Bottom quark mass determination from bottomonium at N^3LO

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XVII International Conference on Hadron Spectroscopy and Structure – September 25^{th}-29^{th} 2017

VNiVERSiDAD D SALAMANCA

Outline

1. Introduction
2. Theoretical Framework
3. Determination of $\overline{m}_b$
4. Conclusions

Outline

Build on top of

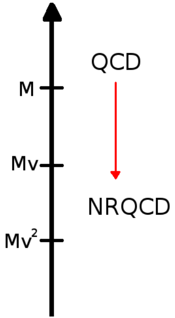
Previous studies:

- Brambilla-Sumino-Vairo, PRD 65, 034001 (2002).
  - $\overline{m}_b$ from $\Upsilon(1S)$ mass at N^{2}LO. $\overline{\mathrm{MS}}$-scheme.
  - Include dominant effects of nonzero charm quark mass.
- Kiyo-Sumino, PLB 752, 122 (2016).
  - $\overline{m}_b$ from $\Upsilon(1S)$ and $\eta_b(1S)$ mass at N^{3}LO. $\overline{\mathrm{MS}}$-scheme.
  - Include finite charm mass effects.
  - α_s^4-term in $(m_{\mathrm{pole}} - \overline{m})$ expansion estimated.
- Ayala-Cvetic-Pineda, J. Phys: Conf. Ser. 762, 012063 (2016).
  - $\overline{m}_b$ from $\Upsilon(1S)$ mass at N^{3}LO. RS-scheme.
  - Include finite charm mass corrections.
- Our goal: Explore the determination of $\overline{m}_b$ from $b\bar{b}$ spectrum at N^{3}LO with:
  - A different short-distance mass scheme (MSR).
  - Higher $b\bar{b}$ states.
  - Careful examination of perturbative errors.

Outline

NRQCD

- $Q\bar{Q}$ bound state $\rightarrowtail m_Q \gg \Lambda_{\mathrm{QCD}}$, $\alpha_s(m_Q) \ll 1$.
- Heavy quarks move slowly $v \ll 1$
- Non-relativistic system $\rightarrowtail$ Different scales:
  - $m_Q \gg m_Q v \gg m_Q v^2 \gtrsim \Lambda_{\mathrm{QCD}}$



- Remaining degrees of freedom:
  - Heavy quark: Soft ($E \sim mv^2$, $k \sim mv$) and Ultra-soft ($E \sim k \sim mv^2$).
  - Gluon: Ultra-soft
  - Light quarks: Ultra-soft

Static $V_{\rm QCD}(q)$ potential

$$V_C(r) = V_C^{\rm LO}(r) + V_C^{\rm NLO}(r) + V_C^{\rm NNLO}(r) + V_C^{\rm NNNLO}(r)$$

where

- $V_C^{\rm LO}(r) = -\frac{C_F \alpha_s}{r} \rightarrowtail$ Coulomb potential.
- $V_C^{\rm NLO}(r)$:
  - Massless $m_c \to$ Appelquist-Dine-Muzinich, '77, '78
  - Massive $m_c \to$ Fischler '77, Billoire '80
- $V_C^{\rm NNLO}(r)$:
  - Massless $m_c \to$ Peter '97, Schroder '99
  - Massive $m_c \to$ Melles '00, Hoang '00, Recksigel-Sumino '02
- $V_C^{\rm NNNLO}(r)$:
  - Massless $m_c \to$ Anzai-Kiyo-Sumino '09, Smirnov-Steinhauser '09

$Q\bar{Q}$ state binding energy

- n_l (massless) active flavors only
- State with (n, j, l, s) quantum numbers:

$$E_X(\mu, \alpha_s(\mu), m_Q^{\rm pole}) = 2m_Q^{\rm pole}\left[1 - \frac{C_F^2 \alpha_s^{(n_l)}(\mu)^2}{8n^2}\sum_{i=0}^{\infty}\left(\frac{\alpha_s^{(n_l)}(\mu)}{\pi}\right)^i \varepsilon^{i+1} P_i(L_{nl})\right]$$

with $L_{nl} = \log\left(\frac{n\mu}{C_F \alpha_s(\mu) m_Q^{\rm pole}}\right) + \sum_{k=1}^{n+l}\frac{1}{k}$.

