

# $\eta$ -mesic nuclei

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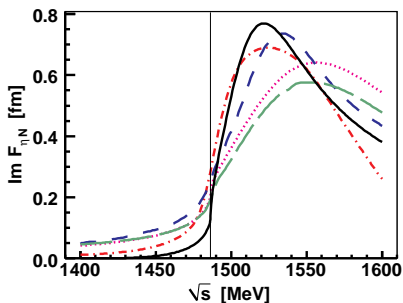
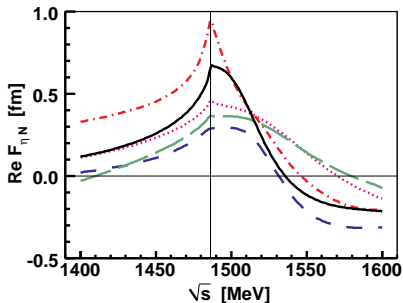
*PLB 725 (2013) 334; NPA 925 (2014) 126; PLB 747 (2015) 345, PLB 771 (2017) 297*



- Haider, Liu (PLB 172 (1986) 257, PRC 34 (1986) 1845)  
moderate attractive  $\eta N$  interaction with scattering length  $a_{\eta N} \sim 0.27 + i0.22$  fm  $\Rightarrow \exists$  of  $\eta$  nuclear bound states (starting  $^{12}\text{C}$ )
- Numerous studies since then yielding  $\text{Re}a_{\eta N}$  from 0.2 fm to 1 fm  
chiral coupled channel models -  $\text{Re}a_{\eta N} < 0.3$  fm;  
K matrix methods fitting  $\pi N$  and  $\gamma N$  reaction data in the  $N^*(1535)$  resonance region -  
 $\text{Re}a_{\eta N} \sim 1$  fm  $\rightarrow$  bound states already in He isotopes
- Strong final-state interaction have been noted in  $p-$  and  $d-$ initiated  $\eta$  production (COSY-ANKE, COSY-GEM, LNS-SPES2,3,4)
- $^{25}_{\eta}\text{Mg}$  ? (COSY-GEM, PRC 79 (2009) 012201(R))  
 $p + ^{27}\text{Al} \rightarrow ^{25}_{\eta}\text{Mg} + ^3\text{He}; \quad ^{25}_{\eta}\text{Mg} \rightarrow (\pi^- + p) + X$   
 $B_{\eta} = 13.1 \pm 1.6$  MeV and  $\Gamma_{\eta} = 10.2 \pm 3.0$  MeV.
- But **NO** decisive experimental evidence so far.  
(negative results for  $^3_{\eta}\text{He}$  (photoproduction on  $^3\text{He}$  - MAMI, PLB 709 (2012) 21;  
 $pd \rightarrow \eta^3\text{He}$  reaction - Xie, Liang, Oset, Moskal, Skurzok, Wilkin, PRC 95 (2017) 015202)  
and for  $^4_{\eta}\text{He}$  ( $dd \rightarrow ^3\text{He}N\pi$  - WASA@COSY PRC 87 (2013)035204; NPA 959 (2017) 102.)

# $\eta N$ scattering amplitudes

- $\eta N$  amplitude for various models:
- Strong energy dependence of the scattering amplitudes !



line	$a_{\eta N}$ [fm]	model
dotted	$0.46+i0.24$	<i>N. Kaiser, P.B. Siegel, W. Weise, PLB 362 (1995) 23</i>
short-dashed	$0.26+i0.25$	<i>T. Inoue, E. Oset, NPA 710 (2002) 354 (GR)</i>
dot-dashed	$0.96+i0.26$	<i>A.M. Green, S. Wycech, PRC 71 (2005) 014001 (GW)</i>
long-dashed	$0.38+i0.20$	<i>M. Mai, P.C. Bruns, U.-G. Meißner, PRD 86 (2012) 094033 (M2)</i>
full	$0.67+i0.20$	<i>A. Cieply, J. Smejkal, Nucl. Phys. A 919 (2013) 334 (CS)</i>

Faddeev (AGS) calculations of  $\eta NNN$  and  $\eta NNNN$  systems

A. Fix, O. Kolesnikov, PLB 772 (2017) 663

Variational calculations

- in hyperspherical basis

N. Barnea, E. Friedman, A. Gal, PLB 747 (2015) 345

- within Stochastic Variational Method (SVM)

N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35;

