Tensor *and scalar* meson contributions to the $\tau \to \pi\pi\pi\nu_\tau$ AFF

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O. Shekhovtsova and SC [arXiv:1707.01137 [hep-ph]]

Motivation

- Impact of J=2 & J=0 resonances in the $\pi\pi\pi$ AFF

- Previously existing analyses for T [x] and S [+]

- Our work incorporates:

  - Chiral Symmetry ➔ **Isospin symmetry** & PCAC
    ➔ Right **low-energy** behaviour (χPT)
    ➔ **Transversality** of the current in the χ limit **(at any q^2)**

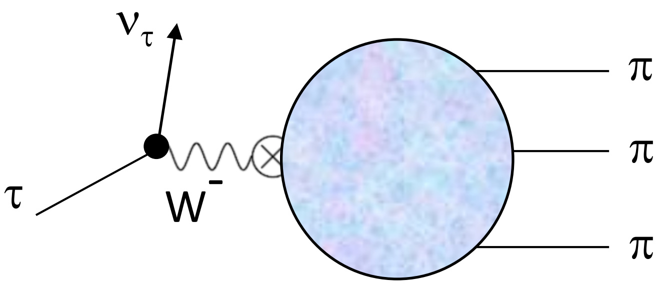
  - Brodsky-Lepage **high-energy** behaviour

- **Ultimate goal:** improved implementation in Tauola MC for future analyses of Belle's data

(x) CLEO, PR61 (2000) 012002
(x) Castro,Muñoz, PRD 83 (2011) 094016

(+) Nugent,Przedzinski,Roig,Shekhovtsova, PRD88 (2013) 093012

$\pi^0\pi^0\pi^-$ and $\pi^-\pi^-\pi^+$ AFFs



$$
\begin{aligned}
\langle\, 3\pi|\bar{d}\gamma^\mu\gamma_5 u|0\,\rangle \;&=\; H^{3\pi}(q^2,s_1,s_2)^\mu \\
&=\; i\,P_T^{\mu\nu}(q)\Big[\mathcal{F}_1(s_1,s_2,q^2)\,(p_1-p_3)_\nu \;+\;\mathcal{F}_2(s_1,s_2,q^2)\,(p_2-p_3)_\mu \\
&\qquad \cancel{+\;\mathcal{F}_3(s_1,s_2,q^2)\;(p_1-p_2)_\mu}\Big] \;+\; \underbrace{i\,q_\mu\;\mathcal{F}_P(s_1,s_2,q^2)}_{\propto\; m_\pi^2/q^2}\,,
\end{aligned}
$$

NOT independent from F_1, F_2

with $q=p_1+p_2+p_3,\; s_1=(p_2+p_3)^2,\; s_2=(p_3+p_1)^2$ and $s_3=(p_1+p_2)^2$ $\qquad\mathbf{P_T(q)^{\mu\nu}=g^{\mu\nu}-q^\mu q^\nu/q^2}$

- Bose symmetry:

$$
\begin{aligned}
\mathcal{F}_1(s_1,s_2,q^2) \;&=\; \mathcal{F}_2(s_2,s_1,q^2)\,,\\
\mathcal{F}_P(s_1,s_2,q^2) \;&=\; \mathcal{F}_P(s_2,s_1,q^2)\,,
\end{aligned}
$$

• Isospin symmetry *, **: $H_\mu^{--+}(p_1,p_2,p_3) = H_\mu^{00-}(p_3,p_2,p_1) + H_\mu^{00-}(p_3,p_1,p_2)$

$$\boxed{\begin{aligned} \mathcal{F}_1^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2) &= \mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_3,q^2) - \mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_2,s_3,q^2) - \mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_3,s_2,q^2)\,, \\ \mathcal{F}_P^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2) &= \mathcal{F}_P^{\pi^0\pi^0\pi^-}(s_1,s_3,q^2) + \mathcal{F}_P^{\pi^0\pi^0\pi^-}(s_2,s_3,q^2)\,. \end{aligned}}$$

