Study of the DKK and $DK\bar{K}$ systems

Vinícius Rodrigues Debastiani

Instituto de Física Corpuscular,
Universidad de Valencia - CSIC (Spain)

27/09/2017
Hadron, Salamanca (Spain)

Outline

1. Introduction
   - The Faddeev Equations
   - DK Cluster: The $D_{s0}^*(2317)$
   - Two-Body Amplitudes
2. Formalism
   - The Fixed Center Approximation
   - $DK\bar{K}$ System
   - Chiral Unitary Approach
   - DKK System
   - Energy Distribution
3. Results
   - $DK\bar{K}$ System
   - DKK System
4. Summary

- L. D. Faddeev, Sov. Phys. JETP 12, 1014 (1961), [Zh. Eksp. Teor. Fiz. 39, 1459 (1960)].
- Variational Method in the study of $\bar{K}\bar{K}N$
  Y. Kanada-En'yo and D. Jido,
  Phys. Rev. C **78**, 025212 (2008).
- $K\bar{K}N$ [from $K\Lambda(1405)$] $\rightarrow N^*$ around 1920 MeV, made mostly of $Na_0(980)$
  A. Martinez Torres and D. Jido,
  Phys. Rev. C **82**, 038202 (2010).
- $\phi K\bar{K}$ $\rightarrow$ reproduce the properties of the $\phi(2170)$
  A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale and E. Oset,
  Phys. Rev. D **78**, 074031 (2008).
- $KK\bar{K}$ $\rightarrow$ associated with the $K(1460)$
  A. Martinez Torres, D. Jido and Y. Kanada-En'yo,
  Phys. Rev. C **83**, 065205 (2011).
- $\pi K\bar{K}$ $\rightarrow$ associated with the $\pi(1300)$
  A. Martinez Torres, K. P. Khemchandani, D. Jido and A. Hosaka,
  Phys. Rev. D **84**, 074027 (2011).
- $J/\psi K\bar{K}$ $\rightarrow$ associated with the $Y(4260)$
  A. Martinez Torres, K. P. Khemchandani, D. Gamermann and E. Oset,
  Phys. Rev. D **80**, 094012 (2009).

- $\eta K\bar{K}$ and $\eta' K\bar{K}$
  W. Liang, C. W. Xiao and E. Oset, Phys. Rev. D **88**, no. 11, 114024 (2013).
- $K^- pp$ system: calculations (including *charge exchange* diagrams)
  T. Sekihara, E. Oset and A. Ramos, "On the structure observed in the in-flight ^{3}He(K$^-$, Λp)n reaction at J-PARC," PTEP **2016**, no. 12, 123D03 (2016).
- Experiment from J-PARC
  Y. Sada *et al.* [J-PARC E15 Collaboration], "Structure near $K^- + p + p$ threshold in the in-flight ^{3}He(K^-, Λp)n reaction," PTEP **2016**, no. 5, 051D01 (2016).
- $\rho B^* \bar{B}^*$ M. Bayar, P. Fernandez-Soler, Z. F. Sun and E. Oset, Eur. Phys. J. A **52**, no. 4, 106 (2016).
- $\pi \bar{K} K^* \rightarrow$ associated with the $\pi_1(1600)$
  X. Zhang, J. J. Xie and X. Chen, Phys. Rev. D **95**, no. 5, 056014 (2017).

Our work:

V. R. Debastiani, J. M. Dias and E. Oset,
"Study of the DKK and $DK\bar{K}$ systems,"
Phys. Rev. D **96**, no. 1, 016014 (2017).

Similar study of the $DK\bar{K}$ system using two different methods:
QCD Sum Rules and full Faddeev equations:

A. Martinez Torres, K. P. Khemchandani, M. Nielsen and F. S. Navarra,
"Predicting the Existence of a 2.9 GeV $Df_0(980)$ Molecular State,"
Phys. Rev. D **87**, no. 3, 034025 (2013).

The $D^*_{s0}(2317)$ molecule will act as the cluster in both systems we have studied: the DKK and the $DK\bar{K}$.