- ε : Parameter to organize perturbative expansion $\rightarrowtail \mathcal{O}(\Lambda_{\rm QCD})$ renormalon cancellation.
- $P_i(L_{nl})$ denotes an $i-th$ degree polynomial of L_{nl}:
  - $P_0 = 1$
  - $P_1 = \beta_0^{(n_l)} L_{nl} + c_1$
  - $P_2 = \frac{3}{4}\beta_0^{(n_l)\,2} L_{nl}^2 + \left(-\frac{1}{2}\beta_0^{(n_l)\,2} + \frac{1}{4}\beta_1^{(n_l)} + \frac{3}{2}\beta_0^{(n_l)} c_1\right) L_{nl} + c_2^{(n_l)}$
  - $P_3 = \frac{1}{2}\beta_0^{(n_l)\,3} L_{nl}^3 + \left(-\frac{7}{8}\beta_0^{(n_l)\,3} + \frac{7}{16}\beta_0^{(n_l)}\beta_1^{(n_l)} + \frac{3}{2}\beta_0^{(n_l)\,2} c_1\right) L_{nl}^2 + \left(\frac{1}{4}\beta_0^{(n_l)\,3} - \frac{1}{4}\beta_0^{(n_l)}\beta_1^{(n_l)} + \frac{1}{16}\beta_2^{(n_l)} - \frac{3}{4}\beta_0^{(n_l)\,2} c_1 + 2\beta_0^{(n_l)} c_2^{(n_l)} + \frac{3}{8}\beta_1^{(n_l)} c_1\right) L_{nl} + c_3^{(n_l)}$

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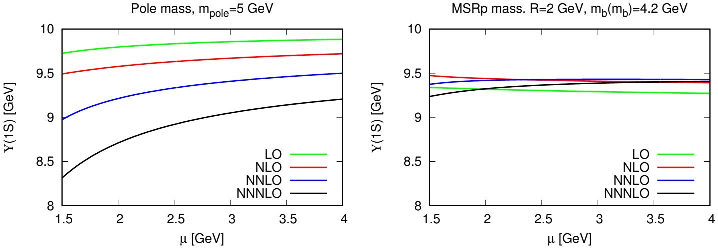

Pole mass scheme

MSRp mass scheme

MSR Mass

- $\overline{\mathrm{MS}}$ scheme $\rightarrowtail$ $m_Q^{\mathrm{pole}} - \overline{m}_Q = \overline{m}_Q \sum_{n=1}^{\infty} a_n^{\overline{\mathrm{MS}}}(n_l, n_h) \left(\frac{\alpha_s^{(n_l+n_h)}(\overline{m}_Q)}{4\pi} \right)^n$
- MSR scheme $\rightarrowtail$ Extension of $\overline{\mathrm{MS}}$ scheme to scales $\ll m_Q$.
- Two ways of implementing series in terms of $\alpha_s^{(nl)}$:
  A. Hoang, PRL101, 151602 (2008), hep-ph/1704.01580 (2017).
    - *Practical MSR mass* $\rightarrowtail$ Use threshold relation for α_s to express the series in terms of $\alpha_s^{(nl)}$:

$$m_Q^{\mathrm{pole}} - m_Q^{MSRp}(R) \quad = \quad R \sum_{n=1}^{\infty} a_n^{MSRp}(n_l) \left(\frac{\alpha_s^{(n_l)}(R)}{4\pi} \right)^n$$

    - *Natural MSR mass* $\rightarrowtail$ Integrate out m_Q virtual loop corrections:

$$m_Q^{\mathrm{pole}} - m_Q^{MSRn}(R) \quad = \quad R \sum_{n=1}^{\infty} a_n^{\overline{\mathrm{MS}}}(n_l, 0) \left(\frac{\alpha_s^{(n_l)}(R)}{4\pi} \right)^n$$

