# Two-pole (two-state) structure of *D* ∗  $_0^*(2400)$

Based on: Phys. Lett. B 767, 465 (2017) [arXiv:1610.06727]

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UNIVERSIDAD DE **MURCIA** 

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In collaboration with: P. Fernández-Soler, F. K. Guo, J. Nieves



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- **o** [Theoretical interpretations](#page-4-0)
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# [Introduction:](#page-2-0) Why is  $D_0^*(2400)$  interesting?

- Lightest open-charm  $J^P = 0^+$  states:  $\begin{cases} D_{50}^*(2317) & , & (S, I) = (1, 0) \\ D_{10}^*(2400) & , & (S, I) = (0, 1) \end{cases}$  $D_0^*(2400)$ ,  $(S, I) = (0, \frac{1}{2})$
- Lightest systems to test ChPT with heavy mesons, besides *D* <sup>∗</sup> → *Dπ*.
- *Dπ* interactions (where it shows up) are relevant, since *Dπ* appears as a final state  $\bullet$ in many reactions that are being considered now (*i.e.*, *Z<sup>c</sup>* (3900) and *D*̄<sup>∗</sup>*Dπ*)
- o Problems with simple quark models expectations:
	- $D_{s0}^*(2317)$  is around 150 MeV below the predicted mass.
		- Godfrey, Isgur, Phys. Rev. D 32, 189 (1985); Godfrey, Moats, Phys. Rev. D 93, 034035 (2016) Lakhina, Swanson, Phys. Lett. B 650, 159 (2007); Ortega *et al.*, Phys. Rev. D 94, 074037 (2016)
	- Naively, one would expect *D*<sub>š0</sub> (∼ *c*s̄) to be <mark>heavier</mark> than *D*<sub>õ</sub> (∼ *cī*).
- $D_0^*(2400)$  is important in **weak interactions and CKM** parameters:

Flynn, Nieves, Phys. Rev. D 76, 031302 (2007)

MA, P. Fernandez-Soler, F.K. Guo, J. Nieves, D.L. Yao, *in preparation*

See De-Liang Yao's talk [Thursday, 15:35, Aula 2.3]

- It determines the shape of the scalar form factor  $f_0(q^2)$  in semileptonic  $D\to\pi$  decays.
- Relation to  $|V_{cd}|$ :  $f_+(0) = f_0(0)$  and d $\Gamma \propto |V_{cd}f_+(q^2)|^2$ .
- Even more interesting: the bottom analogue |*Vub*|.

<span id="page-4-0"></span>

#### [Introduction:](#page-2-0) [Theoretical interpretations](#page-4-0)

#### *cq*̄states

Dai *et al.* Phys. Rev. D 68, 114011 (2003) Narison, Phys. Lett. B 605, 319 (2005) Bardeen *et al.*, Phys. Rev. D 68, 054024 (2003) Lee *et al.*, Eur. Phys. J. C 49, 737 (2007) Wang, Wan, Phys. Rev. D 73, 094020 (2006)

#### *cq*̄+ tetraquarks or meson–meson

Browder *et al.*, Phys. Lett. B 578, 365 (2004) van Beveren, Rupp, Phys. Rev. Lett. 91, 012003 (2003)

#### Pure tetraquarks

Cheng, Hou, Phys. Lett. B 566, 193 (2003) Terasaki, Phys. Rev. D 68, 011501 (2003) Chen, Li, Phys. Rev. Lett. 93, 232001 (2004) Maiani *et al,*, Phys. Rev. D 71, 014028 (2005) Bracco *et al.*, Phys. Lett. B 624, 217 (2005) Wang, Wan, Nucl. Phys. A 778, 22 (2006)

#### Heavy-light meson–meson molecules

Barnes *et al.*, Phys. Rev. D 68, 054006 (2003) Szczepaniak, Phys. Lett. B 567, 23 (2003) Kolomeitsev, Lutz, Phys. Lett. B 582, 39 (2004) Hofmann, Lutz, Nucl. Phys. A 733, 142 (2004) Guo *et al.*, Phys. Lett. B 641, 278 (2006) Gamermann *et al.*, Phys. Rev. D 76, 074016 (2007) Faessler *et al.*, Phys. Rev. D 76, 014005 (2007) Flynn, Nieves, Phys. Rev. D 75, 074024 (2007) Albaladejo *et al.*, Eur. Phys. J. C 76, 300 (2016)

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## [Introduction:](#page-2-0) [Experimental situation, LQCD simulations](#page-5-0)

#### Experimental situation:



#### Lattice QCD:

Masses larger than the physical ones if using *cs*̄interpolators only.