N. Barnea, B. Bazak, E. Friedman, A. Gal, PLB 771 (2017) 297

## interactions involved in variational calculations

- $NN$ : Argonne AV4' potential, Minnesota MN potential
- $\eta N$ : complex energy-dependent local potential derived from the full chiral coupled-channels model:

$$v_{\eta N}(E, r) = -\frac{4\pi}{2\mu_{\eta N}} b(E) \rho_{\Lambda}(r),$$

$$\text{where } E = \sqrt{s} - \sqrt{s_{\text{th}}}, \quad \rho_{\Lambda}(r) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)$$

$b(E)$  fitted to phase shifts  $\delta$  derived from  $F_{\eta N}(E)$  in GW and CS models;  
scale  $\Lambda$  inversely proportional to the  $v_{\eta N}$  range

- $\pi$ -less EFT at LO

nuclear dynamics generated from two  $NN$  + one  $NNN$  contact terms (CT);

one  $\eta N$  + one  $\eta NN$  CT (  $\text{CT}(\eta NN) = \text{CT}(NNN)$  or  $\text{CT}(\eta NN)$  fitted to  $B_{\eta}(\eta NN) = 0$ ;

$v_{\eta N}$  = regulated contact term in  $\pi$ -less EFT

## Energy dependence of $v_{\eta N}(\sqrt{s})$

- $A$  nucleons +  $\eta$  meson:

$$s = (\sqrt{s_{\text{th}}} - B_{\eta} - B_N)^2 - (\vec{p}_{\eta} + \vec{p}_N)^2 \leq s_{\text{th}}$$

$$\text{where } \sqrt{s_{\text{th}}} = m_N + m_{\eta}$$

- near threshold approximated by:

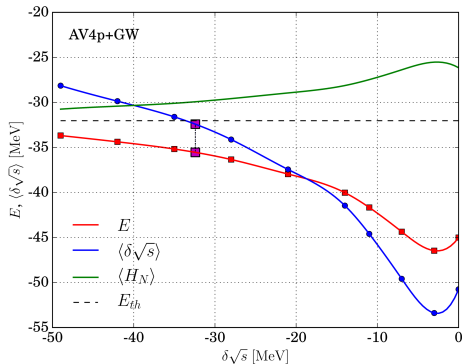
$$\sqrt{s} = \sqrt{s_{\text{th}}} + \delta\sqrt{s}, \quad \delta\sqrt{s} < 0!$$

$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} - \frac{A-1}{A} B_{\eta} - \xi_N \frac{A-1}{A} \langle T_{N:N} \rangle - \xi_{\eta} \left( \frac{A-1}{A} \right)^2 \langle T_{\eta} \rangle,$$

where  $B$  = total binding energy,  $\xi_{N(\eta)} = m_{N(\eta)}/(m_N + m_{\eta})$ ,  
 $T_{\eta} = \eta$  kin. energy,  $T_{N:N} =$  pairwise  $NN$  kin. energy

- $\langle \delta\sqrt{s} \rangle \Rightarrow$  selfconsistency

- Energy dependence  $\Rightarrow$  selfconsistency



The  ${}^4_\eta\text{He}$  g.s. energy  $E + \langle \delta\sqrt{s} \rangle + \langle H_N \rangle$ , for AV4 and GW potentials with  $\Lambda = 4 \text{ fm}^{-1}$

## $\eta$ in 3- and 4-body systems

- Conversion widths  $\Gamma$  of  $\eta$  nuclear few-body systems

perturbative estimate:  $\Gamma = -2\langle\Psi_{\text{gs}}|\text{Im}V_{\eta N}|\Psi_{\text{gs}}\rangle$

Results:

- $\eta NN$  - unbound
- $\eta^3\text{He}$

	$NN$ int.	$\delta\sqrt{s_{\text{sc}}}$	$B_\eta$	$\Gamma_{\text{gs}}$
GW, $\Lambda = 2$	MN	-9.385	0.099	1.144
	AV4'	-11.478	-0.028	0.769
GW, $\Lambda = 4$	MN	-13.392	0.990	3.252
	AV4'	-14.881	0.686	2.438
CS, $\Lambda = 2$	MN	-8.388	-0.217	0.057
CS, $\Lambda = 4$	MN	-8.712	-0.161	0.227