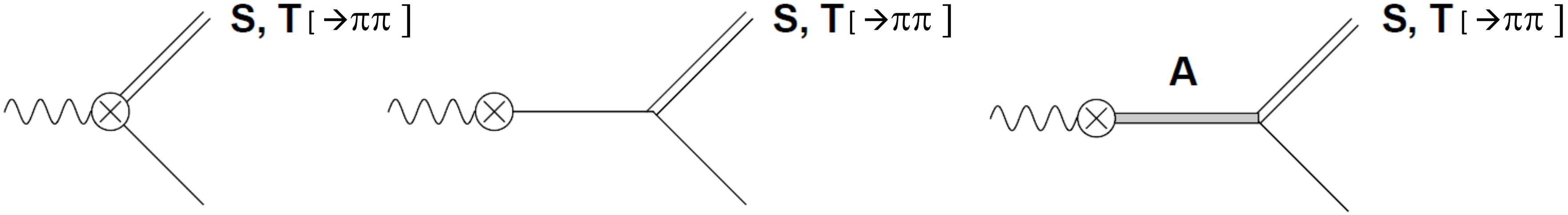
• All the information of both neutral and charged channels contained in

$\boxed{\mathcal{F}_1(s_1,s_2)^{\pi 0\pi 0\pi -}}$ & $\mathcal{F}_P(s_1,s_2)^{\pi 0\pi 0\pi -}$ *(chirally suppressed by m_π^2)*

* Girlanda,Stern, NPB 575 (2000) 285

** Colangelo,Finkemeier,Urech, PRD 54 (1996) 4403
** Pais, Annals Phys. 9 (1960) 548
** CLEO, eConf C0209101 (2002) TU05, [arXive:hep-ex/0210039]
** Schmidtler, NPB Proc.Suppl. 76 (1999) 271
** Hinson (CLEO), PhD thesis, Purdue Universty, 2001.

Decay via resonances: $R\pi$–AFF + $R\to\pi\pi$



- **Previous Works incorporated non-R + V** (*) in Tauola MC (x)
- In this work, **improve previous BW scalar S** (x) + new incorporation T
- G-parity requires that S & T resonances have $G = C\,(-1)^{I} = +1$ → **S & T isosinglets**

(*) Dumm,Roig,Pich,Portolés, PLB 685 (2010) 158
(x) Nugent,Przedzinski,Roig,Shekhovtsova, PRD88 (2013) 093012

RχT prediction

- Lagrangian w/ Resonances + chiral (pseudo) Goldstones
- χ symmetry invariance ← Low-energy behaviour
- General R-operators with lowest order of derivatives [O(d^2)] ← High-energy behaviour

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}_{\mathrm{non-R}} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{R\,R'} + \ldots$$

Scalar contribution

- Sπ-AFF:

$$\langle\, S_{I=0}(k)\pi^-(p)|\bar{d}\gamma^\alpha\gamma_5 u|0\,\rangle \;=\; -2iP_T(q)^{\alpha\nu}\, p_\nu\, \mathcal{F}^a_{S\pi}(q^2;k^2) \;+\; i\, q^\alpha\, \mathcal{H}^a_{S\pi}(q^2;k^2)$$

$$q = k+p$$

- Relevant RχT Lagrangian*:

$$\mathcal{L}^{(2)}_{\mathrm{non-R}} = \frac{F^2}{4}\langle\, u_\mu u^\mu + \chi_+\,\rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}}\langle\, A_{\mu\nu} f_-^{\mu\nu}\,\rangle\,,$$

$$\mathcal{L}_S = c_d\langle\, S u_\mu u^\mu\,\rangle + c_m\langle\, S\chi_+\,\rangle\,,$$

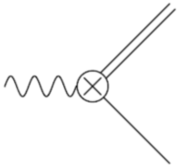
$$\mathcal{L}_{AS} = \lambda_1^{AS}\langle\,\{\nabla_\mu S, A^{\mu\nu}\}u_\nu\,\rangle$$

