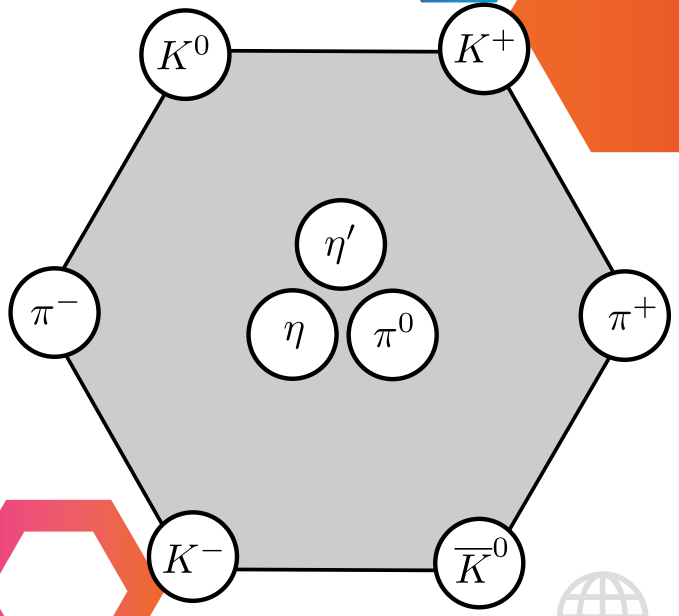


Hadron phenomenology from Dyson-Schwinger equations

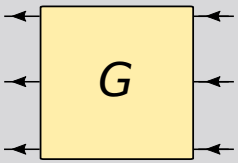
Richard Williams
University of Gießen

Bundesministerium für Bildung und Forschung

HIC for FAIR
Helmholtz International Center

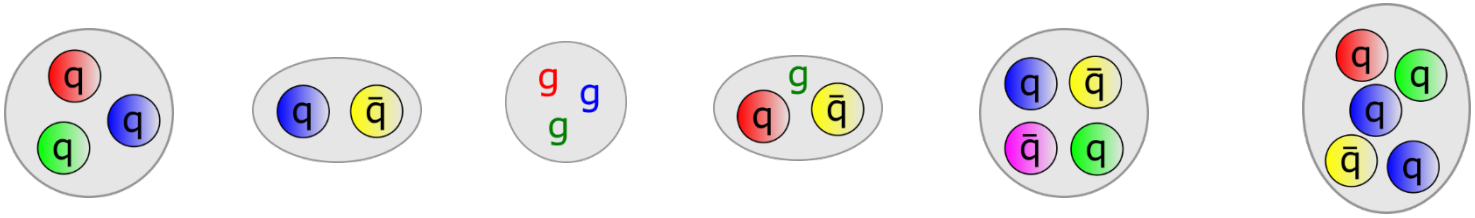
Green's Functions

See: C. S. Fischer



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0| T\psi_\alpha\psi_\beta\psi_\gamma\bar{\psi}_{\alpha'}\bar{\psi}_{\beta'}\bar{\psi}_{\gamma'} |0\rangle$$

- Everything we need to know is encoded in n-point functions.
- **In particular**: bound-states appear as poles.

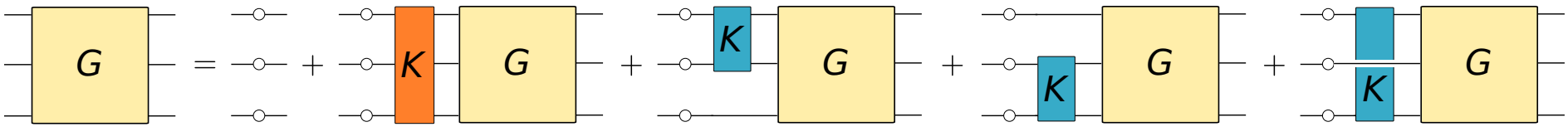


baryons mesons glueballs hybrids tetraquarks pentaquarks

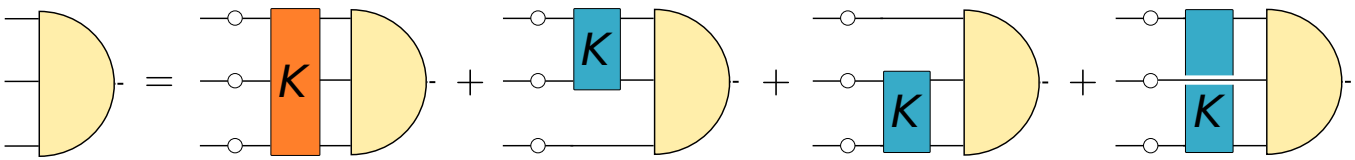
- Select, calculate and analyze the appropriate object.
- Need "G"
- But "G" is *a priori* unknown.

Green's Functions

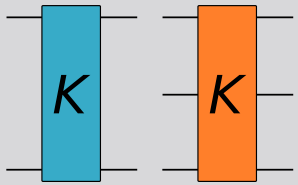
Trade one unknown **G**, for another unknown **K**



on-shell corresponds to homogeneous Bethe-Salpeter equation



Ingredients:

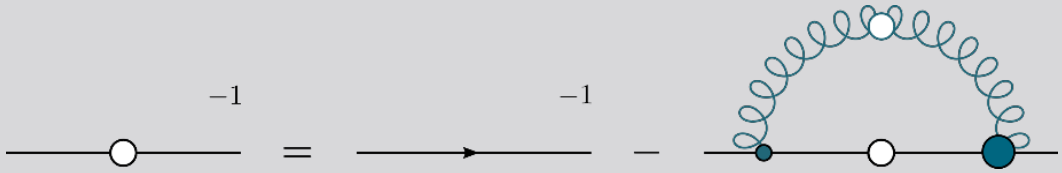


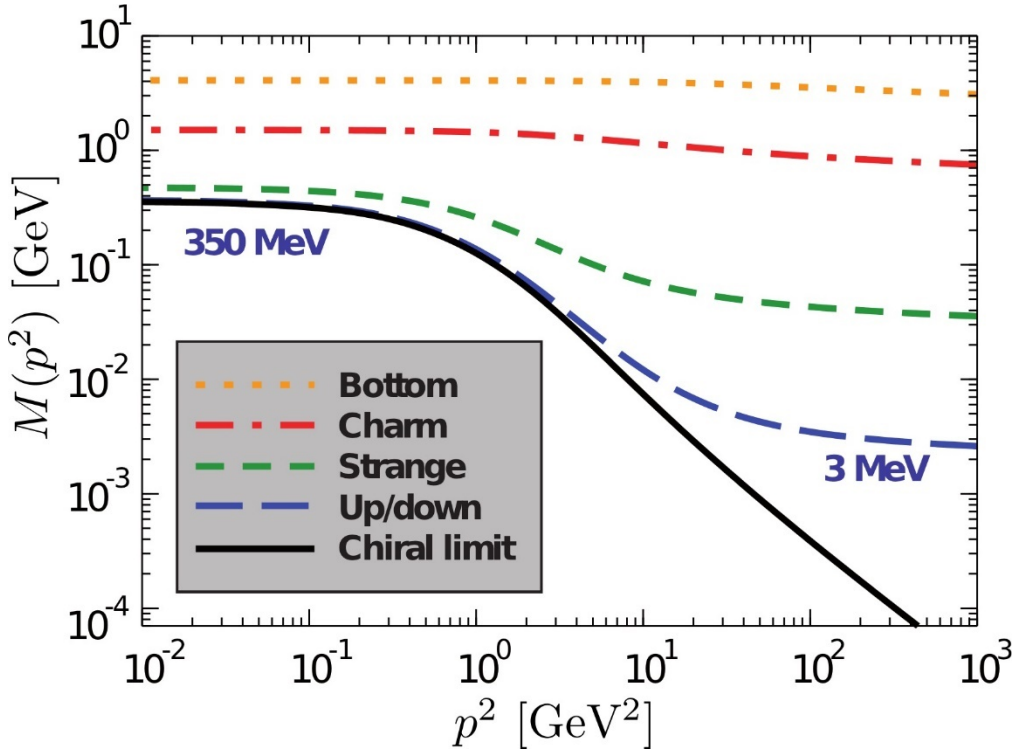
Irreducible 2-, 3-, 4-body kernels (**define** equation)

Dressed particle constituents: Green's functions

Constituents (quark)

Dyson-Schwinger Equation





Strong QCD:

- Mass function runs
- Coupling runs
- Vertices run

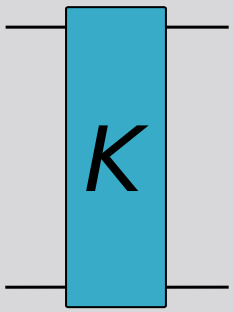
Everything runs!

Input:

- Gluon
- Quark
- Quark-Gluon Vertex

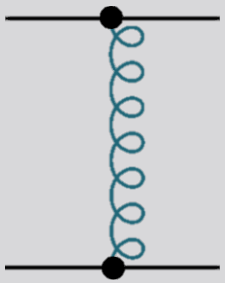
Not directly measurable.

Interaction Kernel

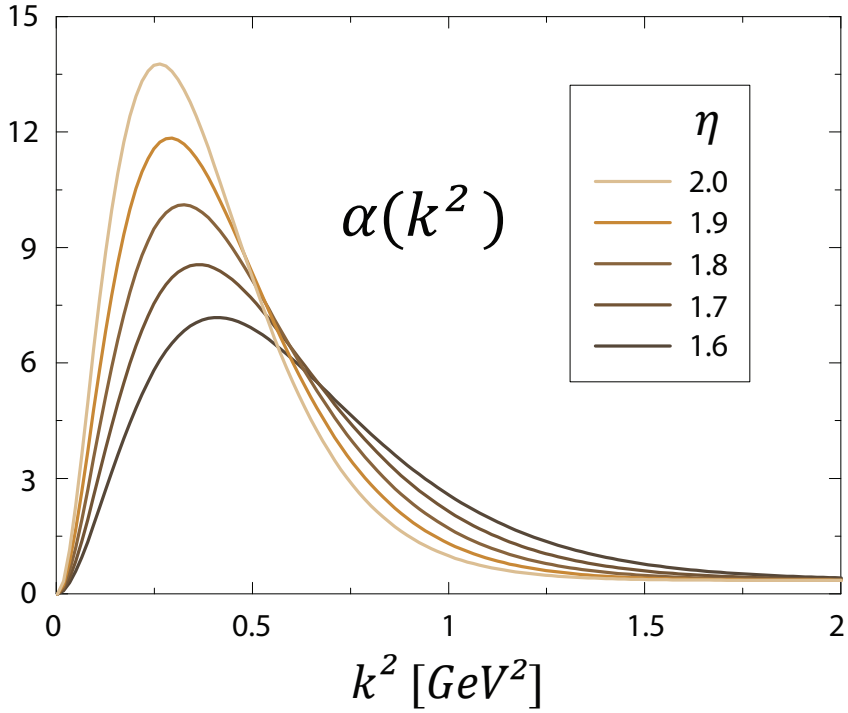


Simplest choice:
One-gluon exchange

Rainbow-ladder



Bare vertices,
dressed gluon
Phenomenological



e.g. Maris-Tandy interaction

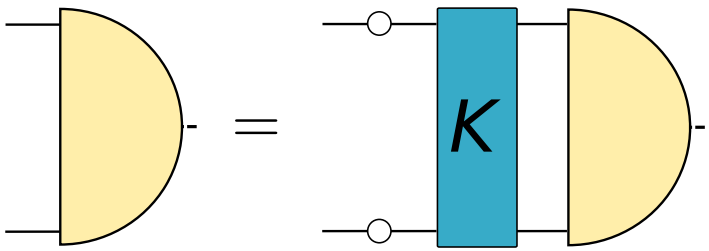
Good for heavy quarks

Reliable in DCSB channels

Known shortcomings

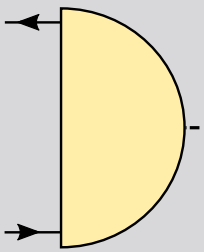
[Maris, Tandy PRC 60 (1999) 055214]

