Study of the $D KK$ and $DK \bar{K}$ systems

Vinícius Rodrigues Debastiani

Instituto de Física Corpuscular,
Universidad de Valencia - CSIC (Spain)

27/09/2017
Hadron, Salamanca (Spain)
Outline

1 Introduction
   - The Faddeev Equations
   - $DK$ Cluster: The $D_{s0}^*(2317)$
   - Two-Body Amplitudes

2 Formalism
   - The Fixed Center Approximation
   - $DK\bar{K}$ System
   - Chiral Unitary Approach
   - $DKK$ System
   - Energy Distribution

3 Results
   - $DK\bar{K}$ System
   - $DKK$ System

4 Summary
The Faddeev Equations

Introduction


- Variational Method in the study of $\bar{K}K^*N$
  Y. Kanada-En'yo and D. Jido,

- $K\bar{K}N$ [from $K\Lambda(1405)] \rightarrow N^*$ around 1920 MeV, made mostly of $Na_0(980)$
  A. Martinez Torres and D. Jido,

- $\phi K\bar{K}$ → reproduce the properties of the $\phi(2170)$
  A. Martinez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale and E. Oset,

- $KK\bar{K}$ → associated with the $K(1460)$
  A. Martinez Torres, D. Jido and Y. Kanada-En'yo,

- $\pi K\bar{K}$ → associated with the $\pi(1300)$
  A. Martinez Torres, K. P. Khemchandani, D. Jido and A. Hosaka,

- $J/\psi K\bar{K}$ → associated with the $Y(4260)$
  A. Martinez Torres, K. P. Khemchandani, D. Gamermann and E. Oset,
The Fixed Center Approximation

Introduction

- $\eta K \bar{K}$ and $\eta' K \bar{K}$

- $K^{-} pp$ system: calculations (including charge exchange diagrams)

- Experiment from J-PARC
  Y. Sada et al. [J-PARC E15 Collaboration], “Structure near $K^- + p + p$ threshold in the in-flight $^3\text{He}(K^-, \Lambda p)n$ reaction,” PTEP 2016, no. 5, 051D01 (2016).

- $\rho B^* \bar{B}^*$

- $\pi \bar{K} K^*$ → associated with the $\pi_1(1600)$
The $DKK$ and $DK\bar{K}$ systems

Introduction

Our work:

V. R. Debastiani, J. M. Dias and E. Oset,
“Study of the $DKK$ and $DK\bar{K}$ systems,”

Similar study of the $DK\bar{K}$ system using two different methods:
QCD Sum Rules and full Faddeev equations:

A. Martinez Torres, K. P. Khemchandani, M. Nielsen and F. S. Navarra,
“Predicting the Existence of a 2.9 GeV $Df_0(980)$ Molecular State,”
The $D_{s0}^*(2317)$ molecule will act as the cluster in both systems we have studied: the $DKK$ and the $DK\bar{K}$.

- The $D_{s0}^*(2317)$ is strongly bound (about 45 MeV of binding energy), and very narrow, with almost zero width.
- Dynamically generated from the $DK$ interaction in isospin 0.
- Its existence has already been established both from chiral unitary approach and Lattice simulations:
  - A. Martinez Torres, E. Oset, S. Prelovsek and A. Ramos, “Reanalysis of lattice QCD spectra leading to the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$,” JHEP 1505, 153 (2015).
The three-body amplitude will be written in terms of the two-body interactions:

- **$DK\bar{K}$** system, described through the interaction of the external $\bar{K}$ with the components of the cluster [$DK$]:
  
  $K\bar{K}$ Using the chiral unitary approach [1].
  (Generates the $a_0(980)$ in $I = 1$ and the $f_0(980)$ in $I = 0$).

  $D\bar{K}$ obtained from the same formalism of the $DK$ interaction [2].
  (Repulsive in isospin 1; Attractive in isospin 0; No bound state in the energy region corresponding to the $DK\bar{K}$ interaction).


The three-body amplitude will be written in terms of the two-body interactions:

- **DKK** system, described through the interaction of the external $K$ with the components of the cluster $[DK]$:
  
  \[ KK \] which we derive from the same approach used in the $K\bar{K}$ [1].
  
  (Repulsive in isospin 1).

  \[ DK \] which is the same used to generate the $D_{s0}^*(2317)$ [2].
  
  (Attractive in isospin 0).


