Global analysis on determination of fracture functions considering sea quark asymmetries in the nucleon

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in collaboration with

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Outline

❖ A new look to inclusive processes

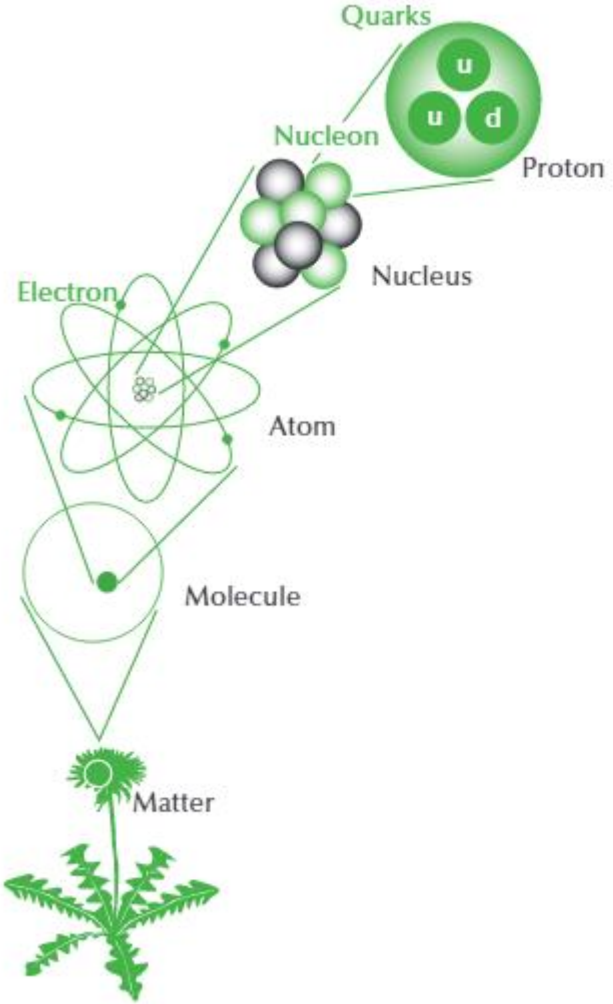
➢ Inclusive Deep Inelastic Scattering

➢ Semi Inclusive Deep Inelastic Scattering

- Fracture Functions
- Forward Baryons in HERA
- DGLAP Eqs & Semi-inclusive Structure Functions

❖ QCDNUM & MINUIT program

❖ Results



A new look to inclusive processes

- Let us reformulate, in a slightly different way, some well known QCD results. Inclusive hard processes can be divided in two classes: **totally inclusive and semi – inclusive ones**.

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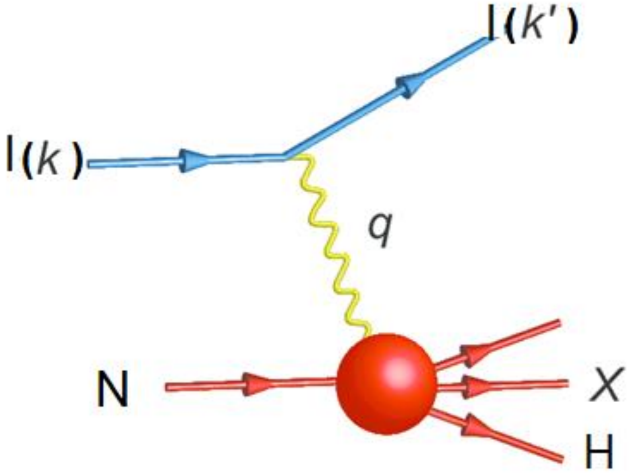

- The cross-section for some hard process H is written as :

$$\sigma_{l+N\to l'+H+X} = \sum_j \int_0^1 \frac{dx}{x} F_N^j(x,Q)\sigma_H^j(x,Q)$$

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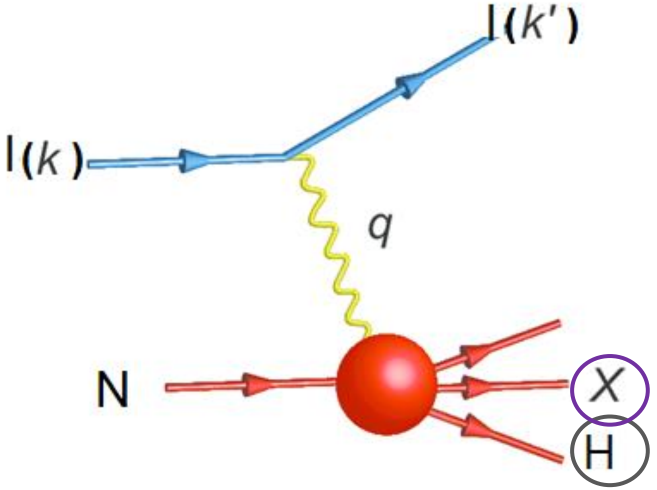

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structure function of the target N.

the final state consists of two well separated clusters of particles:

- H can represent any hard process as a jet, many jets, a photon and a jet, two heavy quarks, etc... They are coming from the subsequent hard interaction of the active parton with the lepton.
- X is originating from the target fragmentation and from the evolution of the active parton.

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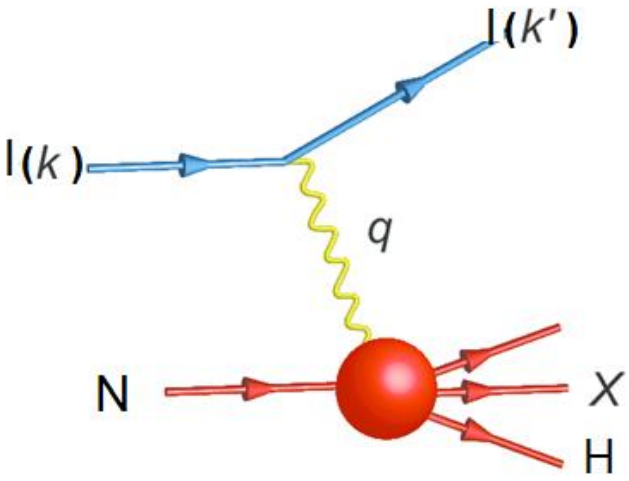

- The cross-section for some hard process H is written as :

$$\sigma_{l+N \to l' + H + X} = \sum_j \int_0^1 \frac{dx}{x} F_N^j(x, Q) \sigma_H^j(x, Q)$$

In this process no particular hadronic final state is singled out.

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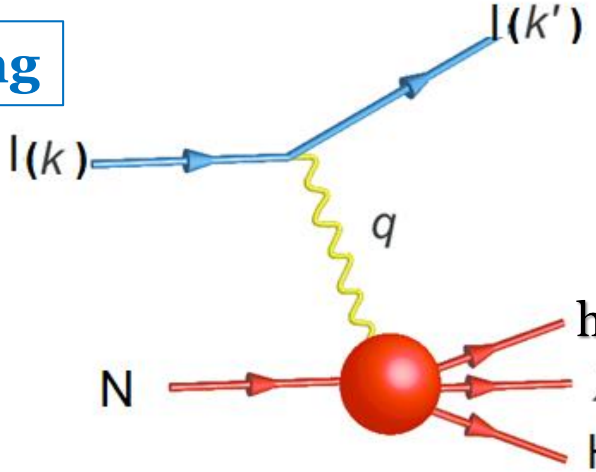
Semi-Inclusive Deep Inelastic Scattering



✓ If at least a single hadron is detected in the final state, The process $l + N \rightarrow l + h + H + X$, will receive contributions from two well separated kinematical regions for the produced hadron **h**:

$$\sigma_{l+A\rightarrow l'+h+H+X} = \sigma_{current} + \sigma_{target}$$

$$= \sigma_{l+A\rightarrow l'+(h+H')+X} + \sigma_{l+A\rightarrow l'+H+(h+X')} \ .$$

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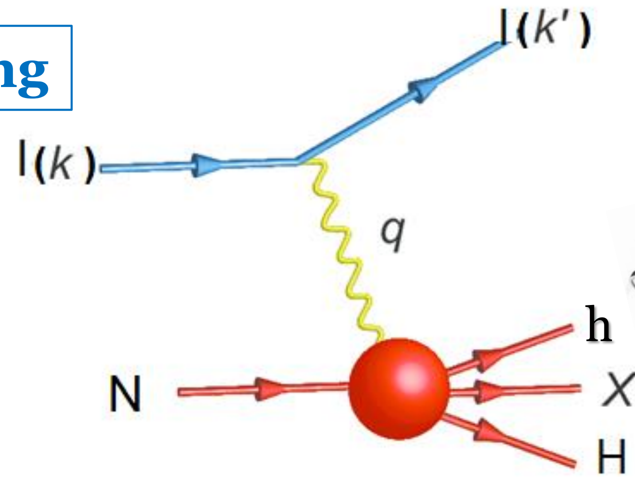

