Light-Meson Spectroscopy in Strong Magnetic Field

Possible sources of the collapse and its prevention.

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Abstract

The spectra of charged and neutral ρ and π-mesons in uniform homogeneous magnetic field (MF) are discussed in the framework of the path integral formalism and vacuum correlator method. The spectra of all 12 spin-isospin s-wave meson states were obtained analytically using the relativistic Hamiltonian for quarks with confinement potential in strong magnetic field. The states have 3 different types of asymptotics in strong MF: two of them are growing with MF and the last one tends to be a constant (zero mode). The mass of the zero mode becomes small in MF which can be the source of the meson collapse. It was shown that the potential collapse has two different sources (color Coulomb and hyperfine interactions) and it doesn't occur for the MF $< 2GeV^2$. The analytic data presented is in a good agreement with lattice calculations.

Motivation

High intense magnetic fields up to $eB \sim \Lambda^2_{QCD} \sim 10^{19}\ G$ are generated during the early stages of heavy ion collisions at RHIC and LHC. What happens with meson mass in strong MF $eB \sim m^2_\pi,\ m^2_\rho$? If one takes ρ as an elementary particle it should collapse in MF when $m^2_\rho + eB(1-g_\rho) < 0$. However, one should take to the account internal meson structure, when quark Larmor radius $l_B = \frac{1}{\sqrt{eB}}$ reaches the meson size $\sim \frac{1}{\sqrt{\sigma}}$. We calculated mass spectrum of the light mesons through the relativistic Hamiltonian was obtained with Vacuum Correlator Method (VCM) to see an interplay between the MF and confinement.

Stochastic Vacuum Correlators Method

Feynman-Fock-Schwinger representation for the meson Green's function in MF

$$G_{q_1\bar{q}_2}(x,y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz^{(1)})(Dz^{(2)})\langle \hat{T} W_\sigma(A)\rangle_A \times$$
$$\exp\left(ie_1\int_y^x A_\mu^{(e)} dz_\mu^{(1)} - ie_2\int_y^x A_\mu^{(e)} dz_\mu^{(2)}\right)\times \qquad (1)$$
$$exp\left(e_1\int_0^{s_1} d\tau_1(\boldsymbol{\sigma}\mathbf{B}) - e_2\int_0^{s_2} d\tau_2(\boldsymbol{\sigma}\mathbf{B})\right)\times \exp(-K_1-K_2)$$

QCD vacuum is filled by the Euclidean stochastic gluonic field(Gaussian noise). Wilson loop $\langle \hat{T} W_\sigma(A)\rangle_A$ after the averaging over the vacuum background

$$\langle W_\sigma(A)\rangle_A = \exp\left(-\int_0^{\tau_E} dt_E\left[\sigma|\mathbf{z}_1-\mathbf{z}_2| - \frac{4}{3}\frac{\alpha_s}{|\mathbf{z}_1-\mathbf{z}_2|}\right]\right) \qquad (2)$$

Proper times s_1, s_2 for each quark are considered as fluctuations around the Euclidean monotonous time t_E (in absence of pair production $\sim \alpha^4$ (Z-graphs) in the leading order)

$$z_4(\tau) = t_E(\tau) + \Delta z_4(\tau),\ \omega_i = \frac{T}{2s_i},\ T = |x_4 - y_4| \qquad (3)$$

Green's function in terms of relativistic Hamiltonian $H_{q\bar{q}}$ after the averaging over time fluctuations

$$G_{q_1\bar{q}_2}(x,y) = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\frac{d\omega_2}{\omega_2^{3/2}}\left\langle \mathbf{x}\left| Tr(\hat{T}e^{-H_{q_1\bar{q}_2}T}\right|\mathbf{y}\right\rangle \qquad (4)$$

We are intersted in ground state energy, $T\to\infty$ leads to the stationary point analysis for integrals over ω_i

$$\hat{H}\psi = M_n\psi,\ \frac{\partial M_n(\omega_i)}{\partial\omega_i} = 0 \qquad (5)$$

M_n is non-perturbative mass spectrum, ω_i - "dynamical" quark masses (constituent mass analog for heavy quarks).

