XVII International Conference on Hadron Spectroscopy and Structure

Mass spectra of triply beauty Ω_{bbb} baryon

Zalak U. Shah

Department of Applied Physics,
S. V. National Institute of Technology, Surat, Gujarat, India.

28-Sept-2017

Particle Physics is a study of **Matter** and **Force**. Everything in the universe is found to be made from a few basic building blocks called fundamental particles, governed by four fundamental forces. All of these are encapsulated in the Standard Model(SM).

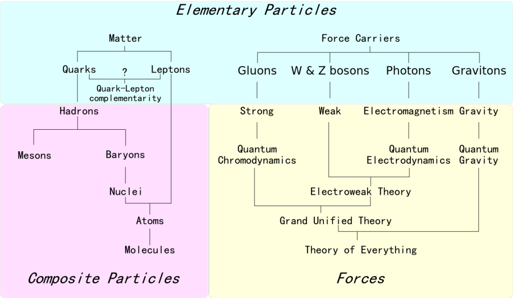


Figure : The Standard Model

Hadrons

The flavored quarks combine together in various aggregates called *hadrons*.
Hadrons are defined as strongly interacting composite particles made from quarks and/or gluons and bound together by their strong interactions.

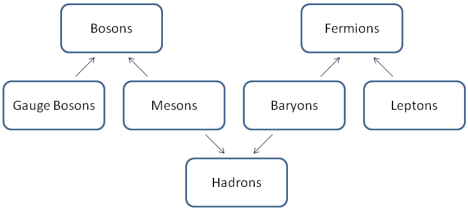


Figure : The classification of particles

Baryons

Baryons are composed of bound states of three quarks. The flavor wave functions of baryon states can then be constructed to be members of SU(3) multiplets as,

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A.$$

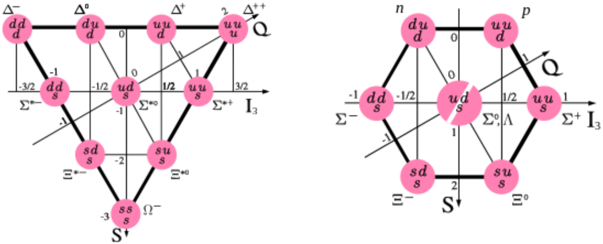


Figure : The symmetric 10 (left) and mixed symmetric 8 (right) of SU(3)

Heavy Baryons I

- The experiments like LHCb, B factories(BARBAR and BELLE) and CLEO have been the main source of producing the heavy baryons. The future experiments at J-PARC, PANDA and Belle II are also expected to give further information on heavy baryons in near future.
- Fortunately, all the ground states with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ of singly heavy charm and bottom baryons have been detected experimentally (except Ω_b^{*0}) [1].
- Many excited states of heavy baryons are also known so far. New states in heavy baryon spectroscopy are also observed by LHCb experiment recently [2, 3, 4]
- Traveling from light sector baryons to heavy sector baryons we need baryons with a heavy quark(s) combination. Any s quark(s) of hyperon baryons can be replaced by heavy quark (c, b) in heavy baryon particles.

Heavy Baryons II

The larger SU(4) group that includes all of the baryons containing zero, one, two or three heavy quarks. The corresponding multiplet structure for SU(4) is

$$4 \otimes 4 \otimes 4 = 20_S \oplus 20_M \oplus 20_M \oplus 4_A$$

Within the flavor SU(3) subgroups, the ground-state heavy baryons containing a single heavy quark belong either to a sextet of flavor symmetric states, or an antitriplet of flavor antisymmetric states, both of which sit on the second layer of the mixed-symmetric 20 of SU(4).

$$3 \otimes 3 = 6_s + \bar{3}_A$$

Triplet baryons are anti symmetric under the interchange of the light (u,d and s) quarks; thus, the product of spin and spatial wave functions must be symmetric while the product of spin and spatial wave functions of sextet baryons are required to be anti symmetric due to their symmetric flavor wave functions under the interchange of the two light quarks.

