Glueball and Meson Spectra,
Semileptonic Decay Constants,
Strong Effective Coupling within
Analytic (Infrared) Confinement

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Outline

• Motivation

• Model: Two-particle bound states

• Infrared regularization

• Effective strong coupling

• Glueball: mass, radius

• Mesons: mass spectrum, semileptonic decay constants

• Fermi Coupling within CCQM

• Summary and Outlook
Many novel behaviors are expected in the IR region $\sim 1 \div 5$ GeV.

Perturbation Theory cannot be used $\Rightarrow$ non-PT methods required.

Quark confinement, QCD running coupling, glueball states, etc. requires a correct description of hadron dynamics in IR region.

Construct simple, reliable models and apply in different sections (e.g.):
- **Strong Interaction**: spectrum of mesons in a wide range of scale;
- **Exotic States**: the lowest-state glueball mass, radius, etc.;
- **Semileptonic decay constants**;
- **Strong Effective and Fermi couplings**;

"Conventional" (**quark-antiquark**) states:

Spin: $\frac{1}{2} \otimes \frac{1}{2}$

Ground states (**Mesons**): $J^{PC}=0^{-+}$, $J^{PC}=1^{--}$

"Exotic" (**di-gluon**) states:

Spin: $1 \otimes 1$

Ground state (**Glueball**): $J^{PC}=0^{++}$
Consider a relativistic quantum-field model of quark-gluon interaction.  

\[ L = -\frac{1}{4} \left( F_{\mu\nu}^A - g f^{ABC} A_\mu^B A_\nu^C \right)^2 + \sum_f \left( \bar{q}_f^a \left[ \gamma^\alpha \partial_\alpha - m_f + g \Gamma^\alpha C A_\alpha^C \right]^{ab} q_f^b \right) \]

\[ F_{\mu\nu}^B \equiv \partial_\mu A_\nu^B - \partial_\nu A_\mu^B \quad \Gamma^\alpha \equiv i \gamma^\alpha \gamma^C \]

**Partition Functional** written in terms of quark and gluon variables

\[ Z = \int \int \delta \bar{q} \delta q \int \delta A \exp \left\{ -\int dx L[\bar{q}, q, A] \right\} \]

**LO contributions to quark-antiquark and two-gluon bound states:**

\[ Z_{(\bar{q}q)} = \int \int \delta \bar{q} \delta q \exp \left\{ -\left( \bar{q} S^{-1} q \right) + \frac{g^2}{2} \left\langle (\bar{q} \Gamma A q)(\bar{q} \Gamma A q) \right\rangle_D \right\} \]

\[ Z_{(AA)} = \left\langle \exp \left\{ -\frac{g}{2} \left( f A A F \right) \right\} \right\rangle_D \]

\[ \left\langle (\bullet) \right\rangle_D \equiv \int \delta A \ e^{-\frac{1}{2}(AD^{-1}A)} (\bullet) \]
• Allocate one-gluon exchange between colored currents

\[
L_{qq} = \frac{g^2}{2} \sum_{f_1,f_2} \int \int dx_1 dx_2 J^B_{\mu f_1 f_2}(x_1, x_2) D^{BC}_{\mu \nu}(x_1, x_2) J^C_{\nu f_1 f_2}(x_2, x_1),
\]

\[
J^B_{\mu f_1 f_2}(x_1, x_2) \equiv i \bar{q}_{f_1}(x_1) \gamma_\mu t^B q_{f_2}(x_2).
\]

• Isolate color-singlet combination

• Perform Fierz transformation for spins \((J = S, P, A, V, T)\)

• Introduce orthonormalized system: \([U_Q]\) with quantum numbers \(Q = \{n, l, \ldots\}\):

• Diagonalization on colorless quark currents \(J_N(x)\) with \(N = \{Q, J, f_1, f_2\}\)

