Hadron phenomenology from Dyson-Schwinger equations

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• Everything we need to know is encoded in n-point functions.
• **In particular**: bound-states appear as poles.

\[ G_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0 | T\psi_\alpha \psi_\beta \psi_\gamma \bar{\psi}_{\alpha'} \bar{\psi}_{\beta'} \bar{\psi}_{\gamma'} | 0 \rangle \]

• Select, calculate and analyze the appropriate object.
• Need “G”
• But “G” is *a priori* unknown.

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Hadron 2017, Salamanca, Spain, 25-29 Sep 2017
Green’s Functions

Trade one unknown $G$, for another unknown $K$

on-shell corresponds to homogeneous Bethe-Salpeter equation

**Ingredients:**

Irreducible 2-, 3-, 4-body kernels (**define** equation)

Dressed particle constituents: Green’s functions
Constituents (quark)

Dyson-Schwinger Equation

Strong QCD:
- Mass function runs
- Coupling runs
- Vertices run

Everything runs!

Input:
- Gluon
- Quark
- Quark-Gluon Vertex

Not directly measurable.
Interaction Kernel

Simplest choice: One-gluon exchange

Rainbow-ladder

Bare vertices, dressed gluon

Phenomenological

\[ \alpha(k^2) \]

- \( \eta \)
  - 2.0
  - 1.9
  - 1.8
  - 1.7
  - 1.6

\[ k^2 [GeV^2] \]

\text{e.g. Maris-Tandy interaction}

\textbf{Good} for heavy quarks

\textbf{Reliable} in DCSB channels

\textbf{Known} shortcomings

[Maris, Tandy PRC 60 (1999) 055214]
BS Amplitude

\[ P = (-1)^{L+1} \]

Not quark model:

Covariantly, "exotic" quantum numbers can be \( q\bar{q} \)

\[ J^{PC} = 0^{++}, 0^{--}, 1^{--}, 0^{+-}, 0^{--}, 1^{--} \]

(possible to have; doesn’t necessarily exist)

Partial wave-basis known for all J (\( \gamma_5 \) to change parity)

\[ J = 0 : \{1, \hat{P}\}\{1, \gamma\} \]

\[ J = 1 : \{1, \hat{P}\}\{\gamma^\mu, n^\mu, \gamma^\mu_\perp \gamma - n^\mu, n^\mu \gamma - \frac{1}{3} \gamma^\mu_\perp\} \]

s-waves  p-waves  d-waves
Mesons

$m [GeV]$

<table>
<thead>
<tr>
<th>Mass</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>$\pi(1300)$</td>
</tr>
<tr>
<td>1.4</td>
<td>$\rho(1450)$</td>
</tr>
<tr>
<td>1.5</td>
<td>$a_0(1450)$</td>
</tr>
<tr>
<td>1.2</td>
<td>$b_1(1235)$</td>
</tr>
<tr>
<td>1.1</td>
<td>$a_1(1260)$</td>
</tr>
<tr>
<td>1.0</td>
<td>$\rho$</td>
</tr>
<tr>
<td>0.7</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

Sensitive to interaction details
Rainbow-Ladder unsuitable

$0^{--}$ $0^{++}$ $1^{--}$ $1^{+-}$ $1^{++}$ $q\bar{q}$ exotics

PDG
RL

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
Truncation

Rainbow-Ladder

- Structure too simple
- States are artefact of this simplicity

Options:

- Systematic "vertex expansion"
- nPI effective action techniques
- Phenomenological models
- (gauge-fixed) lattice data as input

Calculate diagrammatic corrections beyond Rainbow-Ladder

[3PI effective action]

Here: use 3PI effective action

[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[RW, Fischer, Heupel PRD 93 (2016) 034026]
Coupled Equations

Ghost/Gluon fixed to lattice data
Unquenched Quark

\[ S^{-1}(p) = A(p^2) \left( -i\not{p} + M(p^2) \right) \]
Quark-Gluon Vertex

Eight tensor structures
– not one

- Non-trivial scale dependence
- Sizeable corrections!
  (tree-level ~ 1)

\[ \Gamma_\mu^a (l, k) = h_1 \gamma_\mu^l + h_2 l_T^\mu \gamma \cdot l + h_3 i l_T^\mu + h_4 (l \cdot k) \frac{i}{2} [\gamma_\mu^l, \gamma \cdot l] + h_5 \frac{i}{2} [\gamma^l, \gamma \cdot k] + h_6 \frac{1}{6} [\gamma^l, \gamma \cdot l, \gamma \cdot k] + h_7 t_{(kl)}^{\mu\nu} (l \cdot k) \gamma^\nu + h_8 t_{(kl)}^{\mu\nu} [\gamma^\nu, \gamma \cdot l] \]
BS equation

Solve in time-like region (analytic continuation)

\[ -1 \left\langle \right\rangle = -1 \left\langle \right\rangle - \left\langle \right\rangle \]

Bound-state equation features a gluon exchange with two dressed vertices

Rich interaction structure possible as compared to rainbow-ladder
Conventional Mesons
• Sensitivity to interaction exasperated in light sector
• Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]
Beyond RL

\( m \text{ [GeV]} \)

\[
\begin{align*}
\pi(1300) & & \rho(1450) & & a_0(1450) & & b_1(1235) & & a_1(1260) & & \pi_1(1600) \\
\pi_1(1400) & & & & & & & & & & & & & & \rho & & \pi & & & & & & & & & & & & & & \pi
\end{align*}
\]

Scalar:
- 2PI-2L (RL) and 2PI-3L too light
- \( \rho - a_1 \) splitting: 2PI-2L (RL) and 2PI-3L too small
- \( a_1 - b_1 \) splitting: 2PI-2L (RL) and 2PI-3L non-degenerate

[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[RW, Fischer, Heupel PRD 93 (2016) 034026]
Non-Conventional
Non-Conventional

- Glueballs
- Hybrids
- Tetraquarks

Mixing
- Dominant component

Decay channels

Truncation dependence
- Is state even bound?
Summary

Mesons with exotic quantum numbers can (formally) be $q\bar{q}$

- Such states found in rainbow-ladder
- Artefact of the simple interaction

Bethe-Salpeter equations can be solved beyond rainbow-ladder

- Diagrammatic expansion of kernel, 3PI effective action, chiral symmetry
- “Fixes” scalar and axial-vectors. Impact on exotic mesons

Next steps

- Explore relationship to non-conventional mesons
- Phenomenology: mixing, decays. Relevance of sub-leading partial waves.
Thank you

Reviews


Sanchis-Alepuz, RW (in preparation)