* Ecker et al., NPB 321 (1989) 311
* Ecker et al., PLB 223 (1989) 425.
* Pich,Rosell,SC, JHEP 0807 (2008) 014
* Shekhovtsova,SC, [1707.01137 [hep-ph]]

- Prediction for the Sπ-AFF *,(x):

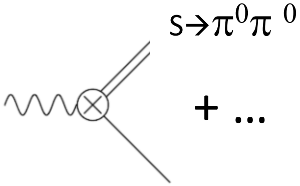
$$\mathcal{F}^a_{S\pi}(q^2;k^2) = \frac{2c_d}{F_\pi} \quad + \quad \frac{\sqrt{2}F_A\lambda_1^{AS}}{F_\pi}\frac{q^2}{M_A^2-q^2}\,,$$

$$\mathcal{H}^a_{S\pi}(q^2;k^2) = \frac{4}{F_\pi}\,\frac{m_\pi^2}{q^2(q^2-m_\pi^2)}\,\left[c_d(qp)+c_m q^2\right]\,,$$



+ ...

- Contribution to the 3π-AFF via S (x):



+ ...

$$\mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_S = \frac{2}{3}\mathcal{F}^a_{S\pi}(q^2;s_3)\,\mathcal{G}_{S\pi\pi}(s_3)\,,$$

$$\mathcal{F}_P^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_S = \mathcal{H}^a_{S\pi}(q^2;s_3)\,\mathcal{G}_{S\pi\pi}(s_3)\,,$$

$$\mathcal{G}_{S\pi\pi}(s_3) = \frac{\sqrt{2}}{F_\pi^2}\,\frac{1}{M_S^2-s_3}\,\left[c_d(s_3-2m_\pi^2)+2c_m m_\pi^2\right]$$

- High-energy constraints $\mathcal{F}^a_{S\pi}(q^2) \longrightarrow 0$ for $q^2\to\infty$ *, (x):

$$F_A\lambda_1^{AS} = \sqrt{2}c_d$$

$$\mathcal{F}^a_{S\pi}(q^2;s_3) = \frac{2c_d}{F_\pi}\,\frac{M_A^2}{M_A^2-q^2}$$

* Pich,Rosell,SC, JHEP 0807 (2008) 014
(x) Shekhovtsova,SC, [1707.01137 [hep-ph]]

• Further refinements (*):

- $\sigma - f_0(980)$ mixing angle ϕ_S =- 8° (xx)

- σ width à la Gounaris-Sakurai / Chew-Mandelstam (x) *[successful for $\eta' \to \eta\pi\pi$ (**)]*:

Previously Breit-Wigner in Tauola (*) ➔ *width + real log (large)*

$$\boxed{\frac{1}{M_\sigma^2 - s} \longrightarrow \frac{1}{M_\sigma^2 - s - f_\sigma(s) - iM_\sigma\Gamma_\sigma(s)}}$$

$$f(s) = c_\sigma s^k \,\mathrm{Re}\overline{B}_0(s, m_\pi^2, m_\pi^2) = \frac{c_\sigma\, s^k}{16\pi^2}\left[2 - \rho_\pi(s)\ln\frac{\rho_\pi(s)+1}{1-\rho_\pi(s)}\right],$$

$$M_\sigma\Gamma_\sigma(s) = c_\sigma s^k \,\mathrm{Im}\overline{B}_0(s, m_\pi^2, m_\pi^2) = \frac{c_\sigma\, \rho_P(s)\, s^k}{16\pi},$$

$$\overline{B}_0(s, m_P^2, m_P^2) = \frac{1}{16\pi^2}\left[2 - \rho_P(s)\ln\frac{\rho_P(s)+1}{\rho_P(s)-1}\right]$$

$$\rho_P(s) \equiv \lambda(s, m_P^2, m_P^2)^{\frac{1}{2}}/q^2 = \sqrt{1 - 4m_P^2/s}$$

(++) Caprini,Colangelo,Leutwyler, PRL96 (2006) 132001

$$\sqrt{s^\sigma_{\rm pole}} = [(441^{+16}_{-8}) - i(544^{+18}_{-25})/2]\ \mathrm{MeV}$$

$$k = 1$$

$$M_\sigma = 806.4\ \mathrm{MeV\ and}\ c_\sigma = 76.12$$

NOT POLE MASS, ONLY PARAMETERS!!!

