Studies of mesic atoms and nuclei

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Motivation: application of meson-baryon chiral phenomenology to mesic atoms and to mesic nuclei

• Partial restoration of chiral symmetry from $\pi^-$ atoms. Update in NPA 928 (2014) 128.

• $\bar{K}$-nucleus potentials from $K^-$ atoms; 1N vs. 2N absorption: NPA 959 (2017) 66.

• Search for $\bar{K}, \eta, \eta', \phi$ nuclear states; review by Metag-Nanova-Paryev, PPNP (in press). See also Barnea et al. (2017) Onset of $\eta$ nuclear binding.
Experimentally determined meson-nucleus potential $V_0 + iW_0$

Metag-Nanova-Paryev, PPNP (in press)
## Hadronic (h) atom scenarios

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Optical model analyses of hadronic atom data

- Handle large data sets across periodic table.
- Identify characteristic entities, thereby linking microscopic approaches to experiments.

Tools of the trade: optical potential variants

- Make $V_{\text{opt}}$ functional of the nuclear density $\rho$.
- Respect the low-density limit $V_{\text{opt}}(\rho) \to t_{hN} \ast \rho$.
- For pions, consider $\rho_n - \rho_p$ dependence of $b_1$ using $r_n - r_p \approx \gamma \frac{N-Z}{A} + \delta$ with $\gamma \approx 1.0 \pm 0.1$ fm.
- Introduce self consistently medium effects, particularly subthreshold hN kinematics.
Self-consistency in mesic-atom & nuclear calculations
Cieplý-Friedman-Gal-Gazda-Mareš, PLB 702 (2011) 402

\[ s_{hN} = (\sqrt{s_{th}} - B_h - B_N)^2 - (\vec{p}_h + \vec{p}_N)^2 < s_{th} \]

\[ \sqrt{s_{hN}} \rightarrow E_{th} - B_N - B_h - \xi_N \frac{p_N^2}{2m_N} - \xi_h \frac{p_h^2}{2m_h} \]

\[ \xi_N(h) = \frac{m_{N(h)}}{(m_N + m_h)} \quad \frac{p_h^2}{2m_h} \sim -V_h - B_h \]

in-medium \( \pi^- \) energy shift \quad in-medium \( K^- \) energy shift
In-medium $\hbar N$ amplitudes

Friedman-Gal-Mareš, PLB 725 (2013) 334

Cieplý-Friedman-Gal-Mareš, NPA 925 (2014) 126

- KG equation and self-energies:

$$\begin{bmatrix} \nabla^2 + \tilde{\omega}_h^2 - m_h^2 - \Pi_h(\omega_h, \rho) \end{bmatrix} \psi = 0$$

$$\tilde{\omega}_h = \omega_h - i\Gamma_h/2, \quad \omega_h = m_h - B_h$$

$$\Pi_h(\omega_h, \rho) \equiv 2\omega_h V_h = -4\pi \sqrt{s} f_{hN}(\sqrt{s}, \rho) \rho$$

- Pauli blocking (Waas-Rho-Weise NPA 617 (1997) 449):

$$f_{WRW}^{hN}(\sqrt{s}, \rho) = \frac{f_{hN}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/m_N)f_{hN}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2} I(\tilde{\omega}_h)$$

- $\sqrt{s}$: $\Lambda^*(1405) \Rightarrow f_{K^-N}(\sqrt{s}), \quad N^*(1535) \Rightarrow f_{\eta N}(\sqrt{s})$.

In medium $\Rightarrow$ go subthreshold: $\delta\sqrt{s} = \sqrt{s} - \sqrt{s_{th}}$

$$\delta\sqrt{s} \approx -B_N \frac{\rho}{\rho_0} - \xi_N B_h \frac{\rho}{\rho_0} - \xi_N T_N(\frac{\rho}{\rho_0})^{2/3} + \xi_h \text{Re} V_h(\sqrt{s}, \rho)$$

- A self-consistency cycle in $\delta\sqrt{s}$ for given $\rho$. 
Partial restoration of chiral symmetry in/from pionic atoms

