Abstract
The spectra of charged and neutral p- and π-mesons in uniform homogeneous
magnetic field (MF) are discussed in the framework of the path integral formalism
and vacuum correlator method. The spectra of all 12 spin-isospin
s-wave meson states were obtained analytically using the relativistic Hamiltonian
for quarks with confinement potential in a strong magnetic field. The states have
5 different types of asymptotics in strong MF: two of them are growing
with MF and the last three tend to a constant (zero-mode). The mass
of the zero mode becomes small in MF which can be the source of the meson collapse.
It was shown that the potential collapse has two different sources (color Coulomb and
hyperfine interactions) and it doesn’t occur for the MF < 2 GeV. The analytic data
presented is in a good agreement with lattice calculations.

Motivation
High intense magnetic fields up to $B \sim 10^5 - 10^7$ T are generated
during the early stages of heavy ion collisions at RHIC and LHC.
What happens with meson mass in strong MF $eB \sim m^2_\rho$, $m^2_\pi$? If one takes $\rho$ as an elementary particle, it should collapse in MF when
$m^2_\rho + eB(1 - y_B) < 0$. However, one should take into account internal
meson structure, when quark Larmor radius $r_B = \frac{1}{eB}$ reaches the meson size $\frac{1}{\sqrt{\pi}}$. We calculated mass spectrum of the light mesons
through the relativistic Hamiltonian was obtained with Vacuum Correlator
Method (VCM) to see an interplay between the MF and confinement.

Stochastic Vacuum Correlators Method
Feynman-Fock-Schwinger representation for the meson Green’s function
in MF
\[
G_{\psi}(x|y; A) = \int_0^\infty \frac{dK}{2\pi} \frac{e^{-iK \int_0^\infty dx z(x)|\psi(0)|^2}}{\xi^2 \xi^2 S_F A} \times \frac{1}{\sinh(Kr_B)}
\]
(1)
QCD vacuum is filled by the Euclidean stochastic gluonic field (Gaussian noise).
Wilson loop $\langle \mathcal{W}(A)\rangle_A$ after averaging the vacuum background
\[
\langle \mathcal{W}(A)\rangle_A = \exp \left( -\int d^4 x \xi^2 \left[ |\psi(x)|^2 - z(x) + \frac{1}{2} \xi^2 |\psi(x)|^2 \right] \right)
\]
(2)
Proper times $\xi^{a}_{\nu}$ for each quadrants are considered as fluctuations around
the Euclidean monotonous time $r_B$ in the absence of pair production $\sim \xi^4$ (Z-graphs) in the leading order
\[
\xi^4 (r_B + \Delta r_B) \leq \frac{\alpha}{4} - \frac{\alpha}{4} \left[ |\psi(x)|^2 - z(x) \right] \geq \frac{\alpha}{4}
\]
(3)
Green’s function in terms of relativistic Hamiltonian $R_{\psi}^{(2)}$ after averaging
over time fluctuations
\[
G_{\psi}(x|y; A) = \int_0^\infty \frac{dK}{2\pi} \frac{e^{-iK \int_0^\infty dx z(x)|\psi(0)|^2}}{\xi^2 S_F A} \times \frac{1}{\sinh(Kr_B)}
\]
(4)
We are interested in ground state energy, $T \rightarrow \infty$ leads to the stationary
point analysis for integrals over $\omega$.
\[
\frac{d}{d\omega} \left( \frac{d}{d\omega} \right) = 0
\]
(5)
$M_{dual}$ is non-perturbative mass spectrum, $\omega_0$ - "dynamical" quark masses
(constant mass analog for heavy quarks).
Total meson mass
\[
M_{total} = M_0 + \left( \frac{1}{\Omega} \frac{1}{\Omega} \right) M_{dual} + \left( \frac{1}{\Omega} \frac{1}{\Omega} \right) \Delta M_{dual}
\]
(6)
\[
\omega_0 = \left( \frac{1}{\Omega} \frac{1}{\Omega} \right) M_{dual}
\]
(7)
Spectral problem has analytical solutions in two cases:
• $Q = 0$ neutral mesons - Pseudomomentum technique
\[
K = r_B + \frac{eB}{\xi} \eta
\]
(8)
K is an integral of motion for $T(\beta)$ in MF.
• $Q \neq 0$ charged mesons - Constituent separation procedure
\[
\sigma^{(\rho)}(\rho^{(\rho)}) \rightarrow B^{(\rho)}(\rho^{(\rho)}) \rightarrow \epsilon^{(\rho)}(\epsilon^{(\epsilon)})
\]
(9)
where $\sigma^{(\rho)}(\rho^{(\rho)})$ are modified string tensions for the elongated confinement
string. Quarks become quasi-independent in $B$ plane for $B > \rho$. Confinement survives in $B$ direction.
Final step to calculate spectra - do the stationary point calculation (5)
for $\omega_0$.

Meson Trajectories Splitting
Meson mass trajectories are splitted in MF due to the magnetic moment
operator $B \cdot \sum_{\alpha} \sigma^\alpha \gamma^\alpha$ in (7) and hyperfine term from (6).
$[+\frac{1}{2}, \frac{1}{2}] \rightarrow [\frac{3}{2}, \frac{1}{2}]$, $[+\frac{1}{2}, \frac{1}{2}] \rightarrow [\frac{3}{2}, \frac{1}{2}]$ basis in spin space is convenient for $B > \rho$
asymp analysis. Three types of behaviour are possible - zero field
seeking (ZFS) and two types of low field seeking (LFS1, LFS2).

Pion Chiral Dynamics in MF
$\epsilon^4(\epsilon^4)$ chiral properties were restored with Gell-Mann-Oakes-Renner relation
\[
\frac{\sigma_{\rho}^2}{M_{\rho}^2} = \frac{m_{\sigma}^2}{4f_{\pi}^2}
\]
(12)
where the chiral condensates $\langle \bar{\psi} \psi \rangle \sim eB$ were calculated with VCM method
and effective chiral lagrangian in terms of non-chiral meson spectrum
\[
\langle \bar{\psi} \psi \rangle = M(0) \int d^4 x \left( \frac{m_{\sigma}^2}{2m_{\pi}^2} \right) \left( \frac{m_{\sigma}^2}{2m_{\pi}^2} \right)
\]
(13)
References