Structure of hadron resonance with nearby CDD zero

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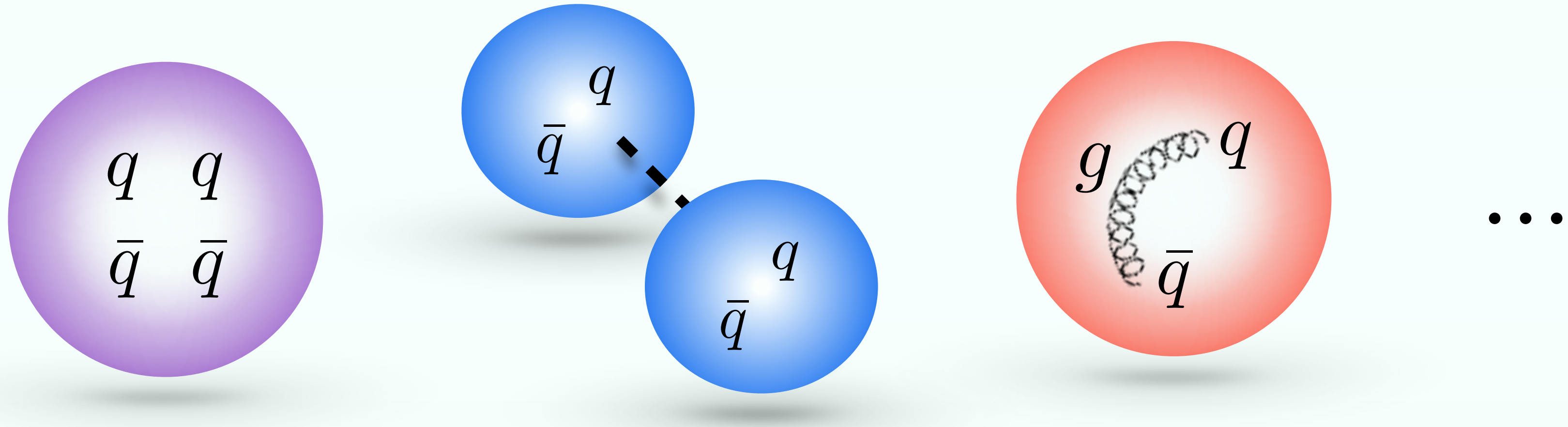
Introduction ~exotic hadrons~

- Exotic hadrons

  Hadrons which do not agree with the predictions of the quark model (qqbar, qqq).

  More complicated internal structure can be expected.

  - tetra quark, penta quark
  - hadron molecule ···



It is important to reveal the internal structure of exotics because we can acquire knowledge of strong interaction in the hadrons!

Introduction ~Methods to study structure~

- Methods to study internal structure from the scattering amplitude
  - Pole counting method

    Qualitative judgement from the positions of the shadow poles.

    D. Morgan, Nucl. Phys. A543, 632 (1992)

    Does a shadow pole lie around the pole representing the eigenenergy?

    Yes —> The focused channel is not the origin of the eigenstate.

    No —> The focused channel is the origin of the eigenstate.

  - Evaluation of compositeness

    Quantitative indicator of the amount of the dynamical fraction of the internal structure

    S. Weinberg, Phys. Rev. 137, B672 (1965).

    - Determination with the eigenenergy and residue of the pole

      T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C 85, 015201 (2012)

      F. Aceti and E. Oset, Phys. Rev. D 86, 014012 (2012)

      T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015)

      Z.-H. Guo and J. A. Oller, Phys. Rev. D 93, 096001 (2016)

    - Determination with Weak-binding relation

      S. Weinberg, Phys. Rev. 137, B672 (1965).

      Y. Kamiya and T. Hyodo, Phys. Rev. C 93, 035203 (2016)

      Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017)

Introduction ~CDD zero~

Castillejo Dalitz Dyson (CDD) Zero

- Defined as the energy point where the scattering amplitude F(E) vanishes.

CDD zero : $\mathcal{F}_{ii}(E_C) = 0$

L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).

- For a coupled-channel problem, both the existence and position depend on the channel.

c. f. The eigenstate pole lies the same position in the every coupled channel.

- Existence indicates the contribution from outside the model space.

G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

- Construction of analytic method including CDD contribution.

J. A. Oller and E. Oset, Phys. Rev. D60, 074023 (1999)

- CDD zero on $\pi\Sigma$c amplitude

CDD zero accompanied by nearby $\pi\Sigma$c thresholds performs the crucial role to reproduce the mass and width of Λc(2595).

Z.-H. Guo and J. A. Oller, Phys. Rev. D93, 054014 (2016), 1601.00862.

→ Can we extract information of the internal structure of the eigenstate from the position of the CDD zero?

Contents

- Introduction
- Zero coupling limit of coupled channel amplitude
- Origin of eigenstate and nearby CDD zero
- Application to $\Lambda(1405)$
- Conclusion

ZCL of coupled channel amplitude

To investigate the origins of the eigenstate,
we consider the zero coupling limit of the coupled channel scattering amplitude.

Zero Coupling Limit (ZCL)

Switch off the inter-channel coupling in V_{ij}

$$V_{ij} = \begin{pmatrix} V_{11} & V_{12} & \cdots & V_{n1} \\ V_{12} & V_{22} & \cdots & V_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{nn} \end{pmatrix} \to \begin{pmatrix} V_{11} & 0 & \cdots & 0 \\ 0 & V_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{nn} \end{pmatrix}$$

Poles and CDD zeros in the ZCL

In the ZCL, the pole exits only in the scattering amplitude of the channel whose interaction is the origin of the eigenstate.

(1) Interaction V_{ii} is the origin of the state. —> The pole remains in F_{ii}.

(2) Interaction V_{ii} is not the origin of the state. —> The pole decouples from F_{ii}.

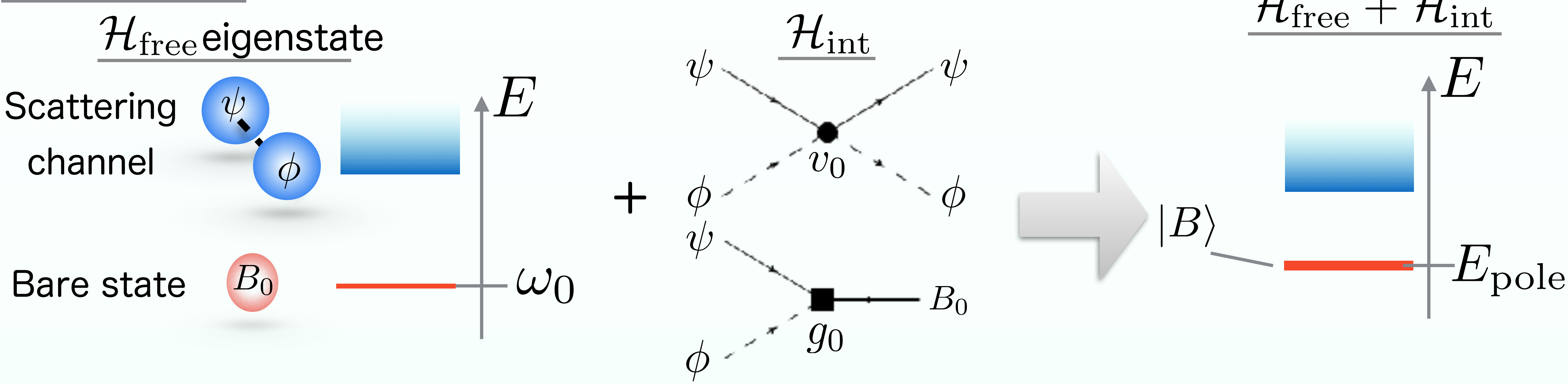
- How does the eigenstate pole decouples from F_{ii}?
- How about the behavior of CDD zero?