Total meson mass

$$M_{total} = M_n + \langle\psi|V_{OGE}|\psi\rangle + \langle\Psi|V_{SS}|\psi\rangle(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + \Delta M_{SE}, \qquad (6)$$

$\langle V_{OGE}\rangle$ - color Coulomb correction, spin-spin colormagnetic interaction $\langle V_{SS}\rangle$ and self-energy ΔM_{SE} are originated from 1-st order perturbative series of vacuum-averaged $\langle(\sigma B)(\sigma B)\rangle_A$ correlators.

Relativistic Hamiltonian for Meson in MF

Relativistic Hamiltonian for meson

$$H_{q\bar{q}} = \sum_{i=1}^{2}\frac{(\mathbf{p}^{(i)} - e_i\mathbf{A}^{(i)})^2 + m_i^2 + \omega_i^2 + e_i\boldsymbol{\sigma}_i\mathbf{B}}{2\omega_i} + \sigma|\mathbf{z}^{(1)} - \mathbf{z}^{(2)}| \qquad (7)$$

Spectral problem has analytical solutions in two cases:

- $Q = 0$ neutral mesons - Pseudomomentum technique

$$\hat{\mathbf{K}} = \hat{\mathbf{P}} + \frac{1}{2}\mathbf{B}\times\boldsymbol{\eta} \qquad (8)$$

K is an integral of motion for (7) in MF.

- $Q \neq 0$ charged mesons - Constituent separation procedure

$$\sigma|\mathbf{z}^{(1)} - \mathbf{z}^{(2)}| \to \sigma_1|\mathbf{z}^{(1)} - \mathbf{z}^{(c.m.)}| + \sigma_2|\mathbf{z}^{(2)} - \mathbf{z}^{(c.m.)}|, \qquad (9)$$

where $\sigma_i(\sigma, e_iB)$ are modified string tensions for the elongated confinement string. Quarks become quasi-independent in $\perp$ B plane for $eB \gg \sigma$. Confinement survives in $\parallel$ B direction.

Final step to calculate spectra - do the stationary point calculation (5) for ω_i.

Meson Trajectories Spltting

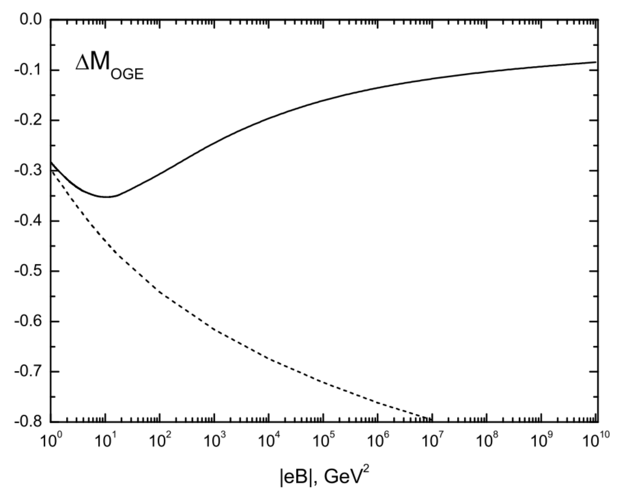
Meson mass trajectories are splitted in MF due to the magnetic moment operator B $\left(\frac{e_1}{2\omega_1}\boldsymbol{\sigma}_1 + \frac{e_2}{2\omega_2}\boldsymbol{\sigma}_2\right)$ in (7) and hyperfine term from (6). $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$ basis in spin space is convenient for $eB \gg \sigma$ asymptotics analysis. Three types of behaviour are possible - zero field seeking (ZFS) and two types of low field seeking (LFS1,LFS2):

- ZFS: $e_1\sigma_1^z > 0,\ e_2\sigma_2^z > 0$: $M_{ZFS}(eB\to\infty) = 2\sqrt{\sigma}$
- LFS1: $e_1\sigma_1^z > 0,\ e_2\sigma_2^z > 0$: $M_{LFS1}(eB\to\infty) = \sqrt{e_1B} + \sqrt{2\sigma}$
- LFS2: $e_1\sigma_1^z < 0,\ e_2\sigma_2^z < 0$: $M_{LFS2}(eB\to\infty) = 2\sqrt{e_1B} + 2\sqrt{e_2B}$

There are 12 mass trajectories for (π,ρ) mesons composed from u, d quarks and antiquarks due to spin-isospin splitting (see Fig. 1,2,3).