BS Amplitude



Not quark model: $\cancel{P = (-1)^{L+1}}$

Meson:



Covariantly, "exotic" quantum numbers can be $q\bar{q}$

$$J^{PC} = 0^{++}, 0^{-+}, 1^{--}, 0^{+-}, 0^{--}, 1^{-+}$$

(possible to have; doesn't necessarily exist)

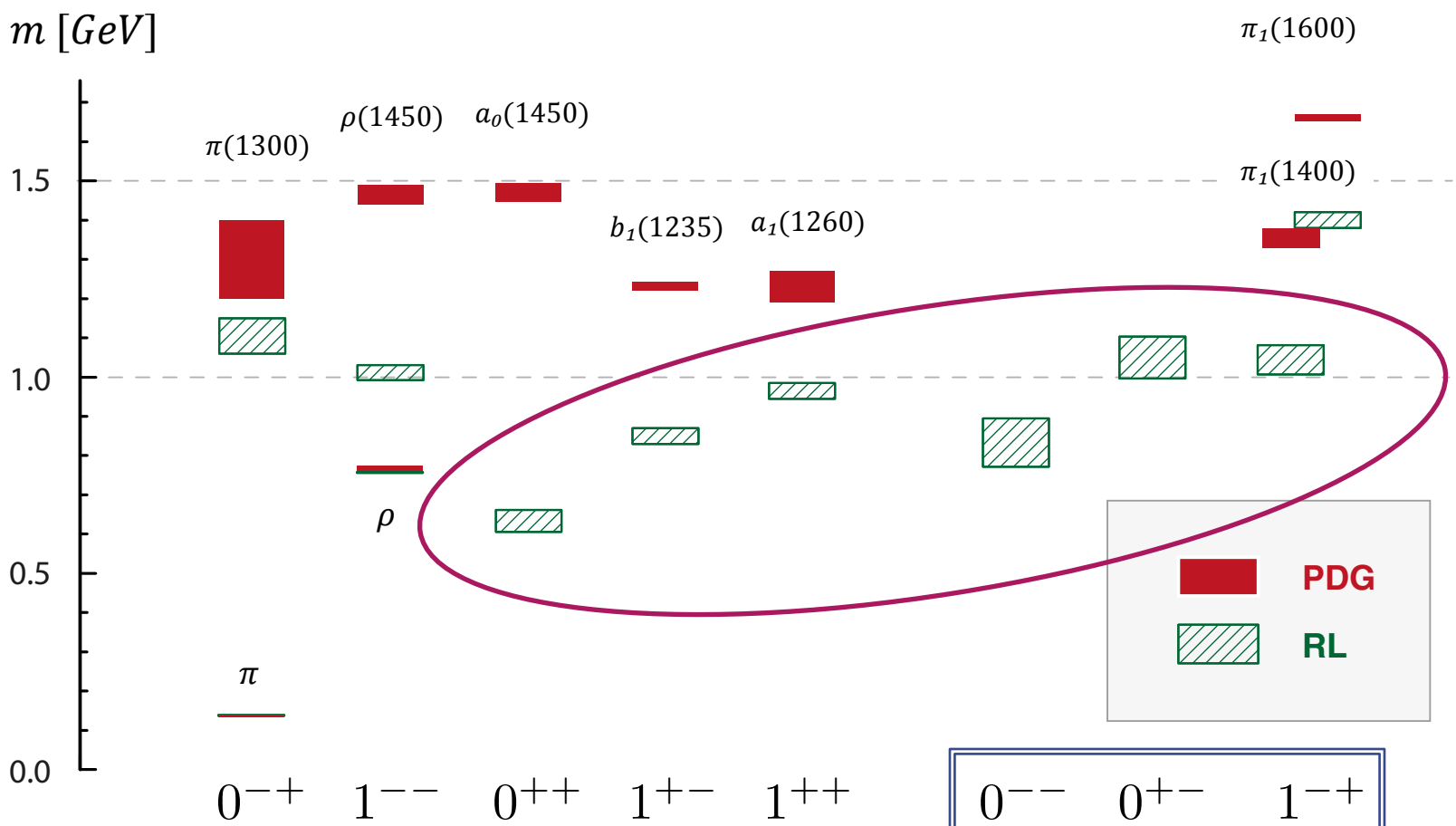
Partial wave-basis known for all J (γ_5 to change parity)

$$J = 0: \ \{\mathbb{1}, \hat{\not{P}}\}\{\mathbb{1}, \not{n}\}$$

$$J = 1: \ \{\mathbb{1}, \hat{\not{P}}\}\{\gamma_\perp^\mu, \ n^\mu, \ \gamma_\perp^\mu \not{n} - n^\mu, \ n^\mu \not{n} - \tfrac{1}{3}\gamma_\perp^\mu\}$$

s-waves **p-waves** **d-waves**

Mesons



0^{--} 0^{+-} 1^{-+}

$q\bar{q}$ exotics

Sensitive to **interaction details**

Rainbow-Ladder **unsuitable**

[Kubrak, Fischer, RW EPJA 50 (2014) 126]

