-x)P^{+},-\mathbf{k})}{\sqrt{1-x}}\left[(A\not{P} + BM_{\pi})\gamma^{5}\right]\frac{u_{h}(xP^{+},\mathbf{k})}{\sqrt{x}} \, .$$

A and B are constants

Vector meson case

For vector mesons, the helicity wavefunction is similar to that of the point-like photon-$q\bar{q}$ coupling, i.e.

$$S^V_{h\bar{h}}(x, \mathbf{k}) = \frac{\bar{v}_{\bar{h}}((1-x)P^+, -\mathbf{k})}{\sqrt{(1-x)}}[\gamma \cdot \epsilon_V]\frac{u_h(xP^+, \mathbf{k})}{\sqrt{x}}$$

where ϵ^μ_V is the polarization vector of the vector meson

- Substituting ϵ^μ_V by the photon polarization vector leads to the well-known photon light-front wavefunctions.
- This assumption for the helicity structure of the vector meson is very common when computing diffractive vector meson production in the dipole model. PRD.94.074018(2016)

Dynamical spin effects

Using the light-front spinors we obtain

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \left\{ AM_{\pi}^2 + B\left(\frac{m_f M_{\pi}}{x(1-x)}\right)\right\}(2h)\delta_{-h\bar{h}} + B\left(\frac{M_{\pi} k e^{i(2h)\theta_k}}{x(1-x)}\right)\delta_{h\bar{h}}$$

with $\mathbf{k} = ke^{i\theta_k}$
If we take $B = 0$, the helicity wavefunction becomes momentum-independent:

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) \to S^{\pi}_{h\bar{h}} = \frac{1}{\sqrt{2}}(2h)\delta_{-h\bar{h}}$$

normalized such that $\sum_{h\bar{h}} |S^{\pi}_{h,\bar{h}}|^2 = 1$
We shall refer to this case as the non-dynamical (momentum-independent) helicity wavefunction.

Spin improved wavefunction

A two-dimensional Fourier transform of our spin-improved wavefunction to impact space gives

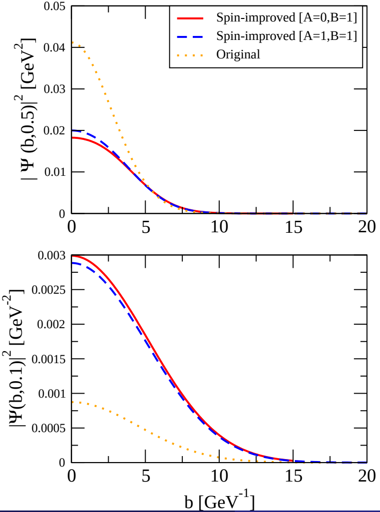
$$\Psi^{\pi}_{h\bar{h}}(x,\mathbf{b}) = \{(Ax(1-x)M_{\pi}^2 + Bm_f M_{\pi})(2h)\delta_{-h\bar{h}} - BM_{\pi} i\partial_b \delta_{h\bar{h}}\}\frac{\Psi^{\pi}(x,\zeta^2)}{x(1-x)}$$

which can be compared to the original holographic wavefunction,

$$\Psi^{\pi[o]}_{h\bar{h}}(x,\mathbf{b}) = \frac{1}{\sqrt{2}} h\delta_{-h\bar{h}}\Psi^{\pi}(x,\zeta^2)$$

$\Psi^{\pi}(x,\zeta^2)$ in both of the above equations, is the holographic wavefunction.