- The $D^*_{s0}(2317)$ is strongly bound (about 45 MeV of binding energy), and very narrow, with almost zero width.
- Dynamically generated from the DK interaction in isospin 0.
- Its existence has already been established both from chiral unitary approach and Lattice simulations:

- F. K. Guo, P. N. Shen, H. C. Chiang, R. G. Ping and B. S. Zou, "Dynamically generated 0+ heavy mesons in a heavy chiral unitary approach," Phys. Lett. B **641**, 278 (2006).
- D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, "Dynamically generated open and hidden charm meson systems," Phys. Rev. D **76**, 074016 (2007).
- A. Martinez Torres, E. Oset, S. Prelovsek and A. Ramos, "Reanalysis of lattice QCD spectra leading to the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$," JHEP **1505**, 153 (2015).

The three-body amplitude will be written in terms of the two-body interactions:

- **DK$\bar{\text{K}}$** system, described through the interaction of the external $\bar{K}$ with the components of the cluster $[DK]$:

  $K\bar{K}$ Using the chiral unitary approach [1].
  (Generates the $a_0(980)$ in $I = 1$ and the $f_0(980)$ in $I = 0$).

  $D\bar{K}$ obtained from the same formalism of the DK interaction [2].
  (Repulsive in isospin 1; Attractive in isospin 0; No bound state in the energy region corresponding to the $DK\bar{K}$ interaction).

[1] J. A. Oller and E. Oset, "Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the σ, $f_0(980)$, $a_0(980)$ scalar mesons," Nucl. Phys. A **620**, 438 (1997), Erratum: [Nucl. Phys. A **652**, 407 (1999)].

[2] D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, "Dynamically generated open and hidden charm meson systems," Phys. Rev. D **76**, 074016 (2007).

The three-body amplitude will be written in terms of the two-body interactions:

- **DKK** system, described through the interaction of the external K with the components of the cluster $[DK]$:

  KK which we derive from the same approach used in the $K\bar{K}$ [1]. (Repulsive in isospin 1).

  DK which is the same used to generate the $D^*_{s0}(2317)$ [2]. (Attractive in isospin 0).

[1] J. A. Oller and E. Oset, “Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the σ, $f_0(980)$, $a_0(980)$ scalar mesons,” Nucl. Phys. A **620**, 438 (1997), Erratum: [Nucl. Phys. A **652**, 407 (1999)].

[2] D. Gamermann, E. Oset, D. Strottman and M. J. Vicente Vacas, “Dynamically generated open and hidden charm meson systems,” Phys. Rev. D **76**, 074016 (2007).

$$
\begin{aligned}
T_1 &= t_1 + t_1\, G_0\, t_2 + t_1\, G_0\, t_2\, G_0\, t_1 + \ldots \\
T_2 &= t_2 + t_2\, G_0\, t_1 + t_2\, G_0\, t_1\, G_0\, t_2 + \ldots
\end{aligned} \tag{1}
$$

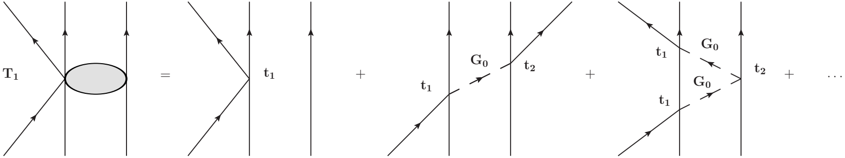


Figure: Diagrams of Fixed Center Approximation to Faddeev equations.

$$
\begin{aligned}
T_1 &= t_1 + t_1\, G_0\, T_2 \\
T_2 &= t_2 + t_2\, G_0\, T_1 \\
T &= T_1 + T_2
\end{aligned} \tag{2}
$$

We use the following channels to account for all the possible interactions in the $DK\bar{K}$ system, where the $[DK]$ acts as the cluster and the $\bar{K}$ will interact with one of its components:

- (1) $K^-[D^+K^0]$, (2) $K^-[D^0K^+]$, (3) $\bar{K}^0[D^0K^0]$, when the $\bar{K}$ interacts with the D of the cluster;
- (4) $[D^+K^0]K^-$, (5) $[D^0K^+]K^-$, (6) $[D^0K^0]\bar{K}^0$, when the $\bar{K}$ interacts with the K of the cluster.
- The channels (3) and (6) are used as intermediate charge-exchange steps

For the $K^-[D^+K^0] \to K^-[D^+K^0]$ amplitude, denoted by $T_{11}^{\rm FCA}$, we have