- $f_0(980)$ width in Flatté form (+)

$$\frac{1}{M_{f_0}^2 - s} \longrightarrow \frac{1}{M_{f_0}^2 - s - iM_{f_0}\Gamma_{f_0}(s)}$$

$$M_{f_0}\Gamma_{f_0}(s) = \frac{c_{f_0} M_{f_0}^2 \rho_K(s)}{16\pi}$$

(*) Shekhovtsova,SC, [1707.01137 [hep-ph]]

(xx) Escribano, PRD 74 (2006) 114020

(**) Escribano,Masjuan,SC, JHEP 1105 (2011) 094

(x) Gounaris and J.J. Sakurai, PRL21 (1968) 244

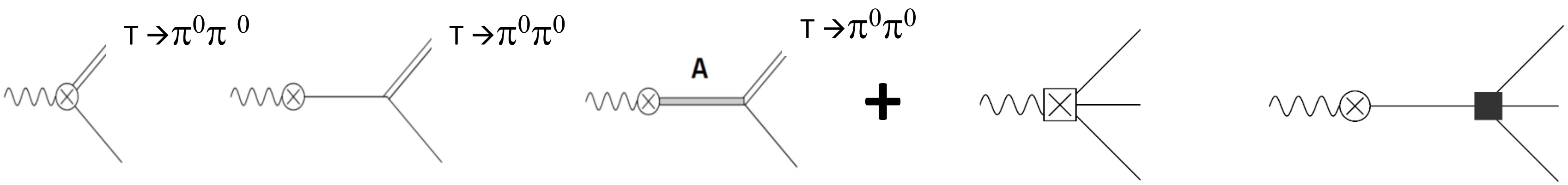
(x) Chew,Mandelstam, PR119 (1960) 467

(+) S. M. Flatte, PLB 63 (1976) 224.

Tensor contribution

- Tπ-AFF: $\langle\, f_2(k,\epsilon)\, \pi^-(p_3)|\, \bar d \gamma^\alpha \gamma_5 u\, |0\,\rangle \,=\, \epsilon^*_{\mu\nu} H^{\alpha,\,\mu\nu}_{T\pi}$

$$= i\,\epsilon^*_{\mu\nu}\,\left[P_T(q)^{\alpha\rho}\, p_3^\nu\, \left(g^\mu_\rho\, \mathcal{F}^a_{T\pi}(q^2;k^2) \,+\, p_{3\,\rho} p_3^\mu\, \mathcal{G}^a_{T\pi}(q^2;k^2)\right) \,+\, p_3^\mu p_3^\nu q^\alpha\, \mathcal{H}^a_{T\pi}(q^2;k^2)\right]$$



non-R interactions (SD terms)

- The relevant RχT Lagrangian with T [most general O(d²)] (+), (x): previous ones + …

$$\mathcal{L}^{(4)}_{\mathrm{non-R}} = L_1^{SD}\langle\, u^\mu u_\mu\,\rangle^2 \,+\, L_2^{SD}\langle\, u^\mu u^\nu\,\rangle\langle\, u_\mu u_\nu\,\rangle \,+\, L_3^{SD}\langle\,(u^\mu u_\mu)^2\,\rangle$$

$$L_2^{SD} = 2L_1^{SD} = -\frac{L_3^{SD}}{2} = -\frac{g_T^2}{M_T^2}$$

for a correct forward ππ scattering High-E behaviour

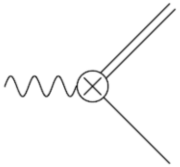
$$\mathcal{L}_T \,=\, g_T\langle\, T_{\mu\nu}\{u^\mu, u^\nu\}\,\rangle$$

$$\mathcal{L}_{AT\pi} \,=\, \lambda_1^{AT}\,\langle\,\{T_{\mu\nu}, A^{\nu\alpha}\}h^\mu_\alpha\,\rangle + \lambda_2^{AT}\langle\,\{A_{\alpha\beta}, \nabla^\alpha T^{\mu\beta}\}u_\mu\,\rangle$$