Results: Wavefunction



Results: Pion radius

The root-mean-square pion radius is given by:

$$\sqrt{\langle r_\pi^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x\mathrm{d}^2\mathbf{b} [b(1-x)]^2 |\Psi^\pi(x, \mathbf{b})|^2 \right]^{1/2}$$

	$\sqrt{\langle r_\pi^2 \rangle}$ [fm]
Original	0.544
Spin-improved ($A = 0, B = 1$)	0.683
Spin-improved ($A = 1, B = 1$)	0.673
Experiment	0.672 ± 0.008

Table: Our predictions for the pion radius using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. The datum is from PDG 2014

Decay constant

$$\langle 0|\bar{\Psi}_d \gamma^\mu \gamma_5 \Psi_u|\pi^+\rangle = f_\pi P^\mu$$

Taking $\mu = +$ and expanding the left-hand-side we obtain

$$\langle 0|\bar{\Psi}_d \gamma^+ \gamma^5 \Psi_u|\pi^+\rangle = \sqrt{4\pi N_c} \sum_{h,\bar{h}} \int \frac{\mathrm{d}^2\mathbf{k}}{16\pi^3} \mathrm{d}x \Psi^\pi_{h,\bar{h}}(x,\mathbf{k}) \left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\}$$

The light-front matrix element in curly brackets can readily be evaluated:

$$\left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\} = 2P^+(2h)\delta_{-h\bar{h}}$$

$$\Rightarrow f_\pi = 2\sqrt{\frac{N_c}{\pi}} \int \mathrm{d}x \{A((x(1-x)M_\pi^2) + Bm_f M_\pi\} \frac{\Psi^\pi(x,\zeta)}{x(1-x)}\bigg|_{\zeta=0} .$$

Prediction for pion's decay constant

	f_π [MeV]
Original	161
Spin-improved ($A = 0, B = 1$)	135
Spin-improved ($A = 1, B = 1$)	138
Experiment	$130.4 \pm 0.04 \pm 0.2$

Table: Our predictions for the pion decay constant using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. The datum is from PDG 2014.

Pion EM form factor

Pion EM form factor defined as

$$\langle \pi^+ : P' | J^\mu_{\text{em}}(0) | \pi^+ : P \rangle = 2(P+P')^\mu F_\pi(Q^2)$$

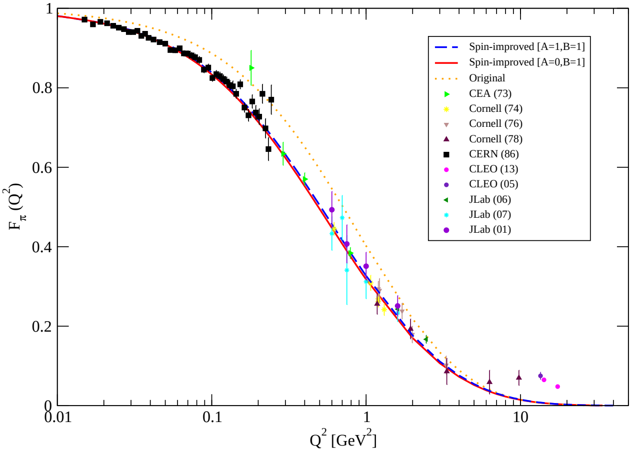
$P' = P + q$, $Q^2 = -q^2$ and the EM current $J^\mu_{\text{em}}(z) = \sum_f e_f \bar{\Psi}(z)\gamma^\mu \Psi(z)$ with $f = \bar{d}, u$ and $e_{\bar{d},u} = 1/3, 2/3$.
The EM form factor can be expressed in terms of the pion LFWF using the Drell-Yan-West formula:

$$F_\pi(Q^2) = 2\pi \int \mathrm{d}x \mathrm{d}b \; b \; J_0[(1-x)bQ] \; |\Psi^\pi(x, \mathbf{b})|^2$$

Note that the above equation implies that $F_\pi(0) = 1$ and that the slope of the EM form factor at $Q^2 = 0$ is related to the mean radius of the pion via

$$\langle r_\pi^2 \rangle = -\frac{6}{F_\pi(0)} \frac{\mathrm{d}F_\pi}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Pion EM form factor-predictions



Conclusion

- We observe improvement in the description of pion data when dynamical spin effects are taken into account.
- This improvement is achieved while using a universal κ and constituent quark masses.