Excited mesons and resonances from lattice QCD

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Hadron Spectrum Collaboration
Excited lattice QCD spectroscopy

Finite-volume energy eigenstates from:

\[ C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}^\dagger_j(0) | 0 \rangle \]

Use many different interpolating operators

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Excited charmonia

$m_\pi = 236$ MeV, one lattice spacing and volume [Cheung et al (HadSpec), JHEP 12 (2016) 089] (similar pattern to older $m_\pi = 391$ MeV, 1 lattice spacing and 3 volumes)
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\[ \eta_c \sim \bar{q} [D_i, D_j] q \sim \bar{q} F_{ij} q \]

\( q\bar{q} \) in L=0, with gluonic 1^{+-}, scale \( \sim 1.2 - 1.3 \text{ GeV} \)

Large overlap with operator

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Excited charmonia

$\eta_c$ \hspace{2cm} $J/\psi$

$\chi_{c2}$

$q\bar{q}$ in $L=1$, with gluonic $1^{+-}$

$q\bar{q}$ in $L=0$, with gluonic $1^{+-}$, scale $\sim 1.2 - 1.3$ GeV

Large overlap with operator

$\sim \bar{q} [D_i, D_j] q \sim \bar{q} F_{ij} q$

Exotic

$m_\pi = 236$ MeV, one lattice spacing and volume [Cheung et al (HadSpec), JHEP 12 (2016) 089]

(similar pattern to older $m_\pi = 391$ MeV, 1 lattice spacing and 3 volumes)
Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons.
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Singularity structure of scattering matrix

Resonance poles on unphysical sheet

\[ M_R \pm i \Gamma/2 \]

\[ \text{Re } E_{\text{cm}} \]

\[ \text{Im } E_{\text{cm}} \]

\[ 2m \]

\[ M_R \]

\[ \Gamma \]

\[ \delta \]

\[ 0^\circ \to 180^\circ \]
Scattering in lattice QCD

**Infinite volume** – contin. spectrum above thresh.

![Diagram of scattering in lattice QCD](image)
Scattering in lattice QCD

**Infinite volume** – contin. spectrum above thresh.

**Finite volume** – discrete spectrum

Non-interacting: \( \vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z) \)

Interacting: \( \vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z) \)

[periodic b.c.s]
Scattering in lattice QCD

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Lüscher method (and extensions): relate **finite-volume energy levels** to infinite-volume scattering **t-matrix**.

Elastic scattering: 1-to-1 correspondence (ignoring partial-wave mixing).

But in general **under-constrained problem** (determinant equ. at each \(E_{cm}\))

→ parameterize \(E_{cm}\) dependence of t-matrix and fit \(\{E_{lat}\}\) to \(\{E_{param}\}\)

Consider many different parameterizations (e.g. K-matrix, eff. range, B.W.)
Infinite volume – contin. spectrum above thresh.

Finite volume – discrete spectrum

Non-interacting: \( \vec{k}_{A,B} = \frac{2\pi}{L} (n_x, n_y, n_z) \)

Interacting: \( \vec{k}_{A,B} \neq \frac{2\pi}{L} (n_x, n_y, n_z) \)

\[ t(E_{\text{cm}}) = \begin{pmatrix} t_{\pi\pi \to \pi\pi}(E_{\text{cm}}) & t_{\pi\pi \to K\bar{K}}(E_{\text{cm}}) \\ t_{K\bar{K} \to \pi\pi}(E_{\text{cm}}) & t_{K\bar{K} \to K\bar{K}}(E_{\text{cm}}) \end{pmatrix} \]

Lüscher method (and extensions): relate finite-volume energy levels to infinite-volume scattering \( t \)-matrix.

Currently limited to hadron-hadron scattering – progress being made on formalism for channels with > 2 hadrons.

But \( \{E_{\text{param}}\} \rightarrow \{E_{\text{lat}}\} \) parameterize \( E_{\text{cm}} \) dependence of \( t \)-matrix and fit.

Consider many different parameterizations (e.g. \( K \)-matrix, eff. range, B.W.)