MSR Mass

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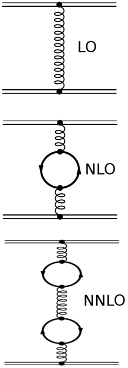
- MSR scheme $\rightarrowtail$ Extension of $\overline{\mathrm{MS}}$ scheme to scales $\ll m_Q$.
- Direct connection with $\overline{\mathrm{MS}}$:
  - *Practical MSR mass* $\rightarrowtail$ $m_Q^{MSRp}(m_Q^{MSRp}) = \overline{m}_Q(\overline{m}_Q)$
  - *Natural MSR mass* $\rightarrowtail$ $m_Q^{MSRn}(\overline{m}_Q) - \overline{m}_Q =$
    $\overline{m}_Q \sum_{k=1}^{\infty} \left[a_k^{\overline{\mathrm{MS}}}(n_l, 1) \left(\frac{\alpha_s^{(n_l+1)}(\overline{m}_Q)}{4\pi} \right)^k - a_k^{\overline{\mathrm{MS}}}(n_l, 0) \left(\frac{\alpha_s^{(n_l)}(\overline{m}_Q)}{4\pi} \right)^k \right]$
- R-evolution:
  - $R \frac{d}{dR} m_Q^{MSR}(R) = -R\gamma^R(\alpha_s(R)) = -R \sum_{n=0}^{\infty} \gamma_n^R \left(\frac{\alpha_s(R)}{4\pi} \right)^{n+1}$
  - Linear in R.
  - γ_n^R : R-anomalous dimension coefficients.
- Express m_Q^{pole} in terms of $\overline{m}_Q$ and R and expand in powers of $\alpha_s(\mu)$.

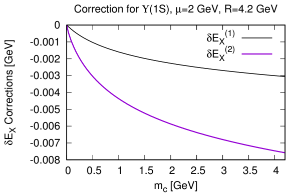
Massive charm quark mass corrections

- Corrections to binding energy:

$$E_X(\mu, \alpha_s(\mu), \overline{m}_b^{\rm pole}, \overline{m}_c) = E_X(\mu, \alpha_s(\mu), \overline{m}_b^{\rm pole}) + \varepsilon^2 \delta E_{m_c}^{(1)} + \varepsilon^3 \delta E_{m_c}^{(2)}$$

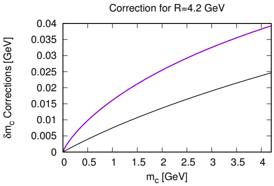
  - $\delta E_{m_c}^{(1)} \rightarrowtail$ Calculated for all quantum numbers.
    Eiras-Soto, PLB491, 101 (2000).
  - $\delta E_{m_c}^{(2)} \rightarrowtail$ Exact formula for $\Upsilon(1S)$
    Hoang, hep-ph/0008102.
    Approximation $\overline{m}_c \to \infty$ for other states.
    Brambilla-Sumino-Vairo PRD65, 034001 (2002):
    $\delta E_{m_c}^{(2)} = E_X(\mu, \alpha_s^{(3)}(\mu), \overline{m}_b^{\rm pole}) - E_X(\mu, \alpha_s^{(4)}(\mu), \overline{m}_b^{\rm pole})$

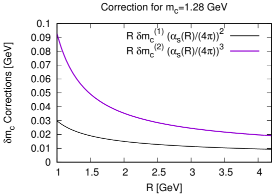


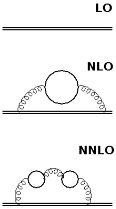


Massive charm quark mass corrections

$$m_Q^{\rm pole} - m_Q^{\rm MSR}(R) = \delta m^{\rm MSR}_{Q\,(\overline{m}_c=0)} + R\left(\varepsilon^2 \delta m_c^{(1)}\left(\tfrac{\overline{m}_c}{R}\right)\left(\frac{\alpha_s(R)}{4\pi}\right)^2 + \varepsilon^3 \delta m_c^{(2)}\left(\tfrac{\overline{m}_c}{R}\right)\left(\frac{\alpha_s(R)}{4\pi}\right)^3\right)$$