Truncation

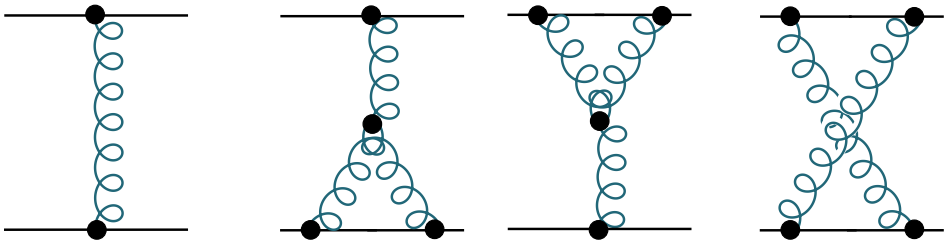
Rainbow-Ladder

- Structure too simple
- States are artefact of this simplicity

Options:

- Systematic "vertex expansion"
- nPI effective action techniques
- Phenomenological models
- (gauge-fixed) lattice data **as input**

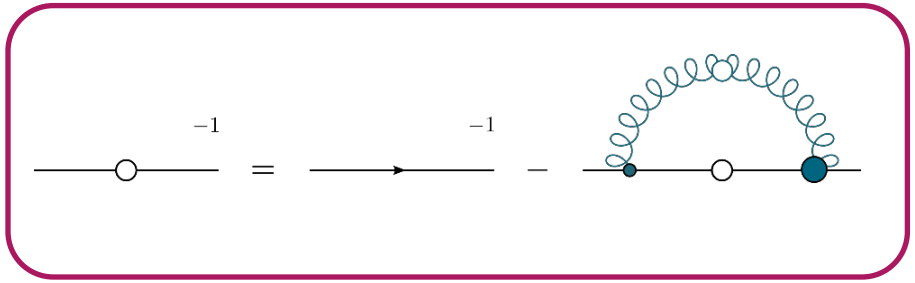
Calculate diagrammatic corrections beyond Rainbow-Ladder

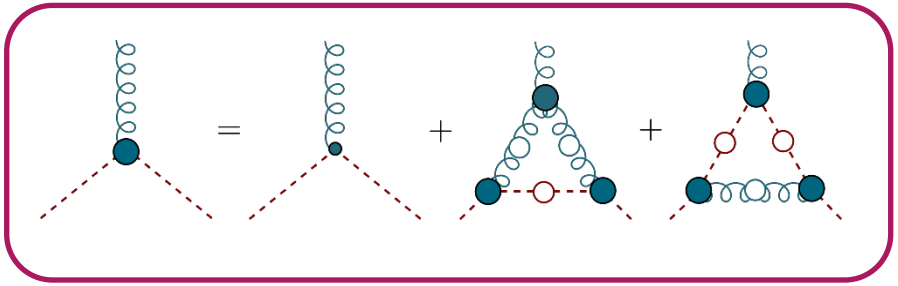


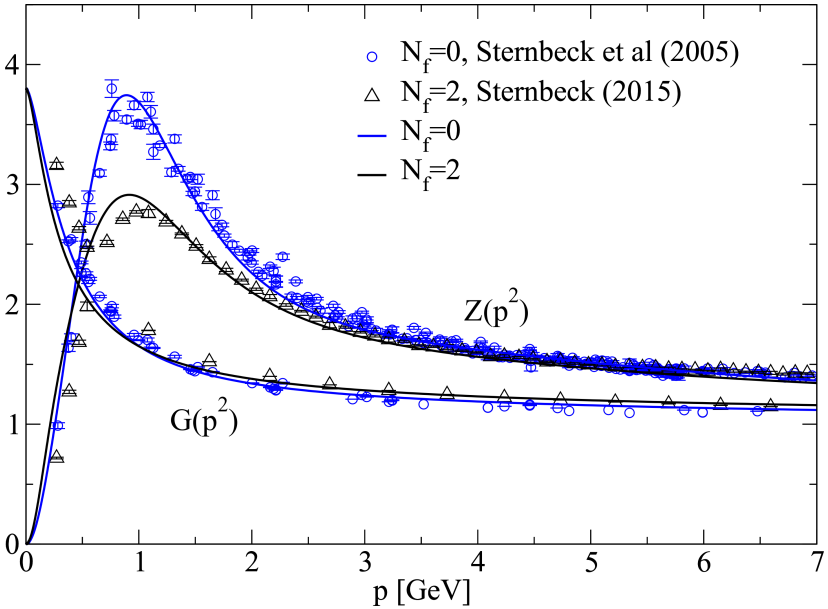
Here: use 3PI effective action

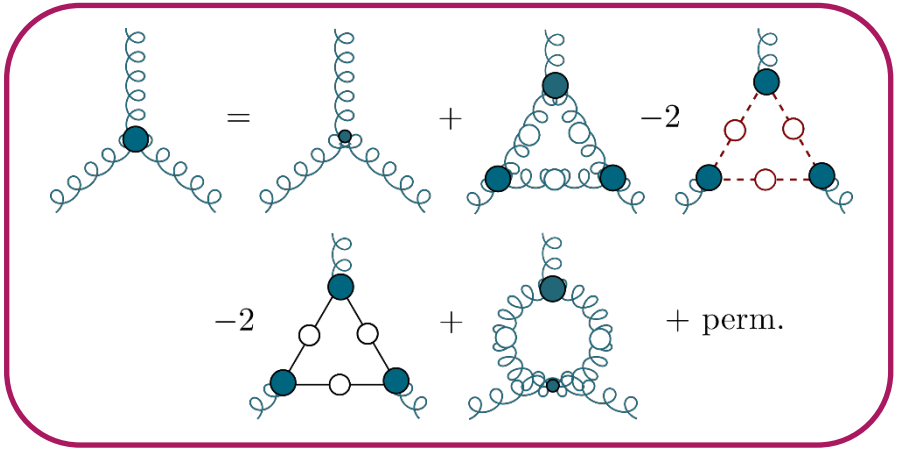
[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