—> Let us examine their behavior with a specific model.

ZCL of coupled channel amplitude

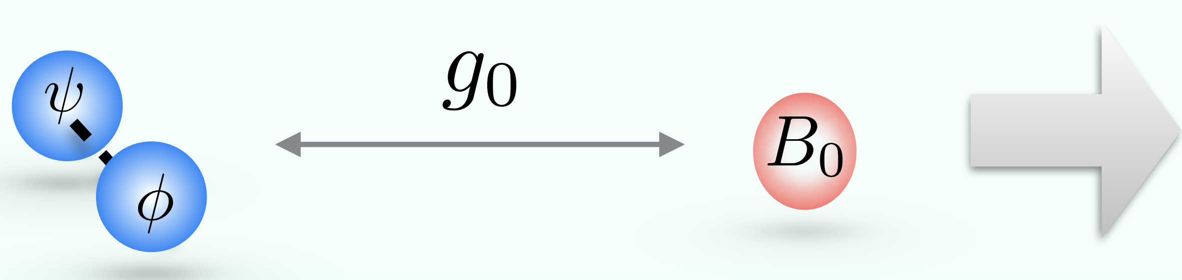
Using non-relativistic field theory as a specific example,
we study the behavior of the pole and CDD zero in the ZCL

Hamiltonian



Zero coupling limit

g_0 gives the coupling between the scattering channel and bare state channel.

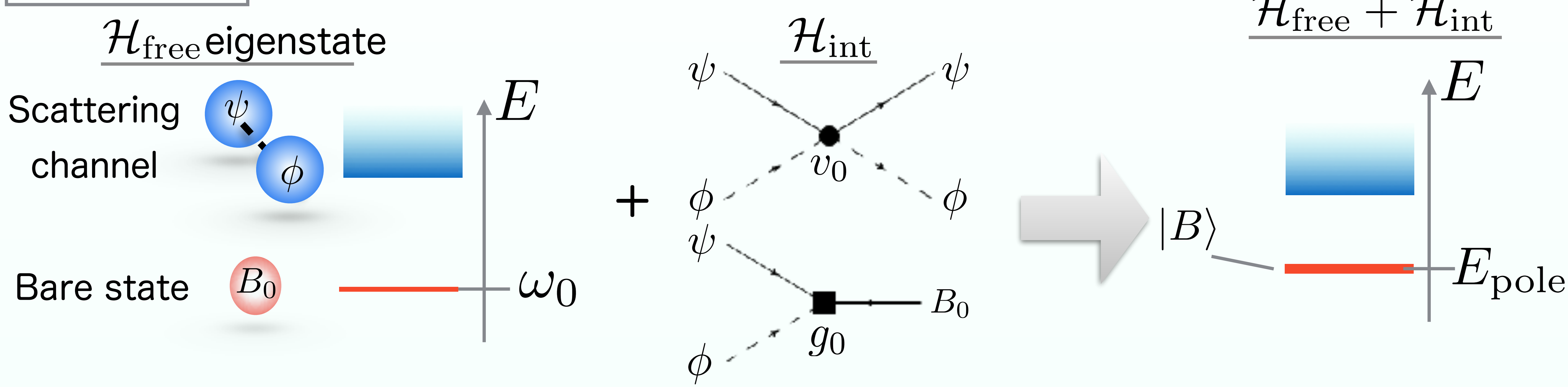


ZCL : $g_0 \to 0$

ZCL of coupled channel amplitude

Using non-relativistic field theory as a specific example,
we study the behavior of the pole and CDD zero in the ZCL

Hamiltonian



T matrix : t(E)

Lippmann-Schwinger Eq. —>

$$t(E) = \frac{(E-\omega_0)v_0 + g_0^2}{(E-\omega_0)(1-v_0 G(E)) - g_0^2 G(E)}$$

Numerator $= 0$ → CDD zero : $E_C = \omega_0 - g_0^2/v_0$

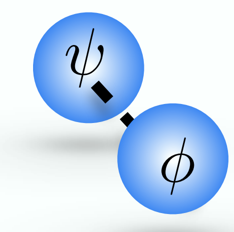
Denominator $= 0$ →

Pole : $(E_{\rm pole} - \omega_0)(1 - v_0 G(E_{\rm pole})) - g_0^2 G(E_{\rm pole}) = 0$

ZCL of coupled channel amplitude

Pole : $(E_{\rm pole} - \omega_0)(1 - v_0 G(E_{\rm pole})) - g_0^2 G(E_{\rm pole}) = 0$

CDD zero : $E_C = \omega_0 - g_0^2 / v_0$



Scenario 1

If the interaction of the scattering channel $\psi\,\phi$ is the origin of the eigenstate,

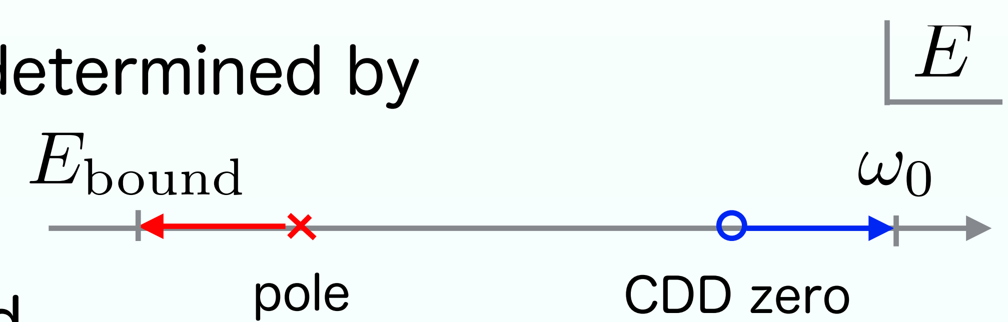
↔ the eigenenergy of the state in the ZCL (g_0 —> 0) does not depend on the bare energy: $E_{\rm pole} \not\to \omega_0$

- Pole

  moves toward the binding energy $E_{\rm bound}$ determined by

  $$1 - v_0 G(E_{\rm bound}) = 0$$

  ↔ The eigenstate is dynamically generated.



- CDD zero

  $E_C \to \omega_0$ (The movement is independent of $E_{\rm bound}$)

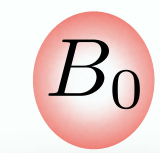
→ The position of CDD zero is independent of that of the pole.

ZCL of coupled channel amplitude

Pole : $(E_{\rm pole}-\omega_0)(1-v_0G(E_{\rm pole}))-g_0^2G(E_{\rm pole})=0$

CDD zero: $E_C=\omega_0-g_0^2/v_0$

Scenario 2



If the bare state B_0 is the origin of the eigenstate,

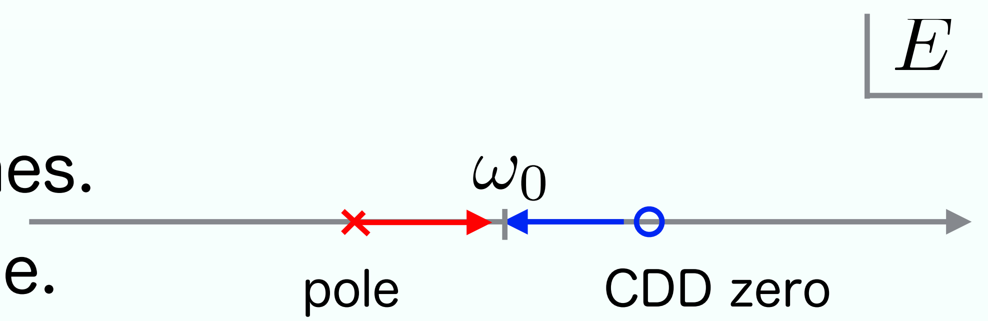
⇔ the pole moves toward the bare state energy ω_0 in the ZCL (g_0—>0) and vanishes in the exact ZCL (g_0=0).