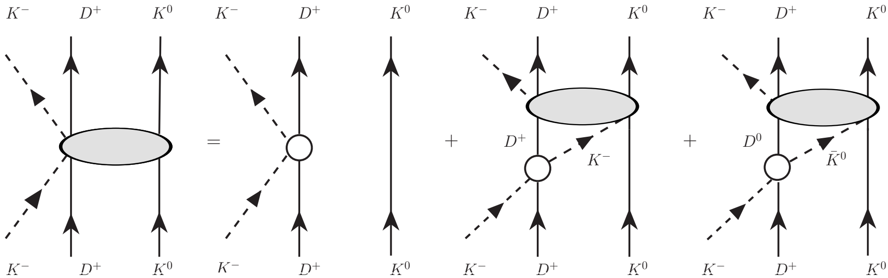


Figure: Diagrams for the K^- multiple scattering of the process $K^-[D^+K^0] \to K^-[D^+K^0]$. The white circles indicate two-body amplitudes of $D\bar{K} \to D\bar{K}$, while the gray bubbles are associated with three-body amplitudes of $DK\bar{K}$.

$$T_{11}^{\rm FCA}(s) = t_1 + t_1\, G_0\, T_{41}^{\rm FCA} + t_2\, G_0\, T_{61}^{\rm FCA}\,, \qquad (3)$$

where t_1 and t_2 are the $D^+K^- \to D^+K^-$ and $D^+K^- \to D^0\bar{K}^0$ two-body scattering amplitudes, respectively.

This way we can systematically write the three-body amplitudes in terms of the two-body amplitudes and the propagator of $\bar{K}$ inside the cluster:

$$T_{ij}^{\rm FCA}(s) = V_{ij}^{\rm FCA}(s) + \sum_{l=1}^{6} \tilde{V}_{il}^{\rm FCA}(s)\, G_0(s)\, T_{lj}^{\rm FCA}(s)\,, \tag{4}$$

where V_{ij} and $\tilde{V}_{il}$ are the elements of the matrices below:

$$V^{\rm FCA} = \begin{pmatrix} t_1 & 0 & t_2 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 & 0 & 0 \\ t_2 & 0 & t_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_6 & t_7 \\ 0 & 0 & 0 & 0 & t_7 & t_8 \end{pmatrix}, \quad \tilde{V}^{\rm FCA} = \begin{pmatrix} 0 & 0 & 0 & t_1 & 0 & t_2 \\ 0 & 0 & 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_2 & 0 & t_4 \\ t_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & t_6 & t_7 & 0 & 0 & 0 \\ 0 & t_7 & t_8 & 0 & 0 & 0 \end{pmatrix}, \tag{5}$$

which are the two-body scattering and transition amplitudes:

$$\begin{aligned} t_1 &= t_{D^+K^- \to D^+K^-} & t_5 &= t_{K^0K^- \to K^0K^-} \\ t_2 &= t_{D^+K^- \to D^0\bar{K}^0} & t_6 &= t_{K^+K^- \to K^+K^-} \\ t_3 &= t_{D^0K^- \to D^0K^-} & t_7 &= t_{K^+K^- \to K^0\bar{K}^0} \\ t_4 &= t_{D^0\bar{K}^0 \to D^0\bar{K}^0} & t_8 &= t_{K^0\bar{K}^0 \to K^0\bar{K}^0}\,, \end{aligned} \tag{6}$$

Isolating the T^{FCA} matrix we get:

$$T_{ij}^{\rm FCA}(s) = \sum_{l=1}^{6} \Big[1 - \tilde{V}^{\rm FCA}(s)\, G_0(s) \Big]^{-1}_{il} V_{lj}^{\rm FCA}(s)\,. \tag{7}$$

Finally, to get the $DK\bar{K} \to DK\bar{K}$ amplitude we need to consider that the DK cluster generates the $D^*_{s0}(2317)$ in isospin 0:

$$|DK(I=0)\,\rangle = (1/\sqrt{2})\,|\,D^+K^0 + D^0K^+\,\rangle, \tag{8}$$

and then sum up all the channels corresponding to initial and final state of $\bar{K}$ interacting with the $|DK(I=0)\,\rangle$ cluster:

$$\begin{aligned} T_{DK\bar{K}} &= \frac{1}{2}\Big(T_{11}^{\rm FCA} + T_{12}^{\rm FCA} + T_{14}^{\rm FCA} + T_{15}^{\rm FCA} + T_{21}^{\rm FCA} + T_{22}^{\rm FCA} + T_{24}^{\rm FCA} + T_{25}^{\rm FCA} \\ &+ T_{41}^{\rm FCA} + T_{42}^{\rm FCA} + T_{44}^{\rm FCA} + T_{45}^{\rm FCA} + T_{51}^{\rm FCA} + T_{52}^{\rm FCA} + T_{54}^{\rm FCA} + T_{55}^{\rm FCA} \Big). \end{aligned} \tag{9}$$

