Pole structure and compositeness

J. A. Oller

Departamento de Física
Universidad de Murcia[1]
Murcia, Spain

Hadron2017
U. Salamanca, September 27th, 2017

[1] Partially funded by MINECO (Spain) and EU, project FPA2016-77313-P

Outline

1. Basic set-up

Highly subjective presentation of the vast literature. My apologizes if you think that important works are missed

Starting point, basic set-up:

Bound state near a two-body threshold.
Non-Relativistic Dynamics

S. Weinberg, PR130,776(1963); PR137,B672(1964)

$$H = H_0 + V$$

Spectrum:

$$H|\psi_\alpha\rangle = E_\alpha|\psi_\alpha\rangle \quad , \quad \text{Continuum spectrum}$$
$$H|\psi_{B_i}\rangle = E_{B_i}|\psi_{B_i}\rangle \quad , E_{B_i} < 0 \ , \quad \text{Discrete Spectrum}$$

Bare spectrum:

$$H_0|\varphi_\alpha\rangle = E_\alpha|\varphi_\alpha\rangle$$
$$H_0|\varphi_n\rangle = E_n|\varphi_n\rangle$$

Elementariness: Z
Compositeness: X

$$\langle \psi_B | \psi_B \rangle = 1 = \underbrace{\sum_n |\langle n | d \rangle|^2}_{Z} + \underbrace{\int d\alpha |\langle \varphi_\alpha | d \rangle|^2}_{X}$$

$$1 = Z + X$$

$$X = 1 - Z = \int d\alpha \, \frac{|\langle \varphi_\alpha | V | \psi_B \rangle|^2}{(E_\alpha - E_B)^2}$$

Wave Function Renormalization: $Z^{1/2}$
There is only **one** "elementary" bare state around E_B S. Weinberg, PRC130,776(1963);PRC131,440(1963)

$$\langle \varphi_0 | \psi_B \rangle = Z^{1/2}$$

2. Different perspective

We focus our attention in the continuum spectrum
The continuum spectrum is common to H and H_0

Let two particles of types A and B
Creation,annihilation operators: $a^\dagger_\alpha a_\alpha$, $b^\dagger_\beta b_\beta$
Operator Numbers:

$$H_0 = \int dE_\alpha E_\alpha a^\dagger_\alpha a_\alpha + \int d\beta E_\beta b^\dagger_\beta b_\beta + \sum_n E_n |\varphi_n\rangle\langle\varphi_n|$$

$$N_D = \int d\alpha a^\dagger_\alpha a_\alpha + \int d\beta b^\dagger_\beta b_\beta$$
$$= \int d^3x \left[\psi^\dagger_A(x)\psi_A(x) + \psi^\dagger_B(x)\psi_B(x)\right]$$

$$[H_0, N_D] = 0 \longrightarrow N_D(t) = N_D(0) = N_H(0)$$

New definition of X

$$X = \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle$$

Equivalence to the previous definition

$$|\psi_B\rangle = \int d\gamma C_\gamma |AB_\gamma\rangle + \sum_n C_n|\varphi_n\rangle$$

$$X = \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle = \int d\gamma |C_\gamma|^2 \ .$$

Specially suitable when using perturbative EFT (e.g. ChPT) with nonperturbative techniques

3. QFT-like Calculation

This new definition is suitable for a QFT treatment

Dirac or Interacting Image

$$V \to Ve^{-\varepsilon|t|} \quad , \quad \varepsilon \to 0^+$$

$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0,-\infty)|\varphi_B(-\infty)\rangle$$

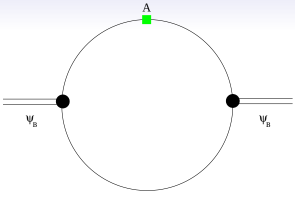
$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0,+\infty)|\varphi_B(+\infty)\rangle$$

$$X = \frac{1}{2}\langle\psi_B|N_D|\psi_B\rangle$$

$$= \frac{1}{2}\langle\varphi_B(+\infty)|U_D(+\infty,0)N_D U_D(0,-\infty)|\varphi_B(-\infty)\rangle$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T/2}^{+T/2} dt\langle\varphi_B(+\infty)|U_D(+\infty,t)N_D(t)U_D(t,-\infty)|\varphi_B(-\infty)\rangle$$

$$U_D(t,-\infty)|\varphi_B(-\infty)\rangle = e^{iH_0 t}e^{-iE_B t}U_D(0,-\infty)|\varphi_B(-\infty)\rangle$$



$$X_{\ell S} = \int \frac{d^3k}{(2\pi)^3} \frac{g_{\ell S}^2(k^2)}{(k^2/2\mu - E_B)^2}$$
$$X = \sum_{\ell S} X_{\ell S}$$