Bali, Phys. Rev. D 68, 071501 (2003) UKQCD Collab., Phys. Lett. B 569, 41 (2003)

Masses consistent with *D*<sub>ŏ</sub><sup>\*</sup>(2400) and *D*<sub>\$0</sub>(2317) obtained when "meson-meson" interpolators are employed.

> Mohler, Prelovsek, Woloshyn, Phys. Rev. D 87, 034501 (2013) Mohler *et al.*, Phys. Rev. Lett. 111, 222001 (2013)

Recent LQCD study with *Dπ*, *Dη* and *DsK*̄coupled-channel scattering. A bound state with large coupling to *Dπ* is identified with  $D_0^*(2400)$ .

Hadron Spectrum Collab., JHEP 1610, 011 (2016)

*cf.* talks: Ch. Thomas on Monday, R. Briceño on Tuesday

## <span id="page-6-0"></span>**Outline**



## <sup>2</sup> [Formalism:](#page-6-0) *T* -matrix for *Dπ*, *Dη*, *DsK*

- [Scattering in infinite volume](#page-7-0)
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## *Dπ*, *Dη*, *DsK* scattering: infinite volume

Coupled channel *T*-matrix: *Dπ*, *Dη*, *D<sub>3</sub>* $\overline{K}$  scattering [ $J^P = 0^+$ , (*S*, *I*) = (0,  $\frac{1}{2}$ )].

• **Unitarity:** 
$$
T^{-1}(s) = V^{-1}(s) - G(s)
$$

Normalization:  $-i\rho_{ii}(s)T_{ii}(s) = 4\pi\sqrt{s}(\eta_i(s)e^{2i\delta_i(s)} - 1).$ 

- $\mathcal{G}_{ii}(\mathsf{s}) = G(\mathsf{s},m_i,M_i)$ , regularized with a subtraction constant  $a(\mu)$  $(\mu = 1 \text{ GeV}).$
- Analytical continuations: Riemann sheets (RS) denoted as (*ξ*1*ξ*2*ξ*<sup>3</sup> ):





# *Dπ*, *Dη*, *DsK* scattering: infinite volume (II)

**Chiral symmetry** used to compute the  $O(p^2)$  potential:

$$
f^{2}V_{ij}(s,t,u)=C_{\text{LO}}^{ij}\frac{s-u}{4}+\sum_{a=0}^{5}h_{a}C_{a}^{ij}(s,t,u)
$$

Guo *et al.*, Phys. Lett. B 666, 251 (2008) Liu *et al.*, Phys. Rev. D 87, 014508 (2013)

- $\circ$  Values of  $h_a$  and  $a(\mu)$  have been **previously fixed**:
	- Fitted to reproduce scattering lengths obtained in a LQCD simulation
	- o They are not fitted in this work. We make "predictions"



<span id="page-9-0"></span>

# *Dπ*, *Dη*, *DsK* energy levels in a finite volume

- Periodic boundary conditions imposes momentum quantization
- **Q** Lüscher formalism:

Commun. Math. Phys. 105, 153 (1986) Nucl. Phys. B 354, 531 (1991)



 $\circ$  In practice, changes in the *T*-matrix:  $T(s) \rightarrow \widetilde{T}(s,L)$ :

Döring *et al.*, Eur. Phys. J. A 47, 139 (2011)

$$
G_{ii}(s) \rightarrow \tilde{G}_{ii}(s,L) = G_{ii}(s) + \lim_{\Lambda \to \infty} \left( \frac{1}{L^3} \sum_{\tilde{h}}^{|\tilde{q}| < \Lambda} l_i(\tilde{q}) - \int_0^{\Lambda} \frac{q^2 dq}{2\pi^2} l_i(\tilde{q}) \right),
$$
  

$$
V(s) \rightarrow \tilde{V}(s,L) = V(s),
$$
  

$$
T^{-1}(s) \rightarrow \tilde{T}^{-1}(s,L) = V^{-1}(s) - \tilde{G}(s,L),
$$

**Free** energy levels:  $E_{n,\text{free}}^{(i)}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$ 

Interacting energy levels  $E_n(L)$ :  $\widetilde{T}^{-1}(E_n^2(L), L) = 0$  (poles of the  $\widetilde{T}$ -matrix).