• Gaussian representation: a new path integration over auxiliary fields \(B_N\):

\[
e^{\frac{1}{2} e^2 g^2 (J_N^+ J_N)} = \int \int \delta B_N^+ \delta B_N \exp \left\{-\sum_N (B_N^+ B_N) + g \sum_N [(B_N^+ J_N) + (J_N^+ B_N)]\right\}
\]

• Explicit path-integration over quark variables and write the effective action

• Hadronization Ansatz: \(B_N\) fields are identified as meson fields with \(N\)
• $Z_N$ rewritten in terms of $B_N$ and all quadratic field configurations are isolated:

$$Z_{(\bar{q}q)} \rightarrow Z_N = \int \prod_N \delta B_N \exp \left\{ -\frac{1}{2} (B_N [1 + g^2 \text{Tr}(V_N S_{m_1} V_N S_{m_2})] B_N) + W_{\text{resid}} [B_N] \right\}$$

• Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system $\{U_N\}$

$$g^2 \text{Tr}(V_N S_{m_1} V_N' S_{m_2}) = (U_N \lambda U_N') = \lambda_N (-p^2) \delta^{JJ'} \delta^{QQ'}$$

• Symmetric Bethe-Salpeter kernel is defined:

$$\alpha \cdot \lambda_j (-p^2) = \frac{4 g^2 C_J}{9} \int \frac{d^4 k}{(2\pi)^4} \{V(k)\}^2 \cdot \Pi_j (k, p)$$

$$\Pi_j (p, k) = -\frac{1}{4!} \text{Tr} \left\{ \Gamma_j \tilde{S}_{m_1} \left( \hat{k} + \xi_1 \hat{p} \right) \Gamma_j \tilde{S}_{m_2} \left( \hat{k} - \xi_2 \hat{p} \right) \right\}$$

$$V_j (k) = \int dx \sqrt{D(x)} U_j (x) e^{ikx}$$

G.Ganbold @ Hadron-2017 (27 Sep 2017)
UV Regularization

• Renormalization:

\[ U_{\text{REN}}(x) \equiv \sqrt{-\alpha \lambda_N (M_N^2)} \cdot U_N(x) \]

\[
\langle U_N \left| 1 + \alpha \lambda_N (-p^2) \right| U_N \rangle = \left\langle U_N \left| 1 + \alpha \lambda_N (M_N^2) - \alpha \lambda_N (M_N^2)(p^2 + M_N^2) \right| U_N \right\rangle
\]

\[ = \left\langle U_{\text{REN}} \left| (p^2 + M_N^2) \right| U_{\text{REN}} \right\rangle \]

Meson Mass Equation

\[ 1 + \alpha \lambda_N (M_N^2) = 0, \quad p^2 = -M_N^2 \]

• Infrared (IR) divergences appear in the loop and vertex functions:

\[ \Pi_j(p,k) = -\frac{1}{4!} \text{Tr} \left\{ \Gamma_j \tilde{S}_{m_1} \left( \hat{k} + \xi_1 \hat{p} \right) \Gamma_j \tilde{S}_{m_2} \left( \hat{k} - \xi_2 \hat{p} \right) \right\} \]

\[ V_j(k) = \int dx \sqrt{D(x)} U_j(x) e^{ikx} \]
**IR Regularization**

- “**Infra-red**” regularizations of *propagators* remove these divergencies:

\[
\tilde{S}_m(\hat{p}) = \frac{1}{-i\hat{p} + m} = (i\hat{p} + m) \cdot \int_{0}^{\infty} dt \exp \left\{ -t \cdot (p^2 + m^2) \right\}
\]

\[
\Rightarrow \tilde{S}_{IR}(\hat{p}) = (i\hat{p} + m) \cdot \int_{0}^{\infty} dt \exp \left\{ -t \cdot (p^2 + m^2) \right\}
\]

\[
D(x) = \frac{1}{4\pi^2 x^2} = \int_{0}^{\infty} ds \ e^{-sx^2} \Rightarrow D_{IR}(x) = \int_{0}^{\infty} ds \ e^{-sx^2}
\]

- Another “**Infra-red-Loop**” regularization of *whole loop* has been used in

G. Ganbold, T. Gutsche, M. Ivanov, V. Lubovitsky
QCD Effective Coupling

**THEORY:** QCD predicts a dependence of $\alpha_s(Q)$ on energy scale $Q$. This dependence is described theoretically by the Renorm Group equations.