- Pole

  $E_{\rm pole}\to\omega_0$

  In this limit, the residue of the pole vanishes.

  —> The pole decouples from the amplitude.



- CDD zero

  $E_C\to\omega_0$ (The behavior is the same as the Scenario 1)

The pole and CDD zero encounter with each other at E=ω_0.

⇒ The pole and CDD zero cancels out with each other to decouple from the scattering amplitude.

Origin of eigenstate and nearby CDD zero

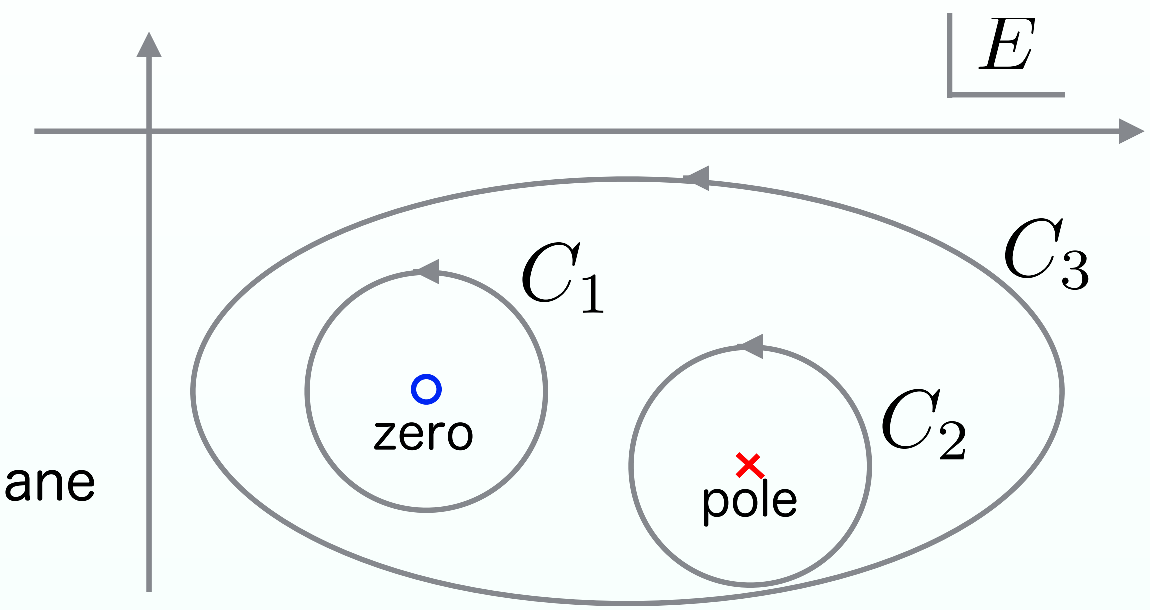
More general consideration on the behavior of poles and CDD zeros in the ZCL

- Principle of argument of scattering amplitude

$$n_C = \frac{1}{2\pi}\oint_C dz \frac{d}{dz} \arg \mathcal{F}(z)$$

F(z) : Partial-wave scattering amplitude

C : Closed integration path in the complex energy plane (No poles and zeros lie on Path C)



$$\bullet\ n_C = (\#\text{ of CDD zeros in } \mathcal{C}) - (\#\text{ of poles in } \mathcal{C}) \in \mathbb{Z}$$

$n_{C_1} = 1$

$n_{C_2} = -1$

$n_{C_3} = 0$

- Topological invariant ($\pi_1(U(1)) \cong \mathbb{Z}$)

—> nc is invariant under the continuous variation of amplitude

・Sudden vanishment of a pole or zero (nc : ±1 —> 0) is prohibited in the change.

・The pair annihilation of a pole and a CDD zeros does not change nc.

Pole and CDD zero must encounter with each other to decouple from the scattering amplitude.

Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

  Interaction of channel i V_{ii} is origin of the state —> Pole remains in F_{ii}.

  Otherwise —> Pole decouples from F_{ii}.

- To decouple from the amplitude $F_{ii}(E)$, pole must meet CDD zero.
- Pole and CDD zero move continuously in the continuous change of amplitude.

Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

  Interaction of channel i V_{ii} is origin of the state —> Pole remains in F_{ii}.

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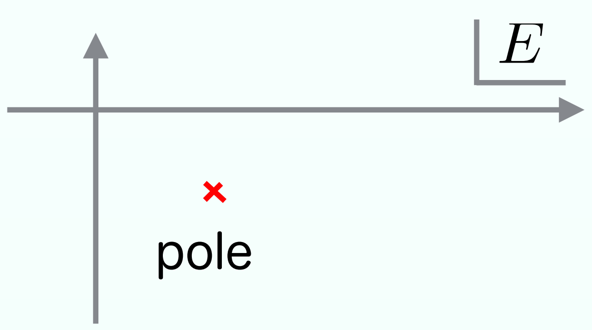
- To decouple from the amplitude $F_{ii}(E)$, pole must meet CDD zero.

- Pole and CDD zero move continuously in the continuous change of amplitude.

⇒ We can study the origin of the eigenstate from the position of poles and CDD zeros on the complex energy plane!

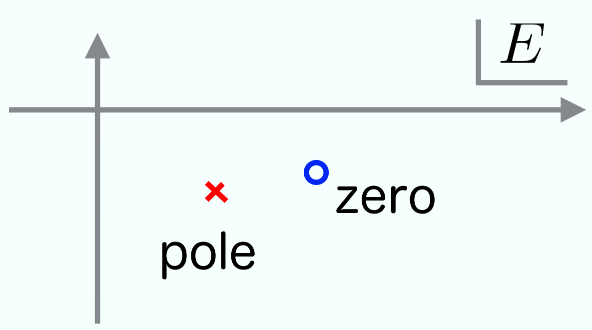
Before taking the ZCL…

(1) CDD zero does not lie around the pole.



⇒ The eigenstate is dynamically generated in channel i.

(2) pole has a nearby CDD zero.



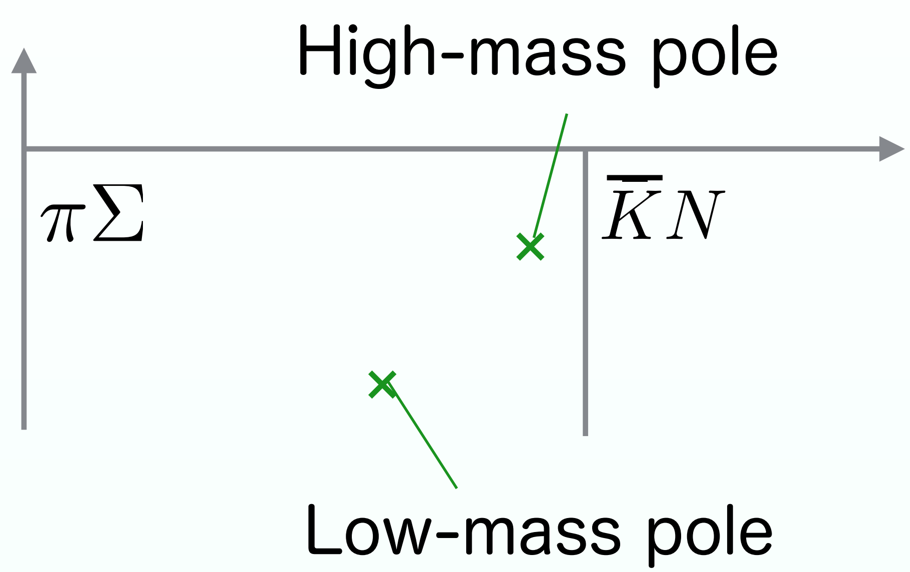
⇒ The origin of the eigenstate is not in channel i.