The propagator of $\bar{K}$ between the particles of the cluster, denoted by G_0, is calculated by

$$G_0(s) = \frac{1}{2M_{D^*_{s0}}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{F_R(\mathbf{q})}{(q^0)^2 - \omega_K^2(\mathbf{q}) + i\epsilon}\,, \qquad q^0 = \frac{s - m_K^2 - M_{D^*_{s0}}^2}{2M_{D^*_{s0}}}\,, \tag{10}$$

where $M_{D^*_{s0}}$ is the mass of the cluster $D^*_{s0}(2317)$, and $\omega_K^2(\mathbf{q}) = \mathbf{q}^2 + m_K^2$, while q^0 is the energy carried by $\bar{K}$ in the cluster rest frame. The form factor $F(\mathbf{q})$, describing an S-wave bound state [1], is given by

$$F_R(\mathbf{q}) = \frac{1}{N} \int_{|\mathbf{p}|,|\mathbf{p}-\mathbf{q}|<\Lambda} d^3\mathbf{p}\; \frac{1}{M_{D^*_{s0}} - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \frac{1}{M_{D^*_{s0}} - \omega_D(\mathbf{p}-\mathbf{q}) - \omega_K(\mathbf{p}-\mathbf{q})}\,, \tag{11}$$

where $\omega_D(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_D^2}$ and the normalization factor N is

$$N = \int_{|\mathbf{p}|<\Lambda} d^3\mathbf{p} \left(\frac{1}{M_{D^*_{s0}} - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \right)^2 . \tag{12}$$

The upper integration limit Λ has the same value of the cutoff used to regularize the DK loop, adjusted in order to get the $D^*_{s0}(2317)$ molecule from the DK interaction.

[1] F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

We use an effective chiral Lagrangian where the pseudoscalar mesons are the degrees of freedom:

$$\mathcal{L}_2 = \frac{1}{12\, f_\pi^2}\mathrm{Trace}[\ (\partial_\mu \Phi\, \Phi - \Phi\, \partial_\mu \Phi)^2 + M\Phi^4\ ]\,, \tag{13}$$

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \ ; \quad M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}\,, \tag{14}$$

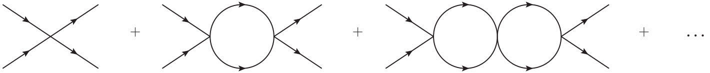


Figure: Diagrams representing meson-meson loops.

J. A. Oller and E. Oset, "Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the σ, $f_0(980)$, $a_0(980)$ scalar mesons," Nucl. Phys. A **620**, 438 (1997), Erratum: [Nucl. Phys. A **652**, 407 (1999)].

From this Lagrangian we extract the kernel of each channel which are then inserted into the Bethe-Salpeter equation, summing the contribution of every meson-meson loop.

$$T_{ij} = V_{ij} + \sum_l V_{il} G_l T_{lj} \qquad \Longrightarrow \qquad T = (1 - VG)^{-1} V\,, \tag{15}$$

where G is the loop-function, which we regularize with a cutoff. In these works we use $q_{max} \sim 600$ MeV. After the integration in q^0 and $\cos\theta$ we have:

$$G = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [(P^0)^2 - (\omega_1 + \omega_2) + i\epsilon]}\,, \tag{16}$$

$$\omega_i = \sqrt{q^2 + m_i^2}, \quad (P^0)^2 = s \tag{17}$$

Each contribution is projected in S-wave and a normalization factor is included when identical particles are present.

The matrix T gives us the scattering amplitude and transitions between each channel, which in charge basis are: 1) $\pi^+\pi^-$, 2) $\pi^0\pi^0$, 3) K^+K^-, 4) $K^0\bar{K}^0$, 5) $\eta\eta$ and 6) $\pi^0\eta$.

