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Heavy meson potential from a modified Schwinger-Dyson strong coupling

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References

C. Ayala, P. González, V. Vento : JPG 43, 125002 (2016).
V. Vento : EPJA 49, 71 (2013).
P. González, V. Mathieu ,V. Vento : PRD 84, 114008 (2011)

Motivation

The heavy quark potential is written in terms of the QCD running coupling $\alpha_V\left(Q^2\right)$.

QCD approaches (SDE , lattice…) provide information on the QCD running coupling $\alpha_s\left(Q^2\right)$.

$\alpha_s\left(Q^2\right)$ and $\alpha_V\left(Q^2\right)$ should be connected.

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Preliminaries

A. Deur, S. J. Brodsky, G. F. de Téramond: PPNP 90, 1 (2016)

Weak coupling regime of QCD : PQCD

$$\alpha_P\left(Q^2\right) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

$$\beta_0 = 11 - \frac{2}{3}n_f$$

$$\Lambda_{\overline{MS}}\ (n_f = 3) \simeq 358 \pm 22\ \mathrm{MeV}$$

$$\Lambda_{\overline{MS}}\ (n_f = 4) \simeq 303 \pm 21\ \mathrm{MeV}$$

Strong coupling regime of QCD : Approaches

Some definitions of the coupling in the IR attempt to generalize the PQCD definition to include confining effects.

Quenched Heavy Quark-Antiquark Potential

Nonrelativistic Static potential

$$V(r) = -\frac{4}{3}\frac{\chi(r)}{r} + \sigma r$$

$$\frac{1}{r} \to \frac{4\pi}{Q^2} \qquad r \to -\frac{8\pi}{Q^4} \qquad (Q^2 = \vec{q}^{\,2})$$

QCD coupling: Effective charge

$$V(Q^2) \equiv -\frac{4}{3}\frac{\alpha_V(Q^2)}{Q^2}$$

For $Q^2 \to \infty$: $\alpha_V(Q^2) \to \alpha_P(Q^2)$

Example: Richardson coupling (ansatz)

J. L. Richardson : PLB 82, 272 (1979)

$$\left(\alpha_V\left(Q^2\right)\right)_{Rich}=\frac{4\pi}{\beta_0\ln\left(1+\frac{Q^2}{\Lambda^2}\right)}$$

$$\left(V\left(r\right)\right)_{Rich}=\frac{8\pi}{3\beta_0}\left(-\frac{f\left(\Lambda r\right)}{r}+\Lambda^2 r\right)$$

