The Generalized Polarizabilities of the proton

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Proton Polarizabilities

Fundamental structure constants (such as mass, size, shape, ...)

Response of internal structure & dynamics to external EM field

Sensitive to the full excitation spectrum of the nucleon

Accessed experimentally through Compton Scattering processes

Virtual Compton Scattering:

Virtuality of photon gives access to the Generalized Polarizabilities $\alpha_E(Q^2)$ & $\beta_M(Q^2)$

$\Rightarrow$ mapping out the spatial distribution of the polarization densities

Fourier transform of densities of electric charges and magnetization of a nucleon deformed by an applied EM field

<table>
<thead>
<tr>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>N BARYONS</td>
</tr>
<tr>
<td>$(S = 0, , I = 1/2)$</td>
</tr>
<tr>
<td>$p$, $N^+ = uud$, $n$, $N^0 = udd$</td>
</tr>
</tbody>
</table>

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

| Mass $m = 1.00727646681 \pm 0.0000000009$ u |
| Mass $m = 938.272046 \pm 0.000021$ MeV [a] |
| $|m_p - m_n|/m_p < 7 \times 10^{-10}$, CL = 90% [g] |
| $|Q_p|/(Q_n) = 0.99999999991 \pm 0.0000000009$ |
| $|Q_p + Q_n|/e < 7 \times 10^{-10}$, CL = 90% [h] |
| $|Q_p + Q_n|/e < 1 \times 10^{-21}$ [i] |
| Magnetic moment $\mu = 2.792847356 \pm 0.0000000023 \, \mu_N$ |
| $\langle \mu_p + \mu_n \rangle / \mu_p = (0 \pm 5) \times 10^{-6}$ |
| Electric dipole moment $\sigma < 0.54 \times 10^{-33} \, e \cdot cm$ |

Electric polarizability $\alpha = (1.2 \pm 0.4) \times 10^{-4}$ fm$^3$

Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm$^3$ $(S = 1.2)$

Charge radius, $\mu p$ Lamb shift = $0.84087 \pm 0.00039$ fm [i]

Charge radius, $e p$ CODATA value = $0.8773 \pm 0.0051$ fm [e]

Magnetic radius = $0.777 \pm 0.016$ fm

Mean life $\tau > 2.1 \times 10^{39}$ years, CL = 90% [e] (p → invisible mode)

Mean life $\tau > 10^{31}$ to $10^{33}$ years [k] (mode dependent)
Scalar Polarizabilities

Response of internal structure to an applied EM field
Scalar Polarizabilities

Response of internal structure to an applied EM field

“stretchability”
\[ \vec{d}_E \text{ induced} \sim \alpha \vec{E} \]
External field deforms the charge distribution

“alignability”
\[ \vec{d}_M \text{ induced} \sim \beta \vec{B} \]
\[ \beta_{\text{para}} > 0 \]
\[ \beta_{\text{diam}} < 0 \]
Paramagnetic: proton spin aligns with the external magnetic field
Diamagnetic: \( \pi \)-cloud induction produces field counter to the external one
Experimental Landscape

\[ a_E \approx 10^{-3} V_N \] (stiffness / relativistic character)

Data suggest non-trivial \( Q^2 \) evolution of \( a_E \)

Current theoretical calculations not able to describe the enhancement at low \( Q^2 \)

\( Q^2 = 0.33 \) (GeV/c)\(^2\) measured twice at MAMI:

\[ \beta_M \] small \( \Longleftrightarrow \) cancellation of competing mechanisms

Large uncertainties

Higher precision measurements needed

\( \Rightarrow \) Quantify the balance between diamagnetism and paramagnetism

Current situation unsatisfactory:
- more measurements needed (vs \( Q^2 \))
- Higher precision measurements needed
Theoretical Landscape

HBChPT
NRQCM
Effective Lagrangian Model
Linear Sigma Model

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All theoretical calculations predict a smooth fall off for $\alpha_E$
None of the models can account for the non trivial structure of $\alpha_E$ suggested by the data

Lattice QCD

- Currently: $Q^2=0$ calculations exist but at unphysical quark masses
- Near Future: calculations at the physical point for $Q^2=0$
- First calculations for $Q^2 \neq 0$
Spatial dependence of induced polarizations on an external EM field