In analogy to the $DK\bar{K}$ system, we have also studied the DKK System, where instead of an external $\bar{K}$ we added another K meson. Therefore, we will have the following channels:

- (1) $K^+[D^+K^0]$, (2) $K^+[D^0K^+]$, (3) $K^0[D^+K^+]$, when the external K interacts with the D of the cluster;
- (4) $[D^+K^0]K^+$, (5) $[D^0K^+]K^+$, (6) $[D^0K^0]K^0$, when the external K interacts with the K of the cluster.
- As before, the channels (3) and (6) are used as intermediate charge-exchange steps

We use the same cluster, the $D^*_{s0}(2317)$ generated from the DK interaction in isospin 0. Then the procedure to obtain the DKK amplitude is completely analogous.

However, in this case the V^{FCA} and $\tilde{V}^{\mathrm{FCA}}$ matrices will be written in terms of the *DK* and *KK* two-body amplitudes.

$$V^{\mathrm{FCA}} = \begin{pmatrix} \bar{t}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{t}_2 & \bar{t}_3 & 0 & 0 & 0 \\ 0 & \bar{t}_3 & \bar{t}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \\ 0 & 0 & 0 & 0 & \bar{t}_6 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \end{pmatrix}, \quad \tilde{V}^{\mathrm{FCA}} = \begin{pmatrix} 0 & 0 & 0 & \bar{t}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{t}_2 & \bar{t}_3 \\ 0 & 0 & 0 & 0 & \bar{t}_3 & \bar{t}_4 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \\ 0 & \bar{t}_6 & 0 & 0 & 0 & 0 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \end{pmatrix}. \quad (18)$$

$$\begin{array}{ll} \bar{t}_1 = t_{D^+K^+\to D^+K^+} & \bar{t}_4 = t_{D^+K^0\to D^+K^0} \\ \bar{t}_2 = t_{D^0K^+\to D^0K^+} & \bar{t}_5 = t_{K^+K^0\to K^+K^0} \\ \bar{t}_3 = t_{D^0K^+\to D^+K^0} & \bar{t}_6 = t_{K^+K^+\to K^+K^+} \end{array} \quad (19)$$

- This time the *DK* two-body amplitudes are explicitly taken into account due to the interaction of the external *K* with the *D* inside the cluster. This interaction is very attractive, and the *DKK* system could bind.

- On the other hand, now we have the *KK* interaction, which is repulsive.

- The partitions $T_{ij}^{\mathrm{FCA}}(s)$ are written into the three-body center-of-mass energy $\sqrt{s}$.
- While the scattering amplitudes $t_i(s_i)$ in the V^{FCA} and $\tilde{V}^{\mathrm{FCA}}$ matrices are written into the two-body center-of-mass energy $\sqrt{s_i}$.
- We use two sets of transformations to obtain the $\sqrt{s_i}$'s in terms of $\sqrt{s}$:

We call this set by "Prescription I", standard transformation from 3-body kinematics used in most works.

$$s_{DK(D\bar{K})} = m_K^2 + m_D^2 + \frac{1}{2M_{D_{s0}^*}^2}(s - m_K^2 - M_{D_{s0}^*}^2)\,(M_{D_{s0}^*}^2 + m_D^2 - m_K^2), \tag{20}$$

$$s_{KK(K\bar{K})} = 2\,m_K^2 + \frac{1}{2M_{D_{s0}^*}^2}(s - m_K^2 - M_{D_{s0}^*}^2)\,(M_{D_{s0}^*}^2 + m_K^2 - m_D^2)\,. \tag{21}$$

In order to estimate the uncertainties we also use "Prescription II", assuming that the kinetic energy in the DK cluster is of the order of the binding energy.

$$s_{DK(D\bar{K})} = \left(\frac{\sqrt{s}}{M_{D^*_{s0}} + m_K}\right)^2 \left(m_K + \frac{m_D\, M_{D^*_{s0}}}{m_D + m_K}\right)^2 - \mathbf{P}_2^2\,, \tag{22}$$

$$s_{KK(K\bar{K})} = \left(\frac{\sqrt{s}}{M_{D^*_{s0}} + m_K}\right)^2 \left(m_K + \frac{m_K\, M_{D^*_{s0}}}{m_D + m_K}\right)^2 - \mathbf{P}_1^2\,, \tag{23}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ stand for the momenta of the D and K mesons in the cluster.
We take $\mathbf{P}_1^2 = \mathbf{P}_2^2 = 2\tilde{\mu}B_{D^*_{s0}} = 2\tilde{\mu}\,(m_D + m_K - M_{D^*_{s0}})$, with $\tilde{\mu}$ the reduced mass of DK.