$\ell = 0$: one has the Weinberg's equation for $1 - Z$

$$g(k) = \frac{1}{2\pi^2} \int_0^\infty k'^2 dk' V(k,k') \frac{1}{k'^2/2\mu - E_B} g(k')$$

$$g_{\ell S}(-k) = (-1)^\ell g_{\ell S}(k) \qquad [T = V + VGT]$$

4. New equation for X

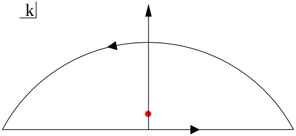
We rewrite symmetrically the integration in k for X

$$X = \left(\frac{\mu}{\pi}\right)^2 \int_{-\infty}^{+\infty} dk k^2 \frac{g^2(k^2)\mathbf{e^{i\varepsilon k}}}{(k^2 - 2\mu E_B)^2}$$

Convergent factor $\mathbf{e^{i\varepsilon k}}$, $\varepsilon \to 0^+$

Analogous to Dimensional Regularization

Notation: $\pm i\gamma = \sqrt{2\mu E_B}$



$$X = \frac{2i\mu^2}{\pi}\frac{\partial}{\partial k}\left[\frac{k^2 g^2(k^2)}{(k+i\gamma)^2}\right]_{k=i\gamma}$$

$$= -\frac{\partial G}{\partial E_B} g^2(-\gamma^2) - \frac{\mu^2\gamma}{\pi}\left.\frac{\partial g^2(k^2)}{\partial k^2}\right|_{k=i\gamma}$$

$$G(E) = -\frac{i\mu}{2\pi}k(E) = \frac{\mu}{\pi^2}\int_0^\infty dk\, k^2 \frac{e^{i\varepsilon k}}{2\mu E_B - k^2}$$

$$X = -\frac{\partial G}{\partial E_B} g^2(-\gamma^2) - \frac{\mu^2\gamma}{\pi}\, \frac{\partial g^2(k^2)}{\partial k^2}\bigg|_{k=i\gamma}$$

The 1st term is model independent

Already a well known contribution Hyodo,Jido,Hosaka PRC85,015201(2012)

The 2nd term is the new one.

E.g. it takes into account that $g^2_{\ell S}(k^2) \propto k^{2\ell}$ for $k \to 0$

Aceti,Oset, PRD86,014012(2012)

The 2nd term depends on V

$$g(k) = \frac{1}{2\pi^2}\int_0^\infty k'^2 dk' V(k,k') \frac{1}{k'^2/2\mu - E_B} g(k')$$

5. Relativistic case

The attention is focused on the wave function renormalization Z

There is a lack of a general framework

Partial results are available:

$0 \leq Z \leq 1$: Lee model Vaughn, Aaron, Amado PRC124,1258(1961); Yukawa type interactions Salam,Nuovo Cim.25,224(1962); Lurié, Macfarlane, PR136,B816(1964)
$Z = 0$ equivalence between 4-Fermi theories and Yukawa theories

Issue: In the relativistic case we can also have multiparticle states. E.g. $\pi\pi$, 4π, $\ldots$, being all of them present in the spectrum of H_0
Nonetheless:

$$[H_0, N_D] = 0$$

Non-Relativistic formalism:

$$|\psi_B\rangle = \int d\gamma C_\gamma |AB_\gamma\rangle + \sum_n C_n |\varphi_n\rangle$$

Relativistic formalism:

$$|\psi_B\rangle = \int d\gamma C_\gamma |AB_\gamma\rangle + \int d\eta D_\eta |AAB_\eta\rangle + \int d\mu\, \delta_\mu |ABB_\mu\rangle + \dots$$
$$+ \int d\eta_\nu F_\nu |CD_\nu\rangle + \dots + \sum_n C_n |\varphi_n\rangle + \sum_n \int d\alpha C_{n\alpha} |A_\alpha \varphi_n\rangle + \dots$$

We provide a new criterion for elementariness of a relativistic bound state

Number operator for each species of particle

$$N_D^A = \int d\alpha a_\alpha^\dagger a_\alpha$$

$$\langle\psi_B|N_D^A|\psi_B\rangle = \int d\alpha |C_\gamma|^2 + 2\int d\eta |D_\eta|^2 + \int d\mu |\delta_\mu|^2 + \dots$$

Criterion for an elementary stable particle

$$\langle N_D^A \rangle \equiv \langle \psi_B | N_D^A | \psi_B \rangle \ll 1$$

For relativistic systems we also have to discard that

$$\langle N_D^E \rangle \equiv \langle \psi_B | N_D^E | \psi_B \rangle > 1$$

N_D^E is the total number operator of bare elementary states.

Criterion for an elementary stable particle

$$\langle N_D^A \rangle \equiv \langle \psi_B | N_D^A | \psi_B \rangle \ll 1$$

We can calculate this expectation value within QFT too

$V \to V e^{-\varepsilon |t|}$

$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0, -\infty)|\varphi_B(-\infty)\rangle$$
$$|\psi_B\rangle = |\varphi_B(0)\rangle = U_D(0, +\infty)|\varphi_B(+\infty)\rangle$$
$$\langle N_D^A \rangle = \langle \psi_B | N_D^A | \psi_B \rangle$$
$$= \langle \varphi_B(+\infty) | U_D(+\infty, 0) N_D^A U_D(0, -\infty) | \varphi_B(-\infty) \rangle$$
$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} dt \langle \varphi_B(+\infty) | U_D(+\infty, t) N_D^A(t) U_D(t, -\infty) | \varphi_B(-\infty) \rangle$$

$$U_D(t, -\infty)|\varphi_B(-\infty)\rangle = e^{iH_0 t} e^{-iE_B t} U_D(0, -\infty)|\varphi_B(-\infty)\rangle$$

