Strongly Interacting Matter Phase Diagram in the presence of Magnetic Fields in an Extended Effective Lagrangian Approach with Explicit Chiral Simmetry Breaking Interactions

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Salamanca, Spain, 26/09/2017

Outline

Introduction

QCD: the Theory of **Strong Interactions**

- Very successfull pQCD at high energy
- Non-perturbative low energy regime requires the use of other tools for instance:
  - lQCD
  - AdS/QCD
  - Dyson-Schwinger
  - FRG
  - Chiral pertubation theory
  - Effective models
    - **Dynamical**/**Explicit** **Chiral Symmetry Breaking** plays a big role in low energy phenomenology

Phase diagram for strongly interacting matter

A clear and present challenge [1]:

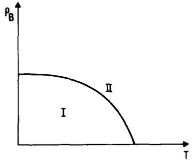

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

[1] N. Cabbibo, G. Parisi Phys.Lett. 59B (1975) 67-69; Kenji Fukushima, Tetsuo Hatsuda Rept.Prog.Phys.74:014001,2011; http://nica.jinr.ru/physics.php

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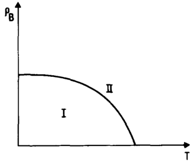

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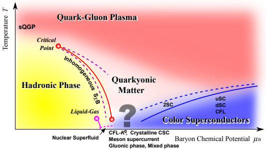

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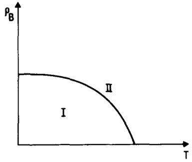

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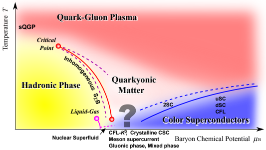

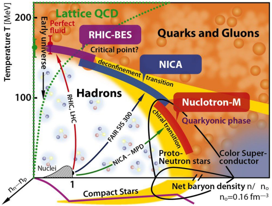

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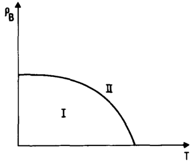

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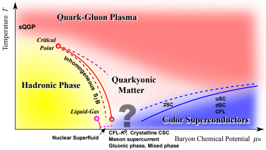

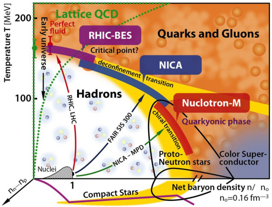

The effect of strong **magnetic fields** on this phase diagram is relevant for instance in:

- **Heavy Ion collisions**
- **Compact stars**
- Early phases of the **Universe**

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Nambu–Jona-Lasinio Model

NJL: effective model for the non-perturbative low energy regime of **QCD** with Dynamical Chiral Symmetry Breaking (**DχSB**)

- **NJL** shares the global symmetries with **QCD**
- Dynamical generation of the **constituent mass**
- Light pseudoscalar as (quasi) **Nambu-Goldstone boson**
- **Quark condensates** as order parameter
- No gluons (no confinement/deconfinement)
- Local and non renormalizable

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q}\imath\partial\!\!\!/ q + \mathcal{L}_m$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a \imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q} \imath \partial\!\!\!/ q + \mathcal{L}_m$$

- **Explicit Chiral symmetry breaking**

$$\mathcal{L}_m = \overline{q} \hat{m} q$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a \imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL}$$

- $\mathcal{L}_m = \overline{q}\hat{m}q$
- **Nambu–Jona-Lasinio** (4 q)

$$\mathcal{L}_{NJL} = G\,\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \overline{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H$$

- $\mathcal{L}_m = \overline{q}\hat{m}q$
- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)$
- **'t Hooft determinant** (6 q)

$$\mathcal{L}_H = \kappa\left(\det\Sigma + \det\Sigma^\dagger\right)$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a \imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\rm eff} = \bar{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q}$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)$
- $\mathcal{L}_H = \kappa\left(\det\Sigma + \det\Sigma^\dagger\right)$
- **Eight quark interaction term**

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \quad \mathcal{L}_{8q}^{(1)} = g_1\left(\mathrm{tr}\,\Sigma^\dagger\Sigma\right)^2, \quad \mathcal{L}_{8q}^{(2)} = g_2\mathrm{tr}\left(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma\right)$$

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\text{eff}} = \bar{q} \imath \partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q}$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
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OZI violation in $\mathcal{L}_H$ and $\mathcal{L}_{8q}^{(1)}$.