**EXPERIMENT:** but its actual value must be obtained from experiment. It is well determined experimentally at relatively high energies $Q > 2$ GeV.

Perturbation theory: one-loop (dashed) and two-loop (solid) approximations to $\alpha_s(Q^2)$, where $Z = Q^2/\Lambda^2$

$\alpha_s(Q < 1) \rightarrow ?$
Effective (Mass-dependent) Strong Coupling

\[ 1 + \tilde{\alpha}_s \lambda_N (M_N^2) = 0 \text{, } p^2 = -M_N^2 \]

\[
U_{n\ell \mu}(x,a) \approx T_{\ell \mu}(ax) \cdot L_n^{l+1/2}(a^2 x^2) \cdot \sqrt{D(x)} \cdot e^{-ax^2}, \quad \sum_{\mu} \int dx \left[ U_{nl\mu}(x,a) \right]^2 = 1
\]

- Particularly, for
  - \( n=l=0, \)
  - \( m_1 = m_2 = M/2: \)

\[
\tilde{\alpha}_s(M) = 1/\tilde{\lambda}(M^2)
\]

Infra-red fixed point:

\[ \tilde{\alpha}_s(0) \approx 1.032 \text{ for any } \Lambda > 0 \]

There exist dispersion relations between time-like and space-like couplings:

\[ \alpha(M) \leftrightarrow \alpha(Q) \]
**Theoretical status:** The existence of glueballs is predicted by QCD because of the self-interaction of gluons.
- Lattice calculations, QCD sum rules, Tube model, Constituent glue model, Holographic QCD model, ...

**Experimental status:** Signatures expected for glueballs:
- enhanced production in gluon-rich channels of radiative decays,
- decay branching fractions incompatible with (q-qbar) states.

\[
J = 0, 1, 2, 3
\]
\[
PC = ++, --, +- , --
\]

\[
M_G \approx 1.6 - 4.9 \text{ GeV}
\]
Scalar Glueball

$J^{PC} = 0^{++}$

Scalar glueball mass:

$\Lambda = 236.0 \text{ MeV}$, \hspace{1cm} $\alpha(M_G) = 0.451$

$M_G \approx 1738.6 \text{ MeV}$

1750±50±80 MeV \hspace{1cm} C.J. Morningstar, M. Peardon (2004).
1710±50±58 MeV \hspace{1cm} QLQCD Infinite volume, continuum limit (Y. Chen 2006)
1710±50 MeV \hspace{1cm} Analytic confinement model (G. Ganbold PRD79, 2009)
1790±50±20 MeV \hspace{1cm} UK-QCD Collaboration (2014)

Scalar glueball “radius”:

$r_G \cdot M_G = \frac{M_G}{2\Lambda} \sqrt{\int d^4 x \cdot x^2 \cdot W(x) \cdot U^2(x)} 
\approx 4.35 \hspace{1cm} \text{UK-QCD Collaboration (2014)}$

Gluon condensate:

$\left\langle \frac{\alpha}{\pi} F^2 \right\rangle = \frac{16 N_c}{\pi} \alpha \Lambda^2 = 0.0214 \text{ GeV}^4$

$\approx 0.0223 \pm 0.0041 \text{ GeV} \hspace{1cm} S. Narison, PLB706 (2012)$

G.Ganbold @ Hadron-2017 (27 Sep 2017)
Meson Spectrum

a) Analytic results

- Asymptotical Regge-type behaviour:

\[ M_{nl}^2 \approx M_{00}^2 + (n+l) \cdot \text{const} \quad \text{for } \{n,l\} \geq 3 \]