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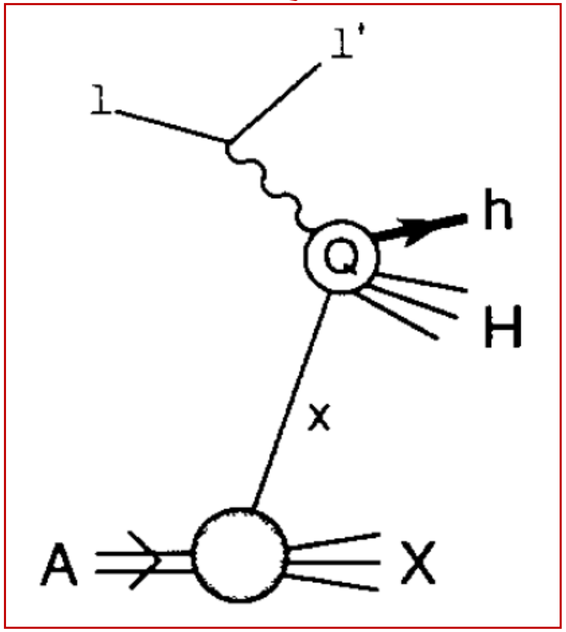

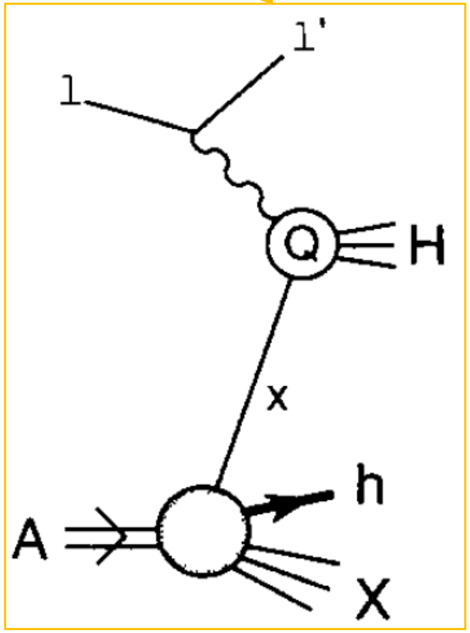

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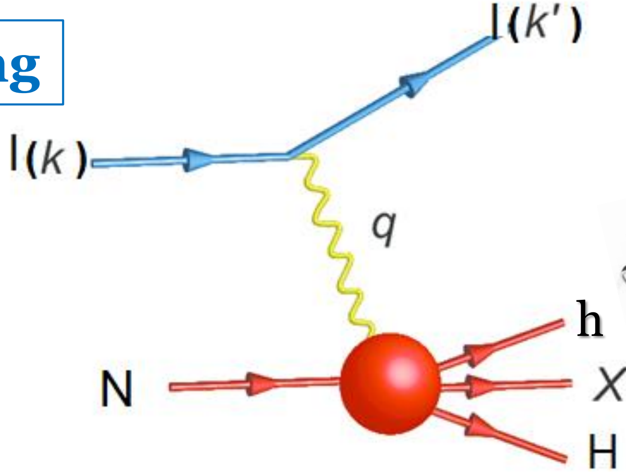

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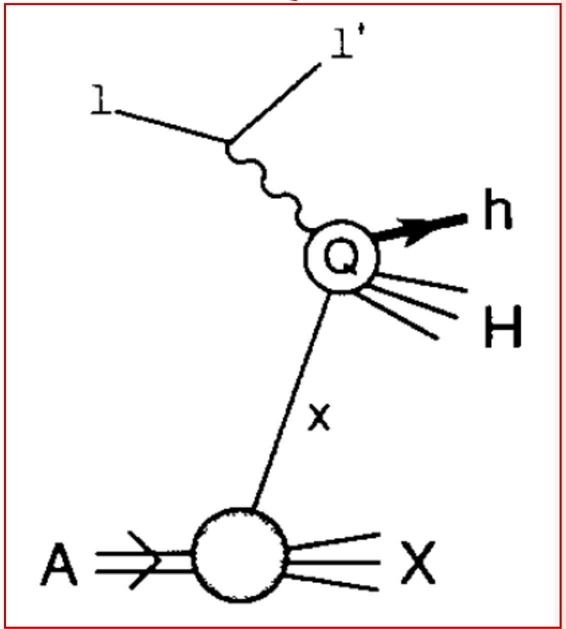

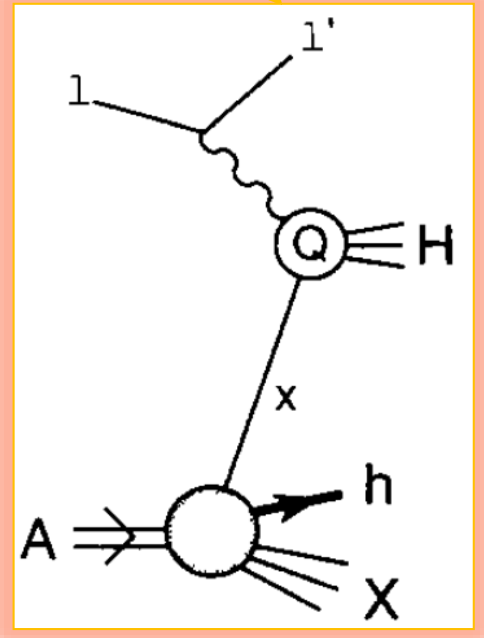

it is necessary to introduce a distribution function that describes hadrons in the target region.
the new distribution functions have been called **"fracture functions"**.

Fracture functions

I. They represents the **conditional probability** of finding at a given scale Q^2 a parton ***i*** with momentum fraction ***x*** of the incoming hadron momentum ***P*** while a hadron ***h*** with momentum fraction ***z*** is detected.

II. They depend upon two hadronic and one partonic label and on two momentum fractions, a Bjorken ***x*** and a ***z*** variable

$$M_i^{h/p}(x, z, Q^2) \qquad z \equiv E_h / E_P$$

III. The scale dependence of fracture functions at $O(\alpha_s)$ in fixed value of ***z*** obey standard DGLAP Eqs:

$$\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(x, z, Q^2)$$
$$= \frac{\alpha_s(Q^2)}{2\pi} \int_{x/(1-z)}^{1} \frac{du}{u} P_j^i(u) M_{j,h/N}\left(\frac{x}{u}, z, Q^2\right)$$
$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_{x}^{x/(x+z)} \frac{du}{(1-u)} \hat{P}_j^{i,l}(u) f_{j/p}\left(\frac{x}{u}, Q^2\right)$$
$$\times D_{h/l}\left(\frac{zu}{x(1-u)}, Q^2\right),$$

- ✓ Fracture functions, as parton distributions in general, are essentially of a nonperturbative nature and have to be extracted from experiment.

DGLAP Eqs (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi):

- describes the evolution of the parton density with Q^2
- For solving this Eqs we need an input in initial scale $Q_0{}^2$
- There are different ways to solve these Eqs:

1) solve the equation directly in x-space
2) solve it for Mellin transforms/Laplas transform of the parton densities and subsequently invert the transforms back to x-space

$$
\begin{aligned}
&\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(x,z,Q^2) \\
&= \frac{\alpha_s(Q^2)}{2\pi} \int_{x/(1-z)}^{1} \frac{du}{u} P_j^i(u) M_{j,h/N}\left(\frac{x}{u}, z, Q^2\right) \\
&+ \frac{\alpha_s(Q^2)}{2\pi} \int_{x}^{x/(x+z)} \frac{du}{(1-u)} \hat{P}_j^{i,l}(u) f_{j/p}\left(\frac{x}{u}, Q^2\right) \\
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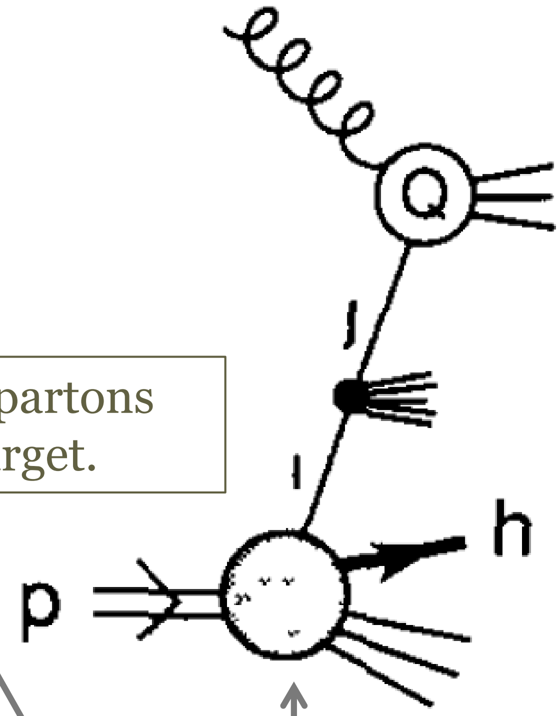