Nucleon form factor data \(\rightarrow\) light-front quark charge densities

Formalism extended to the deformation of these quark densities when applying an external e.m. field:

GP I \(\rightarrow\) spatial deformation of charge & magnetization densities under an applied e.m. field

\textbf{Induced polarization in a proton when submitted to an e.m. field}

\begin{align*}
\textbf{GP I} & \\
\textbf{GP II} & \\
\end{align*}

\textbf{Light (dark) regions} \(\rightarrow\) \textbf{largest (smaller)} values

(photon polarization along x-axis, as indicated)

\textbf{Induced polarization along} \(b_y=0\)


M. Gorchtein, C. Lorce, B. Pasquini, M. Vanderhaeghen
Virtual Compton Scattering

**DR**
- Valid below & above Pion threshold
- Dispersive integrals for Non Born amplitudes
- Spin GPs are fixed
- Scalar GPs have an unconstrained part
- Fit to the experimental cross section at each $Q^2$

**LEX**
- Valid only below Pion threshold
- Structure functions
  \[ d^2\sigma = d^2\sigma^{\text{BH Born}} + q_{\text{em}} \cdot \phi \cdot \Psi_0 + O(q_{\text{em}}^2) \]
  \[ \Psi_0 = v_1 \cdot (P_{LL} - \frac{1}{\epsilon} P_{TT}) + v_2 \cdot P_{LT} \]
- Subtract the spin part
  \[ P_{TT} = [P_{TT \text{ spin}}] \]
  \[ P_{LT} = -\frac{2M}{\alpha_{\text{em}}} \sqrt{\frac{4s}{Q^2}} \cdot \xi^{\mu}_{\text{LM}}(Q^2) \cdot \beta_M(Q^2) + [P_{LT \text{ spin}}] \]
- Utilize DR

Scalar GPs $\alpha_E$ and $\beta_M$
Ongoing Experimental Efforts

MAMI

MAMI A1/1-09 (vcsq2) Fonvieille et al below threshold

MAMI A1/3-12 (vcsdelta) Sparveris et al above threshold

Both experiments utilized the A1 setup at MAMI

Preliminary results were recently released

Analysis is ongoing
Ongoing Experimental Efforts
vcsdelta @ MAMI

Goal 2-fold:
1) Measurement of the electric GP $\alpha_E$
2) First measurement of N-$\Delta$ transition form factors through the $\gamma$ channel

1.1 GeV beam

Measurement at $Q^2 = 0.2 \ (GeV/c)^2$
Ongoing Experimental Efforts
vcsq2 @ MAMI

~ 1.0 GeV beam

\[ Q^2 = 0.1 \text{ (GeV/c)}^2, 0.2 \text{ (GeV/c)}^2, \text{ and } 0.45 \text{ (GeV/c)}^2 \]

Figure 5.8: Setting INP: measured \( ep \rightarrow ep\gamma \) cross section at fixed \( q'_{cm} = 112.5 \text{ MeV/c} \) with respect to \( \varphi_{cm} \) for all the \( \cos(\theta_{cm}) \)-bins. The curves follow the convention of figure 5.6.

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand, 2016
Ongoing Experimental Efforts @ MAMI

MAMI A1/1-09 Fonvieille et al. VCS below threshold data analysis ongoing
MAMI A1/3-12 Sparveris et al. VCS above threshold data analysis ongoing

MAMI setup constraints $Q^2 < 0.45 \text{ (GeV/c)}^2$

Preliminary A1/1-09 (vcsq2)
Preliminary A1/3-12 (vcsdelta)

Data analyzed by 4 PhD students
Jure Bericic (Ljubljana Univ.)
Loup Correa (Clermont-Fd Univ.)
Meriem BenAli (Clermont-Fd Univ.)
Adam Blomberg (Temple Univ.)