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## <sup>3</sup> [Results](#page-10-0)

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<span id="page-11-0"></span>



- LQCD: [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- Bands: [M. A. *et al.*, Phys. Lett. B 767, 465 (2017)]
- *En* (*L*) are provided for *Dπ*, *Dη*, *DsK*̄in a recent LQCD simulation.
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- We compute *E<sup>n</sup>* (*L*) and compare. No fit is performed.
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- Level below threshold, associated with a **bound state** (see below).
- 





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- We compute *E<sup>n</sup>* (*L*) and compare. No fit is performed.
- *E* > 2.7 GeV is probably beyond the range of validity for our *T* -matrix.
- Level below threshold, associated with a **bound state** (see below).
- Second level has large shifts w. r. t. thresholds: Resonance? (see below).

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- For lattice masses, we find a bound state (000) and a resonance (110)  $\bullet$
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- For lattice masses, we find a **bound state** (000) and a **resonance** (110)  $\circ$
- For physical masses:  $\circ$ 
	- The bound state evolves into a resonance (100) above *Dπ* threshold.
	- The resonance varies very little, and is still a resonance (110).
	- For both states, the coupling pattern is similar.
- 





- For lattice masses, we find a bound state (000) and a resonance (110)  $\circ$
- For physical masses:
	- The bound state evolves into a resonance (100) above *Dπ* threshold.
	- The resonance varies very little, and is still a resonance (110).
	- For both states, the coupling pattern is similar.
- PDG includes only one resonance, "suspiciously" lying between both.





- The *D*<sub>ឺ</sub> (2400) structure is actually produced by <mark>two different states</mark> (poles), together with complicated interferences with thresholds.
- This two-state structure was previously reported, and receives now a robust  $\bullet$ support.

Kolomeitsev, Lutz, Phys. Lett. B 582, 39 (2004)

Guo *et al.*, Phys. Lett. B 641, 278 (2006)

Guo *et al.*, Eur. Phys. J. A 40, 171 (2009)

<span id="page-20-0"></span>

### [Results:](#page-10-0) [Amplitudes](#page-20-0)



$$
\circ -i\rho_i(s)T_{ii}(s) = 4\pi\sqrt{s}\left(\eta_i(s)e^{2i\delta_i(s)}-1\right)
$$

\n- Lower pole at 
$$
\sqrt{s} = 2.1 - 0.1
$$
 i GeV:
\n- $|T_{11}(s)|^2$  peaks at  $\sqrt{s} \simeq 2.1$  GeV
\n- $\delta_{11}(s) = \pi/2$  at  $\sqrt{s} \simeq 2.2$  GeV.
\n

• Higher pole at 
$$
\sqrt{s}
$$
 = 2.45 – 0.13 *i* GeV:

- Small enhancement in  $|T_{D_{\Pi}}(s)|^2$ .
- $\circ$  Clear peak in the  $D_{\rm s}\bar{K}$  amplitude. Narrow, non-conventional shape, stretched between thresholds cusps.
- Possible tests in *B* → *DPP*′ decays?

# <span id="page-21-0"></span>**Outline**



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#### <sup>4</sup> *SU*(3[\) light–flavor limit and other predictions \(charm & bottom\)](#page-21-0)

- *SU*(3) [light–flavor limit](#page-22-0)
- [Predictions for other sectors](#page-25-0)

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## *SU*(3[\) light–flavor limit](#page-22-0)

 $\circ$  *SU*(3) flavor limit:  $m_i$  →  $m = 0.49$  GeV,  $M_i$  →  $M = 1.95$  GeV.