The emission of collinear partons from those found in the target.

$$
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&+ \frac{\alpha_s(Q^2)}{2\pi}\int_{\mathsf{x}}^{\mathsf{x}/(\mathsf{x}+\mathsf{z})}\frac{du}{(1-u)}\hat{P}^{i,l}_j(u)f_{j/p}\left(\frac{\mathsf{x}}{u},Q^2\right)\\
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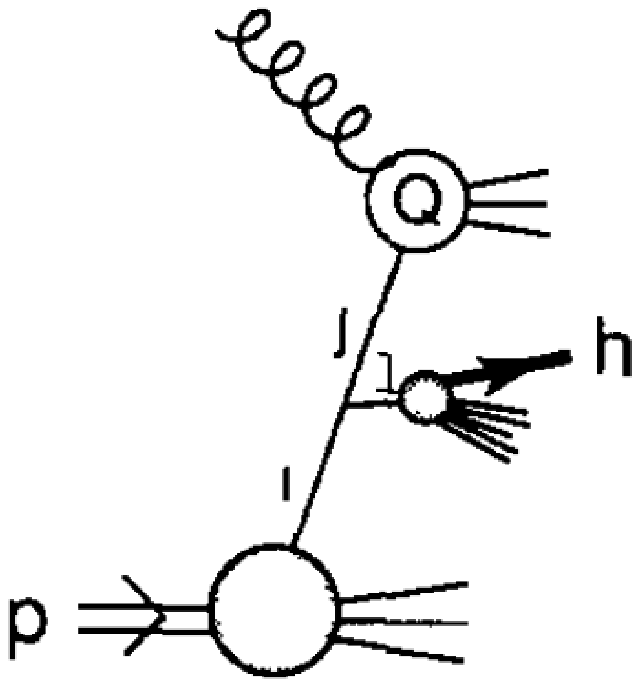

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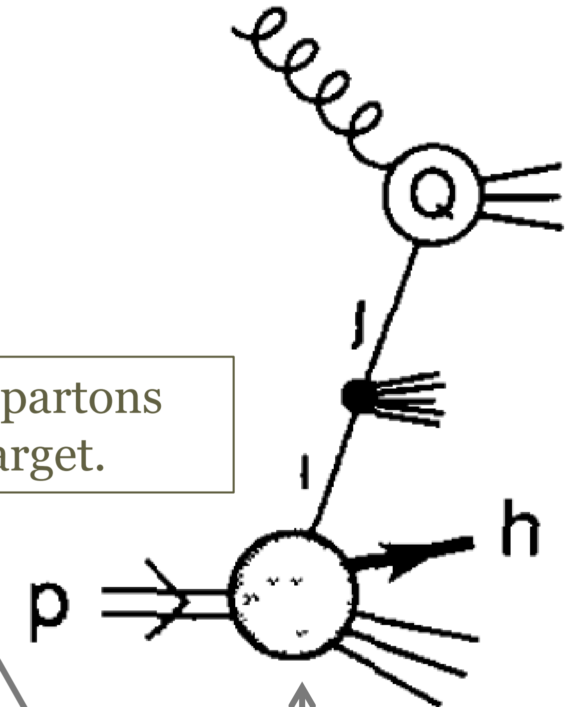

$$\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_{x/(1-z)}^{1}\frac{du}{u}P_j^i(u)M_{j,h/N}\left(\frac{x}{u},z,Q^2\right)$$

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$$\times D_{h/l}\left(\frac{zu}{x(1-u)},Q^2\right),$$

h produced from the evolution of active parton.

Forward Baryons in HERA

ZEUS Collaboration has measured events where neutrons and protons, are produced in the forward direction → **Forward Neutrons & Forward Protons**

- ✓ carrying a sizeable fraction of the available energy .
- ✓ produced at small polar angle with respect to the collision axis ($\theta_B \sim 0.8\ mrad$) in target fragmentation region.
- ✓ their valence-parton composition is almost or totally conserved with respect to the one of initial-state hadrons.

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$$Q^2 = -q^2, \qquad x = \frac{Q^2}{2p\cdot q}, \qquad y = \frac{p\cdot q}{p\cdot k},$$

kinematic variables to describe the inclusive DIS scattering process

$$z = x_L = 1 - \frac{q\cdot(p-p_B)}{q\cdot p} \simeq E_B/E_p$$

longitudinal momentum fraction

$$t = (p-p_B)^2 \simeq -\frac{p_T^2}{x_L} - (1-x_L)\left(\frac{m_B^2}{x_L} - m_p^2\right)$$

The squared four-momentum transfer between the incident proton and the final state baryon

$$\frac{d^4\sigma(ep\to e'BX)}{dx\,dQ^2\,dx_L\,dt} = \frac{4\pi\alpha^2}{xQ^4}\left(1-y+\frac{y^2}{2}\right)F_2^{LB(4)}(x,Q^2,x_L,t) + F_L^{LB(4)}(x,Q^2,x_L,t).$$

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Leading nucleon transverse SF

Leading nucleon longitudinal SF

$$F_k^{LB\,(4)}(x, Q^2, x_L, p_T^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, M_{i/P}^{B}(x, \mu_F^2; x_L, p_T^2) \times C_{ki}\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_R^2)\right) + \mathcal{O}\left(\frac{1}{Q^2}\right).$$

$$\downarrow$$

$$\boxed{M_{i/P}^{B}(x, Q^2, x_L) = \int^{p^2_{T,\max}} dp_T^2\, M_{i/P}^{B}(x, Q^2, x_L, p_T^2)}$$

$$Q^2 \frac{\partial M_{i/P}^{B}(x, Q^2, x_L)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u}\, P_i^j(u)\, M_{j/P}^{B}\left(\frac{x}{u}, Q^2, x_L\right)$$

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hypothesis of limiting fragmentation

- states that target fragmentation is independent of the incident projectile's energy,
- implies that final state baryons emerge from a process which is insensitive to x and Q^2:

$$F_2^{LB(4)}(x, Q^2, x_L, p_T) = f(x_L, p_T) \cdot F(x, Q^2)$$

Baryon variables x_L,p_T

Lepton variables x,Q^2

Eur Phys J C 6 (1999) 587-602

Nuclear Physics B 637 (2002) 3-56

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Our goal

- The goal of this talk is to present Nucleon FFs from QCD analysis of leading-baryon production in the semi-inclusive DIS reaction $ep \to eBX$ at HERA.

$$F_k^{LB\,(4)}(x,Q^2,x_L,p_T^2)=\sum_i \int_X^1 \frac{d\xi}{\xi} M^{B}_{i/P}(x,\mu_F^2;x_L,p_T^2) \times C_{ki}\left(\frac{x}{\xi},\frac{Q^2}{\mu_F^2},\alpha_s(\mu_R^2)\right)+\mathcal{O}\left(\frac{1}{Q^2}\right).$$

$$xM^N_{u_v/P}(x,Q_0^2,x_L)=\mathcal{W}_{u_v}(x_L)\; xu_v^{\mathrm{GJR08}}(x,Q^2)\,,$$

$$xM^N_{d_v/P}(x,Q_0^2,x_L)=\mathcal{W}_{d_v}(x_L)\; xd_v^{\mathrm{GJR08}}(x,Q^2)\,,$$

$$xM^N_{\Delta/P}(x,Q_0^2,x_L)=\mathcal{W}_{\Delta}(x_L)\; x\Delta^{\mathrm{GJR08}}(x,Q^2)\,,$$

$$xM^N_{(\bar d+\bar u)/P}(x,Q_0^2,x_L)=\mathcal{W}_{(\bar d+\bar u)}(x_L)\; x(\bar d+\bar u)^{\mathrm{GJR08}}(x,Q^2)\,,$$

