Calculation of resonances from $K\pi$ scattering.

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Based on:


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Motivation

- $\pi K$ scattering appears as final state in many hadronic processes.

Strange resonances are important:
- By themselves for spectroscopy.
- They appear as intermediate states.

Open issues:
- $K_0^*(800)$ still needs confirmation according to PDG.
- Most of them are not well determined, results come to be incompatible, pole vs Breit-Wigner definition.
- Parameters often extracted from models.
Steps:

1. Study the data to make a set compatible with gaussianity ⇒ Ruiz Arriola.
2. Obtain set of partial waves that describe the data in the region of interest.
3. Construct the amplitudes \( T^l(s, t) = \sum_l (2l + 1) P_l(z_s) f^l_I(s) \).
4. Make them compatible with analyticity ⇒ Peláez, Ruiz de Elvira.
5. Then use a model independent technique to calculate the resonance poles.
Dispersion relations

- We construct the amplitudes using the whole tower of partial waves.
- For $K\pi$ scattering we have two independent amplitudes $T^{1/2}$ and $T^{3/2}$, we study both combinations $T^+$ and $T^-$.
- Room for improvement $\rightarrow$ Constrained fits.
- Above $1.8\text{GeV}$ the discrepancies are too big.

![Graph 1](image1.png)

$\text{Re} T^+(s)$

- Dispersive UFD
- Input UFD

$\text{d}^2 = 3.9$

![Graph 2](image2.png)

$\text{Re} T(s)$

- Dispersive UFD
- Input UFD

$\text{d}^2 = 5.1$
Dispersion relations

- The change in the symmetric amplitude around $1 - 1.2\text{GeV}$ is caused by the change of the $S^{3/2}$-wave.
- The huge change of the antisymmetric one is caused by the Regge $\pi K$ factorization constant.
- Now the amplitudes are fairly compatible with the analyticity requirements.
- The partial waves are now suitable to be used in our analysis.
Constrained Fits

- Almost unchanged below 1.5 GeV.
- Changes above that point, the CFD solution starts to be incompatible with the UFD.
Comparison of the scattering lengths before and after the minimization procedure.

**Table: Scattering lengths.**

<table>
<thead>
<tr>
<th></th>
<th>SL</th>
<th>UFD</th>
<th>CFD</th>
<th>Roy-Steiner result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\pi}a_0^{1/2}$</td>
<td></td>
<td>0.222±0.014</td>
<td>0.218±0.014</td>
<td>0.224±0.022</td>
</tr>
<tr>
<td>$m_{\pi}a_0^{3/2}$</td>
<td></td>
<td>-0.101±0.03</td>
<td>-0.054±0.014</td>
<td>-0.0448±0.0077</td>
</tr>
<tr>
<td>$m_{\pi}a_1^{1/2}$</td>
<td></td>
<td>0.031±0.008</td>
<td>0.024±0.005</td>
<td>0.019±0.001</td>
</tr>
</tbody>
</table>

Dirac collaboration measure the difference between the scalar partial waves.

$$\frac{1}{3} \left( a_0^{1/2} - a_0^{3/2} \right) = 0.11^{+0.09}_{-0.04} m_{\pi}^{-1}, \quad \text{(DIRAC)} \quad (1)$$

Our results are compatible, although we obtain much smaller errors.

$$\frac{1}{3} \left( a_0^{1/2} - a_0^{3/2} \right) = 0.091^{+0.006}_{-0.005} m_{\pi}^{-1}. \quad \text{(CFD)} \quad (2)$$
The partial wave is described by a Padé approximant in the complex plane.

Model dependent techniques are not always suitable for rigorous determinations.

High $L$ or wide resonance poles are not stable when calculated through simple models.

Usual $(q(s)/q(s_r))^L$ and $B_L(q, q_r) \Rightarrow$ deviations in the width.

Rigorous dispersive techniques cannot be applied at high energies (inelastic regions).

We use this Padé approximants to calculate the parameters of the strange resonances.

It is a model independent calculation. No specific functional form.

No a priori relation between residue and pole position.
Padé approximants to extract resonances

- Sequences of Padés used up to a given \( N \).
- Based on its analytic properties, for example, when searching one pole the approximants read

\[
t_1(s) \sim P_1^N(s, s_0) = \sum_{k=0}^{N-1} a_k (s - s_0)^k + \frac{a_N(s - s_0)^N}{1 - \frac{a_{N+1}}{a_N}(s - s_0)}
\]  

(3)

- With a pole located at \( s_p = s_0 + \frac{a_N}{a_{N+1}} \). Where \( a_n = F^{(n)}(s_0)/n! \).
- Which is a simple, but powerful manner of calculating simple resonances.
- The resonance may be surrounded by other non-analytic structures \( \Rightarrow \) more poles have to be included inside our approximants.
- We always truncate the sequence when the difference between the poles is smaller than the experimental error.
The method is suitable for the calculation of both elastic and inelastic resonances. The Padé sequence gives us the continuation to the continuous Riemann Sheet. This method was applied for the first time to study the $\sigma$ and $\rho$ resonances in P. Masjuan and J. J. Sanz-Cillero, Padé approximants and resonance poles, Eur. Phys. J. C 73 (2013) 2594 doi:10.1140/epjc/s10052-013-2594-4.
**K*(892) as a test method**

- The $K^*(892)$ is the simplest resonance that could be determined through padé approximants. Its parameters come out to be compatible with other determinations (pole) as it is a simple Breit-Wigner like resonance. $\sqrt{s_p} = (892 \pm 1) - i(29 \pm 1)$

![Graph](image)

**Figure:** $P^{1/2}$ phase shift.
First we obtain the difference between $s_p^{N-1}$ and $s_p^N$ for the whole energy region of the fit.