▷ Technicalities

In general there are many more diagrams now apart from those in the NR case

One cannot apply the trick associated with the convergent factor in the same way

▷ Other consequences:

If

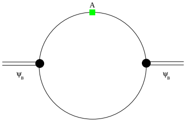
$$\sum_i \langle N_D^{A_i} \rangle = 2 + m \geq 2$$

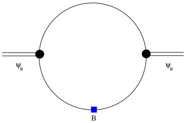
The multiparticle components with $2 + m$ and more particles are important.
Other similar *exclusive* conditional conclusions can be also established

• In the case in which we are close to a two-body threshold (AB) we come back to the NR case for evaluating X_{AB}

- If *there are good reasons* for dominance of two-body channels

$$X_{AB} = \frac{1}{2}(\langle N_D^A \rangle + \langle N_D^B \rangle) \approx \int d\gamma |C_\gamma|^2$$





6. Resonances. NR case

Bogdanova, Hale, Markushin, PRC44,1289(1991);

Baru,Haidenbauer,Hanhart,Kalashnikova,Kudryavtsev,PLB586,53(2004)

Spectral density of the bare state $|\psi_0\rangle$: $\omega(E)$

$$|\psi_0\rangle = \int d\mathbf{k} c_0(k)|\mathbf{k}\rangle$$

$$\omega(E) = 4\pi\mu k|c_0(E)|^2\theta(E)$$

$$\int_0^\infty dE\,\omega(E) = \begin{cases} 1 & \textbf{No bound states} \\ 1-Z & \text{With bound states} \end{cases}$$

How to implement it? Select the *resonant* region around threshold

$$W = \int_{E_-}^{E_+} dE\,\omega(E)$$

Conceptually, it is not *fully* settled as a quantitative estimate of compositeness for resonances

It provides a nice smooth transition from the clear bound states and narrow resonances

It has a clear connection with the pole-counting rule of Morgan NPA543,632(1992), with the presence of nearby CDD poles Kang,Oller, EPJC77,399(2017)

7. Operator-number interpretation

In my developments a resonance follows by analytical continuation from the physical axis

- Let $|\psi_\alpha^+\rangle$ be a two-body in-state

$$
\begin{aligned}
|\psi_\alpha^+\rangle &= U_D(0,-\infty)|\varphi_\alpha\rangle \\
&= |\varphi_\alpha\rangle + \int d\gamma \frac{T_{\gamma\alpha}(E+i\varepsilon)}{E-E_\gamma+i\varepsilon}|\varphi_\gamma\rangle + \sum_n \frac{T_{n\alpha}(E)}{E-E_n}|\varphi_n\rangle
\end{aligned}
$$

S. Weinberg, QFT, Vol.1

$$
\langle\psi_\alpha^+|\underbrace{\int d\gamma a_\gamma^\dagger a_\gamma}_{N_D^A} + \underbrace{\int d\eta b_\eta^\dagger b_\eta}_{N_D^B}|\psi_\alpha^+\rangle = 2\,\langle\varphi_\alpha|\varphi_\alpha\rangle \quad \text{Fine!}
$$

There are cancellations because of unitarity

Problem: This expectation value cannot be analytically continued to the resonance pole

$$\langle\psi_\alpha^+| = \langle\varphi_\alpha| + \int d\gamma \frac{T_{\gamma\alpha}(E-i\varepsilon)}{E-i\varepsilon-E_\gamma}\langle\varphi_\gamma| + \sum_n \frac{T_{n\alpha}(E-i\varepsilon)}{E-i\varepsilon-E_n}\langle\varphi_n|$$

$$T(E\pm i\varepsilon)^\dagger = T(E\mp i\varepsilon)$$

The analytical continuation to $E = M_R - i\Gamma/2$ remains in the 1st or physical Riemann Sheet (RS)

No resonance pole there

The analytical continuation must be done as in the calculation of the S-matrix:

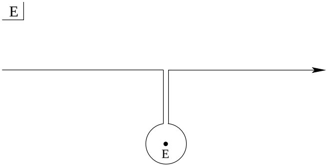
out state $|\psi_\alpha^-\rangle$, $E - i\varepsilon$

$$\langle\psi_\alpha^-|N_D^A + N_D^B|\psi_\alpha^+\rangle$$

$$\langle\psi_\alpha^-| = \langle\varphi_\alpha| + \int d\gamma \frac{T_{\gamma\alpha}(E+i\varepsilon)}{E+i\varepsilon-E_\gamma}\langle\varphi_\gamma| + \sum_n \frac{T_{n\alpha}(E+i\varepsilon)}{E+i\varepsilon-E_n}\langle\varphi_n|$$