$$xM^N_{s^+/P}(x,Q_0^2,x_L)=\mathcal{W}_{s^+}(x_L)\; xs^{+\,\mathrm{GJR08}}(x,Q^2)\,,$$

$$xM^N_{g/P}(x,Q_0^2,x_L)=\mathcal{W}_{g}(x_L)\; xg^{\mathrm{GJR08}}(x,Q^2)\,,$$

hypothesis of limiting fragmentation

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$$xM_{s^+/P}^N(x, Q_0^2, x_L) = \boxed{\mathcal{W}_{s^+}(x_L)}\, xs^{+\,\mathrm{GJR08}}(x, Q^2),$$

$$xM_{g/P}^N(x, Q_0^2, x_L) = \boxed{\mathcal{W}_{g}(x_L)}\, xg^{\mathrm{GJR08}}(x, Q^2),$$

$$\mathcal{W}_{u_v}(x_L) = \mathcal{N}_{u_v}\, x_L^{A_{u_v}}(1-x_L)^{B_{u_v}}(1 + C_{u_v}\, x_L^{D_{u_v}}),$$

$$\mathcal{W}_{d_v}(x_L) = \mathcal{N}_{d_v}\, x_L^{A_{d_v}}(1-x_L)^{B_{d_v}}(1 + C_{d_v}\, x_L^{D_{d_v}}),$$

$$\mathcal{W}_{\Delta}(x_L) = \mathcal{N}_{\Delta}\, x_L^{A_{\Delta}}(1-x_L)^{B_{\Delta}}(1 + C_{\Delta}\, x_L^{D_{\Delta}}),$$

$$\mathcal{W}_{(\bar d+\bar u)}(x_L) = \mathcal{N}_{(\bar d+\bar u)}\, x_L^{A_{(\bar d+\bar u)}}(1-x_L)^{B_{(\bar d+\bar u)}}(1 + C_{(\bar d+\bar u)}\, x_L^{D_{(\bar d+\bar u)}}),$$

$$\mathcal{W}_{s^+}(x_L) = \mathcal{N}_{s^+}\, x_L^{A_{s^+}}(1-x_L)^{B_{s^+}}(1 + C_{s^+}\, x_L^{D_{s^+}}),$$

$$\mathcal{W}_{g}(x_L) = \mathcal{N}_{g}\, x_L^{A_{g}}(1-x_L)^{B_{g}}(1 + C_{g}\, x_L^{D_{g}}),$$

Leading baryon production data used in global analysis

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Leading neutron production in e^+p collisions at HERA

ZEUS Collaboration

S. Chekanov, D. Krakauer, S. Magill, B. Musgrave, A. Pellegrino, J. Repond, R. Yoshida

Argonne National Laboratory, Argonne, IL 60439-4815, USA [48]

M.C.K. Mattingly

Experiment	Observable	$[x_B^{min}, x_B^{max}]$	$[x_L^{min}, x_L^{max}]$	$Q^2(\mathrm{GeV}^2)$	# of points
Zeus02	$F_2{}^{LN}$	$1.1*10^{-5} < x_B < 3.2*10^{-3}$	$0.24 < x_L < 0.92$	$7 < Q^2 < 1000$	300

Leading baryon production data used in global analysis

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S. Chekanov, D. Krak

Argonne Nati

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Nuclear Physics B 658 (2003) 3–46

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Leading proton production in e^+p collisions at HERA

ZEUS Collaboration

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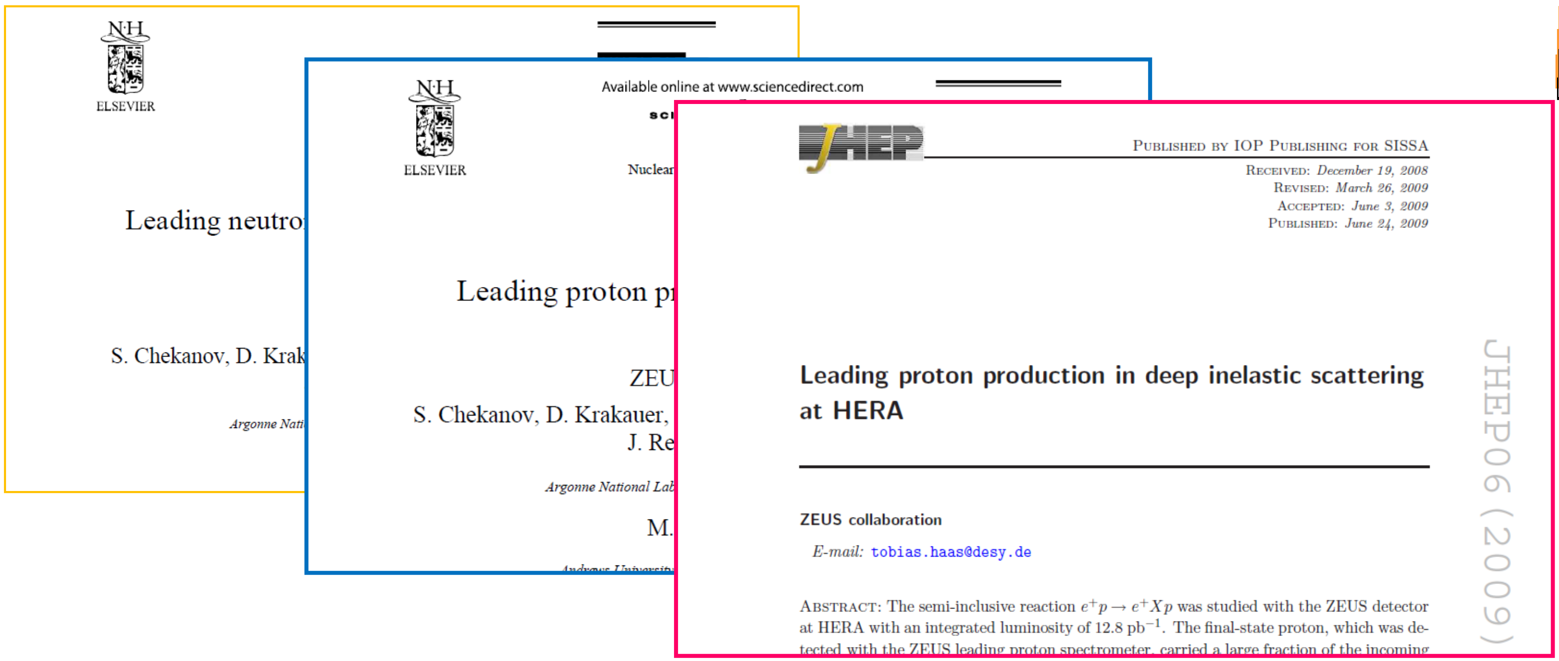
Argonne National Laboratory, Argonne, IL 60439-4815, USA[57]

M.C.K. Mattingly

Experiment	Observable	$[x_B^{min}, x_B^{max}]$	$[x_L^{min}, x_L^{max}]$	$Q^2(\mathrm{GeV}^2)$	# of points
Zeus02	F_2^{LN}	$1.1*10^{-5} < x_B < 3.2*10^{-3}$	$0.24 < x_L < 0.92$	$7 < Q^2 < 1000$	300
Zeus06	$r^{LP} = \frac{F_2^{LP}}{F_2^{P}}$	$8.5*10^{-5} < x_B < 8.2*10^{-2}$	$0.57 < x_L < 0.89$	$3.4 < Q^2 < 377$	226

Leading baryon production data used in global analysis



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Zeus09	$r^{LP} = \frac{F_2^{LP}}{F_2^{P}}$	$9.6*10^{-5} < x_B < 3.2*10^{-2}$	$0.37 < x_L < 0.89$	$4.2 < Q^2 < 237$	168

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QCDNUM

The QCDNUM Home Page

QCDNUM-17-00/08

- Download
- How to install
- Write-up
- Example jobs
- Release history

QCDNUM-17-01/13

- Download
- How to install
- Write-up
- C++ interface
- Example jobs
- Tutorial
- Next release

QCDNUM is a very fast QCD evolution program written in FORTRAN77. The pre-releases 17-01/13 and higher come with a C++ interface.

QCDNUM numerically solves the DGLAP evolution equations on a discrete grid in x and Q2. You can evolve unpolarised parton density functions in NNLO, and polarised pdfs or fragmentation functions in NLO. The jobs example.f and exampleCxx.cc show how to do this in a few lines of code.