Perturbative Corrections and Collapse Prevention

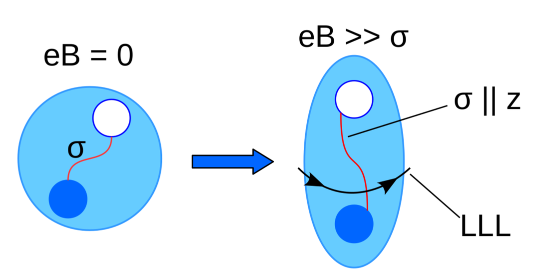
- Color Coulomb interaction - potential danger of the collapse for ZFS states (dashed line). Virtual $q\bar{q}$-pairs in LLL screen the Coulomb potential (solid line) and prevent the collapse.



- Collapse due to the spin-spin interaction

$$V_{SS} = \frac{8\pi\alpha_s}{9\omega_1\omega_2}\delta(\mathbf{r})(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \qquad (10)$$

Ground state wave function for s-wave meson - elongated ellipsoid



$$r_\perp \simeq \sqrt{\frac{2}{eB}} \qquad r_z \simeq \frac{1}{\sqrt{\sigma}}$$

Ellipsoid elongation causes magnetic "focusing" of the wave function $|\psi(0)|^2 \sim eB \to$ boundless decrease of $\langle V_{SS}\rangle$. Collapse prevented due to the natural cutoff parameter in VCM

$$\delta(\mathbf{r}) \to \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\frac{r^2}{\lambda^2}};\ \lambda \sim 1\ GeV^{-1} \qquad (11)$$

λ is correlation length of the stochastic vacuum (gluelump length).

Summary: these two scenarios prevent collapse for all ZFS except π^0. Chiral properties should be consided to prevent π^0 collapse in MF.

Pion Chiral Dynamics in MF

π^0 chiral properties were restored with Gell-Mann-Oaks-Renner relation

$$m^2_\pi f^2_\pi = m_q|\langle\bar{u}u + \bar{d}d\rangle|, \qquad (12)$$

where the chiral condensates $\langle\bar{q}q\rangle \sim eB$ were calculated with VCM method and effective chiral lagrangian in terms of non-chiral meson spectrum

$$\langle\bar{q}q\rangle = N_c M(0)\sum_{n=0}^{\infty}\left(\frac{\psi_n^{(+-)}(0)|^2}{2m_n^{(+-)}} + \frac{\psi_n^{(-+)}(0)|^2}{2m_n^{(-+)}}\right) \qquad (13)$$