Good fit to the quarkonia spectra : $\Lambda=398\ \ \mathrm{MeV}$

$m_c=1491\ \ \mathrm{MeV}$ $m_b=4884\ \ \mathrm{MeV}$

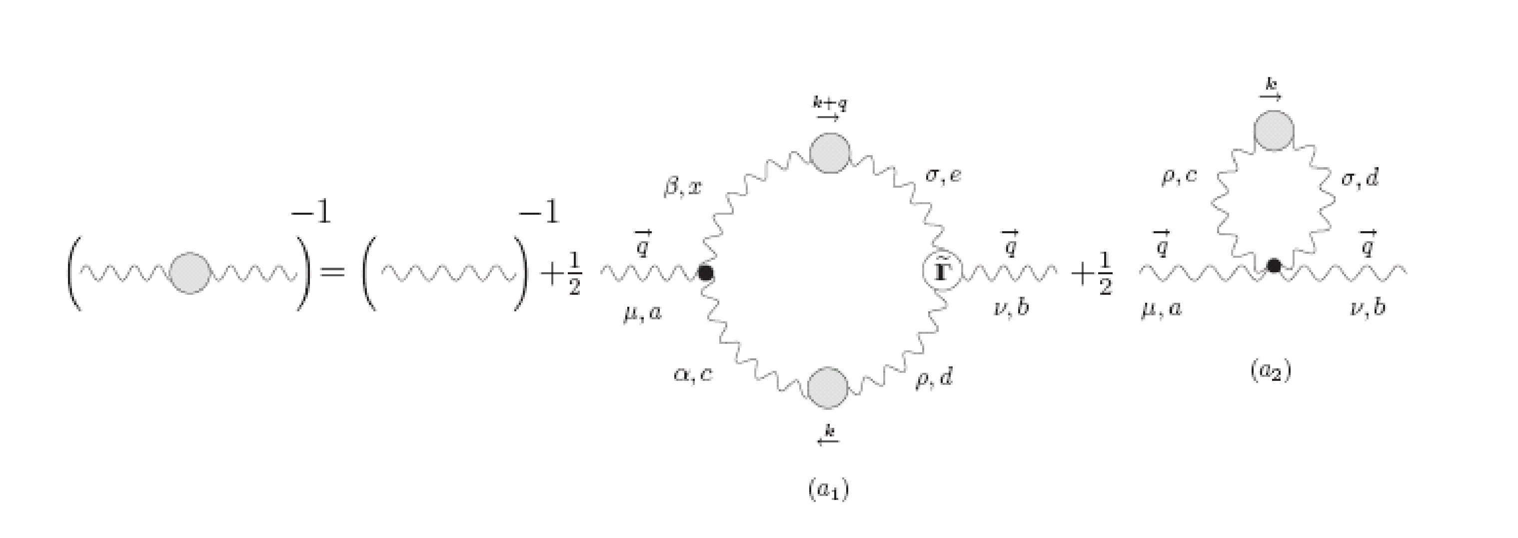
$(n_f=3)$

$$\sigma=\frac{8\pi\Lambda^2}{3\beta_0}=0.15\ \ \mathrm{GeV}^2$$

Quenched Schwinger-Dyson Equations

A QFT is completely characterized by its Green functions which are governed by SDE.

PT – BFM truncation scheme (D. Binosi, J. Papavassiliou, Phys. Rep. 479 (2009) 1-152)



$$(\Delta^{-1})^{ab}_{\mu\nu}(q) = iq^2 g_{\mu\nu}\delta^{ab} - \left[\Pi^{ab}_{\mu\nu}(q)\big|_{a_1} + \Pi^{ab}_{\mu\nu}\big|_{a_2}\right]$$

+ three gluon vertex ansatz

Effective Gluon Mass

Infrared finite quenched solutions of the SDE are obtained. These solutions may be fitted by a "massive" euclidean propagator.

Gauge invariant and renormalization group invariant : $\alpha_s\left(Q^2\right)\Delta\left(Q^2\right)$

$$\Delta\left(Q^2\right)=\frac{1}{Q^2+m_g^2\left(Q^2\right)}$$

$$m_g^2\left(Q^2\right)=\frac{m_g^2\left(0\right)}{1+\left(\frac{Q^2}{M^2}\right)^{1+p}}$$

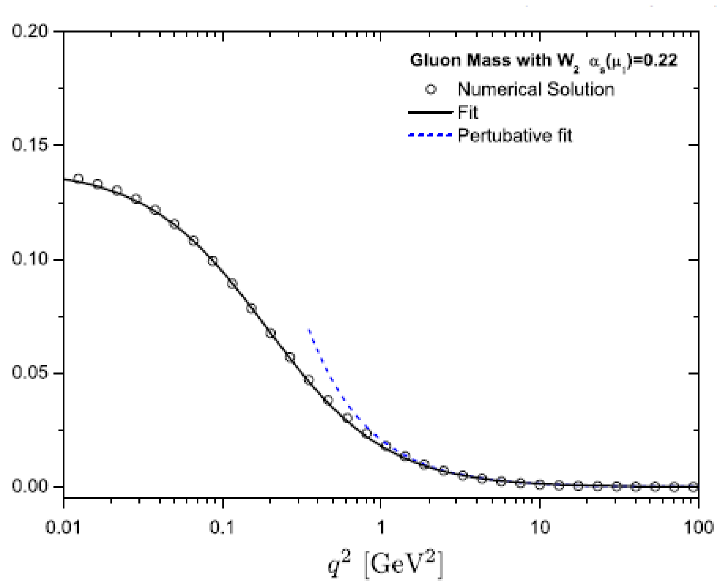
$$\alpha_s\left(Q^2\right)\equiv\alpha_{s(gse)}\left(Q^2\right)=\frac{4\pi}{\beta_0\ln\left(\frac{Q^2+\rho m_g^2\left(Q^2\right)}{\Lambda^2}\right)}$$