QCDNUM supports evolution in the fixed and variable number schemes. To study the scale uncertainties, the renormalisation scale can be varied with respect to the factorisation scale.

QCDNUM also provides a large toolbox where you can enter your own splitting functions and then solve N-fold coupled DGLAP evolution equations (this is available in the pre-releases 17-01/xx). Also provided are routines to convolute pdfs with each other, or with user-defined kernels. With these tools you can write your own evolution routine (e.g. QCD-QED) or calculate parton luminosities and structure functions in both the massless and generalised mass schemes.

With the toolbox you can extend the functionality of QCDNUM as is done by the structure function add-on packages ZMSTF (unpolarised zero-mass

QCDNUM

i. QCDNUM numerically solves the DGLAP evolution equations on a discrete grid in x and Q^2.

ii. QCDNUM supports two evolution schemes:

- FFNS : number of active flavor is kept constant $3 > n_f > 6$ for all Q^2.
- VFNS : number of flavors changes from n_f to $n_f + 1$ at the thresholds ${\mu_c}^2$, ${\mu_b}^2$ and ${\mu_t}^2$.

iii. QCDNUM provides

- Eolution of α_s.
- Evolution of unpolarised parton density functions in NNLO, and polarised pdfs or fragmentation functions in NLO.
- Calculation of the structure functions F_2, F_L.

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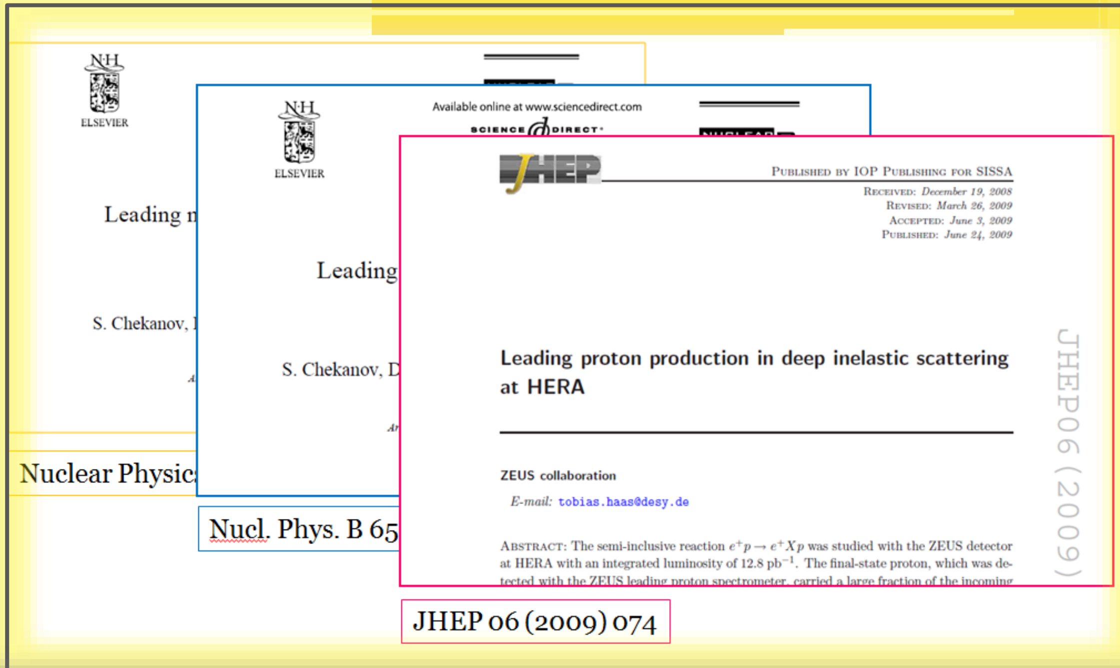

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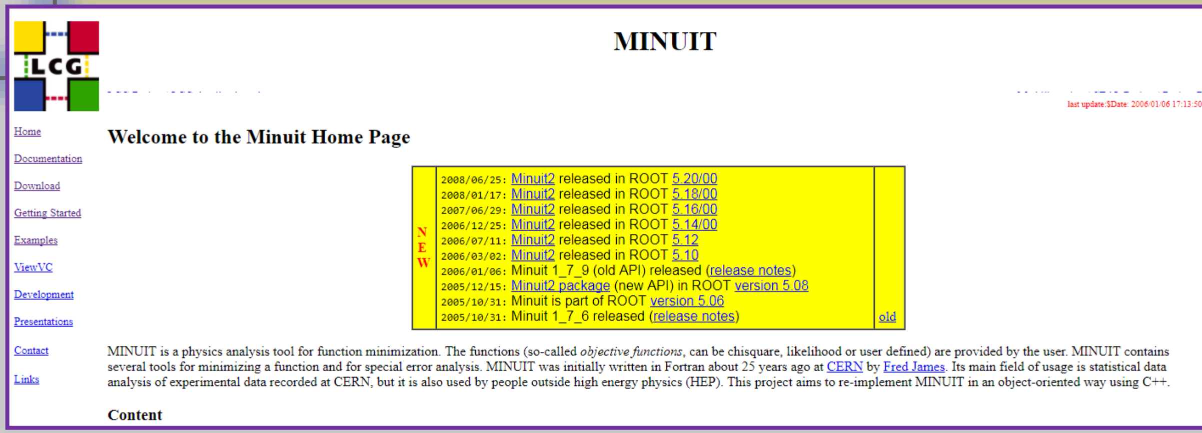

F. James and M. Roos, Comput. Phys. Commun. 10, 343 (1975).

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✓ The χ2 function is minimized by the CERN program library MINUIT.

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Using MINUIT to choose a value for parameters

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Initial PDF in certain Q^2 as an input is ready

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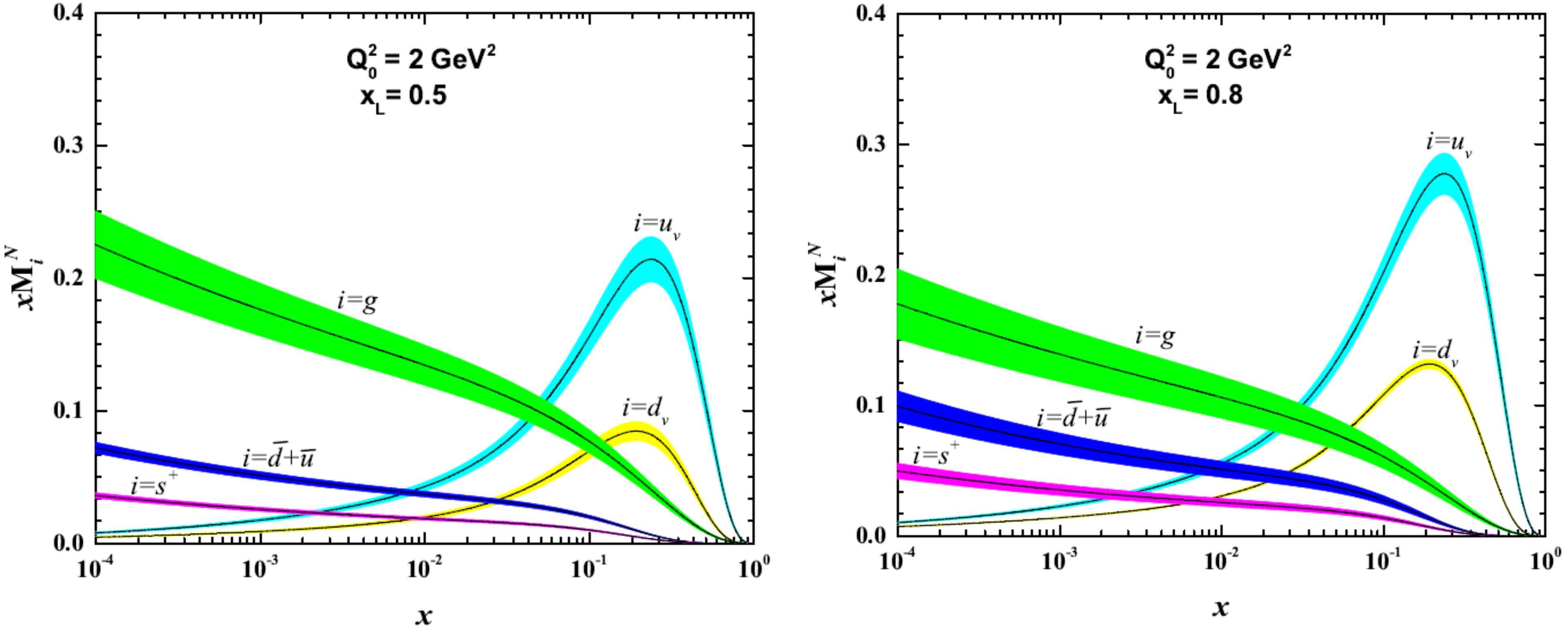
Parameters	$\mathcal{N}$	A	B	C	D
$\mathcal{W}_{u_v}(x_L)$	14.995 ± 0.272	3.866 ± 0.121	1.700 ± 0.037	0.0	0.0
$\mathcal{W}_{d_v}(x_L)$	11.004 ± 0.113	3.866 ± 0.121	1.50*	0.0	0.0
$\mathcal{W}_{\Delta}(x_L)$	47.026 ± 9.734	3.255 ± 0.301	0.948 ± 0.107	0.0	0.0
$\mathcal{W}_{s^+}(x_L)$	0.308 ± 0.025	0.708 ± 0.075	1.544 ± 0.049	17.387 ± 1.988	7.063 ± 0.314
$\mathcal{W}_{g}(x_L)$	1.750 ± 0.269	2.379 ± 0.179	2.426 ± 0.101	38.074 ± 8.965	14.850 ± 1.321
$\alpha_S(Q_0^2)$			0.356*		
$\alpha_S(M_Z^2)$			0.118		
χ^2/N_{pts}			$770.679/677 = 1.138$		