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a \imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Multi-quark interations (u, d and s) [2]

$$\mathcal{L}_{\rm eff} = \bar{q}\imath\partial\!\!\!/ q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q} + \mathcal{L}_\chi$$

- **Extended Explicit Chiral symmetry breaking $\mathcal{L}_\chi$**
- $\mathcal{L}_{NJL} = G\,\mathrm{tr}\left(\Sigma^\dagger\Sigma\right)$
- $\mathcal{L}_H = \kappa\left(\det\Sigma + \det\Sigma^\dagger\right)$
- $\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \quad \mathcal{L}_{8q}^{(1)} = g_1\left(\mathrm{tr}\,\Sigma^\dagger\Sigma\right)^2, \quad \mathcal{L}_{8q}^{(2)} = g_2\mathrm{tr}\left(\Sigma^\dagger\Sigma\Sigma^\dagger\Sigma\right)$

Non canonical explicit chiral symmetry breaking terms

[2] $\Sigma = (s_a - \imath p_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a\imath\gamma_5 q$, and $a = 0, 1, \ldots, 8$

Inclusion of explicit chiral symmetry breaking terms

$$L_\chi = \sum_{i=1}^{10} L_i,$$

$$L_1 = -\kappa_1 e_{ijk} e_{mnl} \Sigma_{im} \chi_{jn} \chi_{kl} + h.c., \qquad L_2 = \kappa_2 e_{ijk} e_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + h.c.,$$

$$L_3 = g_3 \mathrm{tr}\left(\Sigma^\dagger \Sigma \Sigma^\dagger \chi\right) + h.c., \qquad L_4 = g_4 \mathrm{tr}\left(\Sigma^\dagger \Sigma\right) \mathrm{tr}\left(\Sigma^\dagger \chi\right) + h.c.,$$

$$L_5 = g_5 \mathrm{tr}\left(\Sigma^\dagger \chi \Sigma^\dagger \chi\right) + h.c. \qquad L_6 = g_6 \mathrm{tr}\left(\Sigma \Sigma^\dagger \chi \chi^\dagger + \Sigma^\dagger \Sigma \chi^\dagger \chi\right),$$

$$L_7 = g_7 \left(\mathrm{tr}\Sigma^\dagger \chi + h.c.\right)^2 \qquad L_8 = g_8 \left(\mathrm{tr}\Sigma^\dagger \chi - h.c.\right)^2,$$

$$L_9 = -g_9 \mathrm{tr}\left(\Sigma^\dagger \chi \chi^\dagger \chi\right) + h.c. \qquad L_{10} = -g_{10} \mathrm{tr}\left(\chi^\dagger \chi\right) \mathrm{tr}\left(\chi^\dagger \Sigma\right) + h.c.$$

Note: $\kappa_1,\ g_9,\ g_{10}$ can be set to 0 without loss of generality;
$\chi = \mathrm{diag}(m_u, m_d, m_s)$; $\kappa_1, \kappa_2, g_4, g_7, g_8, g_{10}$ OZI violating;

For more on this subject drop by Jorge Morais talk on thursday!

Thermodynamical potential

$$
\begin{aligned}
\Omega =& \mathcal{V}_{st} + \sum_i \frac{N_c}{8\pi^2}\left(J_{-1}\left[M_i, T, \mu_i\right] + C\left[T, \mu_i\right]\right) \\
\mathcal{V}_{st} =& \frac{1}{16}\Big(4G\left(h_i^2\right) + 3g_1\left(h_i^2\right)^2 + 3g_2\left(h_i^4\right) + 4g_3\left(h_i^3 m_i\right) \\
& + 4g_4\left(h_i^2\right)\left(h_j m_j\right) + 2g_5\left(h_i^2 m_i^2\right) + 2g_6\left(h_i^2 m_i^2\right) \\
& + 4g_7\left(h_i m_i\right)^2 + 8\kappa h_u h_d h_s 8\kappa_2\left(m_u h_d h_s + h_u m_d h_s + h_u h_d m_s\right)\Big)\Big|_0^{M_i} \\
\Delta_f =& M_f - m_f \\
=& -Gh_f - \frac{g_1}{2}h_f(h_i^2) - \frac{g_2}{2}(h_f^3) - \frac{3g_3}{4}h_f^2 m_f \\
& - \frac{g_4}{4}\left(m_f\left(h_i^2\right) + 2h_f(m_i h_i)\right) - \frac{g_5+g_6}{2}h_f m_f^2 - g_7 m_f(h_i m_i) - \frac{\kappa}{4}t_{fij}h_i h_j - \kappa_2 t_{fij}h_i m_j \\
J_i\left[M\right] =& 16\pi^2\Gamma(i+1)\int\frac{\mathrm{d}^4 p_E}{(2\pi)^4}\,\hat{\rho}_\Lambda\frac{1}{(p_E^2+M^2)^{i+1}} \\
=& 16\pi^2\int\frac{\mathrm{d}^4 p_E}{(2\pi)^4}\int_0^\infty \mathrm{d}\tau\,\hat{\rho}_\Lambda\tau^i \mathrm{e}^{-\tau(p_E^2+M^2)}. \\
J_{i+1}\left[M\right] =& -\frac{1}{2M}\frac{\partial}{\partial M}J_i\left[M\right]
\end{aligned}
\tag{1}
$$

