TENSOR RESONANCES
IN $\eta\pi$
USING COMPASS DATA

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COMPASS COLLABORATION

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HADRON 2017
SEPTEMBER 25-29, 2017
SALAMANCA, SPAIN
Joint Physics Analysis Center

JPAC is a collaboration between theorists, phenomenologists, and experimentalists to provide phenomenological and data analysis tools for hadron physics.

In 2015, JPAC began working with COMPASS to develop models satisfying S-matrix constraints for data analysis.

\[ \pi p \rightarrow \eta^{(i)} \pi p \]

\[ \pi p \rightarrow (3\pi)p \]

http://www.indiana.edu/~ssrt/
Reaction Theory

Use fundamental physics from relativistic reaction theory (unitarity, analyticity, etc.) to constrain hadronic reaction amplitudes

\[ \mathcal{A}(s + i\epsilon, t) - \mathcal{A}(s - i\epsilon, t) \neq 0 \]

Amplitudes must satisfy these constraints, but the constraints do not fix the dynamics
Causality implies amplitudes are analytic functions of kinematic variables (energy).

Resonance poles lie underneath unitarity cuts on unphysical Riemann sheets.

Understanding of amplitude model important when continuing to complex energies.

\[ \pi \pi \rightarrow \rho \rightarrow \pi \pi \]
**ηπ at COMPASS**

The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons

As first step toward analysis of whole system, test model on $2^{++}$ channel

Expect $a_2(1320)$ (Large peak) to be narrow resonance, and try to determine parameters for the excited $a_2'$

First joint publication between JPAC and COMPASS — Template for future analyses


$J^{PC} = 2^{++}$

C. Adolph [COMPASS], arXiv:1707.02848

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Amplitude Model

\[ \frac{d\sigma}{d\sqrt{s}} \propto I(s) = \int_{t_{\text{min}}}^{t_{\text{max}}} dt \ p \ |a(s, t)|^2 \equiv \mathcal{N} \ p \ |a(s)|^2 \]

Since no \( t \)-dependence, assume flexible parameterization for production mechanism

Kinematics taken for Pomeron with vector coupling

Assume only \( \eta/\pi \) in intermediate state

\[ a(s) = p^2 q \hat{a}(s) \]

\[ \text{Im} \ \hat{a}(s) = \rho(s) \hat{f}^*(s) \hat{a}(s) \]

\[ \text{Im} = \sum_n \]
Amplitude Model

Use N-over-D method for amplitude model

\[ \hat{a}(s) = \frac{n(s)}{D(s)} \]

\(n(s)\) is a flexible model for production mechanism, given by

\[ n(s) = \frac{1}{c_3 - s} \sum_{j=1}^{n_p} a_j T_j(\omega(s)) \]

Production mechanism should be smooth in resonance region

\[ \omega(s) = \frac{s}{s + \Lambda} \]

\[ \Lambda = 1 \text{ GeV}^2 \]
Amplitude Model

Choose CDD parameterization for pole parameters

Form of CDD guarantees no first-sheet-poles

Left-hand-singularities modeled by pole singularity - parameter \( s_R \) corresponds to finite range of strong interactions: \( s_R = 1.5 \text{ GeV}^2 \)

Test different values for \( s_R \) to gauge systematics

\[
D(s) = D_0(s) - \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') N(s')}{s'(s' - s)}
\]

\[
D_0(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s} \quad \rho(s) N(s) = g \frac{\lambda^{5/2}(s, m_{\eta}^2, m_{\pi}^2)}{(s + s_R)^7}
\]
Results of Fit

Fit with $\chi^2$

6 terms in production expansion and 2 CDD poles

Statistical error estimates from bootstrap analysis $\chi^2/dof = 1.9$

<table>
<thead>
<tr>
<th>Denominator parameters</th>
<th>Production parameters [GeV$^{-2}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$ 0.532 ± 0.006</td>
<td>$a_0$ 0.471</td>
</tr>
<tr>
<td>$c_2$ 0.253 ± 0.007</td>
<td>$a_1$ 0.134</td>
</tr>
<tr>
<td>$c_3$ 2.38 ± 0.02</td>
<td>$a_2$ −1.484</td>
</tr>
<tr>
<td>$g$ 113 ± 1</td>
<td>$a_3$ 0.879</td>
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<tr>
<td></td>
<td>$a_4$ 2.616</td>
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<tr>
<td></td>
<td>$a_5$ −3.652</td>
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<tr>
<td></td>
<td>$a_6$ 1.821</td>
</tr>
</tbody>
</table>
Results of Fit