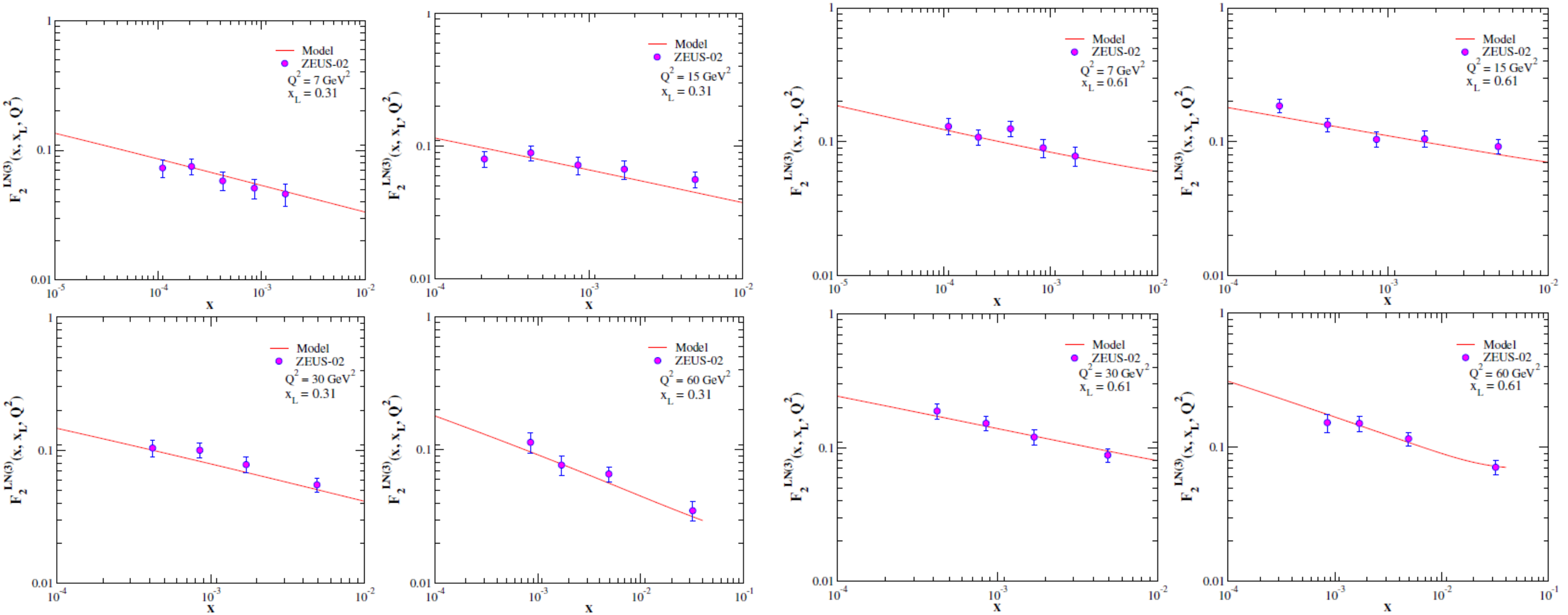
Parameter values for the STKJ17 QCD analysis at the input scale $Q_0^2 = 2\ GeV^2$

The nucleon FFs as a function of x



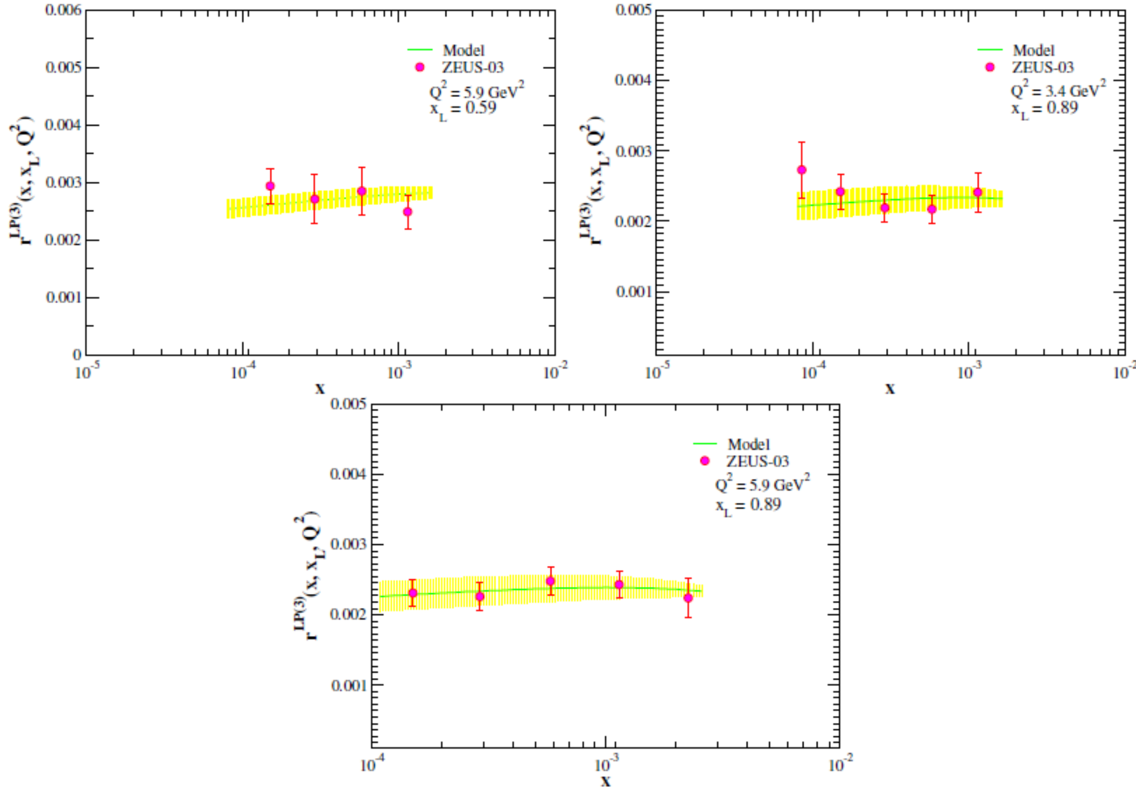
The nucleon FFs $xM_i(x,x_L,Q^2)$ for all parton densities at the input scale $Q_0^2 = 2\ GeV^2$ and for two representative bins of x_L= 0.5, and 0.8.

leading neutron structure functions as a function of x



The tagged-neutron structure function $F_2{}^{LN}$ as a function of x for some selected values of Q^2 at fixed value of x_L = 0.61 , 0.31

The structure function ratio for Forward Proton production



Our theory prediction for the structure function ratio $r^{LP(3)}(x, x_L, Q^2) = \frac{F_2{}^{LP}(x,x_L,Q^2)}{F_2{}^{P}(x,Q^2)}$ as a function of x for some selected values of Q^2 at fixed value of x_L = 0.59 , 0.89

Summary and Conclusions

- ✓ In the recent years, several dedicated experiments at the electron-proton collider HERA have collected high-precision data on the spectrum of leading-baryons carrying a large fraction of the proton’s energy.

- ✓ In addition to these experimental efforts, much successful phenomenology has been developed in understanding the mechanism of leading-baryon productions.