Magnetic field, temperature/chemical potential and regularization

Landau level:

$$\int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} \to \frac{2\pi\,|q|\,H}{(2\pi)^2}\frac{1}{2}\sum_{s=-1,+1}\sum_{m=0}^{+\infty},$$

$$p_\perp^2 \to (2m+1-s)\,|q|\,H$$

Matsubara formalism:

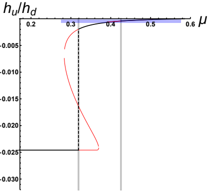
$$p_4 \to \pi T(2n+1) - i\mu$$

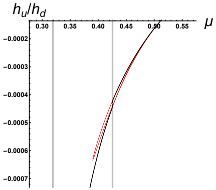
$$\int \mathrm{d}p_4 \to 2\pi T \sum_{n=-\infty}^{+\infty}$$

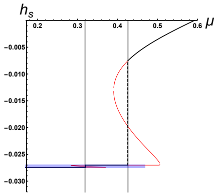
Regularization

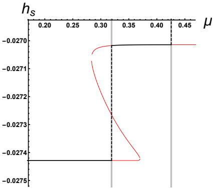
$$\hat{\rho}_{PV}\,[\tau,\Lambda] = 1-(1-\tau\Lambda^2)\mathrm{e}^{-\tau\Lambda^2}$$

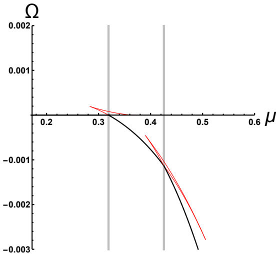
Solutions at $\{T, H|q|\} = \{0\text{GeV}, 0\text{GeV}^2\}$