- $\delta m_c^{(1)} \rightarrowtail$ Exact expression: Gray-Broadhurst-Grafe-Schilcher, ZPC48, 673 ('90).
- $\delta m_c^{(2)} \rightarrowtail$ Exact expression: Bekavac-Grozin-Seidel-Steinhauser, JHEP0710, 006 ('07).
- Finite charm-mass corrections also affects R-evolution:

$$\gamma^R \to \gamma^R + \left(\delta m_c^{(1)}\left(\tfrac{\overline{m}_c}{R}\right) - \tfrac{\overline{m}_c}{R}\,\delta m_c^{(1)\,\prime}\left(\tfrac{\overline{m}_c}{R}\right)\right)\left(\frac{\alpha_s(R)}{4\pi}\right)^2 +$$
$$+\left(\delta m_c^{(2)}\left(\tfrac{\overline{m}_c}{R}\right) - \tfrac{\overline{m}_c}{R}\,\delta m_c^{(2)\,\prime}\left(\tfrac{\overline{m}_c}{R}\right) - \beta_0\,\delta m_c^{(1)}\left(\tfrac{\overline{m}_c}{R}\right)\right)\left(\frac{\alpha_s(R)}{4\pi}\right)^3$$

- Approach that satisfies HQS. Hoang-Lepenik-Preisser, hep-ph/1706.08526

$b\bar{b}$ state binding energy: Schemes

For completeness we will study different cases:

- Two different approaches for MSR mass:
  - *Practical* MSR mass.
  - *Natural* MSR mass.
- Two alternative ways to deal with charm quark:
  - *$(n_l = 4)$-scheme* $\rightarrowtail$ Exhibits massless $m_c \to 0$ limit:

$$E_X\left(n_l = 4, \mu, \alpha_s^{(4)}(\mu), m_b^{\mathrm{MSR}} + \delta m_b^{\mathrm{MSR}} + \varepsilon \delta m_c\right) + \varepsilon^2 \delta E_{m_c}^{(1)} + \varepsilon^3 \delta E_{m_c}^{(2)}$$

    - $n_l = 4$ active massless flavors.
    - Charm mass corrections for non-zero charm mass.
  - *$(n_l = 3)$-scheme* $\rightarrowtail$ Exhibits decoupling $m_c \to \infty$ limit:

$$E_X\left(n_l = 3, \mu, \alpha_s^{(4)}(\mu), m_b^{\mathrm{MSR}}\right) + \varepsilon^2 \delta' E_{m_c}^{(1)} + \varepsilon^3 \delta' E_{m_c}^{(2)},$$

    - $n_l = 3$ active massless flavors.
    - Charm mass corrections for non-infinite charm mass.

$b\bar{b}$ state binding energy: Schemes

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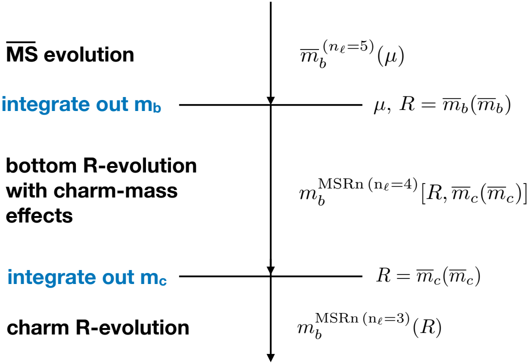


Outline

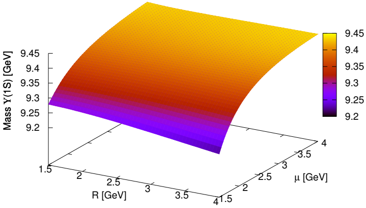
Fitting method

- Aim: Fit $\overline{m}_b$ from experimental $b\bar{b}$ state masses.
- We use different sets of $b\bar{b}$ states: $\{\Upsilon(1S), \eta_b(1S), n=1, n=2, \ldots\}$
- $M_i^{\rm th} = E_X(\mu, \alpha_s(\mu), R, \overline{m}_b, \overline{m}_c) \rightarrow$ Dependent of $\overline{m}_b$ and (μ,R) scales.

