Glueball and Meson Spectra, Semileptonic Decay Constants, Strong Effective Coupling within Analytic (Infrared) Confinement

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Outline

- ***Motivation***
- ***Model: Two-particle bound states***
- ***Infrared regularization***
- ***Effective strong coupling***
- ***Glueball: mass, radius***
- ***Mesons: mass spectrum,***
  semileptonic decay constants
- ***Fermi Coupling within CCQM***
- ***Summary and Outlook***

Motivation

- Many novel behaviors are expected in the IR region ~ 1 ÷ 5 GeV.
- Perturbation Theory cannot be used ⟹ non-PT methods required.
- Quark confinement, QCD running coupling, glueball states, etc. requires a correct description of hadron dynamics in IR region.
- Construct simple, reliable models and apply in different sections (e.g.):
  - Strong Interaction: spectrum of mesons in a wide range of scale;
  - Exotic States: the lowest-state glueball mass, radius, etc.;
  - Semileptonic decay constants;
  - Strong Effective and Fermi couplings;

"Conventional" (***quark-antiquark***) states: Spin: $\frac{1}{2} \otimes \frac{1}{2}$

Ground states (***Mesons***) : $\mathbf{J^{PC}=0^{-+}}$, $\mathbf{J^{PC}=1^{--}}$

"Exotic" (***di-gluon***) states: Spin: $1 \otimes 1$

Ground state (***Glueball***): $\mathbf{J^{PC}=0^{++}}$

Model: Two-particle Bound States

- Consider a relativistic quantum-field model of quark-gluon interaction.

[G.G., PRD79 (2009); PRD81 (2010); Phys.Part.Nucl.(2014)]

$$L = -\frac{1}{4}\left(F^{A}_{\mu\nu} - g f^{ABC} A^{B}_{\mu} A^{C}_{\nu}\right)^{2} + \sum_{f}\left(\bar{q}^{a}_{f}\left[\gamma_{\alpha}\partial_{\alpha} - m_{f} + g\Gamma^{\alpha}_{C} A^{C}_{\alpha}\right]^{ab} q^{b}_{f}\right)$$

$$F^{B}_{\mu\nu} \equiv \partial_{\mu} A^{B}_{\nu} - \partial_{\nu} A^{B}_{\mu} \qquad \Gamma^{\alpha}_{C} \doteq i\gamma_{\alpha} t^{C}$$

- Partition Functional written in terms of quark and gluon variables

$$Z = \iint \delta\bar{q}\,\delta q \int \delta A \;\exp\left\{-\int dx\, L[\bar{q}, q, A]\right\}$$

- **LO** contributions to quark-antiquark and two-gluon bound states:

$$Z_{(\bar{q}q)} = \iint \delta\bar{q}\,\delta q \exp\left\{-\left(\bar{q}\, S^{-1} q\right) + \frac{g^{2}}{2}\left\langle\left(\bar{q}\,\Gamma A q\right)\left(\bar{q}\,\Gamma A q\right)\right\rangle_{D}\right\}$$

$$Z_{(AA)} = \left\langle \exp\left\{-\frac{g}{2}\left(f\,AAF\right)\right\}\right\rangle_{D}$$

$$\left\langle(\bullet)\right\rangle_{D} \doteq \int \delta A \;\; e^{-\frac{1}{2}\left(AD^{-1}A\right)}(\bullet)$$

- Allocate one-gluon exchange between colored currents

$$L_{qq} = \frac{g^2}{2} \sum_{f_1 f_2} \iint dx_1 dx_2 J^B_{\mu\, f_1 f_2}(x_1, x_2) D^{BC}_{\mu\nu}(x_1, x_2) J^C_{\nu\, f_1 f_2}(x_2, x_1),$$

$$J^B_{\mu\, f_1 f_2}(x_1, x_2) \equiv i\, \overline{q}_{f_1}(x_1)\, \gamma_\mu t^B q_{f_2}(x_2).$$

- Isolate color-singlet combination

- Perform Fierz transformation for spins (J = S, P, A, V, T)

- Introduce orthonormalized system: $\{\boldsymbol{U_Q}\}$ with quantum numbers $\boldsymbol{Q=\{n,l, \ldots\}}$:

- Diagonalization on colorless quark currents $\boldsymbol{J_N(x)}$ with $N=\{Q,J,f_1,f_2\}$