- Due to spin effect vector mesons are heavier than pseudoscalars at the same quark contents:

\[ 1 \approx C_J \cdot \exp(M_J^2) \cdot (M_J^2 + \text{const}) \]

\[ 1 = C_P > C_V = 1/2 \]

\[ M_P^2 < M_V^2 \]

- The coupling is bounded from above:

\[ \alpha_s(M) = 1/\lambda_J(M^2) \leq \alpha_s^{\text{max}} \]
## Conventional Meson Mass Estimates

Model parameters fixed by fitting the meson spectrum:

\[ \Lambda = 236.0 \text{ MeV}, \]

\[ m_{ud} = 227.65 \text{ MeV}, \quad m_s = 420.07 \text{ MeV}, \]

\[ m_c = 1521.61 \text{ MeV}, \quad m_b = 4757.20 \text{ MeV}. \]

<table>
<thead>
<tr>
<th>( J^{PC} = 0^{-+} )</th>
<th>PDG-2016</th>
<th>Our est.</th>
<th>( J^{PC} = 1^{--} )</th>
<th>PDG-2016</th>
<th>Our est.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D (~uc)</td>
<td>1869.62</td>
<td>1893.6</td>
<td>( \rho ) (uu)</td>
<td>775.26</td>
<td>774.3</td>
</tr>
<tr>
<td>D_s (~sc)</td>
<td>1968.50</td>
<td>2003.7</td>
<td>( K^* ) (us)</td>
<td>891.66</td>
<td>892.9</td>
</tr>
<tr>
<td>( \eta_c ) (~cc)</td>
<td>2983.7</td>
<td>3032.5</td>
<td>( D^* ) (uc)</td>
<td>2010.29</td>
<td>2003.8</td>
</tr>
<tr>
<td>B (~ub)</td>
<td>5279.26</td>
<td>5215.2</td>
<td>( D_s^* ) (~sc)</td>
<td>2112.3</td>
<td>2084.1</td>
</tr>
<tr>
<td>B_s (~sb)</td>
<td>5366.77</td>
<td>5323.6</td>
<td>( J/\psi ) (~cc)</td>
<td>3096.92</td>
<td>3077.6</td>
</tr>
<tr>
<td>B_c (~cb)</td>
<td>6274.5</td>
<td>6297.0</td>
<td>( B^* ) (~ub)</td>
<td>5325.2</td>
<td>5261.5</td>
</tr>
<tr>
<td>( \eta_b ) (~bb)</td>
<td>9398.0</td>
<td>9512.5</td>
<td>( Y ) (~bb)</td>
<td>9460.30</td>
<td>9526.4</td>
</tr>
</tbody>
</table>

|relative errors| < 1.8%
Semileptonic Decay Constants of Mesons

Important value in particle physics:

$$i f_P p_\mu = \langle 0 | J_\mu (0) | U_{\text{renorm}} (p) \rangle$$

Mass dependence of semileptonic decay constants (Exp. data 2015):

$$\pi^+ \rightarrow \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma$$

$$D_s^+ \rightarrow l^+ \nu$$
\[
if_p p_\mu = \frac{g}{6} \int \frac{d^4k}{(2\pi)^4} \int dx e^{ikx} U_R(x) \sqrt{D(x)} \text{Tr} \left\{ i\gamma_5 \tilde{S}_m (\hat{k} + \xi_1 \hat{p}) i\gamma_5 \gamma_\mu \tilde{S}_m (\hat{k} - \xi_2 \hat{p}) \right\}
\]

**R** – “size” of meson in mass scale

\[
\tilde{U}_R(k) \sim \int_0^1 ds f(s) \cdot \exp \left\{ -\frac{s \cdot k^2}{R^2} \right\}
\]

Model parameters:

\[
\Lambda = 236.0 \text{ MeV},
\]
\[
m_{ud} = 227.65 \text{ MeV}, \quad m_s = 420.07 \text{ MeV},
\]
\[
m_c = 1521.61 \text{ MeV}, \quad m_b = 4757.20 \text{ MeV}.
\]