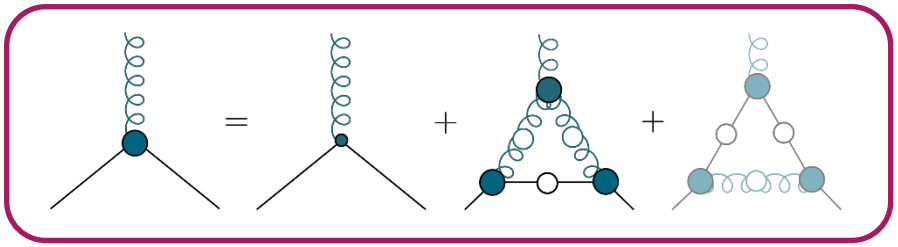
Coupled Equations







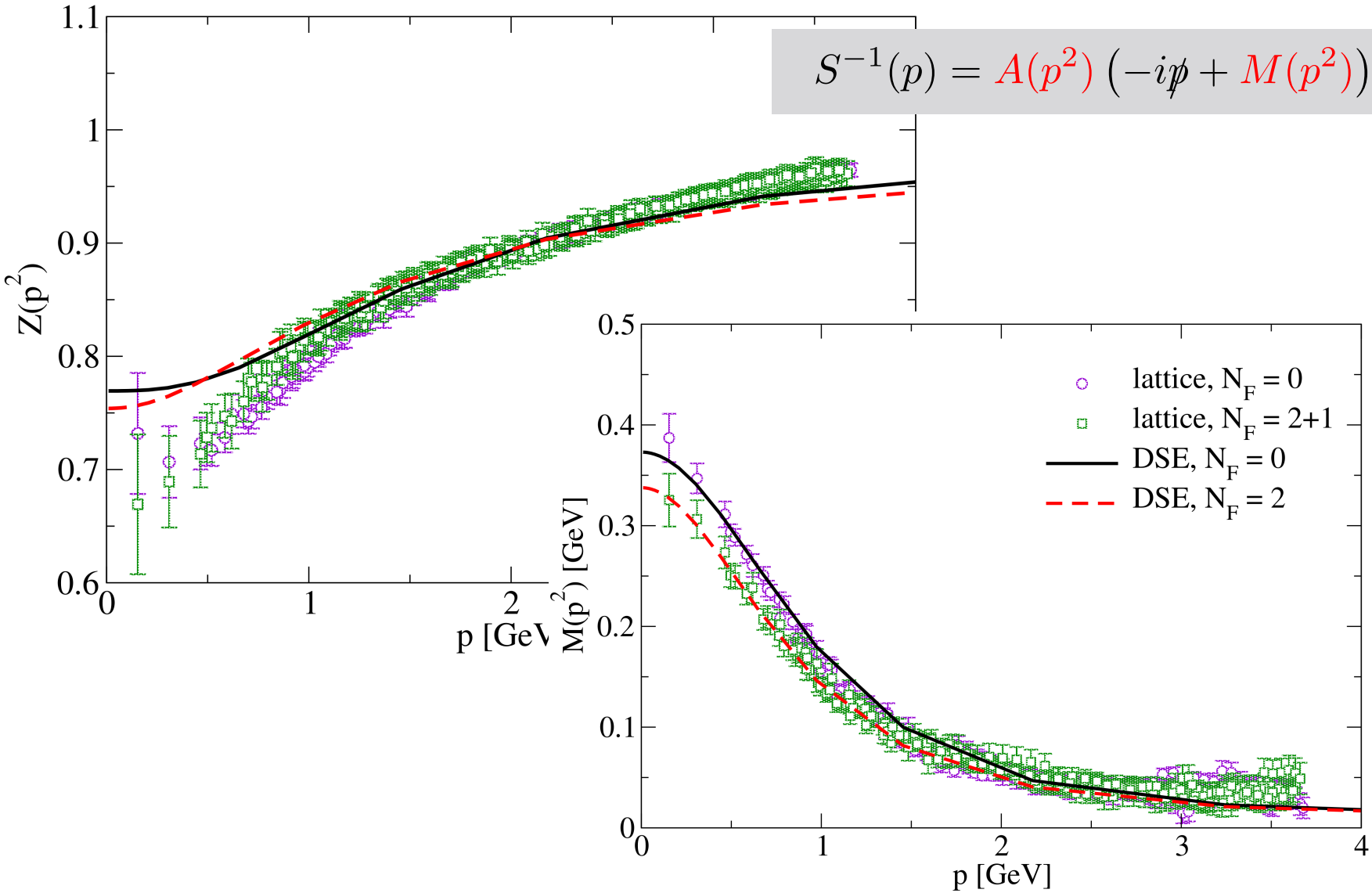




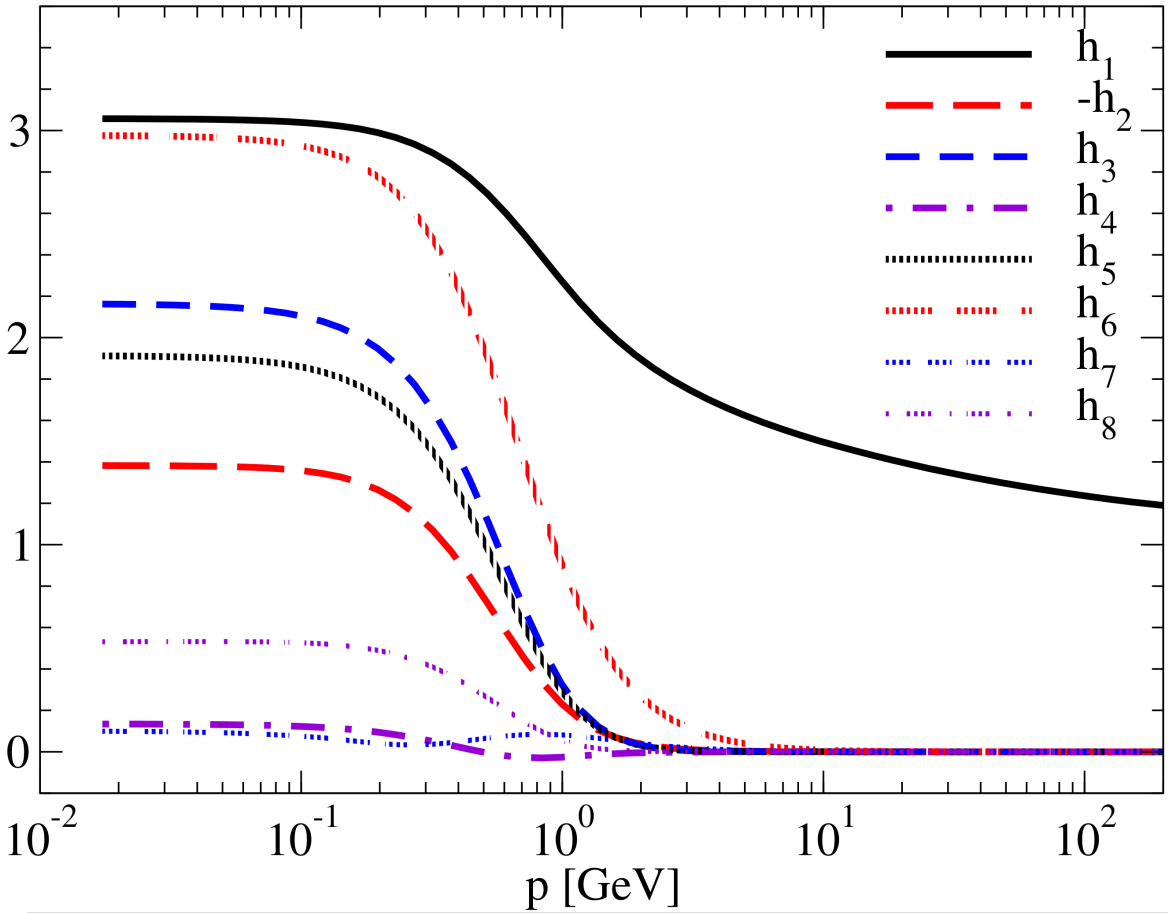
Ghost/Gluon fixed to lattice data

Unquenched Quark

$$S^{-1}(p) = A(p^2)\left(-i\not{p} + M(p^2)\right)$$



Quark-Gluon Vertex



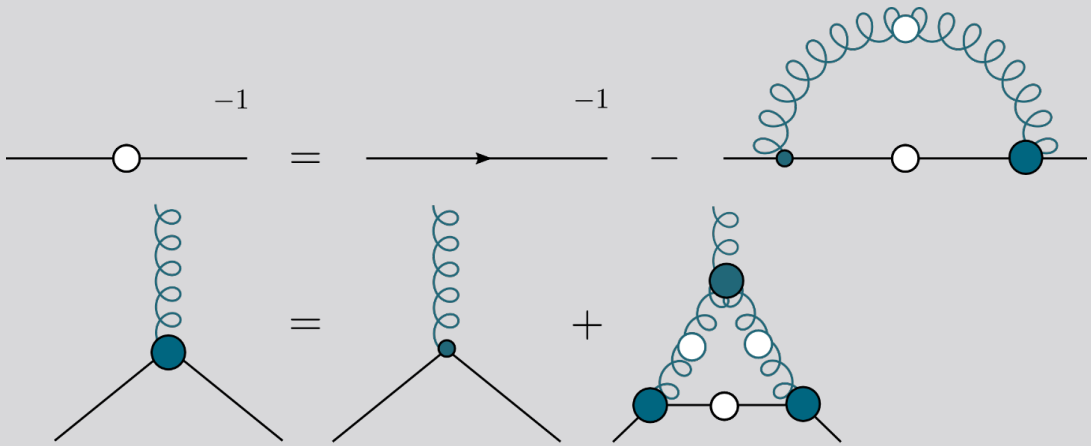
Eight tensor structures – not one

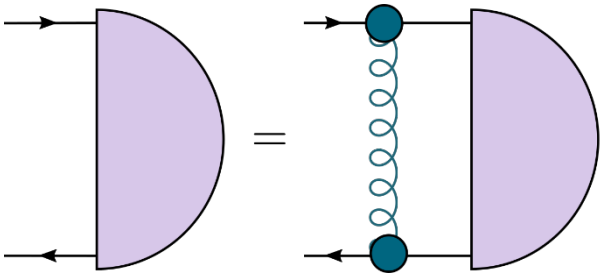
- Non-trivial scale dependence
- Sizeable corrections! (tree-level ~ 1)

$$\Gamma^a_\mu(l,k) = h_1\gamma^\mu_T + h_2 l^\mu_T \gamma\cdot l + h_3 i l^\mu_T + h_4(l\cdot \mathrm{k})\frac{i}{2}[\gamma^\mu_T,\gamma\cdot l] + h_5\frac{i}{2}[\gamma^\mu,\gamma\cdot k]$$
$$+h_6\frac{1}{6}[\gamma^\mu,\gamma\cdot l,\gamma\cdot k] + h_7 t^{\mu\nu}_{(kl)}(l\cdot k)\gamma^\nu + h_8 t^{\mu\nu}_{(kl)}[\gamma^\nu,\gamma\cdot l]$$