- Gaussian representation: a new path integration over auxiliary fields $\boldsymbol{B_N}$:

$$e^{\frac{1}{2} g^2 (J_N^+ J_N)} = \iint \delta B_N^+\, \delta B_N \exp\left\{ -\sum_N (B_N^+ B_N) + g \sum_N [(B_N^+ J_N) + (J_N^+ B_N)] \right\}$$

- Explicit path-integration over quark variables and write the effective action

- Hadronization Ansatz: B_N fields are identified as meson fields with N

- Z_N rewritten in terms of B_N and all quadratic field configurations are isolated:

$$Z_{(\bar{q}q)} \to Z_N = \int \prod_N \delta B_N \exp\left\{ -\frac{1}{2}\left(B_N\left[1+g^2\, Tr(V_N S_{m1} V_N S_{m2})\right] B_N\right) + W_{resid}[B_N] \right\}$$

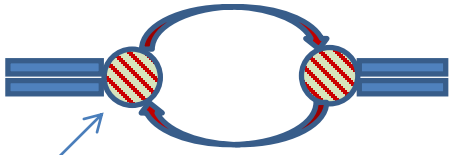
- Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system $\{U_N\}$

$$g^2\, Tr(V_N S_{m1}\, V_{N'} S_{m2}) = \left(U_N\, \lambda\, U_{N'}\right) = \lambda_N(-p^2)\, \delta^{JJ'} \delta^{QQ'}$$

- Symmetric Bethe-Salpeter kernel is defined:

$$\alpha \cdot \lambda_J(-p^2) = \frac{4g^2 C_J}{9} \int \frac{d^4k}{(2\pi)^4} \{V(k)\}^2 \cdot \Pi_J(k,p)$$

$$\Pi_J(p,k) = -\frac{1}{4!} Tr\left\{ \Gamma_J\, \tilde{S}_{m1}\left(\hat{k} + \xi_1 \hat{p}\right) \Gamma_J\, \tilde{S}_{m2}\left(\hat{k} - \xi_2 \hat{p}\right) \right\}$$



$$V_J(k) = \int dx \sqrt{D(x)}\, U_J(x)\, e^{ikx}$$

UV Regularization

- Renormalization: $$U_{REN}(x) \equiv \sqrt{-\alpha\dot{\lambda}_N(M_N^2)} \cdot U_N(x)$$

$$\left\langle U_N \,|1+\alpha\lambda_N(-p^2)|\, U_N \right\rangle = \left\langle U_N \,|1+\alpha\lambda_N(M_N^2) - \alpha\dot{\lambda}_N(M_N^2)(p^2+M_N^2)|\, U_N \right\rangle$$
$$= \left\langle U_{REN} \,|(p^2+M_N^2)|\, U_{REN} \right\rangle$$

Meson Mass Equation

$$1+\alpha\lambda_N(M_N^2) = 0\,, \qquad p^2 = -M_N^2$$

- Infrared (IR) divergences appear in the loop and vertice functions:

$$\Pi_J(p,k) = -\frac{1}{4!} Tr\left\{\Gamma_J\, \tilde{S}_{m1}\left(\hat{k}+\xi_1\hat{p}\right)\Gamma_J\, \tilde{S}_{m2}\left(\hat{k}-\xi_2\hat{p}\right)\right\}$$

$$V_J(k) = \int dx\, \sqrt{D(x)}\, U_J(x)\, e^{ikx}$$

IR Regularization

- "***Infra-red***" regularizations of ***propagators*** remove these divergencies:

$$\tilde{S}_m(\hat{p}) = \frac{1}{-i\hat{p}+m} = (i\hat{p}+m)\cdot\int_0^{\infty} dt \exp\left\{-t\cdot(p^2+m^2)\right\}$$

$$\Rightarrow \tilde{S}_{IR}(\hat{p}) = (i\hat{p}+m)\cdot\int_0^{1/\Lambda^2} dt \exp\left\{-t\cdot(p^2+m^2)\right\}$$

$$D(x) = \frac{1}{4\pi^2 x^2} = \int_0^{\infty} ds\, e^{-s x^2} \quad \Rightarrow \quad D_{IR}(x) = \int_{\Lambda^2/2}^{\infty} ds\, e^{-s x^2}$$