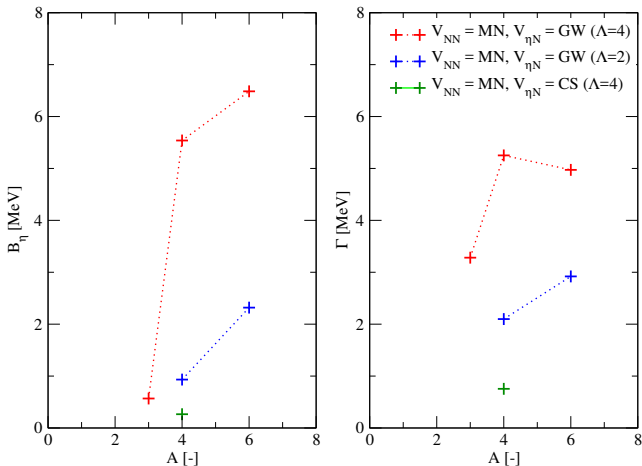


•  $\eta^4\text{He}$ 

	<i>NN</i> int.	$\delta\sqrt{s_{sc}}$	$B_\eta$	$\Gamma_{gs}$
GW, $\Lambda = 2$	MN	-19.477	0.96	1.975
	AV4'	-23.646	0.38	1.207
GW, $\Lambda = 4$	MN	-29.750	4.69	4.5
	AV4'	-32.411	3.51	3.615
CS, $\Lambda = 2$	MN	-16.704	-0.16	0.133
CS, $\Lambda = 4$	MN	-19.246	0.47	0.901

# $\eta$ in 3-, 4- and 6-body systems

- SVM for Minnesota NN model + GW and CS  $\eta N$  models;  
no Coulomb  
calculated by M. Schaefer, preliminary (results from yesterday)!



A. Cieply, E. Friedman, A. Gal, J. Mares, PLB 725 (2013) 334, NPA 925 (2014) 126

- K.-G. equation:

$$\left[ \omega_\eta^2 + \vec{\nabla}^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho) \right] \phi_\eta = 0$$

complex energy  $\omega_\eta = m_\eta - B_\eta - i\Gamma_\eta/2$

- $\Pi_\eta(\omega_\eta, \rho) = 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{E_N} F_\eta N(\sqrt{s}, \rho) \rho$
- $\eta$  in a nucleus  $\Rightarrow$  polarized (compressed)  $\rho \longrightarrow \Pi_\eta(\rho)$   
 $\Rightarrow$  **selfconsistent solution**

- Selfenergy operator

$$\Pi_{\eta}(\omega_{\eta}) = 2 \omega_{\eta} V_{\eta} = -4\pi \frac{\sqrt{s}}{E_N} F_{\eta N}(\sqrt{s}, \rho) \rho$$

- $F_{\eta N} = \eta N$  scattering amplitude with **two-body argument**:

$$\sqrt{s} \quad (s = (\omega_{\eta} + E_N)^2 - (\vec{p}_{\eta} + \vec{p}_N)^2)$$

- $\eta N$  c.m. frame  $\rightarrow$   $\eta$ -nucleus c.m. frame  $\Rightarrow \vec{p}_{\eta} + \vec{p}_N \neq 0$

$$\Rightarrow \sqrt{s} \approx m_{\eta} + m_N - B_{\eta} - B_N - \xi_N \frac{p_N^2}{2m_N} - \xi_{\eta} \frac{p_{\eta}^2}{2m_{\eta}} = E_{\text{th}} + \delta\sqrt{s},$$

$$\delta\sqrt{s} = B_N \frac{\rho}{\bar{\rho}} - \xi_N B_{\eta} \frac{\rho}{\rho_0} - \xi_N T_N \left(\frac{\rho}{\rho_0}\right)^{2/3} + \xi_{\eta} \text{Re} V_{\eta}(\sqrt{s}, \rho)$$

- $\rho =$  nucl. medium density (RMF calculations)
- $V_{\eta}, B_{\eta} \Rightarrow$  self-consistent solution

- Pauli correlations  $\leftarrow$  WRW method

T. Wass, M. Rho, W. Weise, NPA 617 (1997) 449.

$$F_{\eta N}(\sqrt{s}, \rho) = \frac{F_{\eta N}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/E_N)F_{\eta N}(\sqrt{s})\rho},$$

$$\xi(\rho) = \frac{9\pi}{4\rho_f^2} I(\kappa), \quad I(\kappa) = 4 \int_0^\infty \frac{dt}{t} \exp(i\kappa t) j_1^2(t), \quad \kappa = \frac{1}{\rho_f} \sqrt{2m_\eta(B_\eta + i\Gamma/2)}.$$