The Fixed Center Approximation

Formalism

\[ T_1 = t_1 + t_1 G_0 t_2 + t_1 G_0 t_2 G_0 t_1 + \ldots \]
\[ T_2 = t_2 + t_2 G_0 t_1 + t_2 G_0 t_1 G_0 t_2 + \ldots \]  

(1)

\[ T_1 = t_1 + t_1 G_0 T_2 \]
\[ T_2 = t_2 + t_2 G_0 T_1 \]
\[ T = T_1 + T_2 \]  

(2)
We use the following channels to account for all the possible interactions in the $DK\bar{K}$ system, where the $[DK]$ acts as the cluster and the $\bar{K}$ will interact with one of its components:

1. $K^- [D^+ K^0]$, 
2. $K^- [D^0 K^+]$, 
3. $\bar{K}^0 [D^0 K^0]$, when the $\bar{K}$ interacts with the $D$ of the cluster; 

4. $[D^+ K^0] K^-$, 
5. $[D^0 K^+] K^-$, 
6. $[D^0 K^0] \bar{K}^0$, when the $\bar{K}$ interacts with the $K$ of the cluster.

The channels (3) and (6) are used as intermediate charge-exchange steps.
For the $K^- [D^+ K^0] \rightarrow K^- [D^+ K^0]$ amplitude, denoted by $T_{11}^{\text{FCA}}$, we have

\begin{equation}
T_{11}^{\text{FCA}}(s) = t_1 + t_1 G_0 T_{41}^{\text{FCA}} + t_2 G_0 T_{61}^{\text{FCA}},
\end{equation}

where $t_1$ and $t_2$ are the $D^+ K^- \rightarrow D^+ K^-$ and $D^+ K^- \rightarrow D^0 \bar{K}^0$ two-body scattering amplitudes, respectively.
This way we can systematically write the three-body amplitudes in terms of the two-body amplitudes and the propagator of $\bar{K}$ inside the cluster:

$$T_{ij}^{\text{FCA}}(s) = V_{ij}^{\text{FCA}}(s) + \sum_{l=1}^{6} \tilde{V}_{il}^{\text{FCA}}(s) G_0(s) T_{lj}^{\text{FCA}}(s),$$

where $V_{ij}$ and $\tilde{V}_{il}$ are the elements of the matrices below:

$$V^{\text{FCA}} = \begin{pmatrix} t_1 & 0 & t_2 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 & 0 & 0 \\ t_2 & 0 & t_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_6 & t_7 \\ 0 & 0 & 0 & 0 & t_7 & t_8 \end{pmatrix}, \quad \tilde{V}^{\text{FCA}} = \begin{pmatrix} 0 & 0 & 0 & t_1 & 0 & t_2 \\ 0 & 0 & 0 & 0 & t_3 & 0 \\ 0 & 0 & 0 & t_2 & 0 & t_4 \\ t_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & t_6 & t_7 & 0 & 0 & 0 \\ 0 & t_7 & t_8 & 0 & 0 & 0 \end{pmatrix},$$

which are the two-body scattering and transition amplitudes:

$$t_1 = t_{D^+ K^- \to D^+ K^-}, \quad t_5 = t_{K^0 K^- \to K^0 K^-},$$
$$t_2 = t_{D^+ K^- \to D^0 \bar{K}^0}, \quad t_6 = t_{K^+ K^- \to K^+ K^-},$$
$$t_3 = t_{D^0 K^- \to D^0 K^-}, \quad t_7 = t_{K^+ K^- \to K^0 \bar{K}^0},$$
$$t_4 = t_{D^0 \bar{K}^0 \to D^0 \bar{K}^0}, \quad t_8 = t_{K^0 \bar{K}^0 \to K^0 \bar{K}^0},$$
Isolating the $T^{FCA}$ matrix we get:

$$T_{ij}^{FCA}(s) = \sum_{l=1}^{6} \left[ 1 - \tilde{V}^{FCA}(s) G_0(s) \right]^{-1}_{il} V_{lj}^{FCA}(s). \quad (7)$$