Update: Friedman-Gal, NPA 928 (2014) 128
empirical $b_0(E)$ using $b_1$ independent of $\rho$

empirical $b_0(E)$ using $b_1(\rho)$ with $\pi N\sigma$ term

$\gamma$ dependence of fits to 100 data points, Ne to U

More precise determination of $b_1$ than thru DBS

E. Friedman, A. Gal, NPA 928 (2014) 128

PSI results reproduced with $b_1(\rho)$ ansatz (Weise, 2000)

$$b_1(\rho) = - \frac{\mu_{\pi N}}{8\pi f^2_\pi(\rho)}, \quad \frac{f^2_\pi(\rho)}{f^2_\pi} = \frac{<\bar{q}q>_{\rho}}{<\bar{q}q>_{0}} \sim 1 - \frac{\sigma\rho}{m^2_\pi f^2_\pi}$$

Consistency between $\pi^-$ atoms & $\pi^\pm$ scattering deductions.
What do $K^-$ atoms tell us?
$K^-_{\text{atom}}$ widths from Li to U

Lowest $\chi^2$, 84 per 65 data points, in phenomenological potential model F (deep)

J. Mareš, E. Friedman, A. Gal, NPA 770 (2006) 84
Latest kaonic atoms study

E. Friedman - A. Gal, NPA 959 (2017) 66

including capture-at-rest fractions
from bubble-chamber experiments

H. Davis et al., NC 53 (1968) 313

J.W. Moulder et al., NPB 35 (1971) 332

C. Vander Velde-Wilquet et al., NC 39A (1977) 538
Single vs. multi-nucleon absorption

Multinucleon fraction from 3 bubble-chamber experiments:

\[ 0.26 \pm 0.03 \ (C, \ F, \ Br) \quad 0.28 \pm 0.03 \ (Ne) \quad 0.19 \pm 0.03 \ (C) \]

Larger uncertainties from nuclear emulsions

Accept \[ 0.25 \pm 0.05 \] for C \& heavier nuclei, hence single-nucleon fraction \[ 0.75 \pm 0.05 \], to constrain \( K^- \) atomic data fits, using

\[
\Gamma \sim \int |r\psi|^2 \text{Im} V_{K^-} \, dr \quad \text{with} \quad V_{K^-} = V_{K^-}^{1N} + V_{K^-}^{mN}
\]

to separate \( mN \) from \( 1N \) absorption.
Where does absorption take place?

- Overlap peaks at 15-20% \( \rho_0 \) (10-15% \( \rho_0 \)) for lower (upper) state; little higher in C.

- Consistent with BC experiments that yield multinucleon absorption fraction of 0.25±0.05 from carbon on.

- Use it to constrain \( K^- \) atom optical potential fits.

Lower: \( |r\psi|^2 \text{Im} \mathcal{V}_{K^-}^{\text{phenom.}} \)

Upper: nuclear density profile
Six $K^-p$ scattering amplitudes from NLO $\chi$SU(3) CC dynamics

KM=IHW, P=Prague, M1,M2=GO, B1,B2=Bonn

Distinctly different at subthreshold energies

From Cieply,Mai,Meißner,Smejkal, NPA 954 (2016) 17
Six $K^-n$ scattering amplitudes from NLO $\chi$SU(3) CC dynamics

KM=IHW, P=Prague, M1,M2=GO, B1,B2=Bonn
Distinctly different at ALL energies
From Cieply,Mai,Meißner,Smejkal, NPA 954 (2016) 17
Models P & KM 1N absorption fractions

- circles: lower states
- squares: upper states
- solid curves: $\alpha=1$
- dashed curves: $\alpha=2$
- $V_{K^-}^{mn} = -\frac{4\pi}{2\mu_k}B\left(\frac{\rho}{\rho_0}\right)^\alpha \rho$
  with B & $\alpha$ fitted.
- $\alpha_P=1.4\pm1.2$, $\chi^2=125$
- $\alpha_{KM}=1.8\pm0.8$, $\chi^2=122$
  per 65 data points.

exp. 1N fraction 0.75±0.05

P & KM indistinguishable as far as absorption fractions are concerned.
1N absorption fractions in other models

The other models do not make it up.
$K^-$ Ni best-fit in-medium 1N & full amplitudes in model KM

- 1N in-medium amplitude depends little on $\alpha$.
- However, both real and imaginary parts of the FULL amplitude depend strongly on $\alpha$ at $\rho \geq 0.5 \rho_0$. 
$K^-$ Ni best-fit optical potentials in Model KM plus mN term, $\alpha=1\&2$