- Calculate $M_i^{\rm th}(\overline{m}_b)$ for (μ,R) square grid

$$\begin{cases} 1.5\, GeV & \leq \mu \leq & 4\, GeV \\ 1.5\, GeV & \leq R \leq & 4\, GeV \end{cases}$$

- Parameters:
  - $\alpha_s^{(5)}(M_Z) = 0.1181 \pm 0.0011.$
  - $\overline{m}_c = 1.28 \pm 0.03$ GeV.



Fitting method

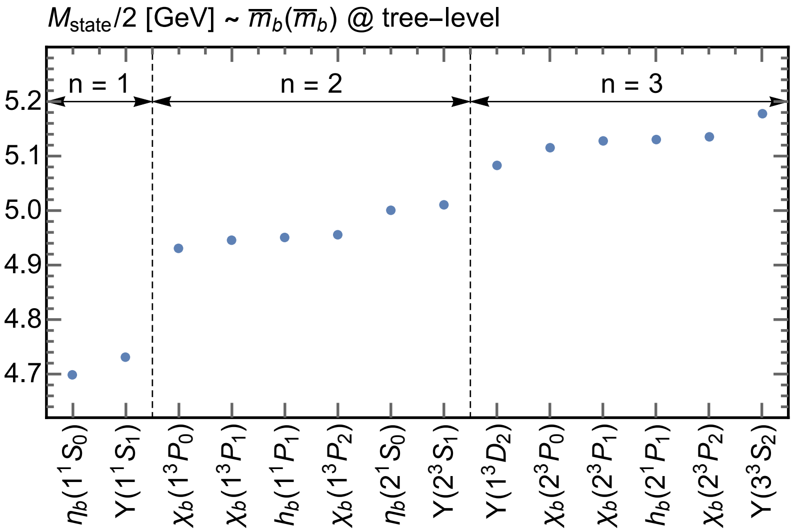
- First approach $\rightarrowtail$ Take average of $M_i^{\rm th}(\overline{m}_b)$ in grid and fit:

$$\chi_A^2 = \sum_i \left(\frac{M_i^{\rm exp} - M_i^{\rm th}(\overline{m}_b)}{\sigma_i^{\rm exp}} \right)^2 \rightarrowtail \overline{m}_b^{BF}$$

- But... Theoretical errors are highly correlated $\rightarrowtail$ D'Agostini bias: $\overline{m}_b^{BF}$ *from a given set is below every* $\overline{m}_b^{BF}$ *from individual states in set.*
- Second approach $\rightarrowtail$ $\overline{m}_{b\,ij}^{BF}$ for each point in (μ, R) grid:
- At each (μ, R) point we obtain $\overline{m}_b^{BF}$:

$$\chi_B^2(\mu, R) = \sum_i \left(\frac{M_i^{\rm exp} - M_i^{\rm th}(\mu, R, \overline{m}_b)}{\sigma_i^{\rm exp}} \right)^2 \rightarrowtail \overline{m}_b^{BF}(\mu, R)$$

- The central value is the average of $\overline{m}_b^{BF}(\mu, R)$ in grid.
- Theoretical uncertainty: $\Delta^{\rm th} = \frac{1}{2}(max(\overline{m}_b^{BF}(\mu, R)) - min(\overline{m}_b^{BF}(\mu, R)))$
- Difference between $(n_l = 3)$ and $(n_l = 4)$ schemes as an additional error $\rightarrowtail$ Ignorance on ε^4 charm-mass corrections.