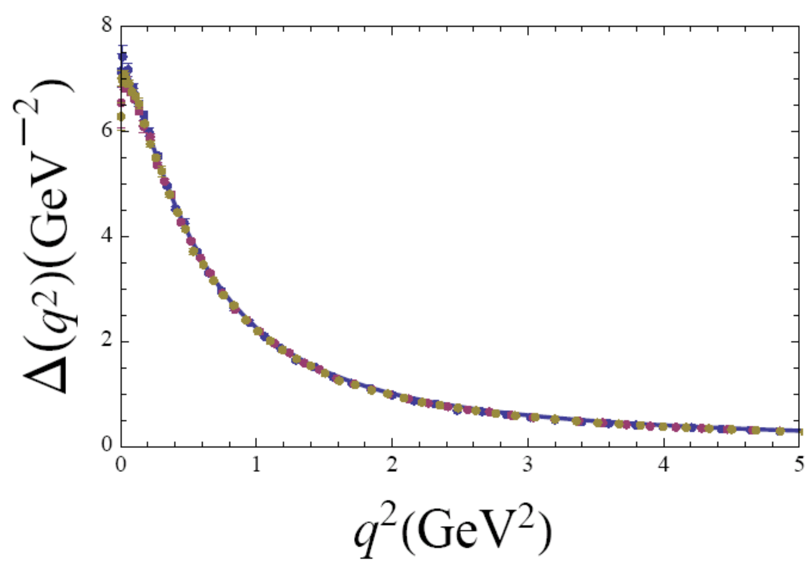
mg(0), M, p > 0, ρ : constants

$$M = 436 \ \ \mathrm{MeV}$$

$$p = 0.15$$

$$m_g\,(0) = 375 \ \ \mathrm{MeV}$$





From $\alpha_s(Q^2)$ to $\alpha_V(Q^2)$

If $\quad \rho m_g^2(Q^2) = \Lambda^2$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2+\rho m_g^2(Q^2)}{\Lambda^2}\right)} \to \frac{4\pi}{\beta_0 \ln\left(1+\frac{Q^2}{\Lambda^2}\right)} = \left(\alpha_V(Q^2)\right)_{Rich}$$

If

$$\rho m_g^2(0) = \Lambda^2 \to \rho m_g^2(Q^2) = \frac{\Lambda^2}{1+\left(\frac{Q^2}{M^2}\right)^{1+p}}$$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2+\rho m_g^2(Q^2)}{\Lambda^2}\right)} \to \frac{4\pi}{\beta_0 \ln\left(\frac{1}{1+\left(\frac{Q^2}{M^2}\right)^{1+p}}+\frac{Q^2}{\Lambda^2}\right)} \equiv \alpha_V(Q^2)$$