Application to Λ(1405)

- $\Lambda(1405)$ ($I=0$ $\bar{K}N$ scattering)
  - $J^P = \frac{1}{2}^-$
  - Analysis with chiral dynamics

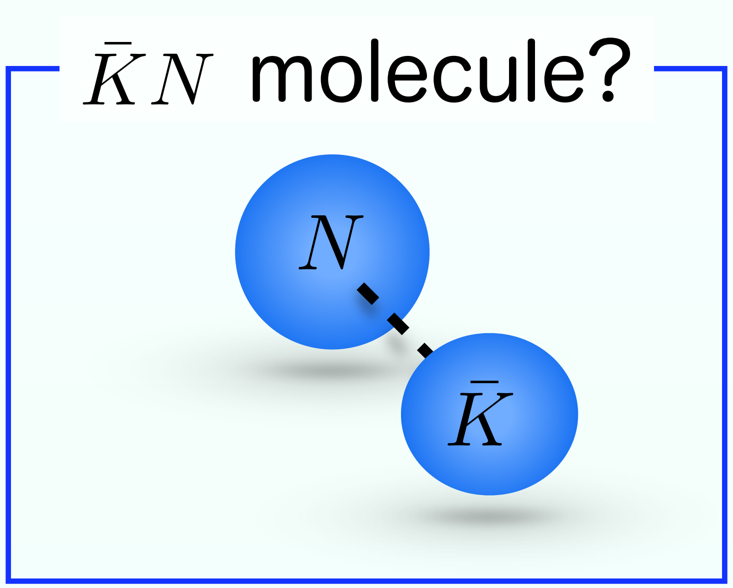
    —> Two pole structure

    D. Jido et al Nucl. Phys. A 725, 181 (2003)

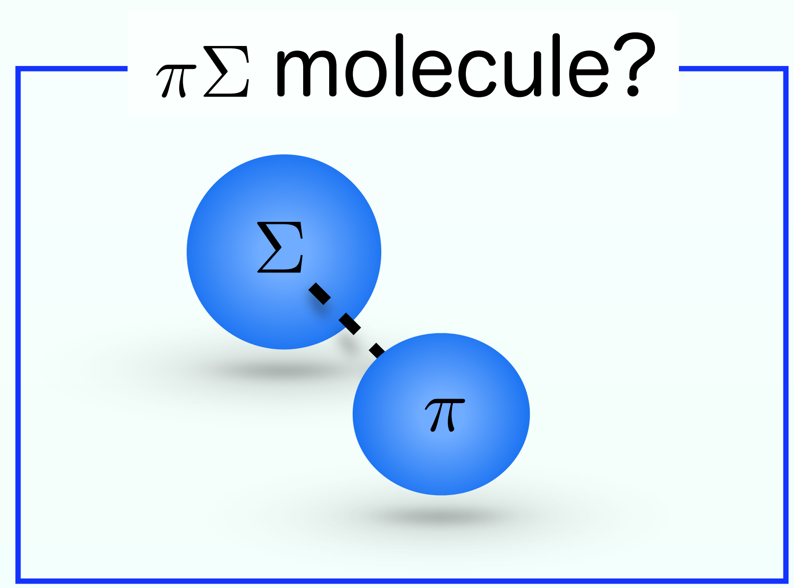


Recent determinations are tabulated in PDG

C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016)



or



Application to Λ(1405)

Effective Tomozawa-Weinberg model

Y. Ikeda, T. Hyodo, and W. Weise, Nucl. Phys. A881, 98 (2012)

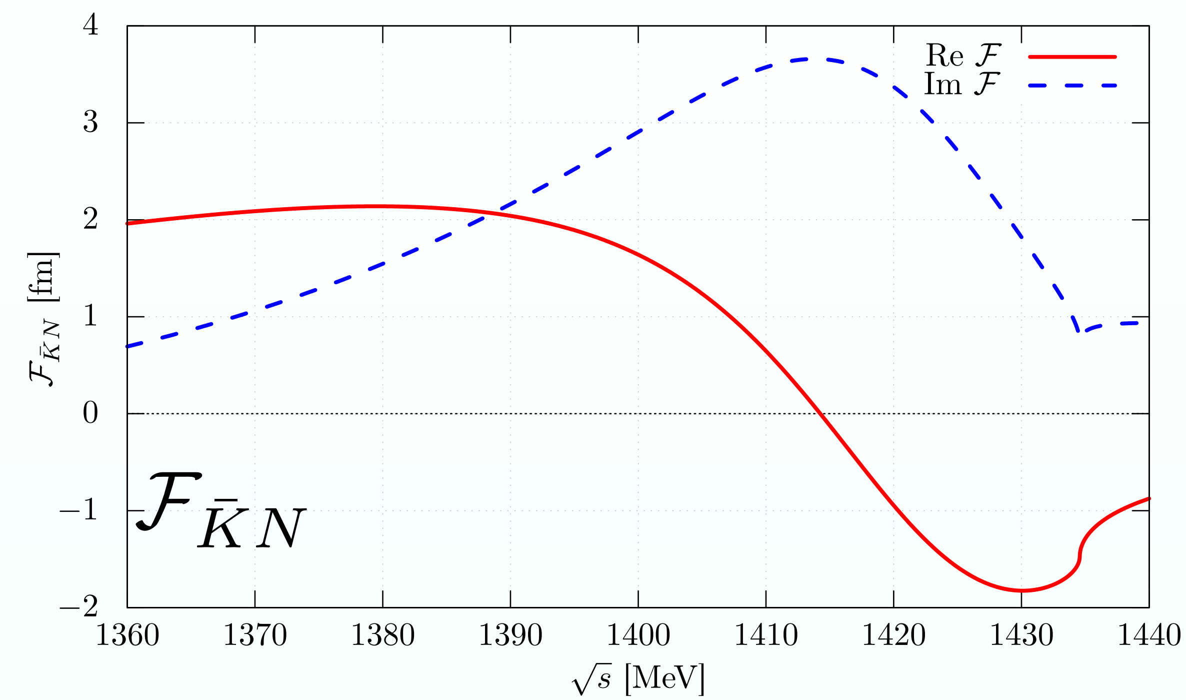
- Isospin basis

  Coupled channel : $\bar{K}N$-$\pi\Sigma$

- interaction : Tomozawa-Weinberg interaction
- Pole position;