Λ – mass scale of IR confinement region

$\Lambda \to 0$ deconfinement

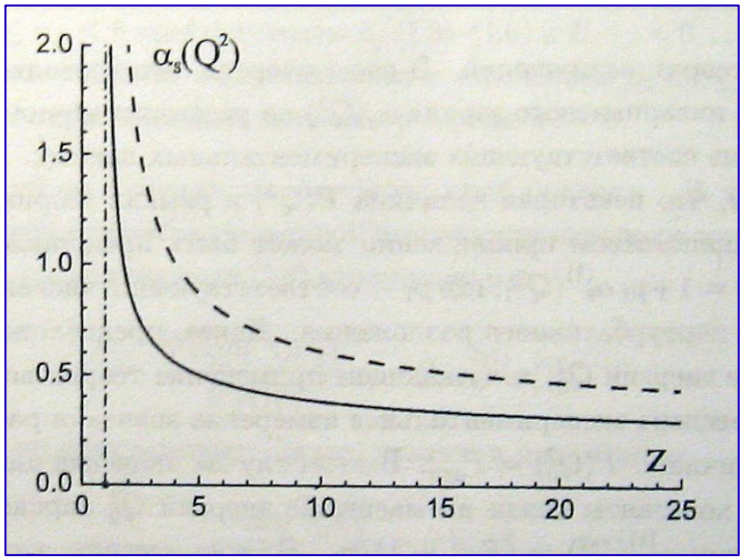
- Another "***Infra-red-Loop***" regularization of ***whole loop*** has been used in

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

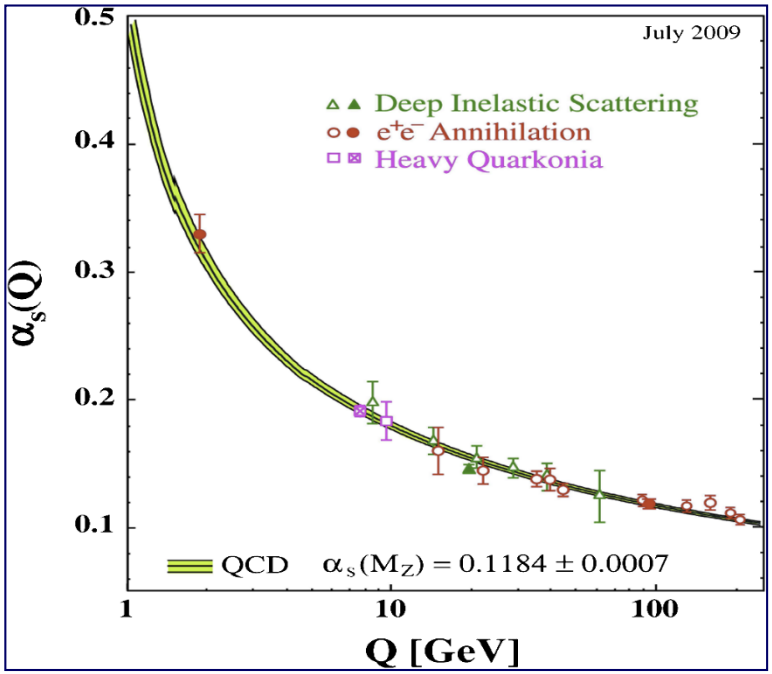
QCD Effective Coupling

THEORY: QCD predicts a dependence of $\alpha_s(Q)$ on energy scale Q.
This ***dependence*** is described theoretically by the Renorm Group equations.

EXPERIMENT: but its ***actual value*** must be obtained from experiment.
It is well determined experimentally at relatively high energies Q >2 GeV.



Perturbation theory: one-loop (dashed) and two-loop (solid) approximations to $\alpha_S(Q^2)$, where $Z=Q^2/\Lambda^2$



$$\alpha_s(Q<1) \rightarrow ?$$

Effective (Mass-dependent) Strong Coupling

$$1+\tilde{\alpha}_s \lambda_N(M_N^2)=0\,, \qquad p^2=-M_N^2$$

$$U_{n\ell\mu}(x,a)\approx T_{\ell\mu}(ax)\cdot L_n^{l+1/2}(a^2x^2)\cdot\sqrt{D(x)}\cdot e^{-ax^2}\,, \qquad \sum_\mu \int dx\left[U_{nl\mu}(x,a)\right]^2=1$$