(*) Ecker,Zauner, EPJC 52 (2007) 315
(x) Shekhovtsova,SC, 1707.01137 [hep-ph[

- Prediction for the $T\pi$-AFF *:

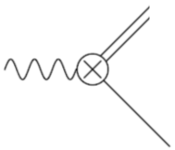
$$\mathcal{F}^a_{T\pi}(q^2;k^2) = -\frac{8g_T}{F_\pi} + \frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{(qp_3)}{M_A^2-q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{(qk)}{M_A^2-q^2},$$
$$\mathcal{G}^a_{T\pi}(q^2;k^2) = -\frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{1}{M_A^2-q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{1}{M_A^2-q^2},$$
$$\mathcal{H}^a_{T\pi}(q^2;k^2) = 0,$$

 + ...

- Contribution to the 3π-AFF via T *:

$$\mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \mathcal{F}_{1,\,(0)}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) + \mathcal{F}_{1,\,(\mathrm{RSD})}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)$$
$$-\frac{4}{9F_\pi^3}\frac{g_T}{M_T^2}\frac{(F_A\lambda_2^{AT}-2\sqrt{2}g_T)}{M_A^2-q^2}\times\Big[s_3q^2 + \frac{M_T^2}{M_T^2-s_3}\left(3(q\Delta p)^2-9(qk)(q\Delta p)-q^2(\Delta p)^2\right)\Big]$$
$$-\frac{8}{3F_\pi^3}\frac{g_T}{M_T^2}\frac{(F_A\lambda_1^{AT}+\sqrt{2}g_T)}{M_A^2-q^2}\times\Big[q^2(kp_3)\left(\frac{2s_3}{3M_T^2}-1\right) + \frac{M_T^2}{M_T^2-s_3}\left((q\Delta p)^2+3(q\Delta p)(qp_3)+\frac{q^2(kp_3)(\Delta p)^2}{3M_T^2}\right)\Big]$$

$T\to\pi^0\pi^0$

 + ...

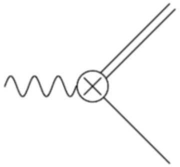
$$\mathcal{F}_P^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \ldots$$

- High-energy constraints $\mathcal{F}^a_{T\pi}(q^2;M_T^2) \stackrel{q^2\to\infty}{\longrightarrow} \mathcal{O}(1/q^2)$ and $\mathcal{G}^a_{T\pi}(q^2;M_T^2) \stackrel{q^2\to\infty}{\longrightarrow} \mathcal{O}(1/q^4)$ *:

$$F_A\lambda_2^{AT} = -2F_A\lambda_1^{AT} = 2\sqrt{2}g_T \;\Longrightarrow\; \boxed{\begin{aligned}\mathcal{F}^a_{T\pi}(q^2;k^2) &= -\frac{8g_T}{F_\pi}\frac{M_A^2}{M_A^2-q^2}\\ \mathcal{G}^a_{T\pi}(q^2;k^2) &= 0.\end{aligned}}$$

(*) Shekhovtsova,SC, [1707.01137 [hep-ph]]

- Prediction for the Tπ-AFF *:



+ ...

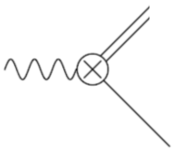
$$\mathcal{F}^a_{T\pi}(q^2;k^2) = -\frac{8g_T}{F_\pi} + \frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{(qp_3)}{M_A^2-q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{(qk)}{M_A^2-q^2}\,,$$

$$\mathcal{G}^a_{T\pi}(q^2;k^2) = -\frac{4\sqrt{2}F_A\lambda_1^{AT}}{F_\pi}\frac{1}{M_A^2-q^2} - \frac{2\sqrt{2}F_A\lambda_2^{AT}}{F_\pi}\frac{1}{M_A^2-q^2}\,,$$

$$\mathcal{H}^a_{T\pi}(q^2;k^2) = 0\,,$$

- Contribution to the 3π-AFF via T *:

T→$\pi^0\pi^0$



+ ...

$$\mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \mathcal{F}_{1,\,(0)}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) + \mathcal{F}_{1,\,(\mathrm{RSD})}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)$$

SD

$$-\frac{4}{9F_\pi^3}\frac{g_T}{M_T^2}\frac{(F_A\lambda_2^{AT}-2\sqrt{2}g_T)}{M_A^2-q^2}\times\Big[s_3q^2 + \frac{M_T^2}{M_T^2-s_3}\left(3(q\Delta p)^2-9(qk)(q\Delta p)-q^2(\Delta p)^2\right)\Big]$$

SD

$$-\frac{8}{3F_\pi^3}\frac{g_T}{M_T^2}\frac{(F_A\lambda_1^{AT}+\sqrt{2}g_T)}{M_A^2-q^2}\times\Big[q^2(kp_3)\left(\frac{2s_3}{3M_T^2}-1\right) + \frac{M_T^2}{M_T^2-s_3}\left((q\Delta p)^2+3(q\Delta p)(qp_3)+\frac{q^2(kp_3)(\Delta p)^2}{3M_T^2}\right)\Big]$$

$$\mathcal{F}_P^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \ldots$$

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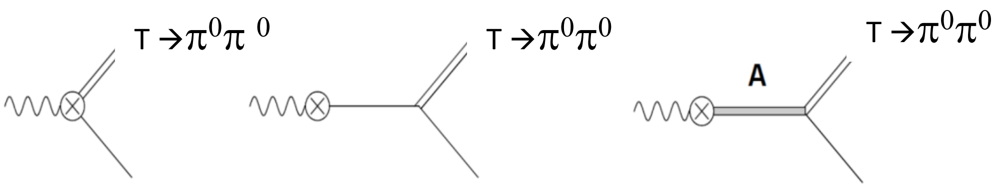
$$F_A\lambda_2^{AT} = -2F_A\lambda_1^{AT} = 2\sqrt{2}g_T$$

$$\boxed{\begin{aligned}\mathcal{F}^a_{T\pi}(q^2;k^2) &= -\frac{8g_T}{F_\pi}\frac{M_A^2}{M_A^2-q^2}\\ \mathcal{G}^a_{T\pi}(q^2;k^2) &= 0\,.\end{aligned}}$$

(*) Shekhovtsova,SC, [1707.01137 [hep-ph]]

• High-energy constraints *(continuation)*

$$\mathcal{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \mathcal{F}_{1,\,(0)}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) + \mathcal{F}_{1,\,(\mathrm{RSD})}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)$$



$$\mathcal{F}_{1,\,(\mathrm{RSD})}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) = -\frac{8\sqrt{2}}{3F_\pi^3}\frac{g_T^2}{M_T^2}\frac{M_A^2}{M_A^2-q^2}\left[(kp_3)+\frac{s_3}{3}\left(1-\frac{2(kp_3)}{M_T^2}\right)\right.$$
$$\left.-\frac{M_T^2}{M_T^2-s_3}\left(3(q\Delta p)+\frac{(\Delta p)^2}{3}+\frac{(kp_3)(\Delta p)^2}{3M_T^2}\right)\right]$$

$$\mathcal{F}_{1,\,(0)}^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2) = \frac{8\sqrt{2}g_T^2}{3F_\pi^3 M_T^2}(2s_1-s_2+s_3-4m_\pi^2)$$

Phenomenology

This is not a fit: preliminary test *[inputs in GeV units but for c_{σ}, c_{f0} –dimensionless–]*