2 independent measurements at $Q^2=0.20 \text{ (GeV/c)}^2$
First measurement of the N-Δ C2 amplitude through the photon channel

Important for cross check to the world data and for cross checking & constraining the model uncertainties
A1/03-12 Preliminary Results

First measurement of the N-Δ C2 amplitude through the photon channel

Important for cross check to the world data and for cross checking & constraining the model uncertainties

Sato Lee
Ongoing Experimental Efforts

JLab

**New Proposal**

E12-15-001 (JLab)

Going from $\varepsilon = 0.6 \rightarrow 0.9$ doubles the sensitivity to the GPs

$\varepsilon = 0.97$ (Jlab)

$\varepsilon = 0.62$ (MAMI)

**Preliminary A1/1-09**

**Preliminary A1/3-12**

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additional +:

Beam energy $\times 4$

Beam current $\times 5$
JLab Hall C with 12 GeV upgrade

- Super High Momentum Spectrometer
  - HB, 3 Quads, Dipole
  - $P \rightarrow 2 - 11$ GeV
  - Resolution: $\delta < 0.1\%$
  - Acceptance: $\delta \rightarrow 30\%, 4$ msr
  - $5.5^\circ < \theta < 40^\circ$
  - Good e/$\pi$/K/$p$ PID
- High Momentum Spectrometer
  - 3 Quads, Dipole
  - $P \rightarrow 7.5$ GeV
  - Resolution: $\delta < 0.1\%$
  - Acceptance: $\delta \rightarrow 18\%, 6.5$ msr
  - $10.5^\circ < \theta < 90^\circ$
  - Good e/$\pi$/K/$p$ PID
- Minimum opening angle $\sim 17^\circ$
- Well shielded detector huts
- 2 beam line polarimeters
- Ideal facility for:
  - Rosenbluth (L/T) separations
  - Exclusive reactions
  - Low cross sections (neutrino level)
Hall C HMS and SHMS

**SHMS:**
- 11-GeV Spectrometer
- Partner of existing 6-GeV HMS

**MAGNETIC OPTICS:**
- Point-to-Point QQGD for easy calibration and wide acceptance.
- Horizontal bend magnet allows acceptance at forward angles (5.5°)

**Detector Package:**
- Drift Chambers
- Hodoscopes
- Cerenkovs
- Calorimeter
- All derived from existing HMS/SOS detector designs

**Well-Shielded Detector Enclosure**

**Rigid Support Structure**
- Rapid & Remote Rotation
- Provides Pointing Accuracy & Reproducibility demonstrated in HMS

SHMS = Super High Momentum Spectrometer
HMS = High Momentum Spectromter
Experiment Setup

**Hall C:** SHMS, HMS
- 4.4 GeV
- 40-85 μA
- Liquid hydrogen 15 cm

**Photons:** e & p detection in coincidence

**Cross sections**

**In-plane azimuthal asymmetries**

\[ A_{\phi_{\gamma*\gamma=0,\pi}} = \frac{\sigma_{\phi_{\gamma*\gamma=0}} - \sigma_{\phi_{\gamma*\gamma=180}}}{\sigma_{\phi_{\gamma*\gamma=0}} + \sigma_{\phi_{\gamma*\gamma=180}}} \]

**Sensitivity to GPs**

**Suppression of systematic uncertainties**
Projected Measurements

$Q^2 = 0.43 \text{ (GeV/c)}^2$

- BH peaks
- Avoid BH peaks
- Stay at $\theta_{\gamma^*\gamma} > 120^\circ$

- $\Phi_{\gamma^*\gamma} = 0^\circ, 180^\circ$
- $\delta \beta_M = 2 \times 10^{-4} \text{ fm}^3$
- $\alpha_E = 5.8 \times 10^{-4} \text{ fm}^3$
- $\alpha_E = 2.4 \times 10^{-4} \text{ fm}^3$
## Kinematical Settings

<table>
<thead>
<tr>
<th>Kinematical Setting</th>
<th>$\theta_{\gamma\gamma}^\circ$</th>
<th>$\theta_\alpha^\circ$</th>
<th>$P_e'(MeV/c)$</th>
<th>$P_p'(MeV/c)$</th>
<th>S/N</th>
<th>beam time (days)</th>
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</thead>
<tbody>
<tr>
<td><strong>Part I</strong></td>
<td></td>
<td></td>
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<tr>
<td>Kin Ia</td>
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<td><strong>Part II</strong></td>
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<tr>
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<td>52.64</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

**SHMS:** one change of setting through Part I  
same position & momentum throughout Part II  
Same beam energy for all settings