**R** – estimated to fit experimental data on meson masses and semileptonic decay constants (in GeV)

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>D_s</th>
<th>(\eta_c)</th>
<th>B</th>
<th>B_s</th>
<th>B_c</th>
<th>(\eta_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>0.93</td>
<td>1.08</td>
<td>1.83</td>
<td>1.73</td>
<td>2.18</td>
<td>3.34</td>
<td>3.80</td>
</tr>
<tr>
<td>K*</td>
<td>0.33</td>
<td>0.38</td>
<td>0.78</td>
<td>0.90</td>
<td>2.40</td>
<td>3.34</td>
<td>2.80</td>
</tr>
<tr>
<td>J/(\Psi)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- Estimated values of the Leptonic decay constants:

<table>
<thead>
<tr>
<th>( J^{PC} = 0^{-+} )</th>
<th>Exp.data</th>
<th>Our estim.</th>
<th>( J^{PC} = 1^{--} )</th>
<th>Exp.data</th>
<th>Our estim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_D )</td>
<td>206.7 ± 8.9</td>
<td>207</td>
<td>( f_\rho )</td>
<td>221 ± 1</td>
<td>221</td>
</tr>
<tr>
<td>( f_{Ds} )</td>
<td>257.5 ± 6.1</td>
<td>257</td>
<td>( f_{K^*} )</td>
<td>217 ± 7</td>
<td>217</td>
</tr>
<tr>
<td>( f_{\eta_c} )</td>
<td>238 ± 8</td>
<td>238</td>
<td>( f_{D^*} )</td>
<td>245 ± 20</td>
<td>245</td>
</tr>
<tr>
<td>( f_B )</td>
<td>192.8 ± 9.9</td>
<td>193</td>
<td>( f_{Ds^*} )</td>
<td>272 ± 26</td>
<td>271</td>
</tr>
<tr>
<td>( f_{Bs} )</td>
<td>238.8 ± 9.5</td>
<td>239</td>
<td>( f_{J/\psi} )</td>
<td>415 ± 7</td>
<td>416</td>
</tr>
<tr>
<td>( f_{Bc} )</td>
<td>489 ± 5</td>
<td>488</td>
<td>( f_{B^*} )</td>
<td>196 ± 44</td>
<td>196</td>
</tr>
<tr>
<td>( f_{\eta_b} )</td>
<td>801 ± 9</td>
<td>800</td>
<td>( f_Y )</td>
<td>715 ± 5</td>
<td>715</td>
</tr>
</tbody>
</table>

+ Agreement between estimated and experimental data is accurate.

+ The meson “size” shrinks \( \sim 1/R \) as the mass grows.
**Fermi Coupling**

- **Yukawa-type model**
  \[ L_Y = \bar{q} (i\hat{\partial} - m) q + \phi_0 (-\partial^\mu \partial_\mu - M_0^2) \phi_0 + g_0 \phi_0 (\bar{q} \Gamma q) \]

- **Fermi-type model**
  \[ L_F = \bar{q} (i\hat{\partial} - m) q + \frac{G}{2} (\bar{q} \Gamma q)^2 \]

Generating functional \[ \Pi(x - y) \equiv -i \cdot tr \{ \Gamma S(x - y) \Gamma S(y - x) \} ; \]

- **if two conditions are fulfilled:**
  \[ -\frac{1}{G} + \tilde{\Pi}(M^2) = 0 \]
  \[ Z \equiv 1 - g_r^2 \cdot \frac{d}{dM^2} \tilde{\Pi}(M^2) = 0 \]

- **then, these two theories are equivalent:**
  \[ L_{F}^{int} = \frac{G}{2} J_H^2 (x) \iff L_Y^{int} = g_H H(x) J_H (x) \]

\[ \phi_0 \equiv Z^{1/2} \phi_r = 0 \]