  High-mass pole ; $1423 - 22i$ MeV

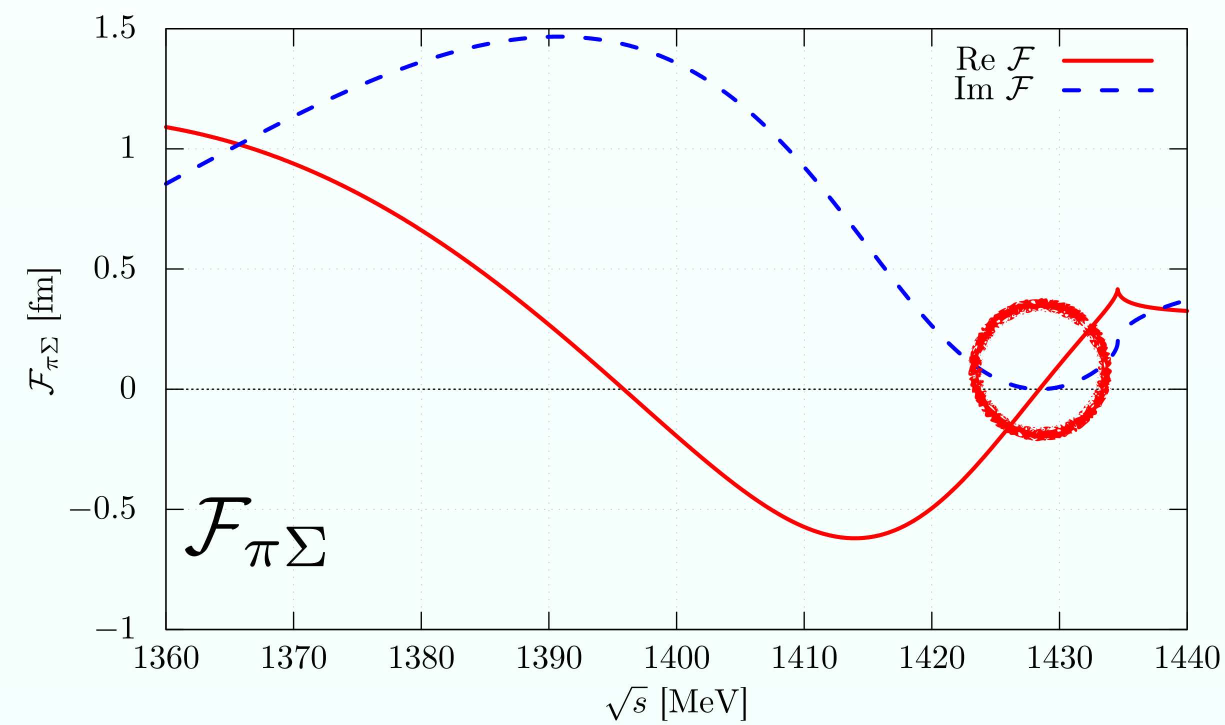
  Low-mass pole ; $1375 - 65i$ MeV



$\mathcal{F}_{\pi\Sigma}$

CDD zero lies near high-mass pole.

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017)



Application to Λ(1405)

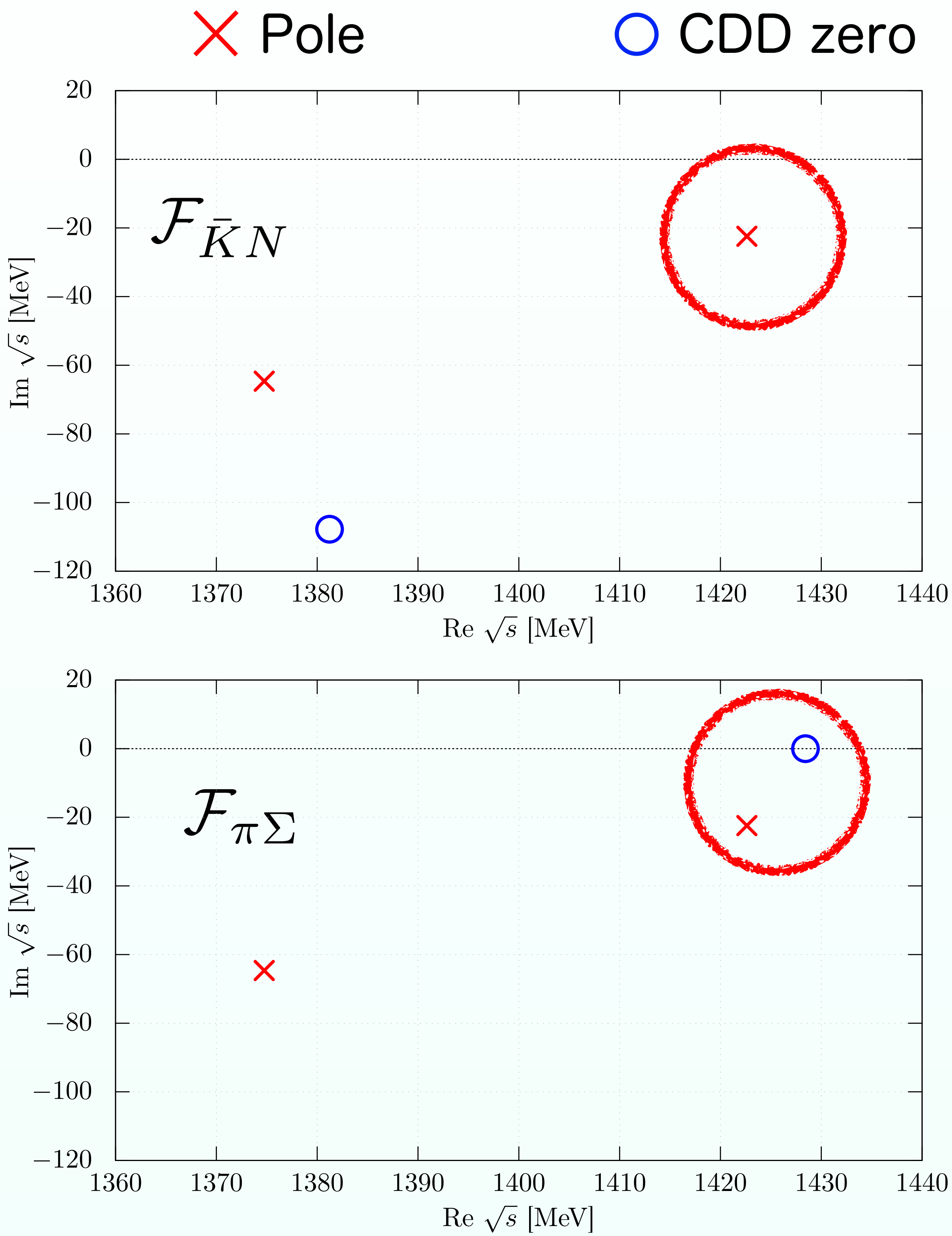
Position of poles and CDD zeros

High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

⇨ Origin is in $\bar{K}N$ channel.



Application to Λ(1405)