**O** Irrep decomposition:  $\overline{3} \otimes 8 = \overline{15 \oplus 6 \oplus 3}$ . *T* and *V* can be diagonalized:

 $V_d(s) = D^{\dagger}V(s)D = \text{diag}(V_{\overline{15}}(s), V_6(s), V_{\overline{3}}(s)) = (A(s) \text{ diag}(1, -1, -3))$  $T_d(s) = \text{diag}(T_{\overline{15}}(s), T_6(s), T_{\overline{3}}(s))$ .

 $T_A^{-1}(s) = V_A^{-1}(s) - G(s, m, M)$  (*e.g.*, each  $T_A(s)$  is "single channel").

 $\overline{\phantom{a}}$  15 is repulsive. 6 and  $\overline{\phantom{a}}$  are attractive. "Curiously",  $\overline{\phantom{a}}$  admits a  $c\bar{q}$  interpretation.





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 $T_A^{-1}(s) = V_A^{-1}(s) - G(s, m, M)$  (*e.g.*, each  $T_A(s)$  is "single channel").

 $\overline{\phantom{a}}$  15 is repulsive. 6 and  $\overline{\phantom{a}}$  are attractive. "Curiously",  $\overline{\phantom{a}}$  admits a  $c\bar{q}$  interpretation.





#### *SU*(3[\) light–flavor limit](#page-22-0) (II) Connecting **physical** ( $x = 0$ ) and **flavor** *SU***(3)** ( $x = 1$ ) limits:

 $m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}})$ , (*m* = 0.49 GeV),  $M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}})$ ,  $(M = 1.95 \text{ GeV})$ .



- The <mark>high *D*↑</mark> connects with a **6 virtual** state (unph. RS, below threshold).
- The low  $\boldsymbol{D}_0^*$  connects with a  $\overline{\mathbf{3}}$  bound state (ph. RS, below threshold).
- The  $\bm{\mathit{D}}_{\mathit{S}0}^{*}$ (**2317)** also connects with the  $\overline{\bm{3}}$ bound state.



- The low  $D_0^*$  and the  $D_{s0}^*(2317)$  are  $SU(3)$  flavor partners.
- This <mark>solves the "puzzle"</mark> of *D*<sub>\$0</sub>(2317) being lighter than *D*<sub>ô</sub>(2400): it is not, the lower  $D_0^*$  pole ( $M = 2105$  MeV) is lighter.

M. Albaladejo (U. Murcia): Two-pole (two-state) structure of  $D_0^*$ 

<span id="page-25-0"></span>

#### [Predictions for other sectors:](#page-25-0) charm



HQSS relates 0 <sup>+</sup> (*D*(*s*)*P*) and 1 <sup>+</sup> (*D* ∗ (*s*)*P*) sectors: similar resonance pattern.

- $\bullet$ Two pole structure: higher *D*<sup>1</sup> pole probably affected by *ρ* channels.
- $D\bar{K}$  [0<sup>+</sup>, ( $-1$ , 0)]: this virtual state (from **6**) has a large impact on the scattering length,  $\mathit{d}_{(-1,0)}^{D\bar{K}}\simeq0.8$  fm. (Rest of scattering lengths are |*a*|  $\simeq0.1$  fm.)



#### [Predictions for other sectors:](#page-25-0) bottom



- Heavy flavour symmetry relates charm (*D*) and bottom (*B*̄) sectors.
- $(0, \frac{1}{2})$ :  $B_0^*$ , two-pole pattern also observed.
- (−1, 0): [*B* (∗)*K*̄]: very close to threshold. Relevant prediction. Can be either **bound or virtual (6)** within our errors.
- (1, 1): [*B*̄ *<sup>s</sup>π*, *BK*̄ , 0 <sup>+</sup>], *X* (5568) channel. No state is found: 15 and 6. If it exists, it is not dynamically generated in *Bsπ*, *BK*̄interactions.