Finally, to get the $\bar{D}K \bar{K} \rightarrow D\bar{K} \bar{K}$ amplitude we need to consider that the $D\bar{K}$ cluster generates the $D_{s0}^*(2317)$ in isospin 0:

$$|DK(I = 0)\rangle = (1/\sqrt{2}) |D^+K^0 + D^0K^+\rangle, \quad (8)$$

and then sum up all the channels corresponding to initial and final state of $\bar{K}$ interacting with the $|DK(I = 0)\rangle$ cluster:

$$T_{DK\bar{K}} = \frac{1}{2} \left( T_{11}^{FCA} + T_{12}^{FCA} + T_{14}^{FCA} + T_{15}^{FCA} + T_{21}^{FCA} + T_{22}^{FCA} + T_{24}^{FCA} + T_{25}^{FCA} 
+ T_{41}^{FCA} + T_{42}^{FCA} + T_{44}^{FCA} + T_{45}^{FCA} + T_{51}^{FCA} + T_{52}^{FCA} + T_{54}^{FCA} + T_{55}^{FCA} \right). \quad (9)$$
The propagator of $\bar{K}$ between the particles of the cluster, denoted by $G_0$, is calculated by

$$G_0(s) = \frac{1}{2M_{D^*_s0}} \int \frac{d^3q}{(2\pi)^3} \frac{F_R(q)}{(q^0)^2 - \omega_K^2(q) + i\epsilon}, \quad q^0 = \frac{s - m_K^2 - M_{D^*_s0}^2}{2M_{D^*_s0}},$$

where $M_{D^*_s0}$ is the mass of the cluster $D^*_s0(2317)$, and $\omega_K^2(q) = q^2 + m_K^2$, while $q^0$ is the energy carried by $\bar{K}$ in the cluster rest frame. The form factor $F(q)$, describing an $S$-wave bound state [1], is given by

$$F_R(q) = \frac{1}{N} \int \frac{d^3p}{|p|, |p-q| < \Lambda} \frac{1}{M_{D^*_s0} - \omega_D(p) - \omega_K(p)} \frac{1}{M_{D^*_s0} - \omega_D(p-q) - \omega_K(p-q)},$$

where $\omega_D(p) = \sqrt{p^2 + m_D^2}$ and the normalization factor $N$ is

$$N = \int \frac{d^3p}{|p| < \Lambda} \left( \frac{1}{M_{D^*_s0} - \omega_D(p) - \omega_K(p)} \right)^2.$$

The upper integration limit $\Lambda$ has the same value of the cutoff used to regularize the $DK$ loop, adjusted in order to get the $D^*_s0(2317)$ molecule from the $DK$ interaction.

We use an effective chiral Lagrangian where the pseudoscalar mesons are the degrees of freedom:

\[ \mathcal{L}_2 = \frac{1}{12 f^2} \text{Trace}[ (\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi)^2 + M\Phi^4 ] , \]  

(13)

\[
\Phi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \\
\pi^- \\
-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \\
K^- \\
\end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix}
m^2_{\pi^0} & 0 & 0 \\
0 & m^2_{\pi^0} & 0 \\
0 & 0 & 2m^2_K - m^2_{\pi^0} \\
\end{pmatrix} , \tag{14}
\]

Figure: Diagrams representing meson-meson loops.

From this Lagrangian we extract the kernel of each channel which are then inserted into the Bethe-Salpeter equation, summing the contribution of every meson-meson loop.

\[
T_{ij} = V_{ij} + \sum_l V_{il} G_l T_{lj} \quad \implies \quad T = (1 - VG)^{-1} V ,
\]

where \( G \) is the loop-function, which we regularize with a cutoff. In these works we use \( q_{\text{max}} \sim 600 \text{ MeV} \). After the integration in \( q^0 \) and \( \cos \theta \) we have:

\[
G = \int_0^{q_{\text{max}}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [(P^0)^2 - (\omega_1 + \omega_2) + i\epsilon]} ,
\]

\[
\omega_i = \sqrt{q^2 + m_i^2}, \quad (P^0)^2 = s
\]

Each contribution is projected in \( S \)-wave and a normalization factor is included when identical particles are present.

The matrix \( T \) gives us the scattering amplitude and transitions between each channel, which in charge basis are: 1) \( \pi^+\pi^- \), 2) \( \pi^0\pi^0 \), 3) \( K^+K^- \), 4) \( K^0\bar{K}^0 \), 5) \( \eta\eta \) and 6) \( \pi^0\eta \).
In analogy to the $DK\bar{K}$ system, we have also studied the $DKK$ System, where instead of an external $\bar{K}$ we added another $K$ meson. Therefore, we will have the following channels:


4. $[D^+K^0]K^+$, 5. $[D^0K^+]K^+$, 6. $[D^0K^0]K^0$, when the external $K$ interacts with the $K$ of the cluster.

As before, the channels (3) and (6) are used as intermediate charge-exchange steps.