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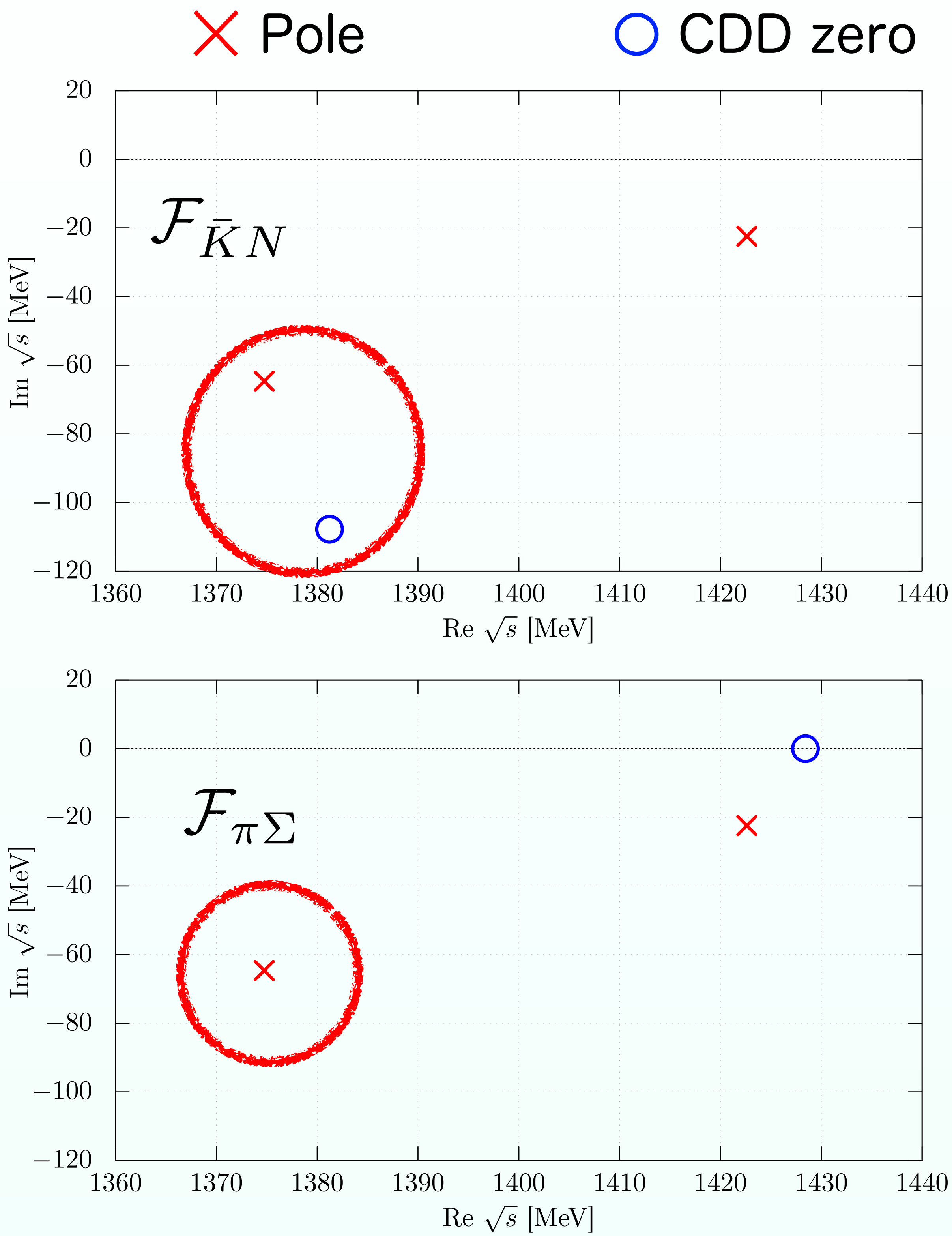


High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

➡ Origin is in $\bar{K}N$ channel.

Low-mass pole

Nearby CDD zero in $\bar{K}N$ amplitude

No Nearby CDD zero in $\pi\Sigma$ amplitude

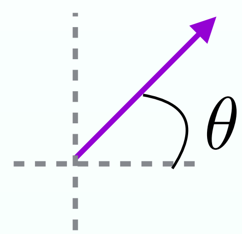
➡ Origin is in $\pi\Sigma$ channel.

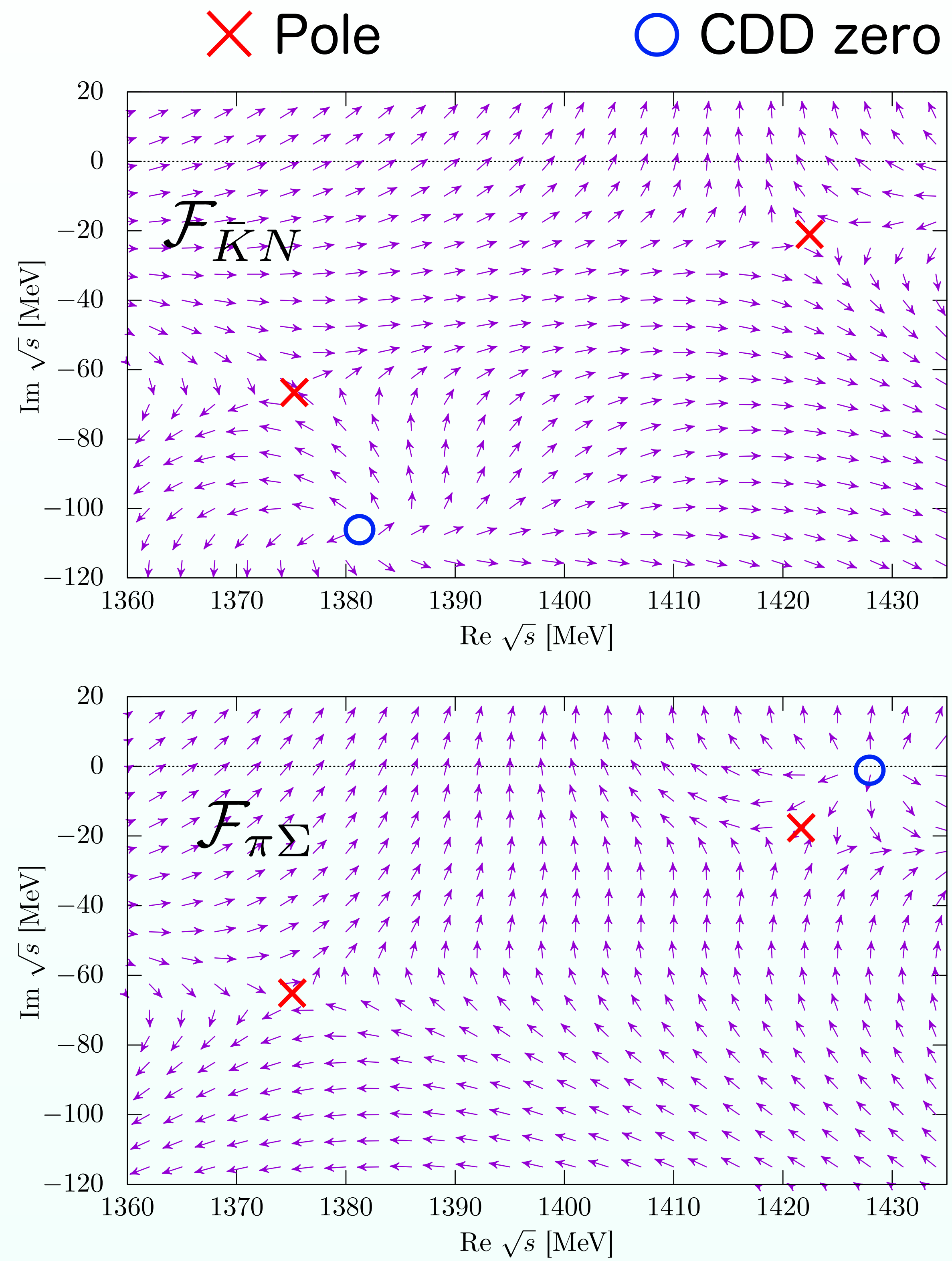
Application to Λ(1405)

Vector map of phase structure of amplitude

The existence of the pole and CDD zero can be confirmed by the phase structure of amplitude.