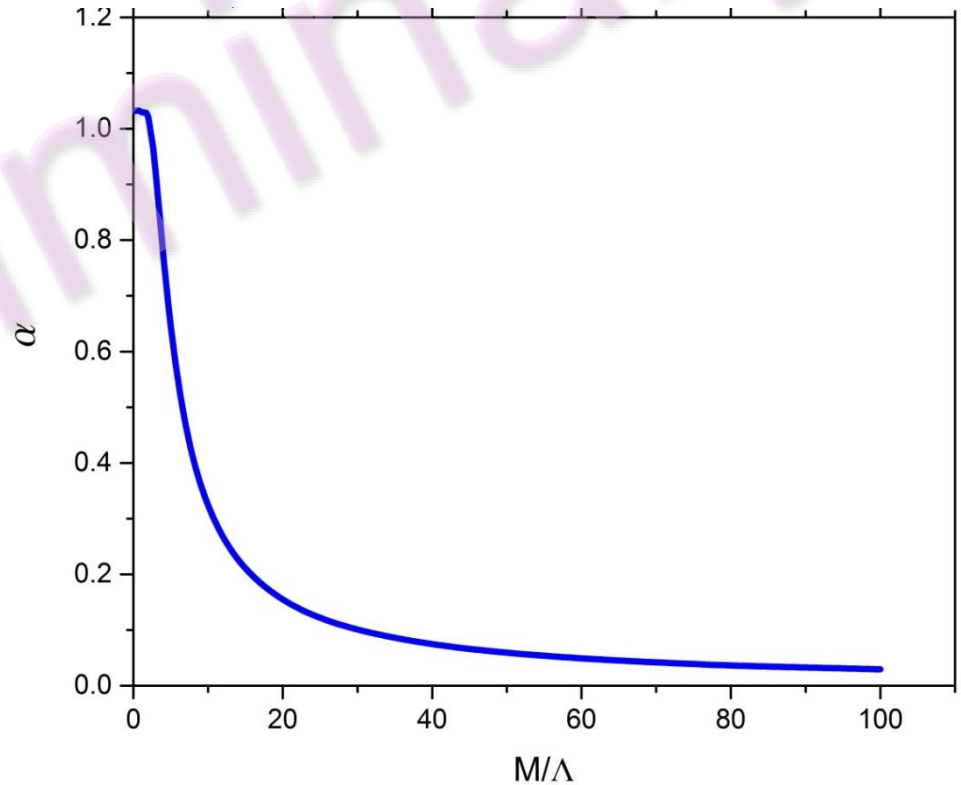
- Particularly, for
  n=l=0,
  $m_1 = m_2 = M/2$:

$$\tilde{\alpha}_s(M)=1/\tilde{\lambda}(M^2)$$

Infra-red fixed point

$$\tilde{\alpha}_s(0)\approx 1.032$$

for any $\Lambda>0$



There exist dispersion relations between time-like and space-like couplings:

α(M) ↔ α(Q)

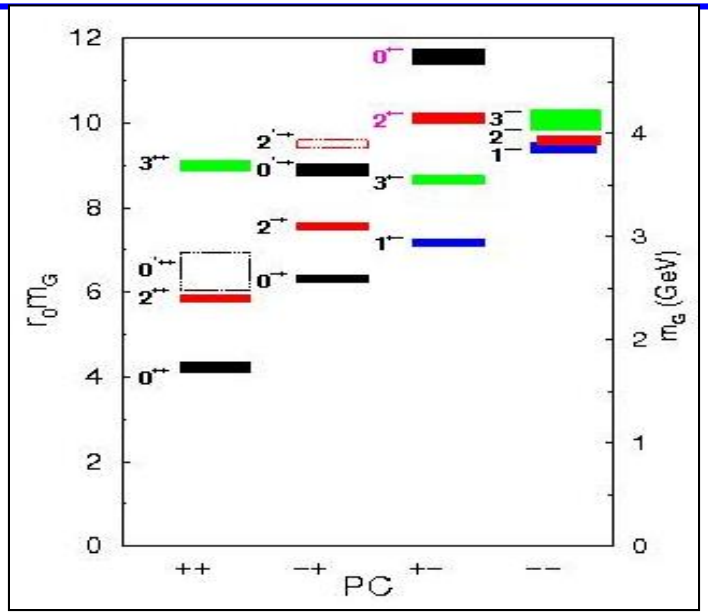
Preliminary

Glueballs (Gluon Bound States)

Experimental status: Signatures expected for glueballs:
-- enhanced production in gluon-rich channels of radiative decays,
-- decay branching fractions incompatible with (q-qbar) states.