Heavy Quark Potential from a modified SD coupling

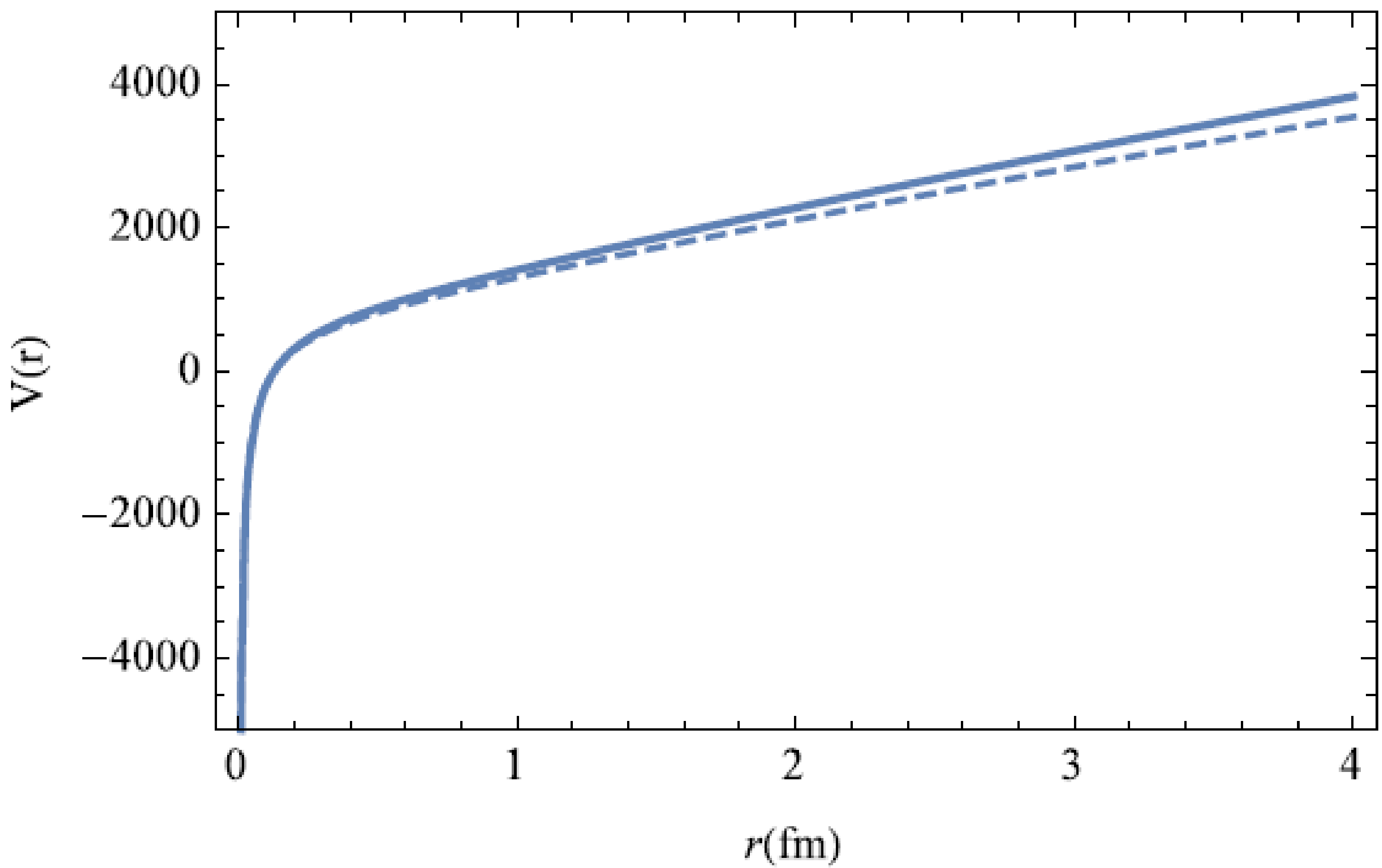
$$V\left(Q^2\right) \equiv -\frac{4}{3}\frac{\alpha_V\left(Q^2\right)}{Q^2}$$

$$V(r) = -\frac{32}{3\beta_0}\int_0^\infty \frac{1}{\ln\left(\dfrac{q^2+\dfrac{\Lambda^2}{1+(q^2/\mathcal{M}^2)^{1+p}}}{\Lambda^2}\right)}\frac{\sin(qr)}{qr}\mathrm{d}q$$