When crossing the real positive energy axis

$$T(E+i\varepsilon) \to T^{II}(E-i\varepsilon)$$



The resonance pole is now reached both for the ket and the bra

8. QFT-like calculation

Dirac or Interacting Image

$$V \to Ve^{-\varepsilon|t|}$$

$$|\psi_R^+\rangle = U_D(0,-\infty)|\varphi_R^+\rangle$$

$$\langle\psi_R^-| = \langle\varphi_R^-|U_D(+\infty,0)$$

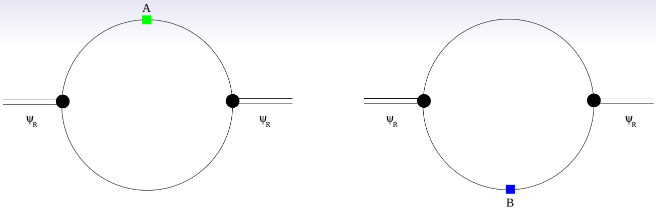
$$X = \frac{1}{2}\langle\psi_R^-|N_D|\psi_R^+\rangle$$

$$= \frac{1}{2}\langle\varphi_R^-|U_D(+\infty,0)N_D U_D(0,-\infty)|\varphi_R^+\rangle$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T/2}^{+T/2} dt\langle\varphi_R^-|U_D(+\infty,t)N_D(t)U_D(t,-\infty)|\varphi_R^+\rangle$$

$$U_D(t,-\infty)|\varphi_R^+\rangle = e^{iH_0 t}e^{-iHt}|\psi_R^+\rangle = e^{-(iM_R+\frac{\Gamma}{2})t}e^{iH_0 t}U_D(0,-\infty)|\varphi_R^+\rangle$$

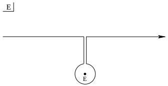
$$\langle\varphi_R^-|U_D(+\infty,t) = \langle\psi_R^-|e^{iHt}e^{-iH_0 t} = \langle\varphi_R^-|U_D(+\infty,0)e^{-iH_0 t}e^{(iM_R+\frac{\Gamma}{2})t}$$



2nd Riemann Sheet: $E_R = \varkappa^2/2\mu$

$$X_{\ell S} = \int \frac{d^3k}{(2\pi)^3} \frac{g_{\ell S}^2(k^2)}{(k^2/2\mu - E_R)^2} + \frac{i\mu^2}{\pi\varkappa} \frac{\partial}{\partial k} \left[k\, g_{\ell S}^2(k^2)\right]_{k=\varkappa}$$

$$X = \sum_{\ell S} X_{\ell S}$$



$$g(k) = \frac{\mu}{\pi^2} \int_0^\infty dk' k'^2 \frac{V(k,k')g(k')}{k'^2 - \varkappa^2}$$
$$+ \frac{i\mu\varkappa V(k,\varkappa)/\pi}{1 - i\mu\varkappa V(\varkappa,\varkappa)/\pi} \frac{\mu}{\pi^2} \int_0^\infty dk' k'^2 \frac{V(\varkappa,k')g(k')}{k'^2 - \varkappa^2}$$

$$g_{\ell S}(-k) = (-1)^\ell g_{\ell S}(k)$$

9. New equation for X

We include the convergent factor for the 2nd RS calculation:

$$X = \frac{\mu^2}{\pi^2}\int_{-\infty}^{+\infty} dk k^2 \frac{g^2(k^2) e^{-i\varepsilon k}}{(k^2-\varkappa^2)^2} + \frac{i\mu^2}{\pi\varkappa}\frac{\partial}{\partial k}\left[k\, g^2(k^2)\right]_{k=\varkappa}$$

$$\begin{aligned} X &= \frac{i\mu^2}{2\pi\varkappa}\frac{\partial}{\partial k}\left[k\, g^2(k^2)\right]_{k=\varkappa} \\ &= -\frac{\partial G^{\mathrm{II}}}{\partial E_R} g^2(\varkappa^2) + \frac{i\mu^2}{2\pi}\left.\frac{\partial}{\partial k} g^2(k^2)\right|_{k=\varkappa} \end{aligned}$$

The novel contribution is the red one
It depends on $V(k,k')$

$$X = \frac{\mu^2}{\pi^2}\int_0^{+\infty} dk^2 \sqrt[\mathrm{II}]{k^2+i\varepsilon}\, \frac{g^2(k^2)}{(k^2-\varkappa^2)^2}$$

Wave function squared:
resonance Gamow state

For an energy-independent potential $X = 1$

Hernández,Mondragón, PRC29,722(1984)

$$V(k,k') = f(k^2)f(k'^2)\,\mathrm{V}$$

In ordinary QM resonances are composite

X is in general complex for a resonance

E.g. for $\mathrm{V} = \mathrm{V}(E)$

$$g(k^2) = \mathrm{V}^{\frac{1}{2}} f(k^2)\left[\frac{\partial(\mathrm{V}\widetilde{G}^{\mathrm{II}})}{\partial E_R}\right]^{-1}$$

$$\begin{aligned} X &= \left[\frac{\partial(\mathrm{V}\widetilde{G}^{\mathrm{II}})}{\partial E_R}\right]^{-1} \frac{-1}{(2\pi)^2}\int_0^\infty dk k^2 \frac{\mathrm{V} f(k^2)^2}{(E_R - k^2/2\mu)^2} \\ &= \left[\frac{\partial(\mathrm{V}\widetilde{G}^{\mathrm{II}})}{\partial E_R}\right]^{-1}\left\{\mathrm{V}\frac{\partial \widetilde{G}^{\mathrm{II}}}{\partial E_R} + \widetilde{G}^{\mathrm{II}}(E_R)\frac{\partial \mathrm{V}(E_R)}{\partial E_R}\right\} \end{aligned}$$