$$\mathcal{F}(\sqrt{s}) = |\mathcal{F}|e^{i\theta}$$





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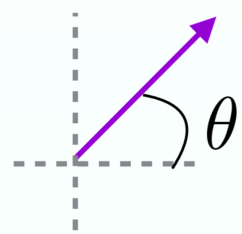
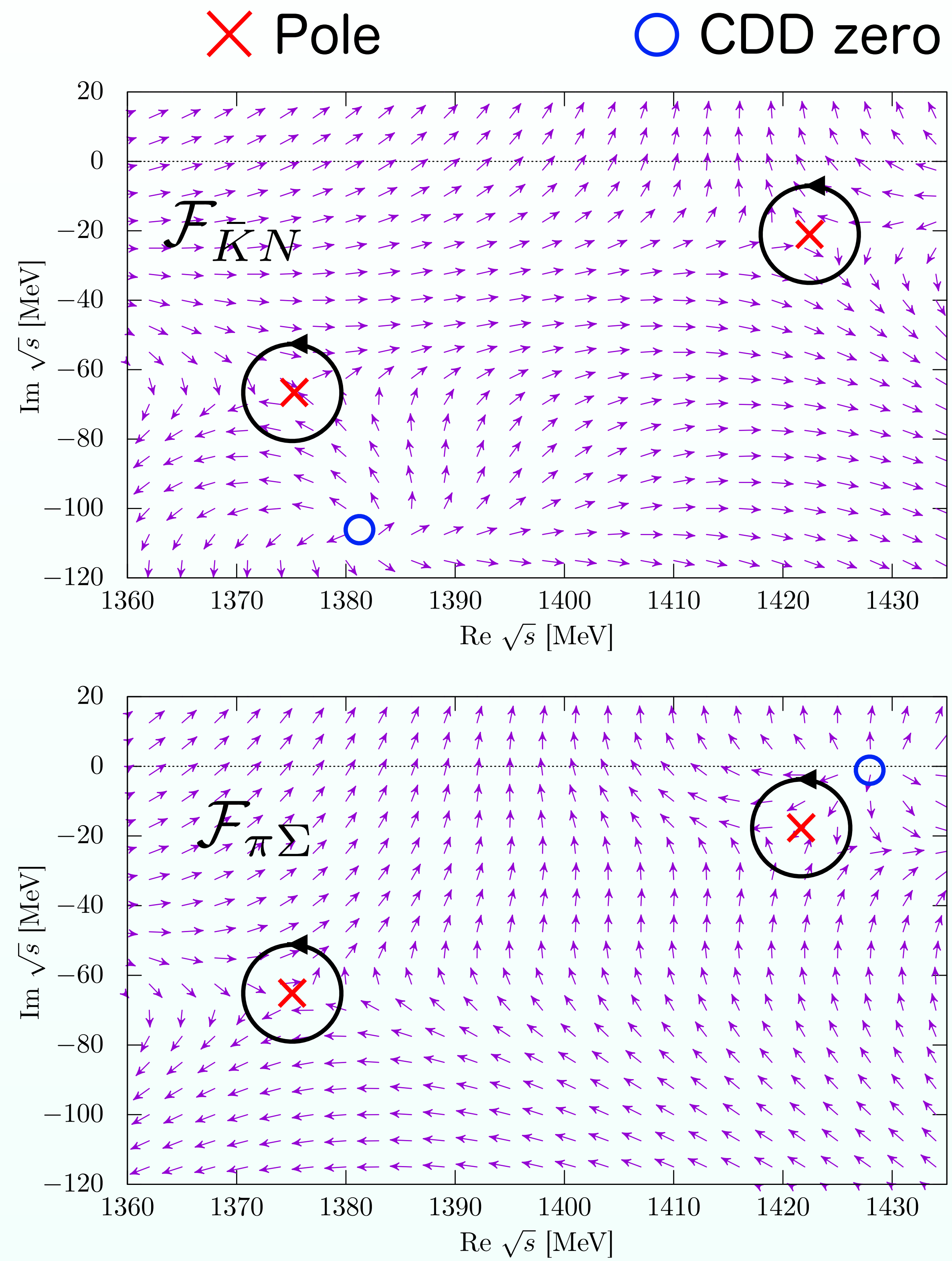


Along contour around pole

The vector of phase turns clockwise.



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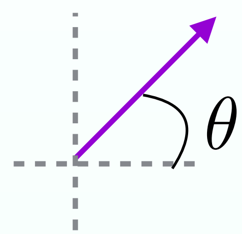
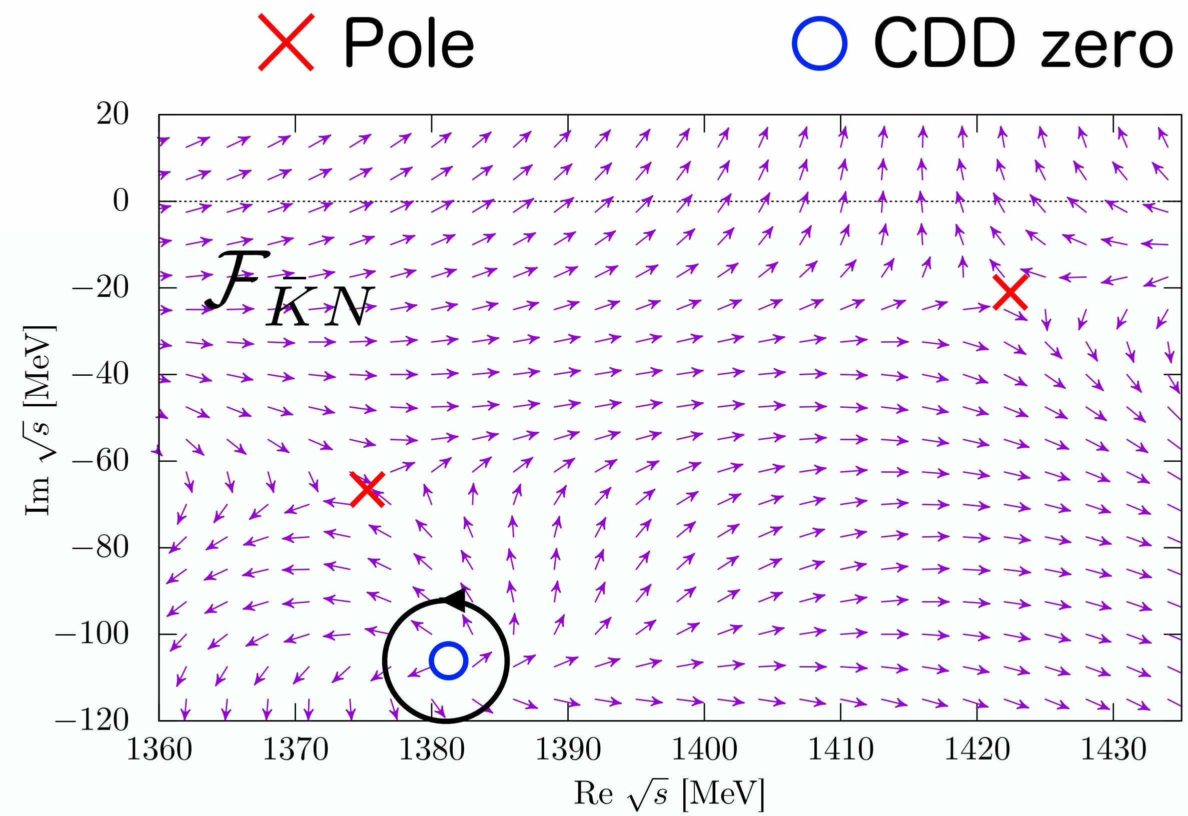