Heavy Baryons III

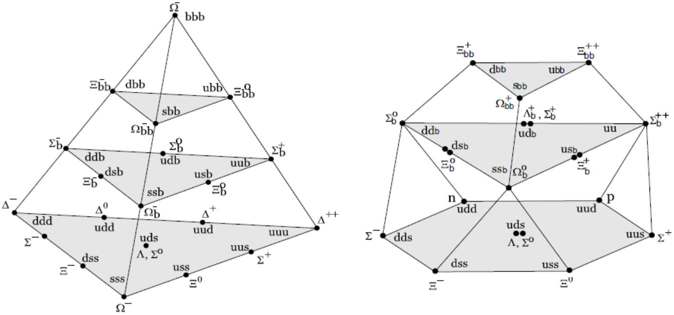


Figure : (a) The symmetric 20_s of SU(4). (b) The mixed-symmetric 20_M. 20_s contains the decuplet and 20_M have the SU(3) octet on the lowest layer.

Singly heavy Baryons:The singly heavy baryon is composed of a charm/beauty quark and two light quarks (u,d,s).

- Σ_c^{++}, Σ_c^{+}, Σ_c^{0}, Ξ_c^{+}, Ξ_c^{0}, Λ_c^{+}, Ω_c^{0}
  Z. Shah et al., Eur. Phys. J. A 52, 313 (2016); Chin. Phys. C 40, 123102 (2016)
- Σ_b^{+}, Σ_b^{-}, Σ_b^{0}, Ξ_b^{+}, Ξ_b^{-}, Λ_b^{-}, Ω_b^{-}
  K. Thakkar et al., Nucl. Phys. A 965,57 (2017)

Doubly heavy Baryons: The doubly heavy baryon is composed of two charm/beauty quarks and a light quarks (u,d,s).

- Ω_{cc}^{+}, Ω_{bb}^{-} and Ω_{bc}^{0} Z. Shah et al., Eur. Phys. J. C, **76**, 530 (2016).
- Ξ_{cc}^{+}, Ξ_{bb}^{-}, Ξ_{bc}^{0}, Ξ_{cc}^{++}, Ξ_{bb}^{0} and Ξ_{bc}^{+} Z. Shah et al., Eur. Phys. J. C, **77**, 129 (2017).

Triply heavy Baryons: The triply heavy baryon is composed of three heavy (charm/beauty) quarks.

- Ω_{ccc}, Ω_{bbb}, Ω_{bbc}, Ω_{ccb}

The Hypercentral Constituent Quark Model I

- The different theoretical approaches such as HQET, QCD sum rules, Regge phenomenology, Lattice QCD, Bag Model and various potential models provide the mass spectrum and decay properties in the field of heavy baryons.
- However, there is no harmony among the theoretical predictions of the properties like hyperfine splitting, the form factors, magnetic moments etc. All these reasons make the study of heavy flavor spectroscopy extremely important and interesting.
- Heavy baryons further provide excellent laboratory to understand the dynamics of light quarks in the vicinity of heavy flavor quarks.
- The chosen potential model is *Hypercentral Constituent Quark Model(hCQM)* .

- The basic idea of the hypercentral approach to three-body systems is very simple. The two relative coordinates are rewritten into a single six dimensional vector and the non-relativistic *Schrödinger* equation in the six dimensional space is solved. The potential expressed in terms of the hypercentral radial co-ordinate, takes care of the three body interactions effectively. A nice description of this model is also given in recent review article [5].

The relevant degrees of freedom for the relative motion of the three constituent quarks are provided by the relative Jacobi coordinates ($\vec{\rho}$ and $\vec{\lambda}$) which is given by [6, 7] as

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \qquad \vec{\lambda} = \frac{m_1\vec{r_1} + m_2\vec{r_2} - (m_1 + m_2)\vec{r_3}}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}} \tag{1}$$

The Hypercentral Constituent Quark Model III

Here m_i and $\vec{r}_i$ (i = 1, 2, 3) denote the mass and coordinate of the i-th constituent quark. The respective reduced masses are given by

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2} \qquad m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1 m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)} \tag{2}$$

The confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential strings increases linearly with a collective radius r_{3q}. Accordingly the effective two body interactions can be written with consideration of only first term which is hypercentral approximation

$$\sum_{i<j} V(r_{ij}) = V(x) + ... \tag{3}$$

The hyper radius x is a collective coordinate and therefore the hypercentral potential contains also the three-body effects.