- **Re** $V_{K^-}$ is undetermined at $\rho \geq 0.25 \rho_0$. Extrapolation to $\rho_0$ gives shallower potentials than best-fit phenom. potentials.
- **Im** $V_{K^-}$ is undetermined at $\rho \geq 0.5 \rho_0$. Extrapolation to $\rho_0$ gives deeper potentials than best-fit phenom. potentials.
$K^-$ nuclear 1N (left) and 2N (right) absorptive potentials, calculated in a chiral unitary model [PRC 86 (2012) 065205] by Sekihara, Yamagata-Sekihara, Jido, Kanada-En’yo. Empirical 2N:1N $\approx$ 1:3 BR is reached at too high density.
Deeply bound atomic states

Saturation of absorption at work

Narrow thru repulsion

Spectrum is insensitive to Re V

Friedman-Gal (1999), PLB 459 43, NPA 658 345
Summary

- Absorption fractions exhibit little dependence on nuclear species and atomic state.
- Absorption fractions favor models P and KM.
- Re $V_{K^-}$ determined only up to 25% $\rho_0$.
- Im $V_{K^-}$ determined up to 50% $\rho_0$.
- Subthreshold energies down to $-30$ MeV reached; no phase-space correction made.
- Good mN interaction models badly needed.
- Are there any narrow $K^-$ nuclear states?

Hrtánková-Mareš (2017)
PLB 770, 342; PRC 96, 015205
Onset of $\eta$ nuclear binding

N. Barnea, B. Bazak, E. Friedman, A. Gal, PLB 771 (2017) 297
N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35
Background & Motivation

- The $\eta N$ s-wave interaction below $N^*(1535)$ is attractive in a $\pi N - \eta N$ model [Bhalelao–Liu (1985)]. Bound states of $\eta(548)$ in $A \geq 12$ nuclei could exist [Haider–Liu (1986)].

- Chiral $N^*(1535)$ meson-nucleon coupled channel models were introduced by Kaiser, Weise et al (1995-1997) and subsequently by Oset et al (2002). These & other models have been used to calculate $\eta$–nuclear quasibound states.

- Exp. searches for such states with proton, pion or photon induced $\eta$ production reactions are inconclusive. For the onset of binding, Krusche & Wilkin (2015) state: “The most straightforward (but not unique) interpretation of the data is that the $\eta d$ system is unbound, the $\eta^4He$ is bound, but that the $\eta^3He$ case is ambiguous.”
Hints from $\eta^3$He production

Fitted $dp \rightarrow \eta^3$He x-sections below 2 MeV vs. experiment. Remarkable energy dependence, suggesting a nearby S-matrix pole could be in action. Deduced $a(\eta^3$He) excludes a quasibound state pole.

Xie-Liang-Oset-Moskal-Skurzok-Wilkin, PRC 95 (2017) 015202

$$a(\eta^3$He) = [-(2.23\pm1.29)+i(4.89\pm0.57)] \text{ fm}$$

- **Would $\eta^4$He be bound?** NOT seen in $dd \rightarrow ^3$He+N+\pi [WASA-at-COSY NPA 959 (2017) 102]. Argued to be more UNBOUND than $\eta^3$He [Fix-Kolesnikov, PLB 772 (2017) 663] owing to a stronger subthreshold suppression in $^4$He.
\( \eta N \) model input

CM s-wave scattering amplitude \( F_{\eta N}(E) \) in two meson-baryon coupled-channel \( N^*(1535) \) models.

\[
\begin{align*}
\mathbf{a}_{\eta N}^{GW} &= 0.96 + i0.26 \text{ fm}, & \mathbf{a}_{\eta N}^{CS} &= 0.67 + i0.20 \text{ fm} \\
\end{align*}
\]

- Derive local, energy dependent potentials \( v_{\eta N}(E;r) \) that reproduce \( F_{\eta N}(E) \) below threshold, for use in solving the \( \eta NN, \eta NNN, \eta NNNN \) few-body Schroedinger equations.
\textbf{F}_{\eta N}(E) \Rightarrow v_{\eta N}(E) \text{ in models GW \& CS}

Strength \( b(E) \) of effective potential \( v_{\eta N}(E) \) at \( E < 0 \)

\[
v_{\eta N}(E;r) = -\frac{4\pi}{2\mu_{\eta N}} \cdot b(E) \left( \frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)
\]

- Scale \( \Lambda \) is inversely proportional to the range of \( v_{\eta N} \).
- \( v_{\eta N} \) is a regulated contact term in \( \pi \)-less EFT which for \( \Lambda \leq m_\rho \approx 4 \text{ fm}^{-1} \) replaces vector-meson exchange.
Energy dependence in $\eta$ nuclear few-body systems

- $N^*(1535)$ makes near-threshold $f_{\eta N}(\sqrt{s})$ & input potential $v_{\eta N}(\sqrt{s})$ strongly energy dependent.
  