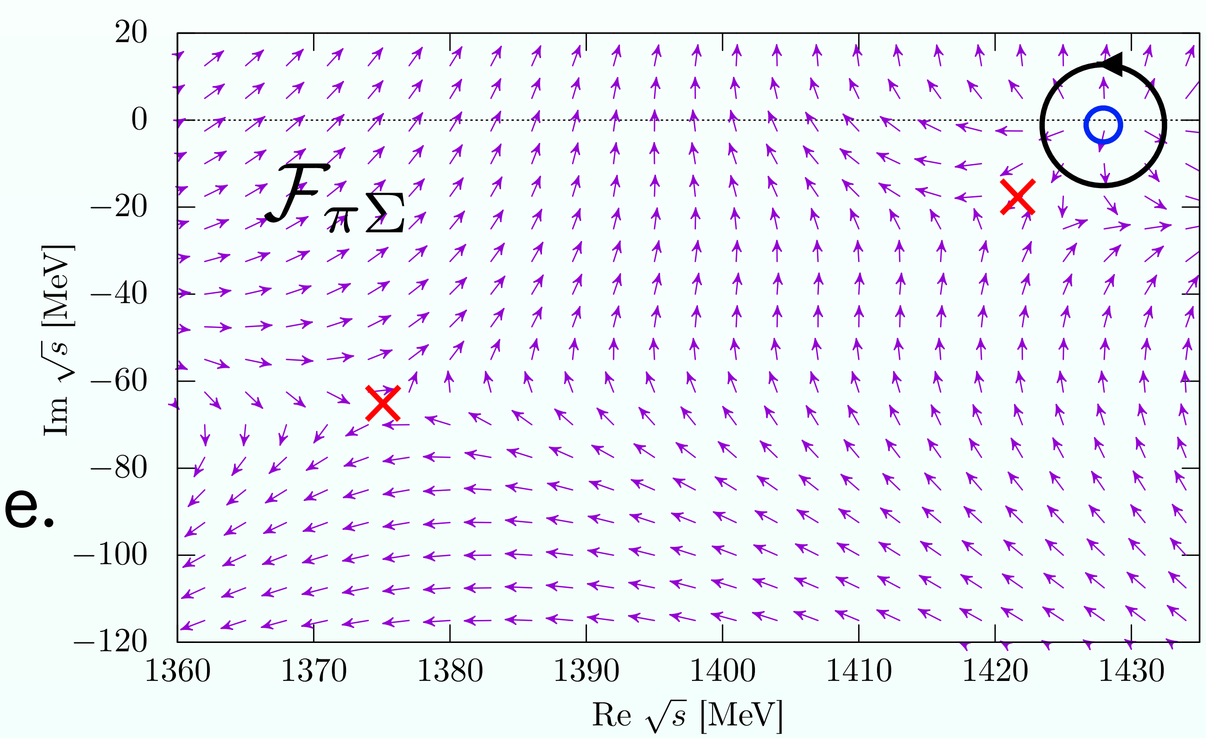


Along contour around pole

The vector of phase turns clockwise.

Along contour around CDD zero

The vector of phase turns counterclockwise.



Application to Λ(1405)

Behavior of poles and CDD zeros in the ZCL

We gradually switch off the $\bar{K}N$-$\pi\Sigma$ coupling.

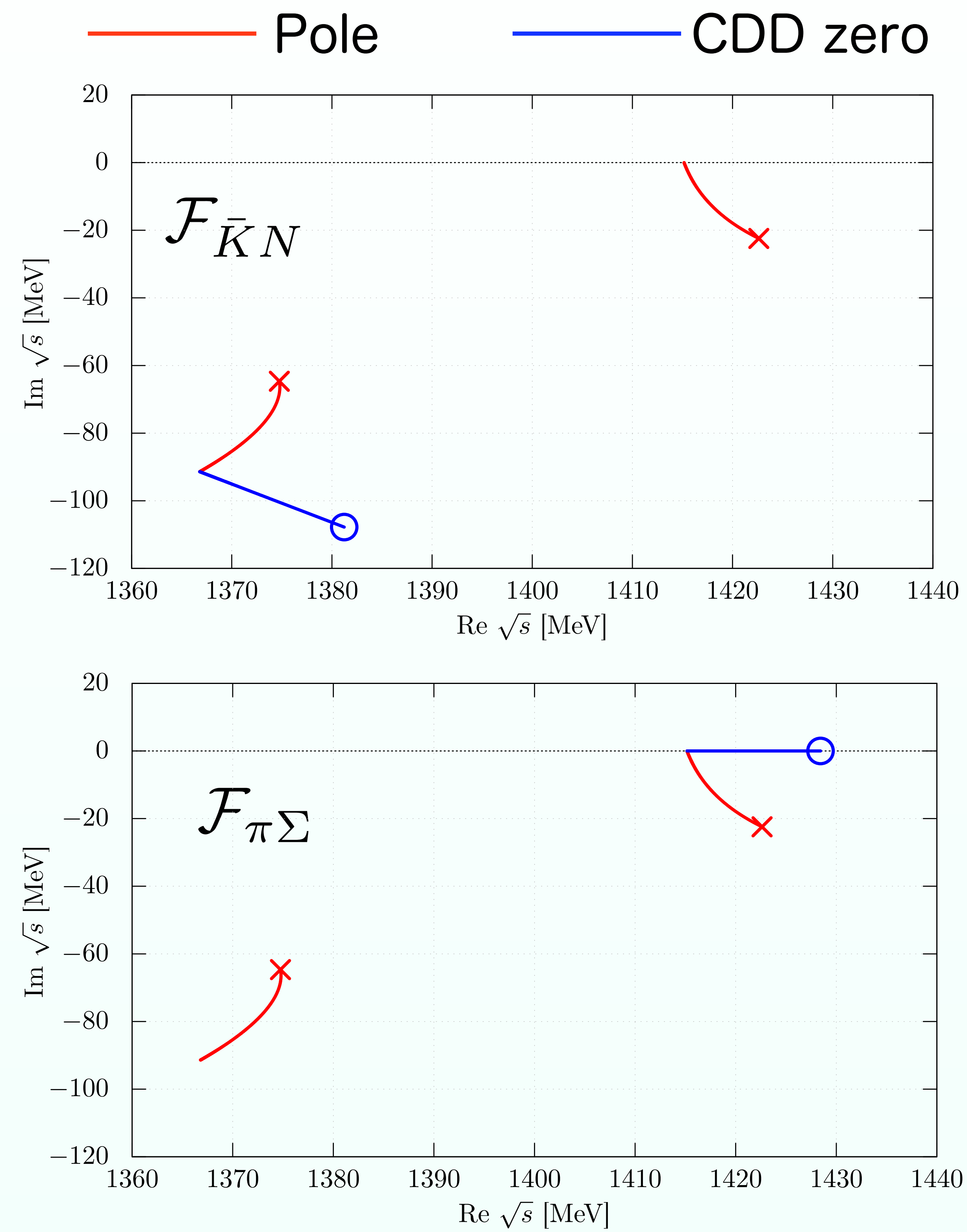
High-mass pole

- remains as a bound state in $\bar{K}N$ amplitude.
- encounters CDD zero and decouples from $\pi\Sigma$ amplitude.

Low-mass pole

- encounters CDD zero and decouples from $\bar{K}N$ amplitude.
- remains as resonance in $\pi\Sigma$ amplitude

⇒ The behavior of poles in the ZCL is consistent with the results from the positions of poles and CDD zeros.



Conclusion

- The eigenstate pole should decouple from the amplitude in the ZCL, if the eigenstate originates in the other channel.
- We show that the pole must annihilate with CDD zero to decouple.
- New method to study the origin of the eigenstate ;

(1) Pole without a nearby CDD zero —> Dynamical origin.

(2) Pole with a nearby CDD zero —> Origin is the other channel.

- Application to Λ(1405)