This prescription is based on another one [1], which shares the binding energy among the three-particles proportionally to their respective masses.

[1] M. Bayar, P. Fernandez-Soler, Z. F. Sun and E. Oset, Eur. Phys. J. A **52**, no. 4, 106 (2016).

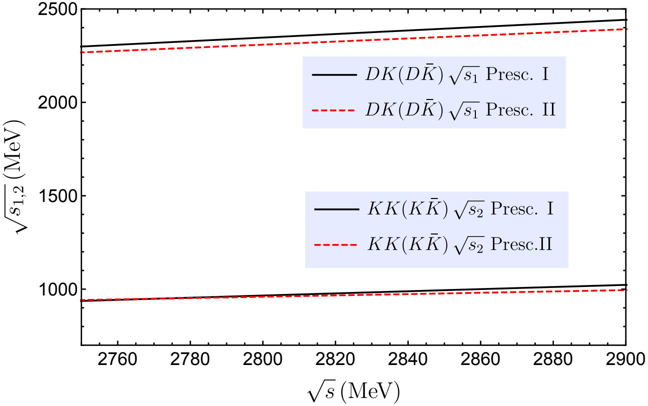


Figure: Energy distribution in the center-of-mass of each two-body system as a function of the total energy of the three-body system, using prescriptions I and II. Here $s_1 = s_{DK(D\bar{K})}$ and $s_2 = s_{KK(K\bar{K})}$. The lower curves are for KK or $K\bar{K}$ and the upper curves are for DK or $D\bar{K}$.

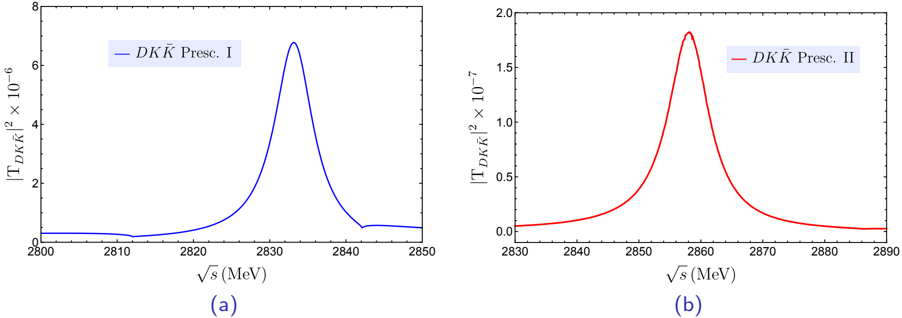
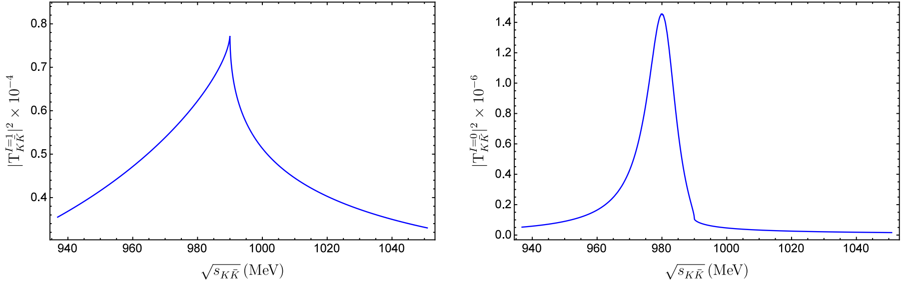


Figure: Results for the total $DK\bar{K}$ amplitude squared using prescriptions I (left) and II (right).

- Narrow bound state around $D\,f_0(980)$ threshold ($\sim$ 2855 MeV) in both prescriptions.



(a) $K\bar{K}$ amplitude in $I = 1$; couples to $a_0(980)$.

(b) $K\bar{K}$ amplitude in $I = 0$; couples to $f_0(980)$.

Figure: Comparison between $K\bar{K}$ amplitude squared in isospin 1 and 0.

- $a_0(980)$ couples to $K\bar{K}$ in $I = 1$, but the coupling to $\pi\eta$ is the dominant one.
- $f_0(980)$ couples strongly to $K\bar{K}$ in $I = 0$, and with less intensity to $\pi\pi$. (The σ meson, $f_0(500)$, is dominated by $\pi\pi$ in $I = 0$).
- In $K\bar{K}$ the $f_0(980)$ amplitude squared is more than **2 orders** of magnitude stronger than $a_0(980)$ around their peaks.