- Chiral coupled-channels model - A. Cieply, J. Smejkal, NPA 919 (2013) 334.

multi-channel L.-Sch. equation:

$$F = V + VGF, \quad F, V \text{ in separable form,}$$

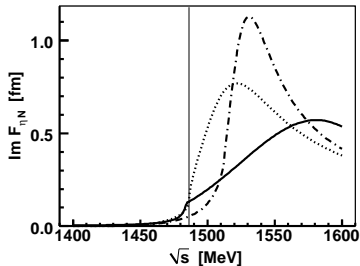
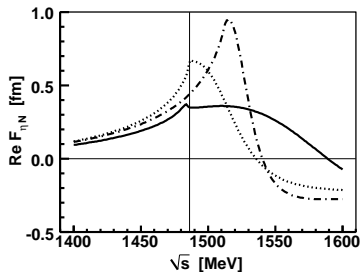
$$G_n(\sqrt{s}; \rho) = -4\pi \int_{\Omega_n(\rho)} \frac{d^3\vec{p}}{(2\pi)^3} \frac{g_n^2(p)}{k_n^2 - p^2 - \Pi_n(\sqrt{s}, \vec{p}; \rho) + i0}.$$

$\Omega_n(\rho) \rightarrow$  intermediate  $N$  energy is above Fermi level (Pauli blocking)

$\Pi \rightarrow$  hadron **self-energies** in  $G$  (+SE option)

$\Rightarrow$  self-consistency

- Energy dependence of  $f_{\eta N}(\sqrt{s})$



chiral CS model (A. Cieply, J. Smejkal, Nucl. Phys. A 919 (2013) 334)

dotted curve: free-space, dot-dashed: Pauli blocked, full: Pauli blocked + hadron selfenergies

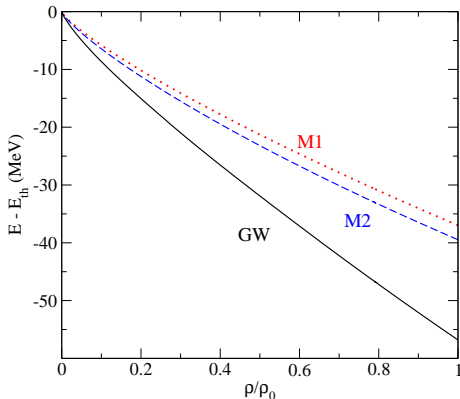
- Nuclear medium reduces the  $\eta N$  attraction at threshold, the amplitude becomes **smaller** when going **subthreshold**

# Energy shift

Energy dependence of  $V_{\eta N}(\sqrt{s}) \leftarrow$  due to  $N^*(1535)$

- In-medium (subthreshold) energy shift:

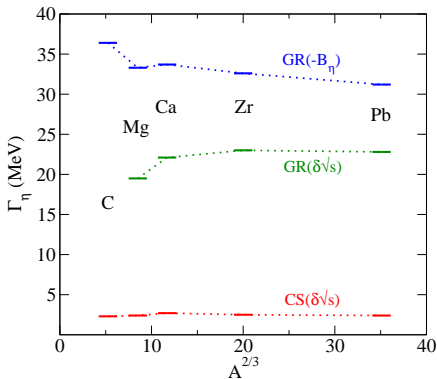
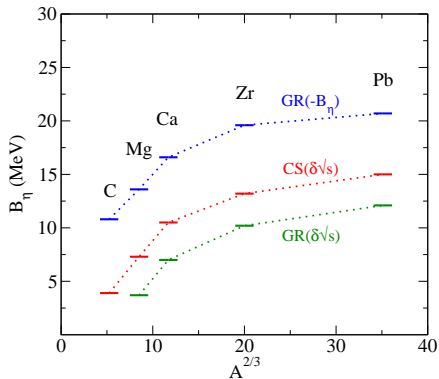
$$\delta\sqrt{s} = -B_N \frac{\rho}{\rho_0} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N \left(\frac{\rho}{\rho_0}\right)^{2/3} + \xi_\eta \text{Re}V_\eta(\sqrt{s}, \rho)$$



- $B_\eta, V_\eta, \rho \Rightarrow$  selfconsistent solution  $\rightarrow$

40 - 60 MeV energy shift at  $\rho_0$  - larger than shift by  $B_\eta$  (GR) or by 30 MeV (Haider, Liu)