  $s = (\sqrt{s_{th}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2 < s_{th}$

- Expanding NR near $\sqrt{s_{th}}$ & evaluating $\langle \delta \sqrt{s} \rangle$:
  
  $\langle \delta \sqrt{s} \rangle = -\frac{B}{A} + \frac{A-1}{2} E_\eta - \xi_N \frac{1}{A} \langle T_A \rangle - \xi_\eta \left( \frac{A-1}{A} \right)^2 \langle T_\eta \rangle$,
  
  $\delta \sqrt{s} \equiv \sqrt{s} - \sqrt{s_{th}}$, $E_\eta = \langle H - H_A \rangle$, $\xi_N(\eta) \equiv \frac{m_N(\eta)}{(m_N + m_\eta)}$.
  
  Agrees to $O(1/A)$ with optical-model limit.

- Self-consistency: output $\langle \sqrt{s} \rangle = \text{input } \sqrt{s}$.

- Near threshold $E_\eta$ & $\langle T_\eta \rangle \to 0$, yet $\langle \delta \sqrt{s} \rangle_{\text{th}} \neq 0$.
  
  Similarly, $\langle \delta \sqrt{s} \rangle_{\text{th}} \neq 0$ in kaonic atoms, starting
  
  with $K^- d$: $\langle \delta \sqrt{s} \rangle_{\text{th}} = -\frac{B_d}{2} - \frac{0.655}{2} \langle T_d \rangle = -4.9$ MeV.
Recent SVM results for $\eta^{3,4}\text{He}$

- Stochastic Variational Method calculations with correlated Gaussian trial wavefunctions, resulting in:
  - $\eta^d$ is definitely unbound in both GW and CS (2015).
  - $\eta^3\text{He}$ is nearly or just bound in GW & unbound in CS.
  - $\eta^4\text{He}$ is bound in GW and just or nearly bound in CS.

Self consistency plot

$\eta^4\text{He}$ bound-state energy $E$, $\langle \delta \sqrt{s} \rangle$ & $\langle H_N = H_A \rangle$, for AV4’ $v_{NN}$ & GW $v_{\eta N}(E)$ with scale $\Lambda=4$ fm$^{-1}$.
Scale dependence; semi-realistic NN

\[ B_\eta \text{ as a function of } \frac{1}{\Lambda} \]

- These bindings will decrease by \( \leq 0.3 \text{ MeV} \) when \( \text{Im } v \) is added. GW just binds \( \eta^3\text{He} \), & definitely binds \( \eta^4\text{He} \).
- AV4p (Argonne) more realistic than MNC (Minnesota).
- CS does not bind \( \eta^3\text{He} \) & is unlikely to bind \( \eta^4\text{He} \).
Scale dependence; pionless EFT at LO

\[ B_\eta(\Lambda) \text{ using } v_{\eta N}^{GW}(E): \text{ with } & \text{ w/o self consistency} \]

- Nuclear dynamics generated from two NN & one NNN contact terms (CT). NNN CT averts \(^3\text{He}\) collapse.
- Add one \(\eta N\) & one \(\eta NN\) CT; given no \(\eta NN\) datum, use CT(\(\eta NN\))=CT(\(\text{NNN}\)) to start with.
Pionless EFT at LO; $\eta_{NN}$ CT

Dependence of $B_\eta(\Lambda)$ on choice of $\eta_{NN}$ CT from Erratum to PLB 771 (2017) 297

- $\eta_{NN}=\text{NNN CTs vs. fitting to assumed } B_\eta(\eta_{NN})=0$.
- Appreciable model dependence for $\Lambda \leq m_\rho \approx 4 \text{ fm}^{-1}$. Need data beyond $\eta N$ modeling.
Summary

• Subthreshold behavior of $f_{\eta N}$ is crucial in studies of $\eta$-nuclear bound states to decide whether (i) such states exist, (ii) can they be resolved (i.e. widths), and (iii) which nuclear targets and reactions to try.

• Binding $\eta^3$He requires a minimum value of $\text{Re } a_{\eta N}$ close to 1 fm, yielding then a few MeV $B_{\eta}(\eta^4\text{He})$. Binding $\eta^4$He requires a lower value of $\text{Re } a_{\eta N}$, roughly exceeding 0.7 fm. Calculated widths of near-threshold states are a few MeV.

Thanks to our collaborators
N. Barnea, B. Bazak, A. Cieplý, J. Mareš