Theoretical status: The existence of glueballs is predicted by QCD because of the self-interaction of gluons.

- **Lattice calculations, QCD sum rules, Tube model, Constituent glue model, Holographic QCD model, …**



$$J = 0, 1, 2, 3$$

$$PC = ++, -+, +-, --$$

$$M_G \approx 1.6 - 4.9 \text{ GeV}$$

Scalar Glueball

$$J^{PC} = 0^{++}$$

Scalar glueball mass:

$$\Lambda = 236.0\ MeV, \quad \alpha(M_G) = 0.451 \quad \rightarrow \quad M_G \approx 1738.6\ MeV$$

1750±50±80 MeV	C.J.Morningstar, M.Peardon (2004).
1710±50±58 MeV	QLQCD Infinite volume, continuum limit (Y.Chen 2006)
1710±50 MeV	Analytic confinement model (G.Ganbold PRD79, 2009)
1790±50±20 MeV	UK-QCD Collaboration (2014)

Scalar glueball "radius":

$$r_G \cdot M_G = \frac{M_G}{2\Lambda}\sqrt{\frac{\int d^4x \cdot x^2 \cdot W(x) \cdot U^2(x)}{\int d^4x \cdot W(x) \cdot U^2(x)}} = 4.41$$

≈ 4.35 UK-QCD Collaboration (2014)

Gluon condensate:

$$\left\langle \frac{\alpha}{\pi} F^2 \right\rangle = \frac{16\, N_C}{\pi} \alpha\, \Lambda^4 = 0.0214\ GeV^4$$

≈ 0.0223 ± 0.0041 GeV S.Narison, PLB706 (2012)

Preliminary

Meson Spectrum

a) **Analytic results**

- Asymptotical Regge-type behavour:

$$M_{nl}^2 \approx M_{00}^2 + (n+l)\cdot const \qquad for \;\; \{n,l\} \geq 3$$

- Due to spin effect vector mesons are heavier than pseudoscalars at the same quark contents:

$$1 \approx C_J \cdot \exp(M_J^2)\cdot(M_J^2 + const)$$
$$1 = C_P > C_V = 1/2$$

$\longrightarrow$ $M_P^2 < M_V^2$

- The coupling is bounded from above:

$$\alpha_s(M) = 1/\lambda_J(M^2) \leq \alpha_s^{\max}$$

Conventional Meson Mass Estimates

Model parameters fixed by fitting the meson spectrum:

$$\Lambda = 236.0\ MeV,$$
$$m_{ud} = 227.65\,MeV, \qquad m_s = 420.07\,MeV,$$
$$m_c = 1521.61\,MeV, \qquad m_b = 4757.20\,MeV.$$

|relative errors| < 1.8%

$J^{PC} = 0^{-+}$	PDG-2016	Our estim.	$J^{PC} = 1^{--}$	PDG-2016	Our estim.
D (~uc)	1869.62	1893.6	ρ (uu)	775.26	774.3
D_s (~sc)	1968.50	2003.7	K* (us)	891.66	892.9
η_c (~cc)	2983.7	3032.5	D* (uc)	2010.29	2003.8
B (~ub)	5279.26	5215.2	D_s* (~sc)	2112.3	2084.1
B_s (~sb)	5366.77	5323.6	J/ψ (~cc)	3096.92	3077.6
B_c (~cb)	6274.5	6297.0	B* (~ub)	5325.2	5261.5
η_b (~bb)	9398.0	9512.5	Y (~bb)	9460.30	9526.4