We use the same cluster, the $D_{s0}^*(2317)$ generated from the $DK$ interaction in isospin 0. Then the procedure to obtain the $DKK$ amplitude is completely analogous.
However, in this case the $V^{FCA}$ and $\tilde{V}^{FCA}$ matrices will be written in terms of the $DK$ and $KK$ two-body amplitudes.

$$V^{FCA} = \begin{pmatrix} \bar{t}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{t}_2 & \bar{t}_3 & 0 & 0 & 0 \\ 0 & \bar{t}_3 & \bar{t}_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \\ 0 & 0 & 0 & 0 & \bar{t}_6 & 0 \\ 0 & 0 & 0 & \bar{t}_5 & 0 & \bar{t}_5 \end{pmatrix}, \quad \tilde{V}^{FCA} = \begin{pmatrix} 0 & 0 & 0 & \bar{t}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{t}_2 & \bar{t}_3 \\ 0 & 0 & 0 & 0 & \bar{t}_3 & \bar{t}_4 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \\ 0 & \bar{t}_6 & 0 & 0 & 0 & 0 \\ \bar{t}_5 & 0 & \bar{t}_5 & 0 & 0 & 0 \end{pmatrix}. \quad (18)$$

$$\bar{t}_1 = t_{D^+K^+ \rightarrow D^+K^+} \quad \bar{t}_4 = t_{D^+K^0 \rightarrow D^+K^0}$$
$$\bar{t}_2 = t_{D^0K^+ \rightarrow D^0K^+} \quad \bar{t}_5 = t_{K^+K^0 \rightarrow K^+K^0}$$
$$\bar{t}_3 = t_{D^0K^+ \rightarrow D^+K^0} \quad \bar{t}_6 = t_{K^+K^+ \rightarrow K^+K^+} \quad (19)$$

- This time the $DK$ two-body amplitudes are explicitly taken into account due to the interaction of the external $K$ with the $D$ inside the cluster. This interaction is very attractive, and the $DKK$ system could bind.

- On the other hand, now we have the $KK$ interaction, which is repulsive.
The partitions $T^{\text{FCA}}_{ij}(s)$ are written into the three-body center-of-mass energy $\sqrt{s}$.

While the scattering amplitudes $t_i(s_i)$ in the $V^{\text{FCA}}$ and $\tilde{V}^{\text{FCA}}$ matrices are written into the two-body center-of-mass energy $\sqrt{s_i}$.

We use two sets of transformations to obtain the $\sqrt{s_i}$'s in terms of $\sqrt{s}$:

We call this set by “Prescription I”, standard transformation from 3-body kinematics used in most works.

\begin{align}
    s_{DK(D\bar{K})} &= m_K^2 + m_D^2 + \frac{1}{2M_{D*}^{s_0}}(s - m_K^2 - M_{D*}^{s_0}^2)(M_{D*}^{s_0} + m_D^2 - m_K^2), \\
    s_{KK(K\bar{K})} &= 2m_K^2 + \frac{1}{2M_{D*}^{s_0}}(s - m_K^2 - M_{D*}^{s_0}^2)(M_{D*}^{s_0} + m_K^2 - m_D^2).
\end{align}

(20)  

(21)
In order to estimate the uncertainties we also use “Prescription II”, assuming that the kinetic energy in the $DK$ cluster is of the order of the binding energy.

$$s_{DK(D\bar{K})} = \left(\frac{\sqrt{s}}{M_{D^*_{s0}} + m_K}\right)^2 \left(m_K + \frac{m_D M_{D^*_{s0}}}{m_D + m_K}\right)^2 - P_2^2,$$

$$s_{KK(K\bar{K})} = \left(\frac{\sqrt{s}}{M_{D^*_{s0}} + m_K}\right)^2 \left(m_K + \frac{m_K M_{D^*_{s0}}}{m_D + m_K}\right)^2 - P_1^2,$$

where $P_1$ and $P_2$ stand for the momenta of the $D$ and $K$ mesons in the cluster. We take $P_1^2 = P_2^2 = 2\tilde{\mu}B_{D^*_{s0}} = 2\tilde{\mu}(m_D + m_K - M_{D^*_{s0}})$, with $\tilde{\mu}$ the reduced mass of $DK$.

This prescription is based on another one [1], which shares the binding energy among the three-particles proportionally to their respective masses.

Energy distribution

Formalism

Figure: Energy distribution in the center-of-mass of each two-body system as a function of the total energy of the three-body system, using prescriptions I and II. Here $s_1 = s_{DK(D\bar{K})}$ and $s_2 = s_{KK(K\bar{K})}$. The lower curves are for $KK$ or $K\bar{K}$ and the upper curves are for $DK$ or $D\bar{K}$. 