The Hypercentral Constituent Quark Model IV

The Hamiltonian of three body baryonic system in the hCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x) \qquad (4)$$

where, $m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$, is the reduced mass .
The hyperradial Schrodinger equation reduces to,

$$\left[\frac{-1}{2m}\frac{d^2}{dx^2} + \frac{\frac{15}{4} + \gamma(\gamma+4)}{2mx^2} + V(x)\right]\phi_\gamma(x) = E\phi_\gamma(x) \qquad (5)$$

The Hypercentral Constituent Quark Model V

The hypercentral potential $V(x)$ [8, 9, 10]

$$V(x) = V^0(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right) V^{(1)}(x) + V_{SD}(x) \quad (6)$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x^\nu \quad \text{and} \quad V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4x^2} \quad (7)$$

$$V_{SD}(x) = V_{SS}(x)\left[S(S+1) - s_\rho(s_\rho+1) - \frac{3}{4}\right] \quad (8)$$

$$+V_{ŁS}(x)(\vec{L}\cdot\vec{S}) + V_T(x)\left[S(S+1) - \frac{3(\vec{S}\cdot\vec{x})(\vec{S}\cdot\vec{x})}{x^2}\right]$$

We solve six dimensional Schrodinger equation numerically using the Mathematica program. We employ the CPP_ν confinement potential for the quarks but extended it in the hypercentral co-ordinates (hCPP_ν).

The Hypercentral Constituent Quark Model VI

The spin-dependent part, $V_{SD}(x)$ contains three types of the interaction terms,

- the spin-spin term $V_{SS}(x)$
- the spin-orbit term $V_{LS}(x)$
- tensor term $V_T(x)$
- The hyper-Coulomb, $\tau = -\frac{2}{3}\alpha_s$
- The Casimir charges of the fundamental and adjoint representation are, $C_F = \frac{4}{3}$ and C_A =3
- m_c=1.275 GeV and m_b=4.67 GeV
- $\vec{S}$=$\vec{S}_\rho$+$\vec{S}_\lambda$ are spin vectors associated with the $\vec{\rho}$ and $\vec{\lambda}$ coordinates.

Triply Heavy Baryons

- The combination of three heavy charm /bottom quarks make them color-singlet bound states. Triply heavy baryons are yet to be observed by experiments and therefore they are not listed in PDG-2016 [1].
- LHCb experiment possibly detect Ω_{bbb}, Ω_{bbc} and Ω^{*}_{bbc} baryons at appropriate integrated luminosity and collision energy in near future. It is also expected from BES III and Belle II (at KEK) to collect the data and possibly provide some information on triply heavy baryons in charm-bottom sector.
- The baryons with three same heavy quarks are spin symmetrical states and thus, we have calculated the ground states and radial excited states (2S-5S) of Ω_{bbb} baryon for $J^P = \frac{3}{2}^+$.
- While for orbital excited states(P, D, F) of triply heavy baryons, S=$\frac{1}{2}$ states can also be considered along with S=$\frac{3}{2}$ states likewise to the light baryons decuplets.

Comparison of Ground state mass (for $J^P = \frac{3}{2}^+$)

Method	Refs.	Mass(GeV)
Hypercentral Model	Our work	14.496
non-relat. quark model	[11]	14.834
Fadeev Approach	[12]	14.276
Variational cornell	[13]	14.398
Regge Phenomology	[14, 15]	14.787
Sum rules	[16]	13.280
Bag Model	[17]	14.300
di-quark Model	[18]	14.370
Sum rule	[19, 20]	14.30
Sum rule	[21]	14.830
Relat. quark model	[22]	14.569
Lattice	[23]	14.366(31)
Lattice	[24]	14.371
Real. Fadeev approach	[25]	13.8
pNRQCD	[26]	14.5

Radial excited states of Ω^{-}_{bbb} baryon (in GeV).