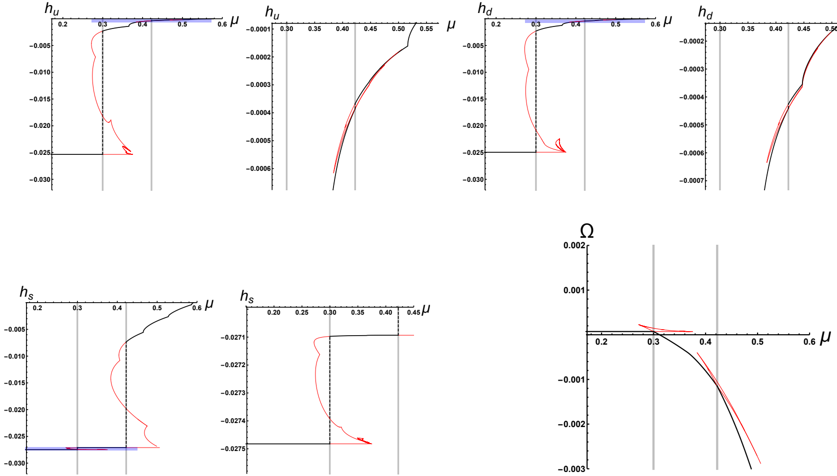




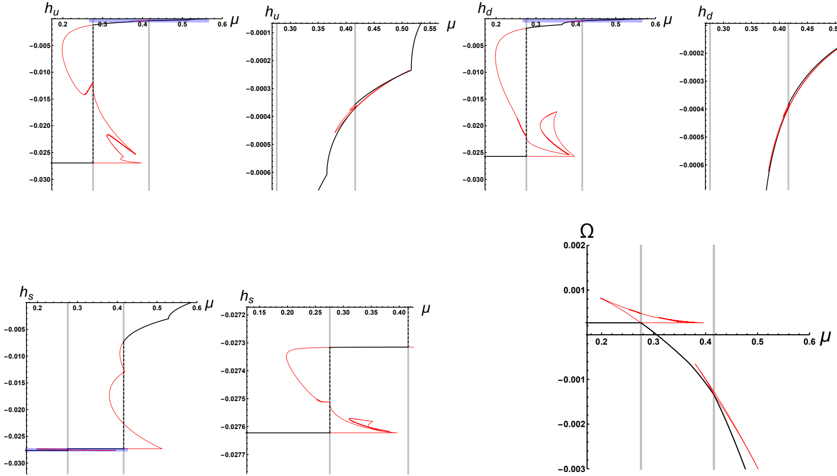




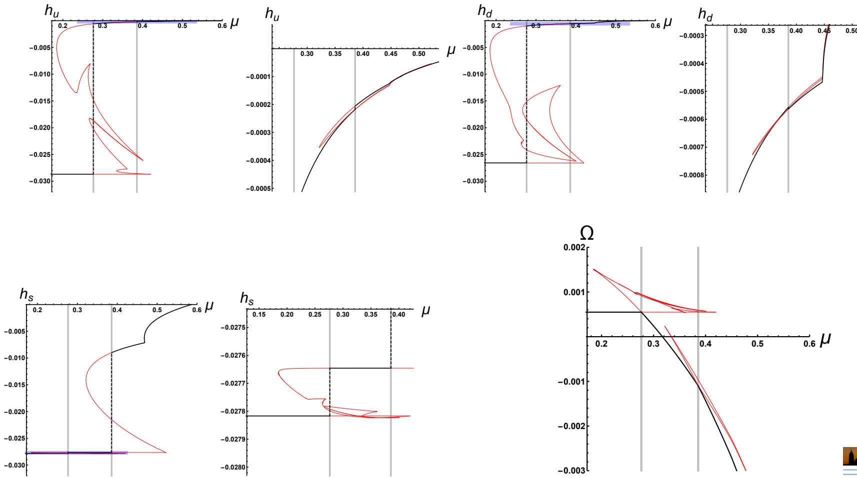
Solutions at $\{T, H|q|\} = \{0\mathrm{GeV}, 0.1\mathrm{GeV}^2\}$



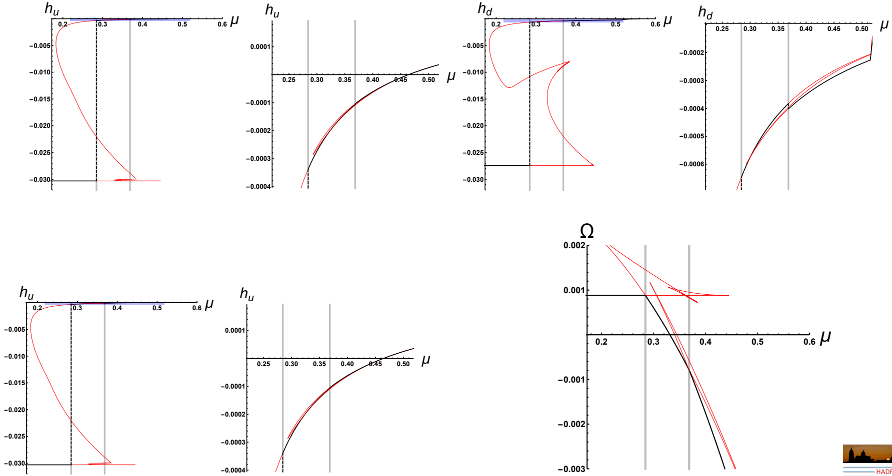
Solutions at $\{T, H|q|\} = \{0\mathrm{GeV}, 0.2\mathrm{GeV}^2\}$



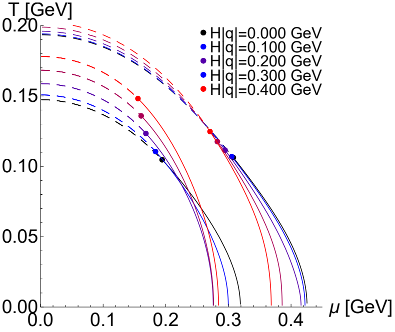
Solutions at $\{T, H|q|\} = \{0, 0.3\}\mathrm{GeV}$

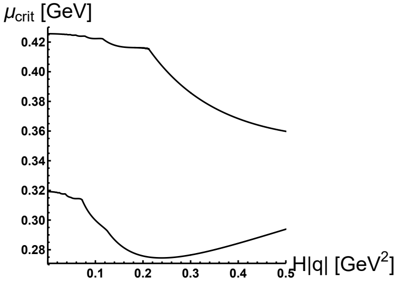


Solutions at $\{T, H|q|\} = \{0\mathrm{GeV}, 0.4\mathrm{GeV}^2\}$



Phase diagram at different field strengths





Conclusions

- Second CEP associated with the strange chiral restoration:
  - Exists even at vanishing magnetic field
  - Location in the PD much less affected by the magnetic field when compared with the other CEP
- *Cleaner* transitions (regularization?)

Acknowledgments and more information

Post-doc grant: SFRH/BPD/110421/2015

FCT Fundação para a Ciência e a Tecnologia

For more information:

- J. Moreira, J. Morais, B. Hiller, A. A. Osipov, A. H. Blin, Phys. Rev. D 91, 116003 (2015) https://arxiv.org/abs/1409.0336
- results with magnetic field are under preparation