- ✓ We have presented the NLO QCD analysis of Nucleon FFs using available data from the forward neutron and forward proton production at HERA.

- ✓ It is shown that an approach based on the fracture functions formalism allows us to phenomenologically parametrize the Nucleon FFs.

Thanks for your attention

Backup

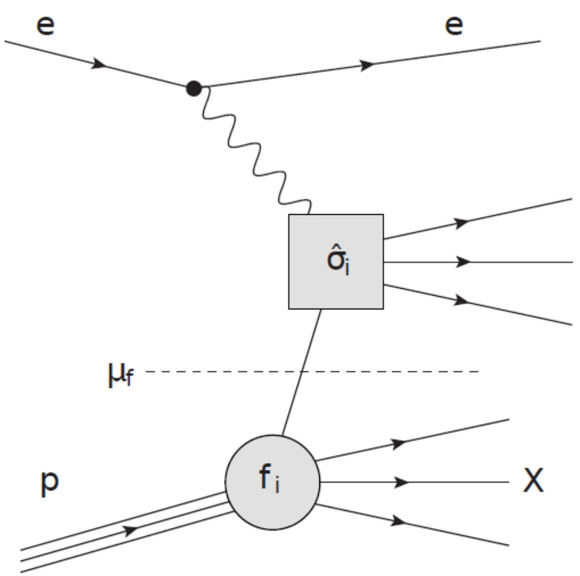
Factorization theorem

- The application of the QCD is limited to the short-distance region, where the perturbative QCD (pQCD) is used.

- At longer distances the DIS cross section cannot be calculated perturbatively due to large higher order corrections.

- The factorisation of the cross section into a short distance hard component calculable in pQCD, $\overline{\sigma}_i$, and long distance non-perturbative soft component f_i $(x,\ \mu_F\ )$ is called the factorisation theorem:

$$\sigma_{ep} = \sum_i f_i(x, \mu_f) \otimes \hat{\sigma}_i.$$



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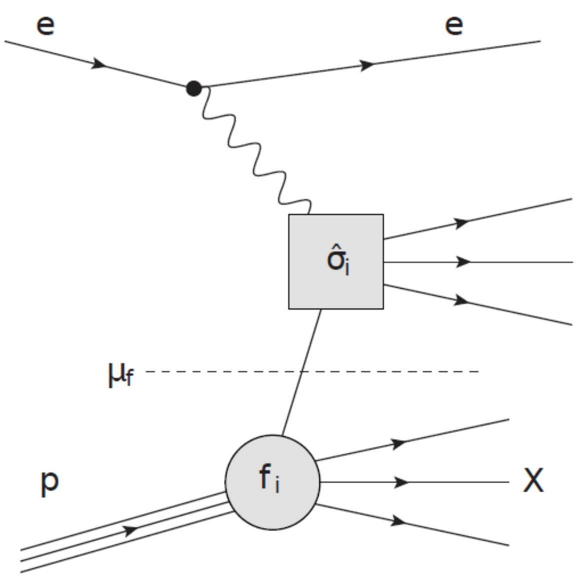
Factorization theorem

- The factorisation scale μ_F defines the border between the long and the short distance part.

- For example soft gluon emission is treated as being part of the (measured) proton structure below an energy scale of μ_F .

- The renormalisation scale μ_r on the other hand is needed to deal with the ultraviolet divergences of the higher orders in perturbation theory.



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a cross section involving hadrons can be decomposed into a *short distance* part and a *long distance* part:

- The short distance or hard scattering part can be calculated perturbatively using the (ultraviolet) renormalisable theory of QCD.
- The long distance part involves the *parton density functions* (PDFs), into which the infrared divergences of QCD are absorbed. The PDFs need to be extracted from experiments.

$$F_2(x, Q^2) = \sum_{i=q,g} \int_x^1 \mathrm{d}z \, C_2^i \left(\frac{x}{z}, \frac{Q^2}{\mu_r^2}, \frac{\mu_f^2}{\mu_r^2}, \alpha_s(\mu_r^2) \right) \, f_i(z, \mu_r^2, \mu_f^2)$$

The coefficient function $C_2{}^i$ is the hard scattering matrix element for an interaction of a photon with a parton i. It can be calculated using a perturbative expansion in α_s.

The parton density function *fi* gives the probability to find a parton with momentum fraction *z* in the proton.

Backup

$$Q^2 \frac{\partial M^{\mathrm{B}}_{i/P}(x, Q^2, x_L)}{\partial Q^2}$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} P_i^j(u)\, M^{\mathrm{B}}_{j/P}\left(\frac{x}{u}, Q^2, x_L\right.$$

$$\boxed{\begin{aligned} &\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(\mathrm{x},\mathrm{z},Q^2) \\ &= \frac{\alpha_s(Q^2)}{2\pi}\int_{\mathrm{x}/(1-\mathrm{z})}^{1}\frac{du}{u}P_j^i(u)M_{j,h/N}\left(\frac{\mathrm{x}}{u},\mathrm{z},Q^2\right) \\ &+ \frac{\alpha_s(Q^2)}{2\pi}\int_{\mathrm{x}}^{\mathrm{x}/(\mathrm{x}+\mathrm{z})}\frac{du}{(1-u)}\hat{P}_j^{i,l}(u)f_{j/p}\left(\frac{\mathrm{x}}{u},Q^2\right) \\ &\times D_{h/l}\left(\frac{\mathrm{z}u}{\mathrm{x}(1-u)},Q^2\right), \end{aligned}}$$

Backup

$$Q^2 \frac{\partial M^{\mathrm{B}}_{i/P}(x, Q^2, x_L)}{\partial Q^2}$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} P_i^j(u)\, M^{\mathrm{B}}_{j/P}\left(\frac{x}{u}, Q^2, x_L\right.$$

$$\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(\mathsf{x},\mathsf{z},Q^2)$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \int_{\mathsf{x}/(1-\mathsf{z})}^1 \frac{du}{u} P^i_j(u) M_{j,h/N}\left(\frac{\mathsf{x}}{u},\mathsf{z},Q^2\right)$$

$$+ \frac{\alpha_s(Q^2)}{2\pi} \int_{\mathsf{x}}^{\mathsf{x}/(\mathsf{x}+\mathsf{z})} \frac{du}{(1-u)} \hat{P}_j^{i,l}(u) f_{j/p}\left(\frac{\mathsf{x}}{u},Q^2\right)$$

$$\times D_{h/l}\left(\frac{\mathsf{z}u}{\mathsf{x}(1-u)},Q^2\right),$$

Backup

$$Q^2 \frac{\partial M^{\mathrm{B}}_{i/P}(x, Q^2, x_L)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u}\, P_i^j(u)\, M^{\mathrm{B}}_{j/P}\left(\frac{x}{u}, Q^2, x_L\right.$$

$p_T^{\ 2}$ of the forward baryons is integrated up to some $p_{T,max}$ ($p_T^{\ 2} < 0.5\ GeV^2$)

$$\frac{\partial}{\partial \ln Q^2} M_{i,h/p}(\mathrm{x},\mathrm{z},Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_{\mathrm{x}/(1-\mathrm{z})}^{1} \frac{du}{u} P^i_j(u) M_{j,h/N}\left(\frac{\mathrm{x}}{u},\mathrm{z},Q^2\right) + \frac{\alpha_s(Q^2)}{2\pi}\int_{\mathrm{x}}^{\mathrm{x}/(\mathrm{x}+\mathrm{z})} \frac{du}{(1-u)} \hat{P}_j^{i,l}(u) f_{j/p}\left(\frac{\mathrm{x}}{u},Q^2\right) \times D_{h/l}\left(\frac{\mathrm{z}u}{\mathrm{x}(1-u)},Q^2\right),$$

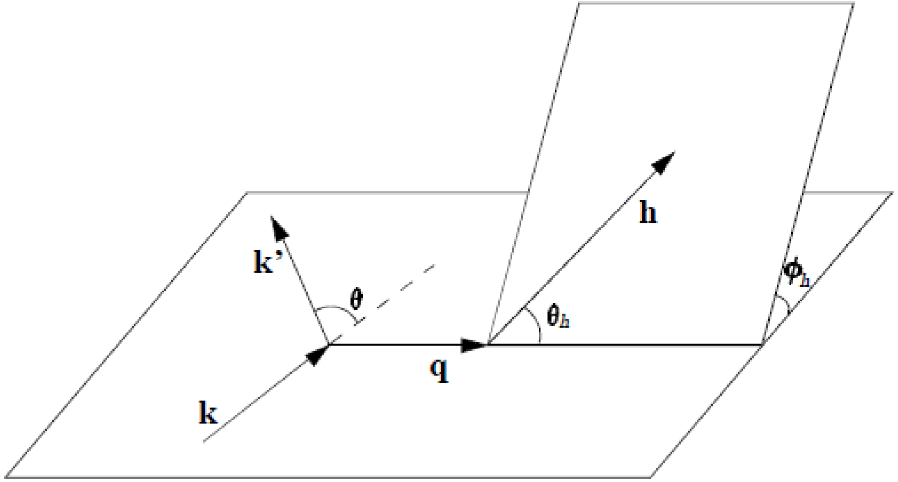
$p_T^{\ 2}$ is integrated over up to values of order Q^2.