$M_{state}/2$ [GeV] ~ $\overline{m}_b(\overline{m}_b)$ @ tree–level
n = 1
n = 2
n = 3
5.2
5.1
5.0
4.9
4.8
4.7
$\eta_b(1^1S_0)$
$\Upsilon(1^1S_1)$
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$\eta_b(2^1S_0)$
$\Upsilon(2^3S_1)$
$\Upsilon(1^3D_2)$
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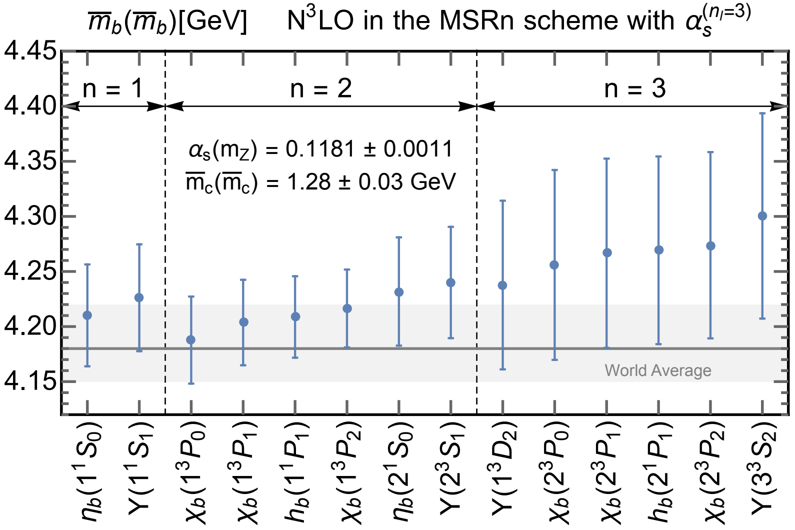
$\Upsilon(3^3S_2)$

Results



$\overline{m}_b(\overline{m}_b)$[GeV]
N^{3}LO in the MSRn scheme with $\alpha_s^{(n_l=3)}$
4.45
4.40
4.35
4.30
4.25
4.20
4.15
n = 1
n = 2
n = 3
$\alpha_s(m_Z) = 0.1181 \pm 0.0011$
$\overline{m}_c(\overline{m}_c) = 1.28 \pm 0.03$ GeV
World Average
$\eta_b(1^1S_0)$
$\Upsilon(1^1S_1)$
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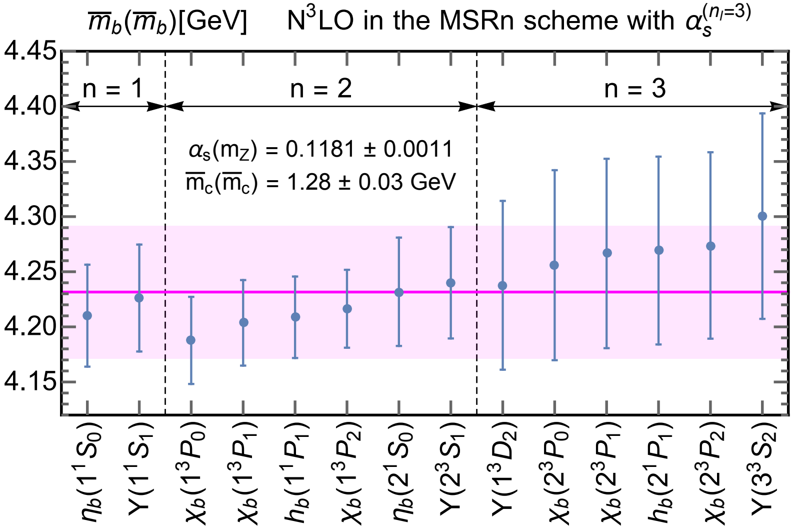