Regularization

$$V(r) = -\frac{32}{3\beta_0}\lim_{\gamma\to 0}\left(\int_\gamma^\infty \frac{1}{\ln\left(\dfrac{q^2+\dfrac{\Lambda^2}{1+(q^2/\mathcal{M}^2)^{1+p}}}{\Lambda^2}\right)}\frac{\sin(qr)}{qr}\mathrm{d}q - I_s(\gamma)\right)$$

$M = 436$ MeV $\quad p = 0.15 \quad \Lambda = 320$ MeV



$m_b = 4450\ \ \mathrm{MeV}$

J^{PC}	nl	$M_{V\,(r)_{\Lambda=320\ \mathrm{MeV}}}$ MeV	M_{PDG} MeV
1^{--}	$1s$	9489	9460.30 ± 0.26
	$2s$	10023	10023.26 ± 0.31
	$1d$	10147	10163.7 ± 1.4
	$3s$	10354	10355.2 ± 0.5
	$2d$	10435	
	$4s$	10621	10579.4 ± 1.2
	$3d$	10681	
	$5s$	10854	10876 ± 11
	$4d$	10903	
$(0,1,2)^{++}$	$1p$	9903	$9899.87 \pm 0.28 \pm 0.31$
$(0,1,2)^{++}$	$2p$	10254	$10260.24 \pm 0.24 \pm 0.50$
$(0,1,2)^{++}$	$3p$	10531	10534 ± 9

$m_c = 1030 \text{ MeV}$

J^{PC}	nl	$M_{V(r)_{\Lambda=320 \text{ MeV}}}$ MeV	M_{PDG} MeV
1^{--}	$1s$	3090	3096.916 ± 0.011
	$2s$	3671	$3686.108^{+0.011}_{-0.014}$
	$1d$	3772	3778.1 ± 1.2
	$3s$	4102	4039 ± 1
	$2d$	4170	4191 ± 5
$(0,1,2)^{++}$	$1p$	3484	3525.30 ± 0.11
2^{++}	$2p$	3940	3927.2 ± 2.6

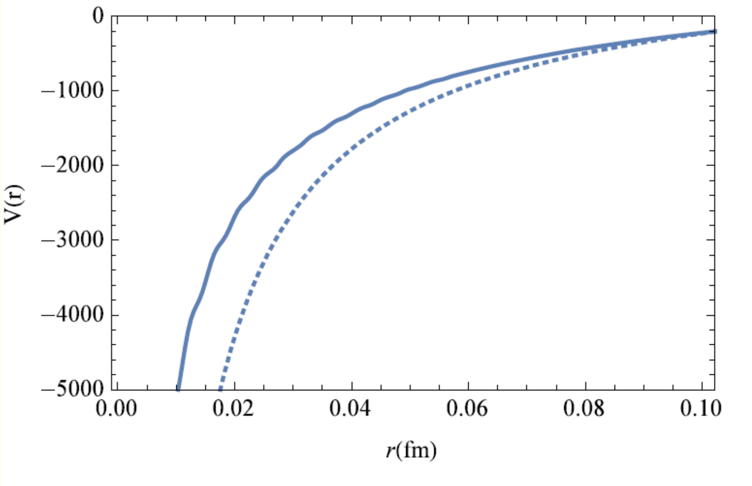
Comparison with other potentials

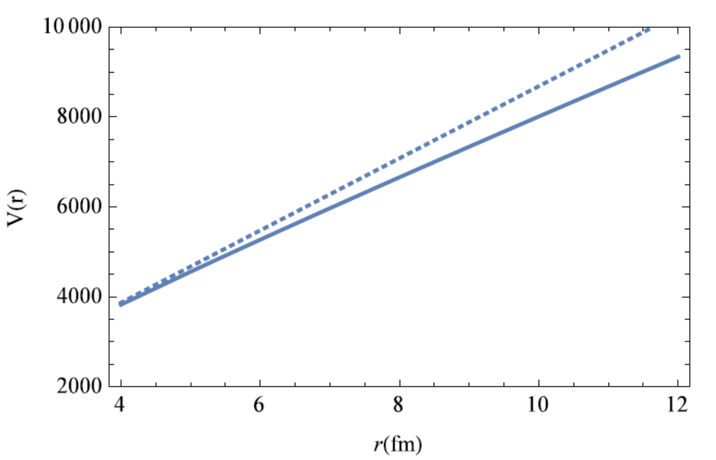
i)Cornell

$$(V^C(r))_{b\bar{b}} = \sigma_{b\bar{b}}\, r - \frac{\chi_{b\bar{b}}}{r} + a_{b\bar{b}}$$