10. Redefinition of "phases"

in-,out-states:
$\eta(E)$ is a complex function with RHC: $\eta(E^*) = \eta(E)^*$

$$|\psi_\alpha^+\rangle \longrightarrow e^{\eta(E_\alpha + i\varepsilon)}|\psi_\alpha^+\rangle$$

$$\begin{aligned}\langle\psi_\alpha^-| &\longrightarrow \langle\psi_\alpha^-|e^{\eta(E_\alpha - i\varepsilon)^*} \\ &= \langle\psi_\alpha^-|e^{\eta(E_\alpha + i\varepsilon)}\end{aligned}$$

Analytical continuation $E_\alpha \to E_R = M_R - i\Gamma/2$

$$\eta(E_\alpha + i\varepsilon) \to \eta^{\mathrm{II}}(E_\alpha - i\varepsilon) \to \eta^{\mathrm{II}}(M_R - i\Gamma/2)$$

An specific fact of resonances; no analogue for bound states.

These phase factors make X_{AB} be positive definite
There could be dependence on the channel, $\eta_{AB}(E)$

$$g^2_{AB}(k^2) \to g^2_{AB}(k^2)\, e^{2\eta^{\rm II}_{AB}(E_R)}$$

$$X_{AB} \to \langle \psi_R^- | N_D^{AB} | \psi_R^+ \rangle\, e^{2\eta^{\rm II}_{AB}(E_R)} \in \mathbb{R}^+$$

Plausible dispersion relation for $\eta(E)$
Narrow-Resonance Case:

$$\eta(E) = \frac{1}{\pi}\int_0^\infty dE' \frac{{\rm Im}\eta(E')}{E' - M_R - i\varepsilon}$$
$$= \frac{1}{\pi}\int_0^\infty dE' \frac{{\rm Im}\eta(E')}{E' - M_R} + i{\rm Im}\eta(E')$$

${\rm Im}\eta(E')$ is smooth and $\eta(M_R) \approx i{\rm Im}\eta(M_R) \to e^{\eta(M_R)}$: Pure phase factor $|e^{\eta(M_R)}| \approx 1$

11. Relativistic case

$$\langle N_D^A\rangle \equiv \langle\psi_R^-|N_D^A|\psi_R^+\rangle$$

The QFT expression can be applied in the relativistic case too

$$V \to Ve^{-\varepsilon|t|}$$

$$\langle\psi_R^-|N_D^A|\psi_R^+\rangle = \langle\varphi_R^-|U_D(+\infty,0)N_D^A U_D(0,-\infty)|\varphi_R^+\rangle$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} dt\langle\varphi_R^-|U_D(+\infty,t)N_D^A(t)U_D(t,-\infty)|\varphi_R^+\rangle$$

Necessary condition for a resonance to be qualified as elementary

$$\langle N_D^A\rangle = 0 \quad , \quad \forall A$$

Criterion for an elementary narrow resonance with respect the open channels

$$\langle N_D^A\rangle = \langle\psi_R^-|N_D^A|\psi_R^+\rangle e^{2i\mathrm{Im}\eta_{\mathrm{A}}^{\mathrm{II}}(E_R)} = \left|\langle\psi_R^-|N_D^A|\psi_R^+\rangle\right| \ll 1$$

12. S-matrix transformations

Introduced in Z.H.Guo, Oller, PRD93,096001(2016)

Example: Narrow resonance case

Laurent series around the resonance pole: $s_P = (M_R - i\Gamma/2)^2$

$$S(s) = \frac{R}{s - s_P} + S_0(s)$$
$$S(s)S(s)^\dagger = I$$

$S_0(s) \to S_0$, constant

$$(s - s_P)(s - s_P^*)S_0S_0^\dagger + (s - s_P)S_0R^\dagger + (s - s_P^*)RS_0^\dagger + RR^\dagger = (s - s_P)(s - s_P^*)$$

$$S_0S_0^\dagger = I$$
$$S_0R^\dagger + RS_0^\dagger = 0$$
$$-s_PS_0R^\dagger - s_P^*RS_0^\dagger + RR^\dagger = 0$$

Solution:

$$S_0 = \mathscr{O}\mathscr{O}^T$$
$$\mathscr{O}\mathscr{O}^\dagger = I$$

Rank 1 Symmetric Projection Operator $\mathscr{A}$:

$$\begin{aligned} R =& i\lambda\mathscr{O}\mathscr{A}\mathscr{O}^T \ , \ \lambda \in \mathbb{R} \\ \mathscr{A}^\dagger =& \mathscr{A} \\ \mathscr{A}^2 =& \mathscr{A} \\ \lambda =& 2\mathrm{Im}\, s_P = -2M_R\Gamma_R \end{aligned}$$