Preliminary

Semileptonic Decay Constants of Mesons

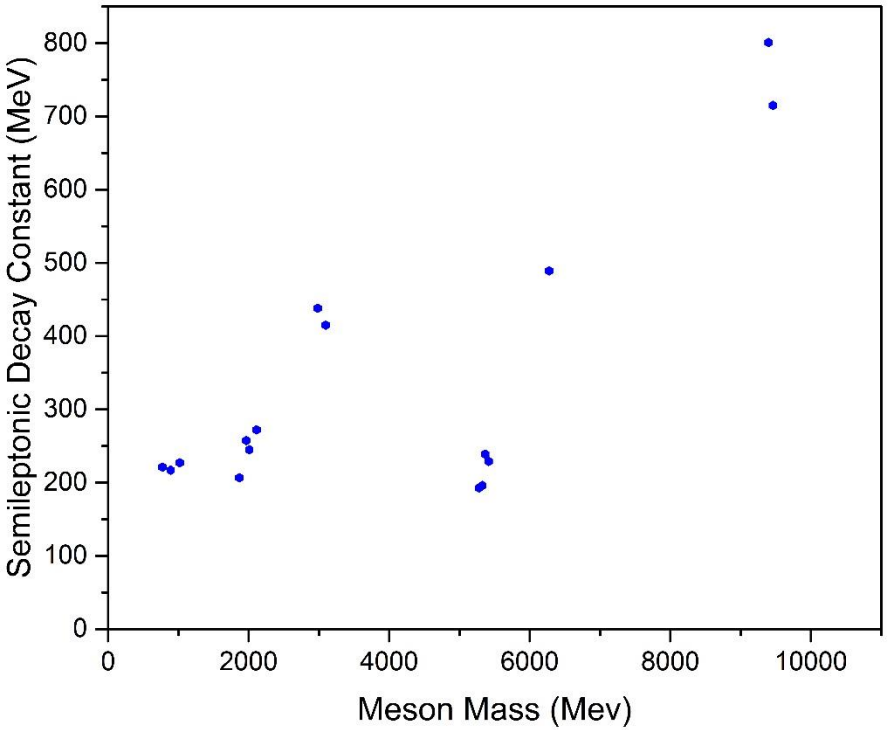
Important value in particle physics:

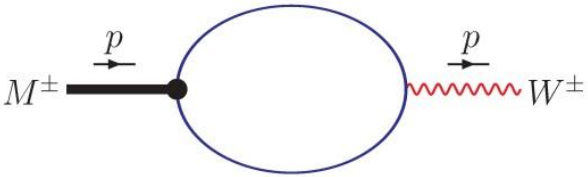
$$i\, f_P\, p_\mu = \langle 0 | J_\mu(0) | U_{renorm}(p) \rangle$$

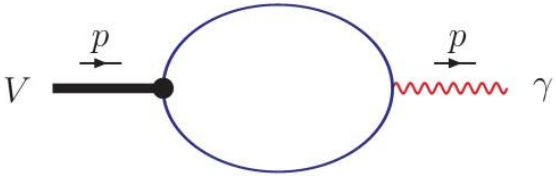
$$\pi^+ \to \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma$$

$$D_s^+ \to l^+ \nu$$

Mass dependence of semileptonic decay constants (Exp. data 2015):







$$i f_p p_\mu = \frac{g}{6}\int \frac{d^4k}{(2\pi)^4}\int dx\, e^{ikx}\, U_R(x)\sqrt{D(x)}\, Tr\left\{ i\gamma_5\, \tilde{S}_{m1}\left(\hat{k}+\xi_1\hat{p}\right) i\gamma_5\gamma_\mu\, \tilde{S}_{m2}\left(\hat{k}-\xi_2\hat{p}\right)\right\}$$

R – "size" of meson in mass scale

$$\tilde{U}_R(k) \sim \int_0^1 ds\, f(s)\cdot \exp\left\{-\frac{s\cdot k^2}{R^2}\right\}$$

Model parameters:

$$\Lambda = 236.0\ MeV,$$

$$m_{ud} = 227.65\, MeV, \qquad m_s = 420.07\, MeV,$$

$$m_c = 1521.61\, MeV, \qquad m_b = 4757.20\, MeV.$$

R – estimated to fit experimental data on meson masses and semileptonic decay constants (in ***GeV***)

D	D_s	η_c	B	B_s	B_c	η_b
0.93	1.08	1.83	1.73	2.18	3.34	3.80
ρ	K*	D*	D_s*	J/Ψ	B*	ϒ
0.33	0.38	0.78	0.90	2.40	3.34	2.80

Preliminary

• ***Estimated values of the Leptonic decay constants:***

$J^{PC} = 0^{-+}$	Exp.data	Our estim.	$J^{PC} = 1^{--}$	Exp.data	Our estim.
f_D	206.7 ± 8.9	207	f_ρ	221 ± 1	221
f_{Ds}	257.5 ± 6.1	257	f_{K*}	217 ± 7	217
$f_{\eta c}$	238 ± 8	238	f_{D*}	245 ± 20	245
f_B	192.8 ± 9.9	193	f_{Ds*}	272 ± 26	271
f_{Bs}	238.8 ± 9.5	239	$f_{J/\psi}$	415 ± 7	416
f_{Bc}	489 ± 5	488	f_{B*}	196 ± 44	196
$f_{\eta b}$	801 ± 9	800	f_Υ	715 ± 5	715