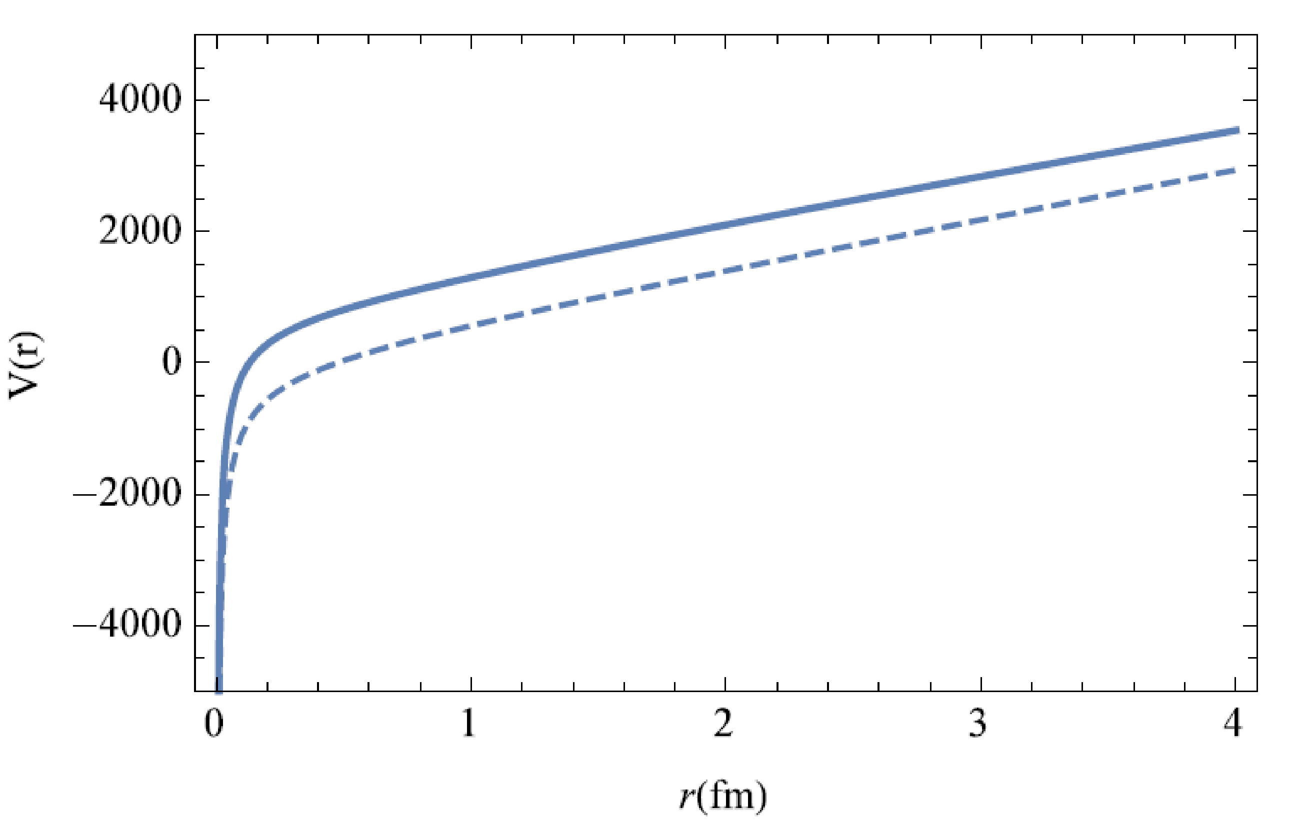
$$\sigma_{b\bar{b}} = 0.16 \quad \mathrm{GeV}^2$$
$$\chi_{b\bar{b}} = 0.5$$
$$a_{b\bar{b}} = 693 \text{ MeV}$$