See plenary by Raul Briceño at 10:10am on Tuesday and recent review [Briceño, Dudek, Young. arXiv:1706.06223]
Charm-light ($D$) and charm-strange ($D_s$) mesons

Some other LQCD studies:

- Mohler et al [PR D87, 034501 (2012)] – $0^+ D\pi$ and $1^+ D^*\pi$ resonances
- Mohler et al [PRL 111, 222001 (2013)] – $0^+ D_{s0}(2317)$ below $D K$ threshold
- Lang et al [PRD 90, 034510 (2014)] – $0^+ D_{s0}(2317)$ and $1^+ D_{s1}(2460)$, $D_{s1}(2536)$
- Bali et al (RQCD) [arXiv:1706.01247] – $0^+ D_{s0}(2317)$ and $1^+ D_{s1}(2460)$
Charm-light (D) and charm-strange (D_s) mesons

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**Dπ, Dη, D_s K (I=½)**

Isospin = ½  
Strangeness = 0  
Charm = 1

\[ P = [0,0,0] \]
\[ J^P = 0^+ [\ell = 0] \]

Reduced sym. \( \rightarrow \) other partial waves can mix in

\[ m_\pi = 391 \text{ MeV} \]
\[ m_D = 1890 \text{ MeV} \]

One lattice spacing  
3 volumes (2 – 3 fm)  
\( (L / \alpha_s = 16, 20, 24) \)

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]
\( \bar{D}_s K (I=\frac{1}{2}) \)

Isospin = \( \frac{1}{2} \)
Strangeness = 0
Charm = 1

\[ P = [0,0,0] \]
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One lattice spacing
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Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]
Dπ, Dη, D_s K (I=½): spectra

Use 47 energy levels for ℓ = 0, 1 and 18 for ℓ = 2

[JHEP 1610, 011 (2016)]
Elastic $D\pi$ ($l=\frac{1}{2}$): $\ell = 0, 1$

$$k \cot \delta_0$$

$$k^3 \cot \delta_1$$
$D\pi, D\eta, D_s\bar{K} (I=\frac{1}{2}): \ell = 0$

$m_\pi = 391$ MeV

$S$-wave

$J^P = 0^+$

47 energy levels (3 volumes)
$D\pi$, $D\eta$, $D_s \bar{K}$ ($I=\frac{1}{2}$): $\ell = 2$

$\ell = 2$

$D$-wave

$J^P = 2^+$

$\delta_{D\pi}$

$\delta_{D_s \bar{K}}$

$\delta_{D\eta}$

$E_{cm}/\text{GeV}$

$m_\pi = 391 \text{ MeV}$

[JHEP 1610, 011 (2016)]
$D\pi, D\eta, D_s\bar{K}$ (I=$\frac{1}{2}$): poles of $t$-matrix

[JHEP 1610, 011 (2016)]
Dπ, Dη, DsK (I=½): poles of t-matrix

1− deeply bound state
m = (2009 ± 2) MeV

c.f. D*(2007)
$D\pi, \ D\eta, \ D_s\bar{K} \ (I=\frac{1}{2})$: poles of $t$-matrix

$m_\pi = 391 \ MeV$

$D\pi$ thresh.

$0^+$ bound state just below thresh.

$m = (2275.9 \pm 0.9) \ MeV$

c.f. $D\pi$ thr. = $(2276.4 \pm 0.9) \ MeV$

c.f. $D_0^*(2400)$
$D\pi, \ D\eta, \ D_s \bar{K} \ (I=\frac{1}{2})$: poles of $t$-matrix

$m_\pi = 391$ MeV

0$^+$ bound state just below thresh.
$m = (2275.9 \pm 0.9)$ MeV

c.f. $D\pi$ thr. = $(2276.4 \pm 0.9)$ MeV

c.f. $D_0^*(2400)$

2$^+$ narrow resonance
$m = (2527 \pm 3)$ MeV

$\Gamma = (8.2 \pm 0.7)$ MeV

c.f. $D_2^*(2460)$ (also couples to $D^*\pi$)

[JHEP 1610, 011 (2016)]
Summary

• **Significant progress** in LQCD calculations of resonances, near-threshold states, etc – **map out scattering amps**.
• Example of some recent work in the open-charm sector
• Other talks with Hadron Spectrum Collaboration results:
  • Raul Briceño [10:10 TUES, plenary] – light hadrons, ...
  • Gavin Cheung [11:40 WEDS, exotics session]
    – charm tetraquarks (exotic-flavour channels)
• Ongoing work on other channels
• Use $m_\pi$ dependence as tool to probe structure
• Extensions of formalism to e.g. 3-hadron scattering
Hadron Spectrum Collaboration

Jefferson Lab, USA:
  Bipasha Chakraborty, Jozef Dudek\(^1\), Robert Edwards, David Richards, Raul Briceño\(^2\)
  \(^{(1} \text{ and W&M, } ^{(2} \text{ and ODU)}\)

Trinity College Dublin, Ireland:
  Mike Peardon, Sinéad Ryan, David Wilson, Cian O’Hara, David Tims

University of Cambridge, UK:
  CT, Graham Moir, Gavin Cheung, Antoni Woss

Tata Institute, India:
  Nilmani Mathur