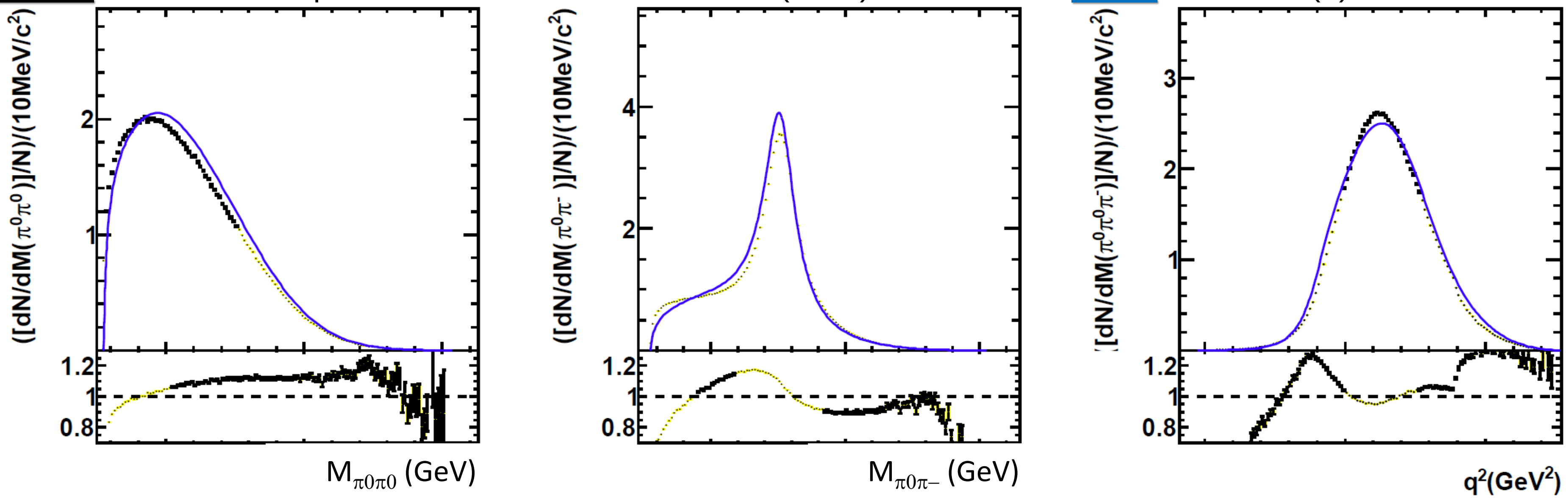
M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	M_{f_2} PDG	Γ_{f_2} PDG	F_π PDG
0.772	1.35	0.448	1.10	0.8064	1.275	0.185	0.0922
F_V	F_A	β_ρ	g_T (*)	c_d (x)	c_σ	M_{f_0}	c_{f_0}
0.168	0.131	-0.32	0.028	0.026	76.12	1.024	17.7

Previous Tauola for A-V resonances

$$s_{f_0}^{\rm pole} = (990 - i70/2)^2 \ {\rm MeV}^2$$

$$\sqrt{s^\sigma_{\rm pole}} = [(441^{+16}_{-8}) - i(544^{+18}_{-25})/2] \ {\rm MeV}$$

BLACK: MC data from param+fit values in CLEO, PR61 (2000) 012002; **BLUE:** this work (+)



(*) Ecker,Zauner, EPJC 52 (2007) 315

(x) Escribano,Masjuan,SC, JHEP 1105 (2011) 094

(+) Shekhovtsova,SC, [1707.01137 [hep-ph]]

- Similar plots (+) for $\pi^-\pi^-\pi^+$ data [Nugent (BaBar) NP Proc.Suppl. 253-255 (2014) 38]

Conclusions

✓ S & T lightest nonet contributions to the $\pi\pi\pi$-AFF in RχT

✓ RχT automatically implements χ symmetry:

Improvements→

- Isospin relation between $\pi^0\pi^0\pi^-$ & $\pi^-\pi^-\pi^+$
- Transversality
- Right low-energy behaviour (χPT)

Additional constraints→

- Right high-energy behaviour for Sπ-AFF & Tπ-AFF

✓ ATπ operators for the 1st time in RχT:

- Extension of Ecker and Zauner T Lagrangian
- Future analogous studies: $e^+ e^- \to a_2[\to\pi\eta]\ \pi$

$\gamma^*\gamma^* \to f_2[\to\pi\pi]\ ,\quad a_2[\to\pi\eta]$

BACKUP

- Choice of the σ parameters: not very dependent on the pole input

(*)

$$\sqrt{s^\sigma_{\rm pole}} = [(441^{+16}_{-\ 8}) - i(544^{+18}_{-25})/2]\ \text{MeV}$$

$$k = 1$$

$$M_\sigma = 806.4\ \text{MeV and } c_\sigma = 76.12$$

(x)

$$\sqrt{s^\sigma_{\rm pole}} = [(457^{+14}_{-13}) - i(558^{+22}_{-\ 14})/2]\ \text{MeV}$$

$$k = 1$$

$$M_\sigma = 804.1\ \text{MeV and } c_\sigma = 70.96$$

(*) Caprini,Colangelo,Leutwyler, PRL96 (2006) 132001
(x) García-Martín,Kaminski,Peláez,Ruíz de Elvira, PRL107 (2011) 072001

• Tensor resonance width *, (x), (+):

$$\frac{1}{M_T^2 - s} \quad \longrightarrow \quad \frac{1}{M_{f_2}^2 - s - iM_{f_2}\Gamma_{f_2}(s)}$$

$$\Gamma_{f_2}(s) = \Gamma_0^{f_2} \frac{s^2}{M_{f_2}^4} \frac{\rho_\pi(s)^5}{\rho_\pi(M_{f_2}^2)^5}$$

Using the PDG central values, $\Gamma^{\rm exp}_{f_2\to\pi\pi} = 157.2$ MeV, $M_{f_2} = 1275.5$ MeV, $m_\pi = 139.57$ MeV and $F_\pi = 92.2$ MeV, one obtains

$$\Gamma_{f_2\to\pi\pi} = \frac{g_T^2 M_{f_2}^3 \rho_\pi(M_{f_2}^2)^5}{40\pi F_\pi^4}$$

$$g_T \simeq 28\,\mathrm{MeV}$$

* Shekhovtsova,SC, [1707.01137 [hep-ph]]

(x) CLEO, PR61 (2000) 012002

(+) Ecker,Zauner, EPJC 52 (2007) 315

• Definition of the chiral tensors in the RχT Lagrangian:

$$A_{\mu\nu} = A^a_{\mu\nu}\lambda^a/\sqrt{2} \qquad\qquad A_{\mu\nu} = \begin{pmatrix} 0 & a_1^+ & 0 \\ a_1^- & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mu\nu} + \ \ldots$$

$$U = u^2 = \exp\{\pi^a\lambda^a/F\}\,, \qquad D_\mu U = \partial_\mu U - i r_\mu U + i U \ell_\mu\,, \qquad u_\mu = i u^\dagger (D_\mu U) u^\dagger$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,, \qquad f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u\,, \qquad \nabla_\mu \cdot = \partial_\mu \cdot + [\Gamma_\mu, \cdot]\,,$$

$$\Gamma_\mu = \frac{1}{2}\left\{ u^\dagger(\partial_\mu - i r_\mu) u + u(\partial_\mu - i\ell_\mu) u^\dagger \right\}\,,$$

$$\chi = 2B_0 \mathrm{diag}(m_u, m_d, m_s) + \ldots \qquad \ell_\alpha = \frac{g}{\sqrt{2}}(W_\alpha^+ T_+ + \mathrm{h.c.}) \qquad r_\alpha = 0$$

$$h_{\alpha\mu} = \nabla_\alpha u_\mu + \nabla_\mu u_\alpha \qquad T_+ = V_{ud}(\lambda^1 + i\lambda^2)/2 + V_{us}(\lambda^4 + i\lambda^5)/2$$

isosinglet scalar $S_{I=0} \sim u\bar{u} + d\bar{d}$,

$$S = \begin{pmatrix} \frac{S_{I=0}}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{S_{I=0}}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \ \ldots$$