$$0.1 \quad \mathrm{fm} < r < 4 \quad \mathrm{fm} \rightarrow V(r) = \left(V^C(r)\right)_{b\bar{b}}$$





ii) Richardson

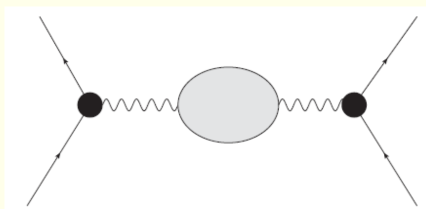


Physical interpretation

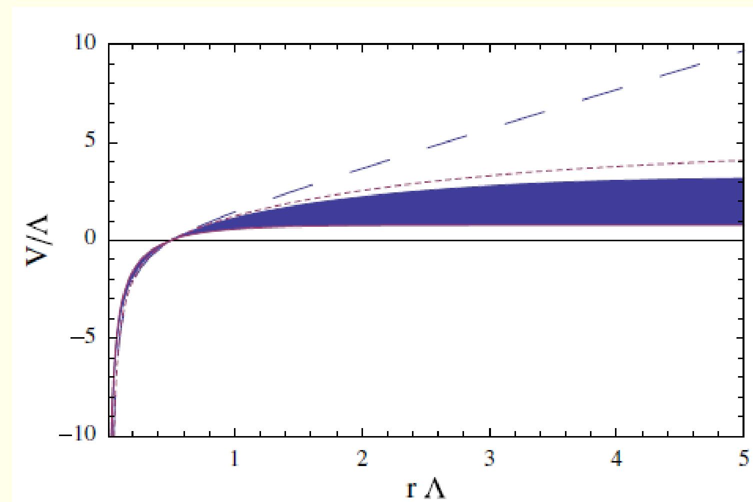
$\rho m_g^2\left(Q^2\right)=\Lambda^2$ and $\rho m_g^2\left(0\right)=\Lambda^2$ give rise to IR divergent couplings.

$\alpha_V\left(Q^2\right)$ comprises effects from one gluon and multigluon exchanges.

OGE



$$V_{OGE}\left(Q^2\right)=-\frac{4}{3}\,\frac{\alpha_s\left(Q^2\right)}{Q^2+m_g^2\left(Q^2\right)}$$



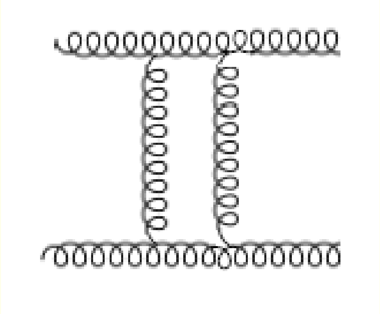
FULL

$$V\left(Q^2\right)=-\frac{4}{3}\,\frac{\alpha_s\left(Q^2\right)}{Q^2+m_g^2\left(Q^2\right)}\left(1+\frac{m_g^2\left(Q^2\right)}{Q^2}\right)$$