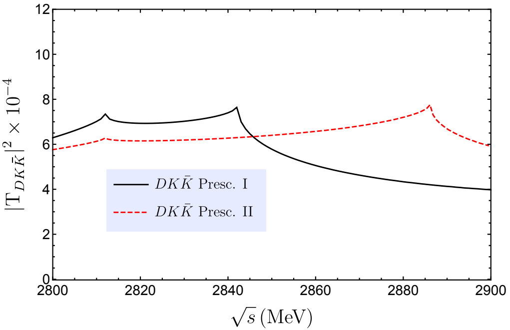


Figure: Results for the $DK\bar{K}$ amplitude squared after removing the $f_0(980)$ contribution, using prescriptions I and II.

Table: Comparison between position and intensity of the peaks found in the $DK\bar{K}$ amplitude.

	Prescription I		Prescription II	
	$\sqrt{s}$	$\|T\|^2$	$\sqrt{s}$	$\|T\|^2$
Total	2833	6.8×10^6	2858	1.8×10^7
$I=1$ only	2842	7.7×10^4	2886	7.8×10^4

Our Result [1]:

$M_{DK\bar{K}} = 2833 - 2858$ MeV.

QCD Sum Rules [2]:
$M_{Df_0} = (2926 \pm 237)$ MeV.

Full Faddeev equations [2]:
$M_{Df_0} = 2890$ MeV.

[1] V. R. Debastiani, J. M. Dias and E. Oset,
"Study of the DKK and $DK\bar{K}$ systems,"
Phys. Rev. D **96**, no. 1, 016014 (2017).

[2] A. Martinez Torres, K. P. Khemchandani, M. Nielsen and F. S. Navarra,
"Predicting the Existence of a 2.9 GeV $Df_0(980)$ Molecular State,"
Phys. Rev. D **87**, no. 3, 034025 (2013).

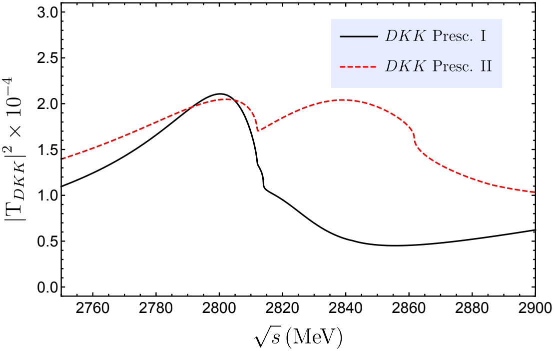


Figure: Results for the total *DKK* amplitude squared using prescriptions I and II.

- Broad, irregular structures. Both amplitudes decrease around 2812 MeV which corresponds to the $D^*_{s0}(2317)K$ threshold.
- *KK* repulsion vs. *DK* attraction.
- Enhancement below threshold with "Prescripiton I".
- Small strength if compared to the $DK\bar{K}$ system. No clear bound state or resonance.

Summary

- Fixed Center Approximation to Faddeev Equations. Charge exchange diagrams included.
- $DK\bar{K}$ and DKK three-body interactions.
- Cluster DK in the $I = 0$: $D^*_{s0}(2317)$.
- Two-body scattering: $D\bar{K}$ and $K\bar{K}$ in $DK\bar{K}$ system; DK and KK in DKK.
- Uncertainties were estimated taking into account two different prescriptions to obtain $\sqrt{s_{DK}}$ and $\sqrt{s_{KK}}$ from the total energy of the system $\sqrt{s}$.

- $DK\bar{K}$ system binds: $I(J^P) = 1/2(0^-)$ state, with mass about $2833 - 2858$ MeV, dominated by a $Df_0(980)$ component (from $K\bar{K}$ in $I = 0$).
- Results compatible with other methods using QCD Sum Rules ($M_{Df_0} = (2926 \pm 237)$ MeV) and full Faddeev equations ($M_{Df_0} = 2890$ MeV.)
- This state could be seen in the $\pi\pi D$ invariant mass distribution (from $f_0(980) \to \pi\pi$ decay).

- DKK does not seem to bind. KK repulsion seems to be of the same magnitude as the attraction on the DK interaction.

THANK YOU!