M. A. *et al.*, Phys. Lett. B 757, 515 (2016); Guo *et al.*, Commun. Theor. Phys. 65, 593 (2016)

(1,0): Our results for  $B_{s0}^*$  and  $B_{s1}$  agree with **other results** from LQCD: Lang *et al.*, Phys. Lett. B 750, 17 (2015); M. A. *et al.* Eur. Phys. J. C77, 170 (2017)

# <span id="page-27-0"></span>**Outline**



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- [Chiral dynamics and two-state structure\(s\)](#page-28-0)
- o [Conclusions](#page-29-0)

## <span id="page-28-0"></span>[Chiral dynamics and two-state structure\(s\)](#page-28-0)

Other famous two-poles structures rooted in chiral dynamics:

*Λ*(1405) [*Σπ*, *NK*̄]

Oller, Meißner, Phys. Lett. B 500, 263 (2001) Jido *et al.*, Nucl. Phys. A 725, 181 (2003) García-Recio *et al.*, Phys. Lett. B 582, 49 (2004) Magas *et al.*, Phys. Rev. Lett. 95, 052301 (2005)

*K*1 (1270)

Roca *et al.*, Phys. Rev. D 72, 014002 (2005) Geng *et al.*, Phys. Rev. D 75, 014017 (2007) García-Recio *et al.*, Phys. Rev. D 83, 016007 (2011)

#### Chiral dynamics:

- Incorporates the *SU*(3) light-flavor structure,
- Determines the strength of the interaction,
- Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.

<span id="page-29-0"></span>

## **[Conclusions](#page-27-0)**

- We have studied *Dπ*, *Dη*, *D<sub>s</sub>* $\bar{K}$  scattering [0<sup>+</sup>, (*S*, *l*) = (0,  $\frac{1}{2}$ )]
- So far only one pole reported experimentally, but...
- We have presented a strong support for the <mark>existence of two *D*\*(2400) states</mark> (different poles), based on a succesful, no-fitting comparison of our *T* -matrix with the energy levels of a recent LQCD simulation.
- A *SU*(3) study shows that *D* ∗ *s*0 (2317) and the lower *D* ∗ 0 (2400) are flavour partners: they complete a  $\overline{3}$  multiplet.
- The lower pole ( $M = 2105^{+6}_{-8}$  MeV) is lighter than  $D^*_{50}(2317)$ , solving this apparent contradiction.
- **Predictions for other sectors** (heavy vectors, bottom sector) have been also given. In particular:
	- o The two-pole structure is also seen in the **bottom sector.**
	- A very near-threshold state (bound or virtual) is predicted for *BK* (*B*̄*K*̄).

# Two-pole (two-state) structure of *D* ∗  $_0^*(2400)$

Based on: Phys. Lett. B 767, 465 (2017) [arXiv:1610.06727]



UNIVERSIDAD DE **MURCIA** 

# Miguel Albaladejo (U. Murcia)

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## Connecting *SU*(3) and physical limits Riemann sheets

Riemann sheets:

 $\mathcal{G}_{ii}(s) \rightarrow \mathcal{G}_{ii}(s) + i$ 

#### *SU*(3) limit:

$$
\frac{\rho_i(s)}{4\pi\sqrt{s}}\xi_i \qquad m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),
$$
  

$$
M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).
$$

- Physical case ( $x = 0$ ): RS specified by ( $\xi_1 \xi_2 \xi_3$ ),  $\xi_i = 0$  or 1.
- *SU*(3) symmetric case (*x* = 1): all channels have the same threshold, so there are only two RS (000) and (111).
- To connect the l**ower pole** with the  $\mathcal{T}_\mathbf{6}$  virtual state,

$$
\xi_3 = x \qquad (1, 1, 0) \to (1, 1, x)
$$

To connect the **lower pole** with the  $T_{\bf \overline{3}}$  bound state,

$$
\xi_1 = 1 - x \qquad (1,0,0) \to (1 - x, 0, 0)
$$

### Lattice QCD parameterization



**Table 11.** The S-wave t-matrix parametrisations used in section 3.3.1 where " $\checkmark$ " denotes a free parameter and "-" a parameter fixed to zero. The channel labels are ordered by increasing mass,  $1 = D\pi$ ,  $2 = D\eta$  and  $3 = D_s\overline{K}$ . The forms shown also included a free P-wave part contributing an additional 3 parameters. Forms with  $\chi^2/N_{\text{dof}} > 1.9$  (shown in italics) were not used in our final analysis as described in the text of section 3.3.1.

THEP10(2016)01  $\overline{\phantom{0}}$