BS equation

Solve in time-like region (analytic continuation)



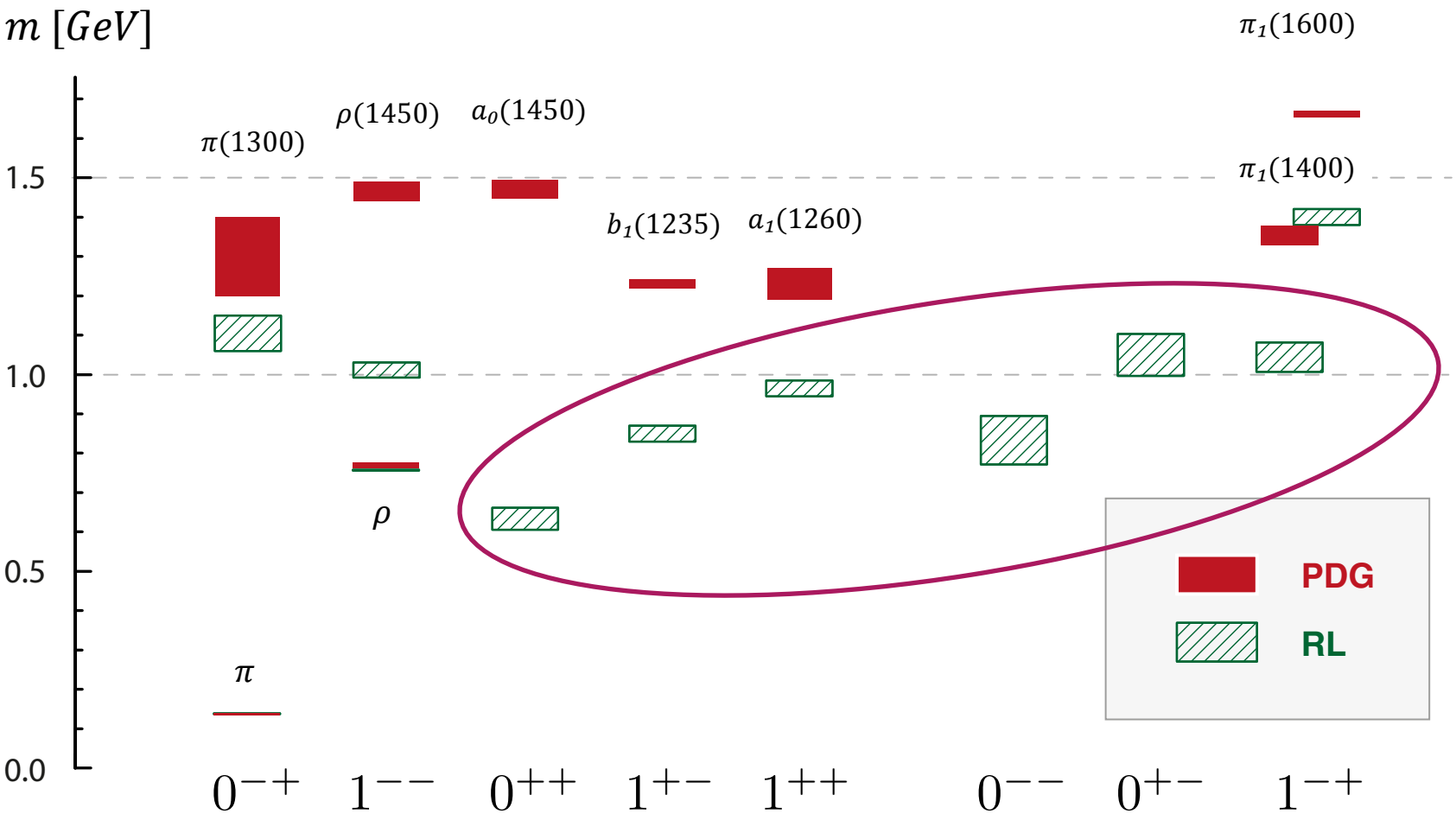


Bound-state equation features a gluon exchange with two dressed vertices

Rich interaction structure possible as compared to rainbow-ladder

Conventional Mesons

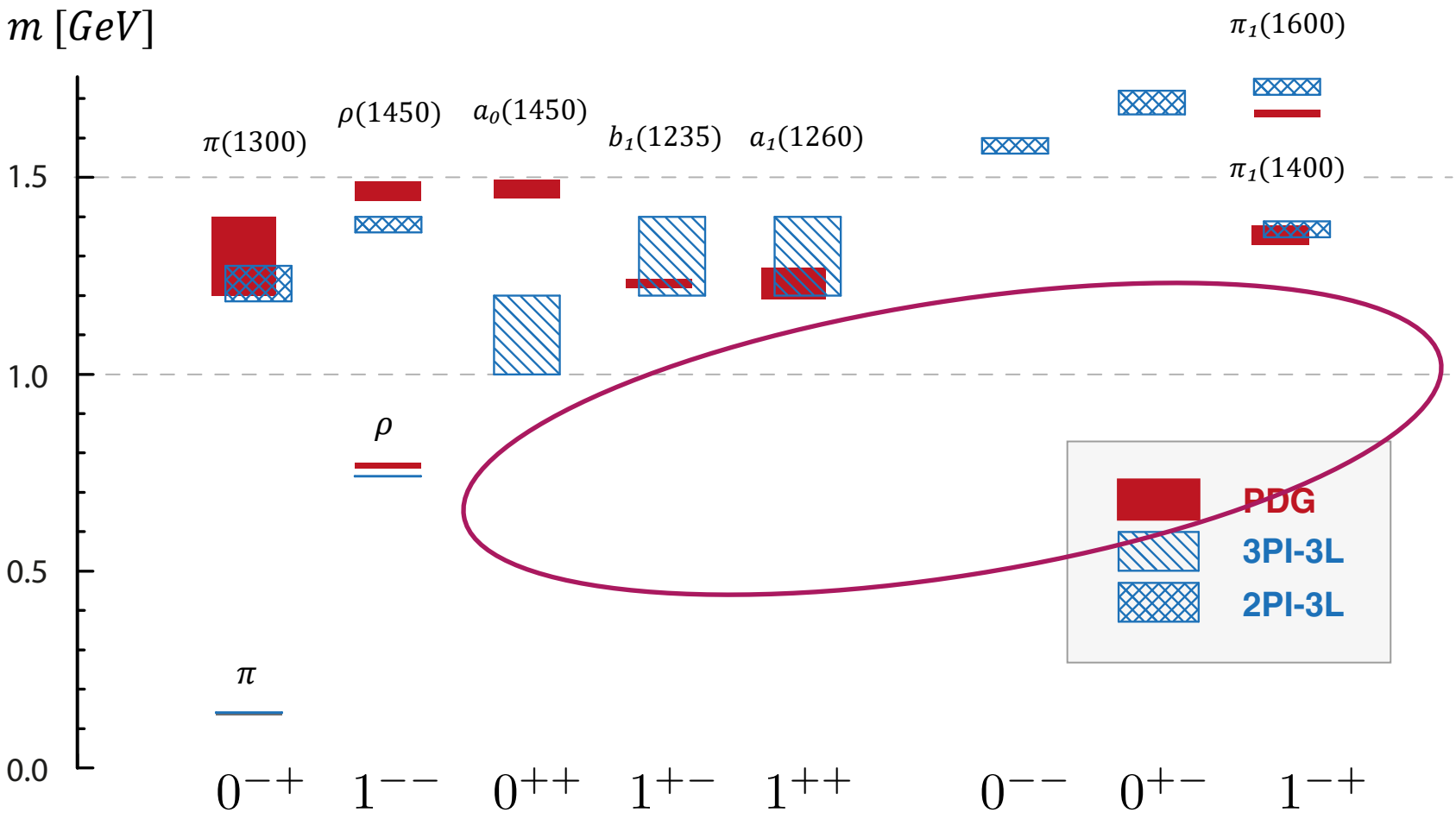
Rainbow-Ladder



- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]

Beyond RL

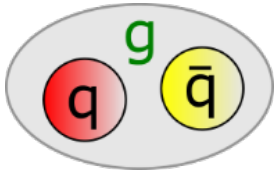


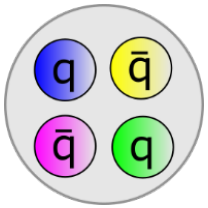
Scalar:	2PI-2L (RL) and 2PI-3L too light	[Sanchis-Alepuz, RW PLB 749 (2015) 592]
$\boldsymbol{\rho - a_1}$ splitting	2PI-2L (RL) and 2PI-3L too small	[RW, Fischer, Heupel PRD 93 (2016) 034026]
$\boldsymbol{a_1 - b_1}$ splitting	2PI-2L (RL) and 2PI-3L non-degenerate	

Non-Conventional

Non-Conventional

- **Glueballs**
- **Hybrids**