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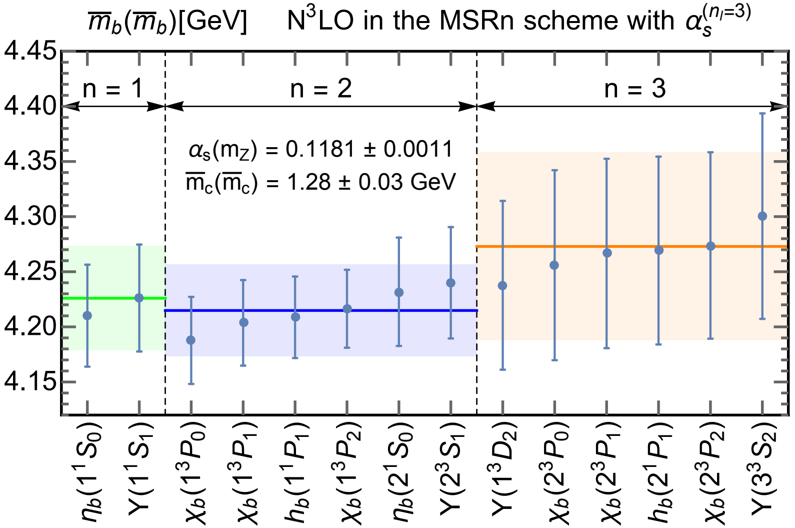

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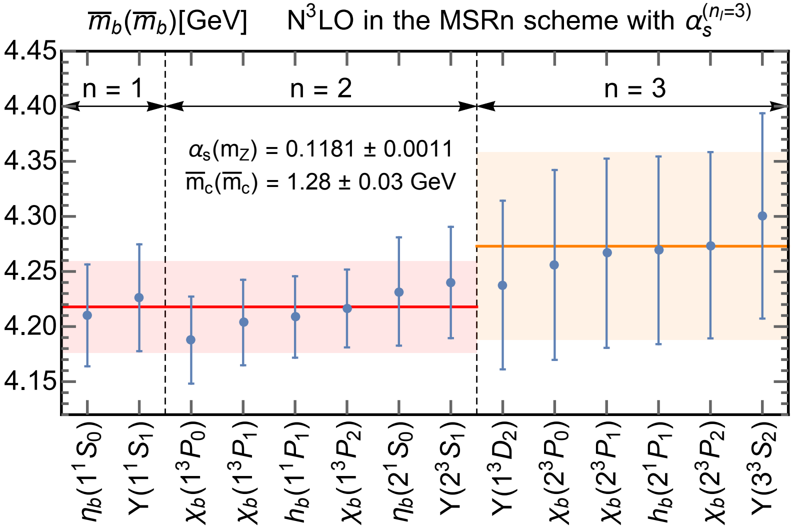

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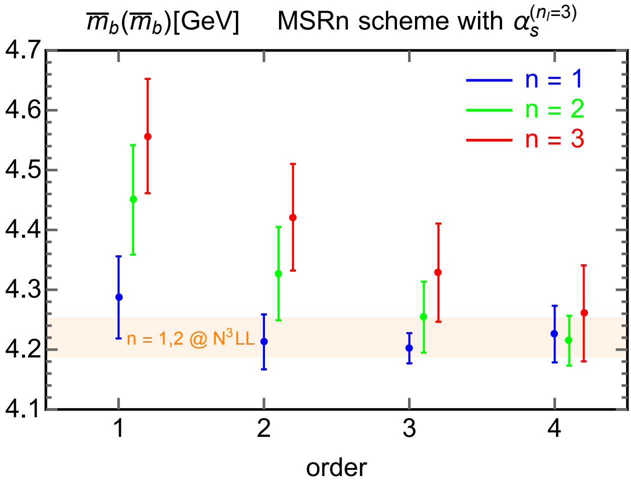
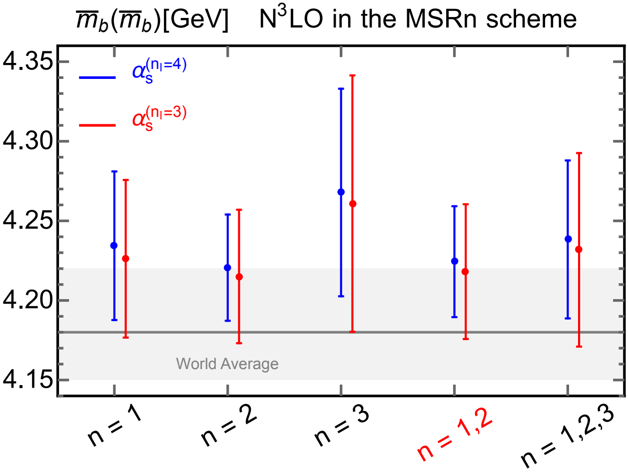
Convergence at different orders of ε expansion