Run a Montecarlo for every fit to calculate the statistical errors of every resonance.

We stop at a $N$ ($N + 1$ derivatives) where the systematic uncertainty is smaller than the statistical one (usually $N = 4$ is enough).

For every fit we search the $s_0$ that gives the minimum difference between $N - 1$ and $N$.
However, the $K_0^*(800)$ is a broad resonance, with more than one non-analytic structure surrounding it due to the distance from the real axis.

Two??

$$P_N^2(s, s_0) = \frac{\sum_{k=0}^{N} (a_k a_N^2 - a_k a_{N-1} a_{N+1} - a_{k-1} a_{N-1} a_{N+1} + a_{k-1} a_N a_{N+2} + a_{k-2} a_{N+1}^2 - a_{k-2} a_N a_{N+2})(s - s_0)^k}{a_N^2 - a_{N-1} a_{N+1} + (a_{N-1} a_{N+2} - a_N a_{N+1})(s - s_0) - (a_N a_{N+2} - a_{N+1}^2)(s - s_0)^2}$$

More?? → $P_N^M(s, s_0) = Q_N(s, s_0) / R_M(s, s_0)$

Instead of calculating $P(s, s_0)$ one can use $P(\omega(s), \omega_0)$ with $\omega(s)$ being a convenient transformation.
As pointed out in I. Caprini, P. Masjuan, J. Ruiz de Elvira and J. J. Sanz-Cillero, Uncertainty estimates of
the $\sigma$-pole determination by Padé approximants, Phys. Rev. D 93 (2016) the determination of broad
resonances is not stable $\Rightarrow$ small variations of the data description
could produce significant changes in the parameters $\Rightarrow$ systematic
uncertainties.

Determination of uncertainties $\Rightarrow$ different models to describe the
data AND dispersion relations.

In this case we use three different parameterizations for
$t'^l = \frac{1}{\sigma(s)(\cot\delta'^l(s)-i)}$

- Conformal mapping $\cot\delta'^l(s) = \sum B_n\omega(s)^n$ where
  \[
  \omega(s) = \frac{\sqrt{y(s) - \alpha\sqrt{y(s_0) - y(s)}}}{\sqrt{y(s) + \alpha\sqrt{y(s_0) - y(s)}}}
  \]
  and $y(s) = (\frac{s-su}{s+su})^2$

- Effective range parameterization $\sigma(s)\cot\delta'^l(s) = \sum B_n q^n$

- Chew-Mandelstan parameterization $\rightarrow$ using $J(s) = \frac{2}{\pi} + \frac{\sigma(s)}{\pi} \log \frac{\sigma(s)-1}{\sigma(s)+1}$
  instead of $\sigma(s)$
All other different parameterizations are almost equal in the real axis but produce different parameters.

Best value → Average between different number of poles including the systematic uncertainties.

Figure: $S^{1/2}$ phase shift.
More than one pole is included inside the approximants, the best value is an average of the results.

For the $K_0^*(800)$ resonance we obtain

$$\sqrt{s_p} = (670 \pm 18) - i(295 \pm 28) \text{MeV}$$

$$\sqrt{s_p} = (682 \pm 29) - i(274 \pm 12) \text{MeV} \text{(PDG)}$$

(5)
$K_0^*(1430)$

- Almost elastic $Br \approx 97\%$.
- Clear peak in scattering.
- Broad, influenced by other structures.
For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19)\text{MeV}$$

$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40)\text{MeV (PDG)}$$

(6)
- Inelastic $Br \simeq 7\%$.
- Smooth shape, not clear structure.
- Close to a threshold.
For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59}) \text{MeV}$$

$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10) \text{MeV (PDG)}$$
For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2)\text{MeV}$$

$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3)\text{MeV} (PDG) \quad (8)$$
For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14) \text{MeV}$$

$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11) \text{MeV (PDG)}$$

(9)
Summary

- Resonance parameters obtained from model-independent analytic approach.
  - Data checked to fulfill gaussianity.
  - DR imposed to the parameterizations.
- $K_0^*(800)$ pole confirmed. Parameters compatible with PDG and Roy-Steiner equations.
- Inelastic resonances are also calculated using this Padé technique.
- Systematic uncertainties also considered.

**TO DO:**
- $f_0(980)$ and $f_0(1370)$ accessible through $\pi\pi$ scattering and $\pi\pi \rightarrow K\bar{K}$.
- $f_0(980)$ appears to be too close from threshold to be accurate.
Thank you for your attention!