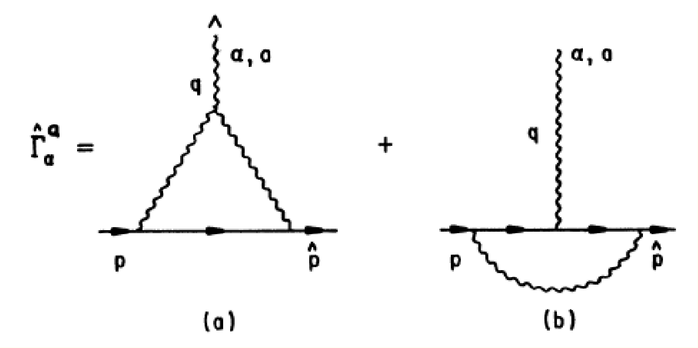
$$\rho m_g^2\left(0\right)=\Lambda^2$$

Options

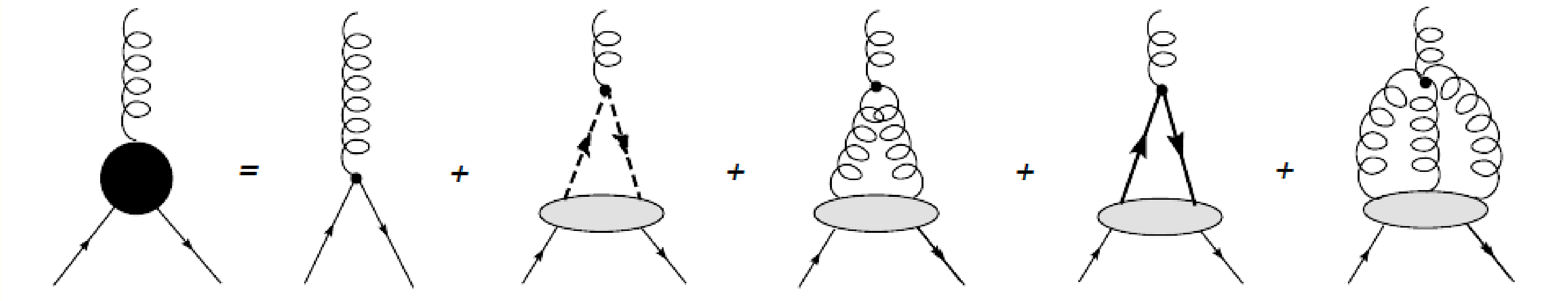
i)Multigluon exchange

IR divergent  + …

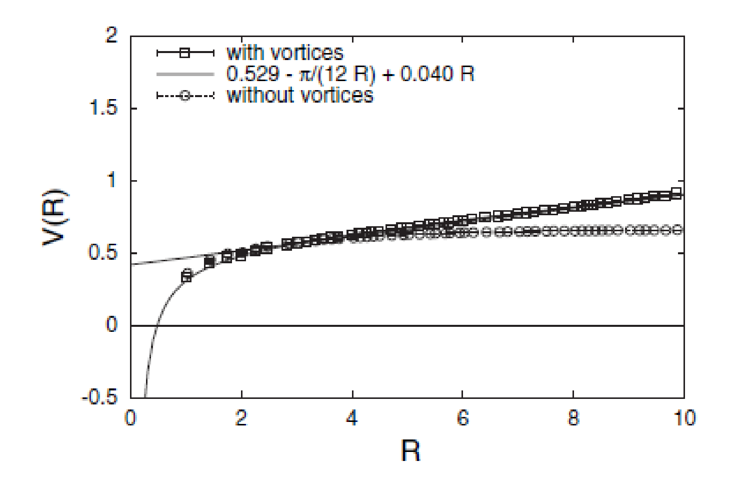
ii) Vertex corrections



iii) Infrared divergence of the quark-gluon vertex due to chiral symmetry breaking.
R. Alkofer, C. S. Fischer, F. J. Llanes-Estrada, K. Schwenzer, AP 324, 106 (2009).



iv) Vacuum structure (center vortices) J. Greensite, S. Olejnik, PRD 67, 094503 (2003).



Summary

i) From the heavy quark-antiquark potential we have defined an effective potential coupling, with the correct asymptotic behavior, that incorporates confining effects.

ii) By assuming for this potential coupling the same momentum dependence as for the Schwinger-Dyson (SD) coupling we have shown that a good description of the heavy quarkonia spectra is feasible.

iii) The IR divergent value of the potential could be either indicating an IR divergent value of the coupling or an IR finite value of the coupling through the presence of IR divergent contributions beyond the OGE.

THE END