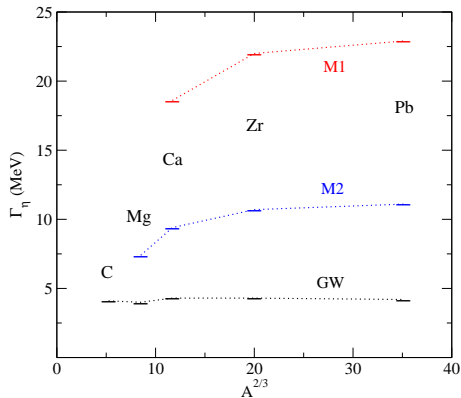
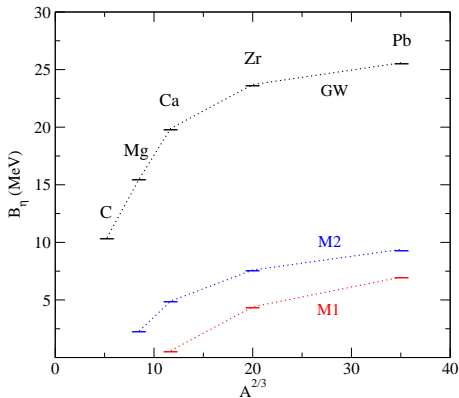
- Sensitivity to the energy shift:  
selfconsistent  $\delta\sqrt{s}$  reduces both  $1s$   $B_\eta$  and  $\Gamma_\eta$



- GR widths too large to resolve  $\eta$  bound states !

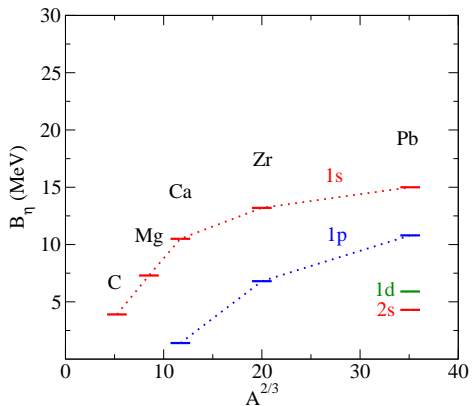
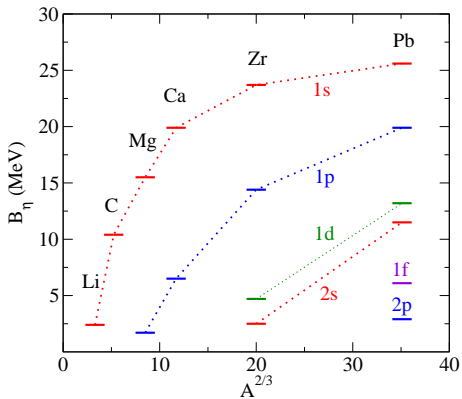


- Model dependence:



- Larger  $\text{Re } a_{\eta N}$  gives larger  $B_\eta$  vs. no relation between  $\text{Im } a_{\eta N}$  and  $\Gamma_\eta$

- Predictions of GW and CS models:  
all states in selected nuclei are shown; both models give small widths



## $\eta$ nuclear bound states:

- Large energy shift and rapid decrease of the  $\eta N$  amplitudes below threshold  $\Rightarrow$  relatively small binding energies and widths of the calculated  $\eta$  nuclear bound states

$\eta d$  unbound in considered models

${}^3_{\eta}\text{He}$  bound state  $\leftarrow \text{Re}a_{\eta N} \approx 1 \text{ fm}$

${}^4_{\eta}\text{He}$  bound state  $\leftarrow \text{Re}a_{\eta N} \geq 0.7 \text{ fm}$

${}^{12}_{\eta}\text{C}$  bound state  $\leftarrow \text{Re}a_{\eta N} \geq 0.5 \text{ fm}$

- Additional width contribution not considered in this work due to  $\eta N \rightarrow \pi\pi N$  and  $\eta NN \rightarrow NN \Rightarrow$  estimated to add few MeV
- Subthreshold behavior of  $F_{\eta N}$  is crucial to decide whether  $\eta$  nuclear states exist, in which nuclei, and if their widths are small enough to be resolved in experiment.

Thank you

Nir Barnea, Aleš Ciepły, Eli Friedman, Avraham Gal and Martin Schaefer!