Preliminary

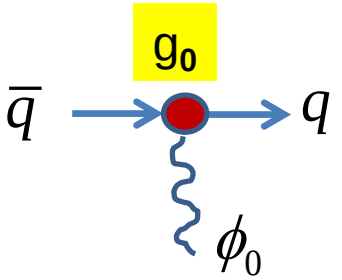
+ Agreement between estimated and experimental data is accurate.

+ The meson “size” shrinks **~1/R** as the mass grows.

Fermi Coupling

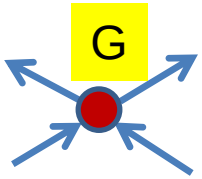
- **Yukawa-type model**

$$L_Y = \bar{q}\,(i\hat{\partial} - m)\,q + \phi_0\,(-\partial^\mu \partial_\mu - M_0^2)\,\phi_0 + g_0 \phi_0\,(\bar{q}\,\Gamma\,q)$$



- **Fermi-type model**

$$L_F = \bar{q}\,(i\hat{\partial} - m)\,q + \frac{G}{2}(\bar{q}\,\Gamma\,q)^2$$



Generating functional $\Pi(x-y) \equiv -i \cdot \mathrm{tr}\left\{\Gamma\, S(x-y)\,\Gamma\, S(y-x)\right\}$;

- **if two conditions are fulfilled:**

$$-\frac{1}{G} + \tilde{\Pi}(M^2) = 0$$

$$Z \equiv 1 - g_r^2 \cdot \frac{d}{dM^2}\tilde{\Pi}(M^2) = 0$$

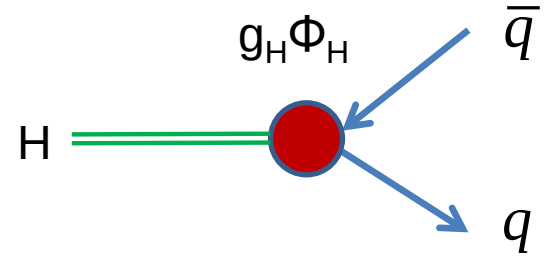
- **then, these two theories are equivalent:**

$$L_F^{\mathrm{int}} = \frac{G}{2} J_H^2(x) \quad \Leftrightarrow \quad L_Y^{\mathrm{int}} = g_H H(x) J_H(x)$$

$$\phi_0 \equiv Z^{1/2}\phi_r = 0$$

Compositeness Condition

CCQM: Meson-Quark Interaction



- Lagrangian:

$$L_{\text{int}} = g_H H(x) J_H(x)$$

- Quark currents (for mesons):

$$J_H(x) = \int dx_1 \int dx_2\, F_H(x; x_1, x_2)\, \bar{q}(x_2) \Gamma_H q(x_1)$$

$$\Gamma_P = i\gamma^5; \quad \Gamma_V = \gamma^\mu$$

- Vertex function (trans. inv.)

$$F_H(x; x_1, x_2) = \delta(x - \omega_1 x_1 - \omega_2 x_2) \cdot \Phi_H(|x_1 - x_2|^2)$$

$$\omega_j = m_j / (m_1 + m_2); \quad \omega_1 + \omega_2 = 1$$

Gaussian form:

$$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2}\right)$$

$1/\Lambda_H$ ~ hadron "size"

- Quark propagator **(**in the Schwinger representation**):**

$$\tilde{S}_{m1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = (m_1 + \hat{p}) \cdot \int_0^\infty ds_1 \exp\left[-s\left(m_1^2 - p^2\right)\right]$$

• **Meson mass equation (1st equation)**:

$$1 = G \cdot \tilde{\Pi}(M^2)$$

● "Infra-red" regularization with parameter **λ** has been used for the ***whole loop.***

• Model parameters:
fixed earlier by fitting semileptonic decay constants and electromagnetic decay rates of mesons.