Resonant S-matrix $S_R(s)$:

$$S(s) = \mathscr{O}\underbrace{\left(I + \frac{i\lambda\mathscr{A}}{s - s_R}\right)}_{S_R(s)}\mathscr{O}^T$$

Origin of phases: Smooth non-resonant terms, $\mathscr{O}$

E.g. Coulomb phases in nuclear physics

In general, do not take the real part in $\langle\psi_R^-|N_D^A|\psi_R^+\rangle$ to make it real!!

The right procedure is doing the phase or S-matrix transformations

The transformed S matrix

$$S_{\mathscr{O}}(s) \equiv \mathscr{O}S(s)\mathscr{O}^T$$

E.g. for the case of only one channel:

$$g_A^2 \to g_A^2 \quad \underbrace{S_0^{-1}}_{\text{Non-Resonant terms}} \quad \underbrace{e^{-i\phi}}_{\text{Resonance Propagator is Complex}}$$

$$\langle\psi_R^-|N_D^A|\psi_R^+\rangle \to |\langle\psi_R^-|N_D^A|\psi_R^+\rangle|$$

13. Finite width resonances

Necessary Condition for still interpreting $|\langle\psi_R^-|N_D^A|\psi_R^+\rangle|$ as an average number of particles Z.H.Guo, Oller, PRD93,096001(2016)

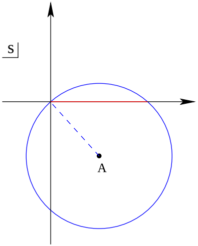
The transformations

$$S_{\mathcal{O}}(s) \equiv \mathcal{O} S(s) \mathcal{O}^T$$
$$\mathcal{O}\mathcal{O}^\dagger = I$$
$$g_A^2 \to g_A^2 \mathcal{O}_{AA}^2$$

make sense only if:

▷The Laurent expansion around s_P is valid in some interval of physical (real values above threshold) for s

$$S(s)S(s)^\dagger = I \quad \text{is meaningful}$$



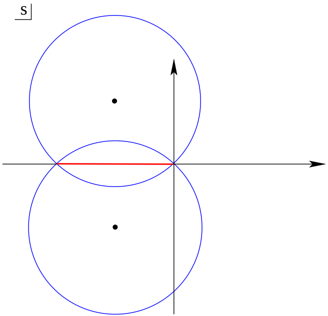
Condition A: $s_n < \mathrm{Re}\, s_P < s_{n+1}$

s_n is the threshold of channel n

Physical idea

- If this condition is fulfilled one can think of a physical process with a clear resonance contribution. E.g. the σ and E791 data on D^+ and D_s^+ decays
- The resonance phenomenon is physically manifest in the open channels
- We preserve $|g_A|$ to the open channels

A resonance is then very different



$$\frac{g^2}{s-s_R}+\frac{g^{2*}}{s-s_R^*}=2\mathrm{Re}\frac{\mathrm{g}^2}{\mathrm{s}-\mathrm{s_R}}$$

Double-pole like virtual state

This could well be the case for the $X(3872)$, at least as a double-like pole. It could also be triple-like, etc. Z.H.Guo, Oller, PRD93,096001(2016)

$\bar{D}^0 D^{*0}$ threshold. Tiny width

An example: S-wave Effective Range Expansion

X.W.Kang,Z.H.Guo,Oller,PRD94,014012(2016)

$$T(k) = \frac{1}{-\frac{1}{a} + \frac{1}{2} r k^2 - ik}$$
$$G(k) = -ik$$

$$\tan\phi = \frac{\Gamma}{2M_R} \longrightarrow 0 \leq \phi \leq \pi/2 \text{ for } M_R \geq 0$$
$$k_R = k_r - i\,k_i = \sqrt{2\mu(M_R - i\Gamma/2)} = |k_R|(\cos\phi/2 - i\sin\phi/2)$$

$$X = -\gamma^2 \frac{dG}{ds} = -\gamma_k^2 \frac{dG}{dk} = i\frac{k_i}{k_r} = i\tan\frac{\phi}{2}$$

$|X| \leq 1 \leftrightarrow k_r \geq k_i \leftrightarrow M_R \geq 0$

($|X| = 1$ for $M_R = 0$ and $\Gamma > 0$)

If the real part is taken then ALWAYS $X = 0$!