**Compositeness Condition**
CCQM: Meson-Quark Interaction

- Lagrangian:
  \[ L_{\text{int}} = g_H H(x) J_H(x) \]

- Quark currents (for mesons):
  \[ J_H(x) = \int dx_1 \int dx_2 F_H(x; x_1, x_2) \bar{q}(x_2) \Gamma_H q(x_1) \]

- Vertex function (trans. inv.):
  \[ F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2) \cdot \Phi_H \left( |x_1 - x_2|^2 \right) \]

  **Gaussian form:**
  \[ \tilde{\Phi}_H(-p^2) = \exp\left( \frac{p^2}{\Lambda_H^2} \right) \]
  \[ 1/\Lambda_H \sim \text{hadron “size”} \]

- Quark propagator (in the Schwinger representation):
  \[ \tilde{S}_{m_1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = (m_1 + \hat{p}) \cdot \int_0^\infty ds \exp\left[ -s \left( m_1^2 - p^2 \right) \right] \]
**Meson mass equation (1st equation):**

\[
1 = G \cdot \widetilde{\Pi}(M^2)
\]

- “Infra-red” regularization with parameter $\lambda$ has been used for the **whole loop**.

- Model parameters: fixed earlier by fitting semileptonic decay constants and electromagnetic decay rates of mesons.

| $\lambda$ = 0.181 GeV, |
| $m_{ud} =$ 0.235 GeV, |
| $m_s =$ 0.442 GeV, |
| $m_c =$ 1.61 GeV, |
| $m_b =$ 5.07 GeV |

The meson “size” $\sim 1/\Lambda_H$ shrinks as the mass grows:

**G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky**

*J.Phys. G 42, 075002 (2015).*

| TABLE III: The fitted values of the size parameters $\Lambda_H$ in GeV. |
|---|---|---|---|---|---|---|---|---|---|
| $\pi$ | $K$ | $D$ | $D_s$ | $B$ | $B_s$ | $B_c$ | $\eta_c$ | $\eta_b$ |
| 0.87 | 1.02 | 1.71 | 1.81 | 1.90 | 1.94 | 2.50 | 2.06 | 2.95 |
| $\rho$ | $\omega$ | $\phi$ | $J/\psi$ | $K^*$ | $D^*$ | $D_s^*$ | $B^*$ | $B_s^*$ | $\Upsilon$ |
| 0.61 | 0.50 | 0.91 | 1.93 | 0.75 | 1.51 | 1.71 | 1.76 | 1.71 | 2.96 |
Fermi G coupling: Comparison with QCD Running Coupling

We compare dimensionless Fermi coupling $1.74 \lambda^2 G$ (red curve) to QCD effective charge $\alpha_s$ (blue curve).

- Despite the different model origins, the behaviors of two curves are very similar each other in the region above $\sim 2$ GeV.
- The values at origin are mostly determined by the different confinement mechanisms. This explains the different behaviors below 2 GeV.
Analytic (Infrared) Confinement combined with QFT methods may serve reasonable frameworks to address simultaneously different sectors in particle physics, such as:

- meson ground state spectrum (except light mesons)
- meson semileptonic decay constants
- running (mass dependent) effective strong coupling
- the lowest glueball mass and radius
- the gluon condensate
- Fermi coupling’s dependence on mass scale

Suggested simple forms of propagators leads to reasonable results.

Our approach gives a new glance at the QCD effective coupling.
Our models can be extended to study:

♣ spectra of other mesons (scalar, isoscalar, …).

♣ admixtures ($qq + gg$, …)

♣ radial excitations (charmoniums and heavy bottomiums).

♣ exotic mesons (tetraquark, $X(3872)$ and $Z(4430)$ …)

♣ baryon decays, resonances ($\Lambda_b \rightarrow \Lambda^* + J/\Psi$, $\Lambda_b \rightarrow \chi_{cc} + J/\Psi$).

♣ higher glueball states ($0^{++}$, $2^{--}$, …)

♣ di-muon resonances and multi-jet channels