<table>
<thead>
<tr>
<th>Part</th>
<th>I</th>
<th>I</th>
<th>I</th>
<th>II</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2$</td>
<td>0.33 (GeV/c)</td>
<td>0.43 (GeV/c)</td>
<td>0.52 (GeV/c)</td>
<td>0.65 (GeV/c)</td>
<td>0.75 (GeV/c)</td>
</tr>
</tbody>
</table>
Phase Space

Part I

\[
\Phi_{\gamma\gamma} = 0^\circ \text{ and } 180^\circ \text{ at } \theta_{\gamma\gamma} = 165^\circ
\]

Part II

\[
\Phi_{\gamma\gamma} = 0^\circ \text{ and } 180^\circ \text{ at } \theta_{\gamma\gamma} = 165^\circ
\]

Phase space binned in \( Q^2, W, \theta_{\gamma\gamma}, \Phi_{\gamma\gamma} \)

Cross section:

DR calculation, B. Pasquini


<table>
<thead>
<tr>
<th>Part</th>
<th>I</th>
<th>I</th>
<th>I</th>
<th>II</th>
<th>II</th>
</tr>
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<tbody>
<tr>
<td>( Q^2 )</td>
<td>0.33 (GeV/c)</td>
<td>0.43 (GeV/c)^2</td>
<td>0.52 (GeV/c)^2</td>
<td>0.65 (GeV/c)^2</td>
<td>0.75 (GeV/c)^2</td>
</tr>
</tbody>
</table>
Projected Measurements

\[ \alpha_E = 4.8 \times 10^{-4} \text{ fm}^3 \]
\[ \alpha_E = 1.5 \times 10^{-4} \text{ fm}^3 \]

\[ \beta_M = 1.6 \times 10^{-4} \text{ fm}^3 \]
\[ \beta_M = 0.4 \times 10^{-4} \text{ fm}^3 \]

\[ Q^2 = 0.65 \left( \text{GeV/c} \right)^2 \]

\[ \sigma < \pm 1.3\% \text{ (stat)} < \pm 3.3\% \text{ (syst)} \]
\[ A \approx \pm 0.7\% \text{ (stat)} \approx \pm 1.1\% \text{ (syst)} \]

Statistical < \pm 1.3%  
Beam energy / scat. Angle \pm 1-2.5%  
Target density \pm 0.5%  
Detector efficiency \pm 0.5%  
Acceptance \pm 0.5%  
Target cell backgr. \pm 0.5%  
Target length \pm 0.3%  
Beam charge \pm 0.3%  
Dead time \pm 0.3%  
Pion contamination in MM \pm 0.3%  
Rad. Corr. \pm 1.5%  
Other \pm 0.5%
Projected Measurements

\[ \delta_{\text{GPS}}^{\text{stat}} \approx 0.7 \delta_{\text{GPS}}^{\text{syst}} \]

\[ \delta_{\text{GPS}}^{\text{syst}} \approx \delta_{\text{GPS}}^{\text{FFs/DR mult}} \]

Statistics
Systematics
FFs, DR multipoles

\( \sigma, A \)
Part I approved in summer 2016 (Jlab PAC 44): (4.4 GeV, 85 µA, Hall C)
Other ongoing efforts

E08-010 (Hall-A/Jlab): \( \gamma \)-channel

parasitic access to VCS at very low \( Q^2 \)

data analysis is ongoing (and very challenging)

\[ Q^2 = 0.04 \,(\text{GeV/c})^2 \, \text{to} \, 0.13 \,(\text{GeV/c})^2 \]
Summary

Intense experimental effort focusing on the measurement of the electric and magnetic GPs
- fundamental structure constants
- internal structure and dynamics of the nucleon
- complementary information to elastic & transition FFs, GPDs, TMDs, ...

New results (MAMI) and a recently approved new experiment (Jlab)
in a region very sensitive to the nucleon dynamics

- improve the precision of $a_E$ and $\beta_M$ by a factor of 2
- GPs $Q^2$ signature
- explore mechanism for the non trivial $Q^2$ dependence of $a_E$
- quantify the balance between paramagnetism and diamagnetism through $\beta_M$
- provide, with high precision, the spatial deformation of charge & magnetization densities under an applied e.m. field (currently a profound structure is suggested in the region 0.5 fm – 1 fm)
- Lattice QCD results will be emerging in the next few years - high precision benchmark data to cross check these calculations

Puzzle w.r.t. $a_E$

Thank you!