- **Tetraquarks**

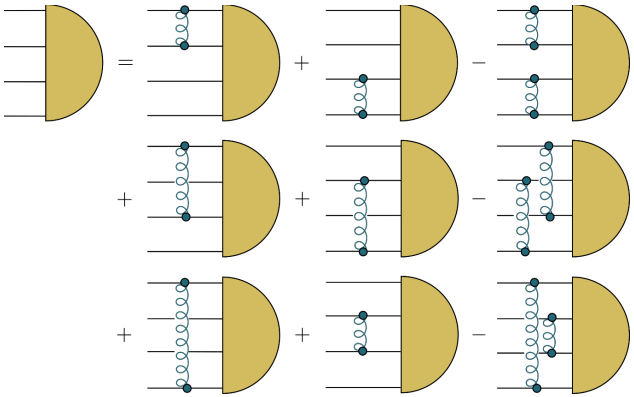




Mixing

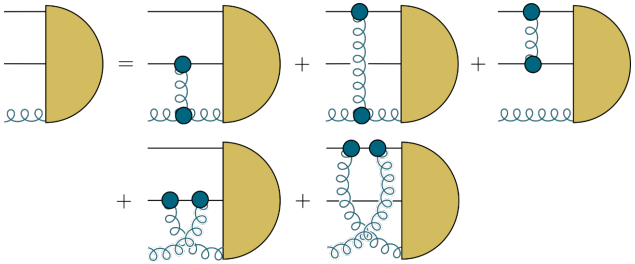
- Dominant component

Decay channels



Truncation dependence

- Is state even bound?



Summary

Mesons with exotic quantum numbers can (formally) be $q\bar{q}$

- Such states found in rainbow-ladder
- Artefact of the simple interaction

Bethe-Salpeter equations can be solved beyond rainbow-ladder

- Diagrammatic expansion of kernel, 3PI effective action, chiral symmetry
- “Fixes” scalar and axial-vectors. Impact on exotic mesons

Next steps

- Explore relationship to non-conventional mesons
- Phenomenology: mixing, decays. Relevance of sub-leading partial waves.

Thank you

Reviews

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 Prog. Part. Nucl. Phys. (in press)

Sanchis-Alepuz, RW (*in preparation)*