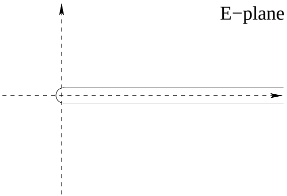
14. Scattering Amplitude $t(E)$

Dispersion Relation for the inverse of $t(E)$

$$\mathrm{Im}\, t(E)^{-1} = -ik$$

One subtraction is needed

$$\oint dz \frac{t(z)^{-1}}{(z-E)(z-C)}$$



The only other structure apart from the threshold that can give rise to a strong distortion in $t(E)^{-1}$ is a pole at M_Z

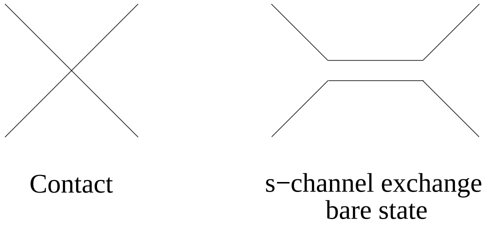
$$t(E) = \frac{1}{\frac{\lambda}{E-M_Z} + \beta - ik}$$

CDD pole Castillejo, Dalitz, Dyson, PR, 101, 453 (1956)

The general formula for a partial-wave without crossed-channel dynamics was deduced in: Oller, Oset PRD60,074023 (1999)

Contact interaction plus s-channel exchange of bare resonances

Kang,Oller, EPJC77,399(2017) study of the $X(3872)$



[BHKKN] Baru,Hanhart,Kalashnikova,Kudryavtsev, EPJA,44,93(2010) *Interplay of quark and meson degrees of freedom in a near-threshold resonance*

[ABK] Artoisenet,Braaten,Kang, PRD,82,014013(2010) *Using line shapes to discriminate between binding mechanisms for the* $X(3872)$

[BHKKN]

$$D_F(E) = E - E_f - \frac{(E-E_f)^2}{(E-M_Z)^2} + \frac{i}{2} g_f k$$

$$t(E) = \frac{g_f}{8\pi^2 \mu D_F(E)}$$

$$t(E) = \frac{1}{4\pi^2\mu} \frac{E - E_f + \frac{1}{2} g_f \gamma_V}{(E-E_f)(\gamma_V + ik) + \frac{i}{2} g_f \gamma_V k}$$

$$g_f = \frac{2\lambda}{\beta^2}$$

$$E_f = M_Z - \frac{\lambda}{\beta}$$

$$\gamma_V = -\beta$$

$\gamma_V = 1/a_V$, a_V scattering length in pure contact-interaction theory.

For $|M_Z| \gg |E_f|$ one recovers the standard Flatté approximation

Limitation of [BHKKN] and [ABK]

- They predict only $\lambda \geq 0$

$$\begin{array}{ll} \text{[BHKKN]} & \text{[ABK]} \\ \lambda = \frac{\gamma_V^2}{2} g_f & \lambda = \frac{2g^2\gamma_0^2(\gamma_1-\kappa_2)^2}{(\gamma_0+\gamma_1-2\kappa_2)^2} \end{array}$$

- Positive effective range r, v_3, v_5, etc, cannot be reproduced with $\lambda \geq 0$:

$$r = -\frac{\lambda}{\mu M_Z^2} < 0$$

$$v_3 = -\frac{\lambda}{8\mu^3 M_Z^4} < 0$$

- $\omega(E) \geq 0 \to \lambda \geq 0$:

$$\omega(E) = \theta(E) \frac{\lambda k/\pi}{|\lambda + (\beta - ik)(E - M_Z)|^2}$$

Constant contact term plus one s-channel bare-pole exchange picture collapses for $\lambda < 0$

14.- Conclusions

- A new perspective on compositeness based on the number operators
- Amenable to calculations employing QFT
- New equation of compositeness for NR systems
- It can be also extended to relativistic systems
- Generalization to resonances
- Phase-factor transformations
- S-matrix transformations
- Universal criterion for a relativistic or non-relativistic bound state to be qualified as elementary
- Necessary condition for a resonance to be elementary
- More work is needed for finite-width resonances.
- CDD & including bare state explicitly

Other methods to study the nature of resonances

- Study of form factors and determination of the corresponding quadratic radius Sekihara,Hyodo,Jido,PRC83,055202(2011); Albaladejo,Oller,PRD86,034003(2012)
- Pole counting rule Morgan NPA543,632(1992). Presence/absence of nearby CDD poles Kang,Oller, EPJC77,399(2017)

Other methods to study the nature of resonances

- Study of form factors and determination of the corresponding quadratic radius Sekihara,Hyodo,Jido,PRC83,055202(2011); Albaladejo,Oller,PRD86,034003(2012)
- Pole counting rule Morgan NPA543,632(1992). Presence/absence of nearby CDD poles Kang,Oller, EPJC77,399(2017)
- Evolution of the pole positions with the increase in the number of color of QCD. E.g. for a $q\bar{q}$ $M = \mathcal{O}(N_C^0)$ and $\Gamma = \mathcal{O}(N_c^{-1})$. Pioneer works Oset,Oller,PRD60,074023(1999); Peláez,PRL92,102001(2004); Hyodo,Jido,Hosaka, PRL97,192002(2006)
- Regge trajectories Londergan,Nebreda,Peláez,Szczepaniak,PLB729,9(2014)
- Dependence on the mass under quark mass variations. Lattice QCD. Ruiz de Elvira,Meißner,Rusetsky,Schierholz,arXiv:1706.09015
- Compare predictions within specific models with experiment,e.g. spectrum, decay properties, etc