$V. R. DEBASTIANI$ (IFIC, UV-CSIC)

Study of the $DKK$ and $DK\bar{K}$ systems

27/09/2017 - Hadron, Salamanca
**DK̅K System: $D_{s0}^*(2317)\,\bar{K}$ or $D\, [a_0(980) / f_0(980)]$?**

**Results**

![Graphs showing the results for the total $DK\bar{K}$ amplitude squared using prescriptions I (left) and II (right).](image)

**Figure:** Results for the total $DK\bar{K}$ amplitude squared using prescriptions I (left) and II (right).

- Narrow bound state around $D\, f_0(980)$ threshold ($\sim 2855$ MeV) in both prescriptions.
$K\bar{K}$ Interaction: The $a_0(980)$ and $f_0(980)$

Results

(a) $K\bar{K}$ amplitude in $I = 1$; couples to $a_0(980)$.

(b) $K\bar{K}$ amplitude in $I = 0$; couples to $f_0(980)$.

Figure: Comparison between $K\bar{K}$ amplitude squared in isospin 1 and 0.

- $a_0(980)$ couples to $K\bar{K}$ in $I = 1$, but the coupling to $\pi\eta$ is the dominant one.
- $f_0(980)$ couples strongly to $K\bar{K}$ in $I = 0$, and with less intensity to $\pi\pi$. (The $\sigma$ meson, $f_0(500)$, is dominated by $\pi\pi$ in $I = 0$).
- In $K\bar{K}$ the $f_0(980)$ amplitude squared is more than 2 orders of magnitude stronger than $a_0(980)$ around their peaks.
**DK\bar{K} System:** mostly $D f_0(980)$.

**Results**

**Figure:** Results for the $DK\bar{K}$ amplitude squared after removing the $f_0(980)$ contribution, using prescriptions I and II.

**Table:** Comparison between position and intensity of the peaks found in the $DK\bar{K}$ amplitude.

<table>
<thead>
<tr>
<th></th>
<th>Prescription I</th>
<th>Prescription II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$</td>
<td>$</td>
<td>T</td>
</tr>
<tr>
<td>Total</td>
<td>2833</td>
<td>$6.8 \times 10^6$</td>
</tr>
<tr>
<td>$l = 1$ only</td>
<td>2842</td>
<td>$7.7 \times 10^4$</td>
</tr>
</tbody>
</table>
**DK\bar{K} System:** mostly $D f_0(980)$.

**Results**

Our Result [1]:

$$M_{DK\bar{K}} = 2833 - 2858 \text{ MeV}.$$  

QCD Sum Rules [2]:

$$M_{Df_0} = (2926 \pm 237) \text{ MeV}.$$  

Full Faddeev equations [2]:

$$M_{Df_0} = 2890 \text{ MeV}.$$  


**Results**

**Figure:** Results for the total $DKK$ amplitude squared using prescriptions I and II.

- Broad, irregular structures. Both amplitudes decrease around 2812 MeV which corresponds to the $D_{s0}^*(2317)K$ threshold.
- $KK$ repulsion vs. $DK$ attraction.
- Enhancement below threshold with “Prescription I”.
- Small strength if compared to the $DK\bar{K}$ system. No clear bound state or resonance.
Summary

- Fixed Center Approximation to Faddeev Equations. Charge exchange diagrams included.
- $DK\bar{K}$ and $DKK$ three-body interactions.
- Cluster $DK$ in the $I = 0$: $D_{s0}^*(2317)$.
- Two-body scattering: $D\bar{K}$ and $K\bar{K}$ in $DK\bar{K}$ system; $DK$ and $KK$ in $DKK$.
- Uncertainties were estimated taking into account two different prescriptions to obtain $\sqrt{s_{DK}}$ and $\sqrt{s_{KK}}$ from the total energy of the system $\sqrt{s}$.

- $DK\bar{K}$ system binds: $I(J^P) = 1/2(0^-)$ state, with mass about $2833 - 2858$ MeV, dominated by a $Df_0(980)$ component (from $K\bar{K}$ in $I = 0$).
- Results compatible with other methods using QCD Sum Rules ($M_{Df_0} = (2926 \pm 237)$ MeV) and full Faddeev equations ($M_{Df_0} = 2890$ MeV.)
- This state could be seen in the $\pi\pi D$ invariant mass distribution (from $f_0(980) \rightarrow \pi\pi$ decay).

- $DKK$ does not seem to bind. $KK$ repulsion seems to be of the same magnitude as the attraction on the $DK$ interaction.
THANK YOU!