Resulting Meson Spectra in MF

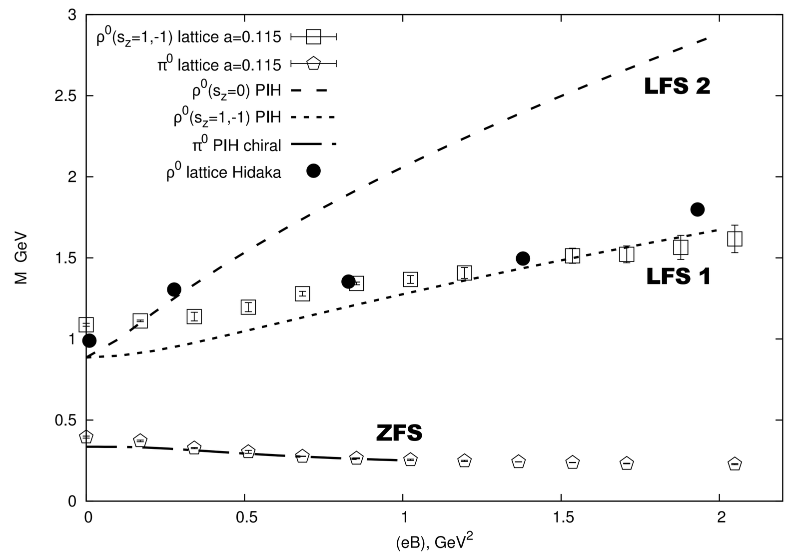


Figure 1: (π^0,ρ^0) trajectories in MF for $u\bar{u}$ quark constituents. Another 4 trajectories for $d\bar{d}$ demonstrate the same behaviour up to the scale factor $\sqrt{2}$ is defined by the q_u/q_d ratio. Lattice data is from [1,5]

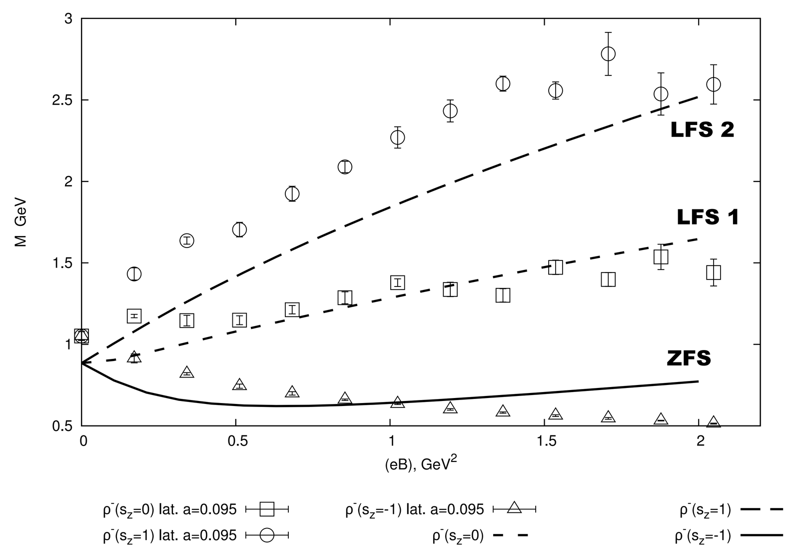


Figure 2: ρ^- meson trajectories in comparison with lattice data from [1]

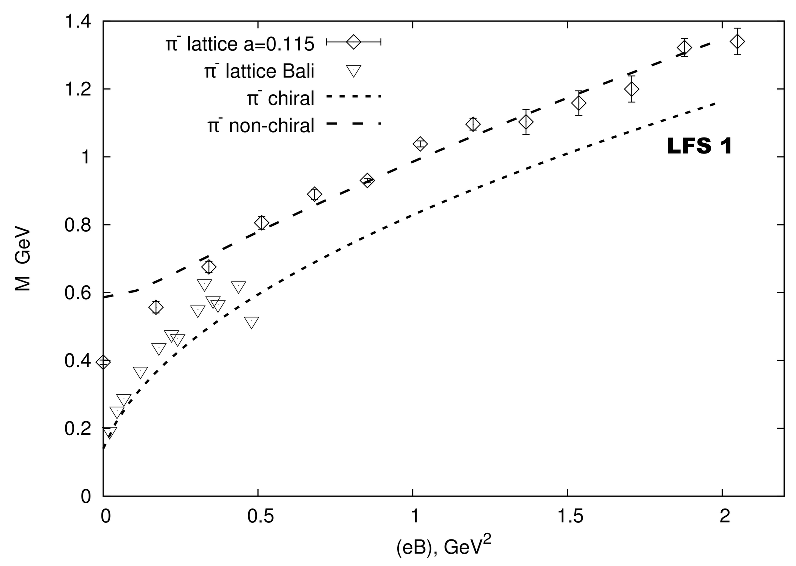


Figure 3: π^- meson loose its chiral properties with MF $eB > 0.5\ GeV^2$. Analytcal results are in good agreement with lattice data from [1,5]

References

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