![Graph showing intensity vs. square root of s (GeV). The graph includes a peak at around 1.5 GeV with a decreasing trend as s increases. The y-axis is labeled as intensity multiplied by 10^3.]
Amplitudes

Production amplitude smooth in resonance region

Low $s$ behavior is suppressed by threshold functions

$$\hat{f} = N(s)/D(s)$$
Pole Positions

Continue amplitude to 2nd sheet to extract pole positions

\[ D_{II}(s) = D(s) + 2i\rho(s)N(s) \]

Pole positions

\[ D_{II}(s_p) = 0 \]

Mass and Width definitions

\[ m = \text{Re} \sqrt{s_p} \]

\[ \Gamma = -2 \text{Im} \sqrt{s_p} \]
\[ |D_{\text{II}}^{-1}(s)| \]

Sheet I

Sheet II
\[ D(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s} \]
Coupled Channels

\( a_2 \rightarrow \rho \pi \) is dominant mode

Fit for \( \eta \pi \) prefers two \( a_2 \) poles

Fit for \( \rho \pi \) prefers one \( a_2 \) pole

Pole positions are consistent for single channel approximation

\[
\hat{a}_j(s) = \sum_k [D(s)]^{-1}_{jk} n_k(s)
\]
Final Pole Positions

Final pole positions and estimated uncertainties for $a_2, a'_2$

\[ m(a_2) = (1307 \pm 1 \pm 6) \text{ MeV}, \quad m(a'_2) = (1720 \pm 10 \pm 60) \text{ MeV}, \]
\[ \Gamma(a_2) = (112 \pm 1 \pm 8) \text{ MeV}, \quad \Gamma(a'_2) = (280 \pm 10 \pm 70) \text{ MeV} \]
Extensions
\[ \pi^- p \rightarrow \pi^- \pi^- \pi^+ p \]

Developed quasi-two-body unitary model
- Assumes isobar is quasi-stable particle

First analysis in \( J^{PC} = 2^{-+} \)

Poles may hide under cuts generated by isobar decay
Exotics at COMPASS

Coupled channel analysis including $J^{PC} = 1^{--}$

Studies with P-wave on-going

Summary

Active collaboration between theorist and experimentalist to construct analytic models

Have constructed analytic amplitude for the fitting of partial wave intensities and the extraction of resonance pole positions for the D-wave $\eta\pi$ system at COMPASS

Have performed systematic studies on model to understand stability and nature of poles found

Pole positions are consistent with previous COMPASS study

This analysis serves as a template for more detailed ongoing analyses
Back-Up
Vector Pomeron Model

Pomeron trajectory intercept $\alpha \approx 1$. As first approximation, treat pomeron as particle with a vector coupling

$$A(\pi p \rightarrow \eta \pi p) = \sum_{\lambda} A_\lambda (\pi \mathbb{P} \rightarrow \eta \pi) V_\lambda (p \rightarrow \mathbb{P} p)$$

$$V_\lambda (p \rightarrow \mathbb{P} p) \sim \epsilon_\mu (\lambda) \bar{u} \gamma^\mu u$$

$$\sim \frac{s_{tot}}{\sqrt{s}} \sin \alpha_{recoil}$$

In Gottfried-Jackson Frame

$s_{tot}, s \rightarrow \infty$ with $s/s_{tot}$ constant

$$\sin \alpha_{recoil} \rightarrow \frac{1}{q}$$

Pion-Pomeron momentum
1 CDD vs 2 CDD

1 CDD pole

2 CDD poles
Residuals

\[ Res_j = \frac{O_j - I(s_j)}{\sigma_j} \]

Performed 1 million fits with random starting values for all parameters. Minimum solution was \( \chi^2 / dof \sim 2 \)

No clear region where deviation seems systematic.
Fixing the Range of Interaction

Preliminary fits at various $s_R$ to determine value

Consistent with range of strong interaction $\sim 1$ GeV