Name	$\sqrt{s_P}$ [MeV]	$X^R_{\pi\pi}$	$X^R_{\bar{K}K}$	$X^R_{\eta\eta}$	$X^R_{\eta\eta'}$	X^R
$f_0(500)$	$442^{+4}_{-4} - i246^{+7}_{-5}$	$0.40^{+0.01}_{-0.01}$	$\cdots$	$\cdots$	$\cdots$	$0.40^{+0.01}_{-0.01}$
$f_0(980)$	$978^{+17}_{-11} - i29^{+9}_{-11}$	$0.02^{+0.01}_{-0.01}$	$0.65^{+0.10}_{-0.16}$	$\cdots$	$\cdots$	$0.67^{+0.11}_{-0.17}$
$f_0(1710)$	$1690^{+20}_{-20} - i110^{+20}_{-20}$	$0.00^{+0.00}_{-0.00}$	$0.03^{+0.02}_{-0.02}$	$0.02^{+0.03}_{-0.03}$	$0.25^{+0.16}_{-0.16}$	$0.30^{+0.17}_{-0.17}$
$\rho(770)$	$760^{+7}_{-5} - i71^{+4}_{-5}$	$0.08^{+0.01}_{-0.01}$	$\cdots$	$\cdots$	$\cdots$	$0.08^{+0.01}_{-0.01}$
		$X^R_{K\pi}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$K_0^*(800)$	$643^{+75}_{-30} - i303^{+25}_{-75}$	$0.94^{+0.19}_{-0.39}$	$\cdots$	$\cdots$	$\cdots$	$0.94^{+0.19}_{-0.39}$
$K^*(892)$	$892^{+5}_{-7} - i25^{+2}_{-2}$	$0.05^{+0.01}_{-0.01}$	$\cdots$	$\cdots$	$\cdots$	$0.05^{+0.01}_{-0.01}$
		$X^R_{\pi\eta}$	$X^R_{\bar{K}K}$	$X^R_{\pi\eta'}$	$\cdots$	
$a_0(1450)$	$1459^{+70}_{-95} - i174^{+110}_{-100}$	$0.09^{+0.02}_{-0.07}$	$0.02^{+0.12}_{-0.02}$	$0.12^{+0.21}_{-0.08}$	$\cdots$	$0.23^{+0.35}_{-0.17}$
		$X^R_{\rho\pi}$				
$a_1(1260)$	$1260 - i250$	0.45				0.45
Hyperon with $I=0$		$X^R_{\pi\Sigma}$	$X^R_{\bar{K}N}$	$\cdots$	$\cdots$	$\cdots$
$\Lambda(1405)$ broad	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$0.73^{+0.16}_{-0.07}$	$\cdots$	$\cdots$	$\cdots$	$0.73^{+0.16}_{-0.07}$
$\Lambda(1405)$ narrow	$1421^{+3}_{-2} - i\,19^{+8}_{-5}$	$0.18^{+0.15}_{-0.06}$	$0.81^{+0.18}_{-0.08}$	$\cdots$	$\cdots$	$0.99^{+0.33}_{-0.14}$
Hyperon with $I=1$		$X^R_{\pi\Lambda}$	$X^R_{\pi\Sigma}$	$X^R_{\bar{K}N}$	$\cdots$	$\cdots$
	$1376^{+3}_{-3} - i\,33^{+5}_{-5}$	$0.04^{+0.01}_{-0.00}$	$0.0^{+0.0}_{-0.0}$	$\cdots$	$\cdots$	$0.04^{+0.01}_{-0.00}$
	$1414^{+2}_{-3} - i\,12^{+1}_{-2}$	$0.03^{+0.00}_{-0.00}$	$0.01^{+0.00}_{-0.00}$	$0.13^{+0.03}_{-0.03}$	$\cdots$	$0.17^{+0.03}_{-0.03}$
		X^R_{DK}	$X^R_{D_s\eta}$	$X^R_{D_s\eta'}$		
$D^*_{s0}(2317)$	2321^{+6}_{-3}	$0.57^{+0.01}_{-0.01}$	$0.12^{+0.01}_{-0.01}$	$0.02^{+0.01}_{-0.01}$	$\cdots$	$0.71^{+0.03}_{-0.03}$
		$X^R_{J/\psi f_0(500)}$	$X^R_{J/\psi f_0(980)}$	$X^R_{Z_c(3900)\pi}$	$X^R_{\omega\chi_{c0}}$	$\cdots$
$Y(4260)$	$4232.8 - i36.3$	0.00	0.02	0.02	0.17	0.21
		$X^R_{\Sigma_c^+\pi^0}$	$X^R_{\Sigma_c^{++}\pi^-}$	$X^R_{\Sigma_c^0\pi^+}$	$\cdots$	$\cdots$
$\Lambda_c(2595)$	$2592.25 - i1.3$	$0.11^{+0.02}_{-0.02}$	$\cdots$	$\cdots$	$\cdots$	$0.11^{+0.02}_{-0.02}$

Table: Z.H.Guo, Oller, PRD93,096001(2016)