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

$$\lambda = 0.181\,\text{GeV},$$
$$m_{ud} = 0.235\,\text{GeV},$$
$$m_s = 0.442\,\text{GeV},$$
$$m_c = 1.61\,\text{GeV},$$
$$m_b = 5.07\,\text{GeV}$$

+ The meson "size" ~1/ Λ_H shrinks as the mass grows:

TABLE III: The fitted values of the size parameters Λ_H in GeV.

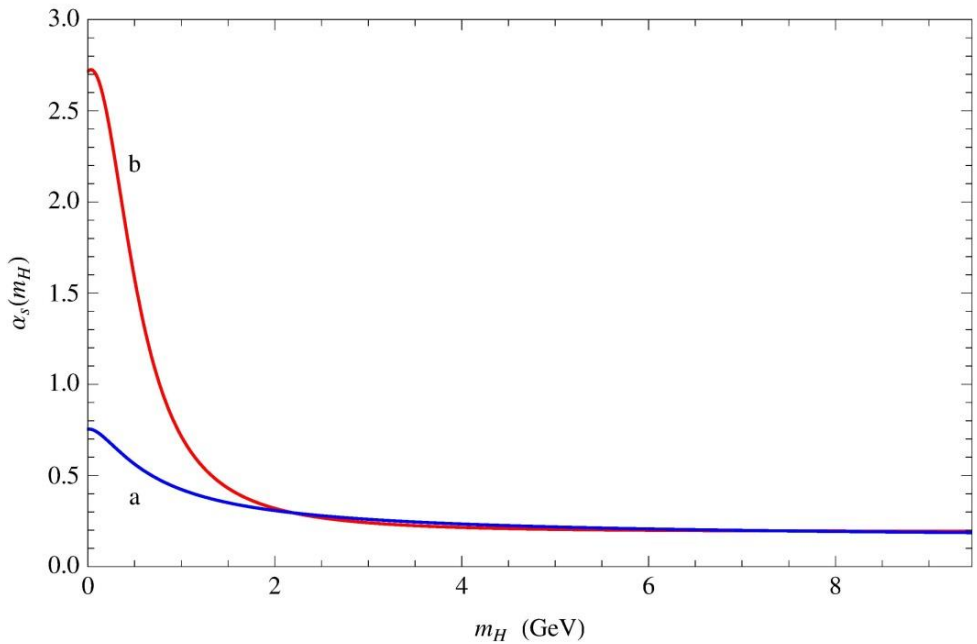
π	K	D	D_s	B	B_s	B_c	η_c	η_b	
0.87	1.02	1.71	1.81	1.90	1.94	2.50	2.06	2.95	
ρ	ω	ϕ	J/ψ	K^*	D^*	D_s^*	B^*	B_s^*	Υ
0.61	0.50	0.91	1.93	0.75	1.51	1.71	1.76	1.71	2.96

Fermi G coupling: Comparison with QCD Running Coupling

We compare dimensionless Fermi coupling 1.74 **λ^2*G (red curve)** to QCD effective charge **α_s (blue curve).**

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky J.Phys. G 42, 075002 (2015).

G.Ganbold, Phys. Rev. D 81, 094008 (2010) Phys. Part. Nucl. 43, 79, (2012) Phys. Part. Nucl. 45, 10, (2014).



- Despite the different model origins, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.
- The values at origin are mostly determined by the different confinement mechanisms. This explains the different behaviors below 2 GeV.

Summary:

- **Analytic** (**Infrared) Confinement** combined with QFT methods may serve reasonable frameworks to address simultaneously **different sectors in particle physics, such as:**

  - meson ground state spectrum (except light mesons)
  - meson semileptonic decay constants
  - running (mass dependent) effective strong coupling
  - the lowest glueball mass and radius
  - the gluon condensate
  - Fermi coupling's dependence on mass scale

- Suggested **simple forms of propagators** leads to reasonable results.

- **O**ur approach gives a new glance at the QCD effective coupling.

Outlook:

Our models can be extended to study:

- spectra of other mesons (scalar, isoscalar, …).
- admixtures (***qq*** + ***gg***, …)
- radial excitations (charmoniums and heavy bottomiums).
- exotic mesons (tetraquark, ***X(3872)*** and ***Z(4430)*** …)
- baryon decays, resonances ($\Lambda_b \to \Lambda^* + J/\Psi$, $\Lambda_b \to \chi_{cc} + J/\Psi$).
- higher glueball states (0^{-+}, 2^{--}, …)
- di-muon resonances and multi-jet channels