Results for different schemes

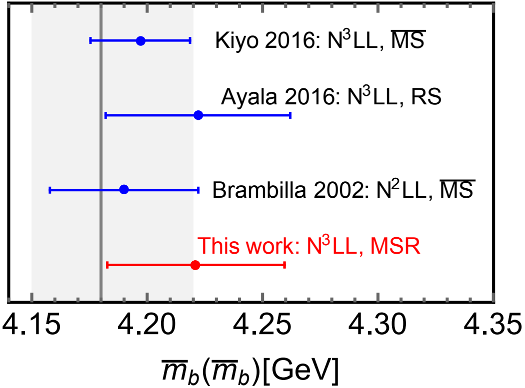


Outline

Conclusions

- Value compatible with World Average $\overline{m}_b = 4.18^{+0.04}_{-0.03}$ GeV and with previous estimations from NRQCD.
- Final value for states with $n = 1, 2$: $\overline{m}_b = 4.221 \pm 0.038\ GeV$

m_b from Bottomonium



For other application of MSR mass scheme see talk by **V. Mateu:**

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- Final value for states with $n = 1, 2$: $\overline{m}_b = 4.221 \pm 0.038\ GeV$
- Splitting the error in different sources of uncertainty:

$$\begin{aligned} \overline{m}_b &= 4.221 \pm 0.034(\mathrm{th}) \pm 0.017(\alpha_s) \pm 0.0006(m_c) \\ &\pm 0.003(\varepsilon^4) \pm 0.00006(\mathrm{exp})\ GeV . \end{aligned}$$

- Theoretical error from variation of (μ, R) scale dominates.

For other application of MSR mass scheme see talk by **V. Mateu:** Monte Carlo Top Quark Mass Calibration, Friday at 12:40

Thanks for your attention.

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Acknowledgements:

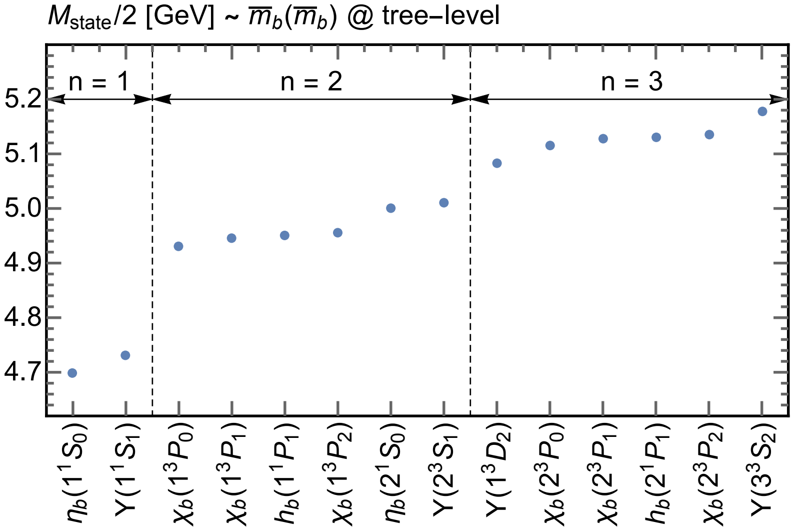
Junta de Castilla y León

European Union
European Regional Development Fund

VNiVERSiDAD D SALAMANCA
CAMPUS DE EXCELENCIA INTERNACIONAL

USal 800 AÑOS 1218 ~ 2018

Bottomonium spectrum



Bottomonium spectrum