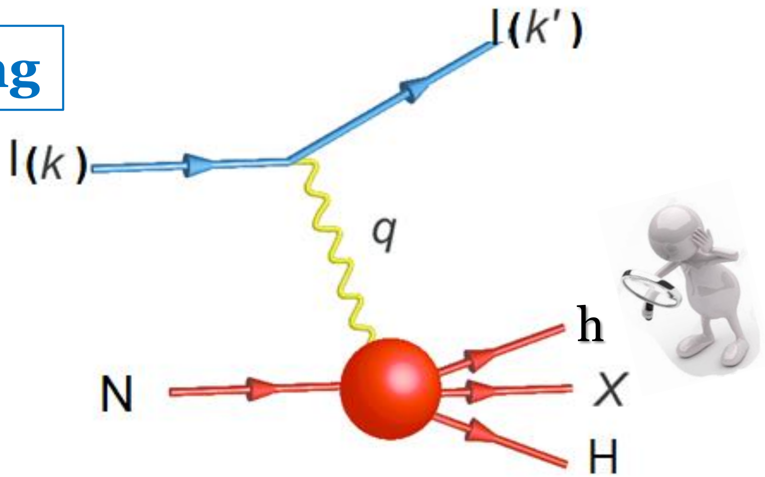
Backup

Semi-Inclusive Deep Inelastic Scattering

$$\sigma_{l+A\to l'+h+H+X} = \sigma_{current} + \sigma_{target}$$

$$= \sigma_{l+A\to l'+(h+H')+X} + \sigma_{l+A\to l'+H+(h+X')} \,.$$





✓ Kinematics in semi-inclusive inelastic scattering of lepton with momentum k with observation of hadron h.

Backup

Semi-Inclusive Deep Inelastic Scattering

$$\sigma_{l+A\to l'+h+H+X} = \sigma_{current} + \sigma_{target}$$

$$= \underline{\sigma_{l+A\to l'+(h+H')+X}} + \sigma_{l+A\to l'+H+(h+X')} \cdot$$

✓ the leading order expression for the cross section becomes:

Phys.Rev. D56 (1997) 426-432

$$\frac{d\sigma_p^h}{dxdydz} = \frac{1+(1-y)^2}{2y^2} \sum_{i=q,\bar{q}} c_i [f_{i/p}(x) D_{h/i}(z) + (1-x) M_{i,h/p}(x,(1-x)z)].$$

$$\boxed{c_i = 4\pi e_{q_i}^2 \alpha^2 / x(P+l)^2}$$

the parton distribution of flavor *i*: the probability to find parton *i* with momentum fraction x in the proton.

the fragmentation function of a hadron *h* from a parton *i*: It describe the collinear transition of a quark *i* into a hadron *h* with a fraction *z* of its momentum.

Backup

Semi-Inclusive Deep Inelastic Scattering

$$\sigma_{l+A\to l'+h+H+X} = \sigma_{current} + \sigma_{target}$$

$$= \sigma_{l+A\to l'+(h+H')+X} + \underline{\sigma_{l+A\to l'+H+(h+X')}} \cdot$$

✓ the leading order expression for the cross section becomes:

Phys.Rev. D56 (1997) 426-432

$$\frac{d\sigma_p^h}{dxdydz} = \frac{1+(1-y)^2}{2y^2} \sum_{i=q,\bar{q}} c_i [f_{i/p}(x) D_{h/i}(z) + (1-x) M_{i,h/p}(x,(1-x)z)]$$

$$\boxed{c_i = 4\pi e_{q_i}^2 \alpha^2 / x(P+l)^2}$$

- in order to describe hadrons produced in the target fragmentation region a new distribution has to be introduced, the so-called **fracture functions**.
- These distributions represent the probability of finding a parton of flavor *i* and a hadron *h* in the target *N*.

Backup

QCDNUM

A full NNLO analysis in ten lines

```
call QCINIT(6,' ')
call SETORD(iord)
call SETALF(as0,r20)
call SETTHR(nfin,q2c,q2b,q2t)
call GXMAKE(xmin,1,1,nxin,nx,iosp)
call GQMAKE(qq,wt,2,nqin,nq)
call FILLWT(0,id1,id2,nw)
call EVOLFF(func,def,iq0,eps)
call ALLPDF(x,q,pdf,0)
call STRFUN(2,proton,x,q,F2p,1,0)
```

Backup

QCDNUM initialization

```
call QCINIT(6,' ')
call SETORD(iord)
call SETALF(as0,r20)
call SETTHR(nfin,q2c,q2b,q2t)
call GXMAKE(xmin,1,1,nxin,nx,iosp)
call GQMAKE(
call FILLWT(
call EVOLFF(
call ALLPDF(
call STRFUN(
```

- Initialize QCDNUM
- Set LO, NLO, NNLO
- Set starting value of α_s
- Set FFNS or VFNS
- Set thresholds in the VFNS

Backup

Grids and weights

- Define (multiple) x grid(s)
- Define spline interpolation order
- Define μ^2 grid

```
call SETTHR(nfin,q2c,q2b,q2t)
call GXMAKE(xmin,1,1,nxin,nx,iosp)
call GQMAKE(qq,wt,2,nqin,nq)
call FILLWT(0,id1,id2,nw)
call EVOLFF(func,def,iq0,eps)
call ALLPDF(x,q,
call STRFUN(2,pr
```

- Partition the internal store
- Calculate weight tables

Backup

NNLO evolution of all PDFs

- User supplied function **func(i,x)** provides $f_i(x)$ at the input scale μ^2 for the gluon and $2n_f$ quark densities
- The input scale is given by the grid point **iq0**
- In the VFNS, **iq0** < charm threshold

```
call FILLWT(0,id1,id2,nw)
call EVOLFF(func,def,iq0,eps)
```

- The flavor decomposition of each input quark density is given in **def(-6:6,12)**

Backup

QCDNUM

Harvest the results….

```
call QCINIT(6,' ')
ca
ca
call SETTHR(nfin,q2c,q2b,q2t)
call G
call G
call F
call E
call ALLPDF(x,q,pdf,0)
call STRFUN(2,proton,x,q,F2p,1,0)
```

➲ Interpolate to x and μ^2 and return the densities g, d, u, …, t in **pdf(-6:6)**

➲ Calculate F_2, F_L, F'_L or xF_3 for a linear combination of quarks and anti-quarks as specified in the input **array(-6:6)**

Backup

MINUIT

- ❖ Program packages such as MINUIT (from CERN, but used by others than those involved in particle physics) by default use a series of

A. a simplex method

Nelder, J.A. & Mead, R. (1965). A simplex method for function minimization. Computer Journal, 7, 308-313.

B. a gradient method.

Variance algorithm for minimization
By William C. Davidon

The gradient method often used is based on a technique by Fletcher and Powell. This in turn is based on a small and nice algorithm called the Davidon's variance algorithm.

Backup

Neutron FFs uncertainties

- Standard Hessian method

Phys.Rev.D65:014013,2001

- For observable $\mathcal{O}$:

$$\Delta\mathcal{O} = \left[\Delta\chi^2_{\mathrm{global}} \sum_{i,j=1}^{k} \frac{\partial\mathcal{O}}{\partial p_i} C_{ij} \frac{\partial\mathcal{O}}{\partial p_j}\right]^{\frac{1}{2}}.$$

- For Neutron FFs :

$$\Delta\beta M(\beta, Q^2, x_L) = \left\{\sum_{i=1}^{k}\left(\frac{\partial\beta M}{\partial p_i}\right)^2 C(p_i, p_i) + \sum_{i\neq j=1}^{k}\left(\frac{\partial\beta M}{\partial p_i}\frac{\partial\beta M}{\partial p_j}\right) C(p_i, p_j)\right\}^{\frac{1}{2}},$$