State	(ν)	A	B
	0.5	15.021	15.024
2S	1.0	15.154	15.163
	1.5	15.266	15.276
	2.0	15.365	15.377
	0.5	15.344	15.350
3S	1.0	15.154	15.654
	1.5	15.897	15.918
	2.0	16.132	16.159
	0.5	15.652	15.661
4S	1.0	16.124	6.149
	1.5	16.560	16.592
	2.0	16.963	17.004
	0.5	15.948	15.960
5S	1.0	16.613	6.644
	1.5	17.245	17.289
	2.0	17.845	17.900

P states of Ω_{bbb}^{-} baryon

State	A	B	[14]	[11]	[19]	[27]
$(1^4P_{1/2})$	4.996	5.012		5.155		
$(1^4P_{3/2})$	4.976	4.991	5.073^{+109}_{-107}	5.160	4.90	5.11
$(1^4P_{5/2})$	4.950	4.965				
$(2^4P_{1/2})$	5.577	5.607				
$(2^4P_{3/2})$	5.554	5.584				
$(2^4P_{5/2})$	5.525	5.553				
$(3^4P_{1/2})$	6.153	6.201				
$(3^4P_{3/2})$	6.134	6.177				
$(3^4P_{5/2})$	6.104	6.145				
$(4^4P_{1/2})$	6.740	6.794				
$(4^4P_{3/2})$	6.715	6.770				
$(4^4P_{5/2})$	6.682	6.738				
$(5^4P_{1/2})$	7.321	7.392				
$(5^4P_{1/2})$	7.296	7.366				
$(5^4P_{5/2})$	7.262	7.330				

D states of Ω_{bbb}^{-} baryon

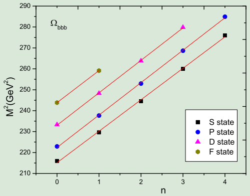
State	A	B	[15]	[11]	[28]	[24]
$(1^4D_{1/2})$	15.287	15.306				
$(1^4D_{3/2})$	15.281	15.300				
$(1^4D_{5/2})$	15.274	15.293		15.101		
$(1^4D_{7/2})$	15.267	15.286	15.318±123	15.101	14.960	14.969
$(2^4D_{1/2})$	15.771	15.797				
$(2^4D_{3/2})$	15.765	15.791				
$(2^4D_{5/2})$	15.758	15.784				
$(2^4D_{7/2})$	15.750	15.776				
$(3^4D_{1/2})$	16.255	16.289				
$(3^4D_{3/2})$	16.250	16.284				
$(3^4D_{5/2})$	16.243	16.276				
$(3^4D_{7/2})$	16.234	16.268				
$(4^4D_{1/2})$	16.741	16.782				
$(4^4D_{3/2})$	16.735	16.776				
$(4^4D_{5/2})$	16.728	16.769				
$(4^4D_{7/2})$	16.720	16.760				

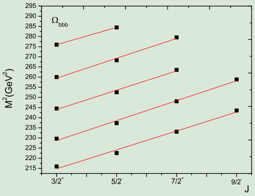
F states of Ω_{bbb}^{-} baryon

State	A	B	[15]
$(1^2F_{5/2})$	5.822	5.862	
$(1^2F_{7/2})$	5.777	5.815	
$(1^4F_{3/2})$	5.871	5.915	
$(1^4F_{5/2})$	5.835	5.877	
$(1^4F_{7/2})$	5.790	5.829	5.520^{+156}_{-154}
$(1^4F_{9/2})$	5.737	5.772	
$(2^2F_{5/2})$	6.396	6.450	
$(2^2F_{7/2})$	6.350	6.402	
$(2^4F_{3/2})$	6.447	6.502	
$(2^4F_{5/2})$	6.410	6.464	
$(2^4F_{7/2})$	6.364	6.416	
$(2^4F_{9/2})$	6.309	6.359	

- Regge theory is concerned with the particle spectrum, the forces between the particles and the high energy behavior of the scattering amplitudes.
- One of the most distinctive features of Regge theory are the Regge trajectories; introduced by T. Regge in 1960s [29].
- Regge trajectories are directly related with mass spectrum of hadrons.
- Using the masses of hadrons , the regge trajectories in (n, M^2) and (J, M^2) planes can be generated. Further, their slopes and intercepts can also be tabulated.

Regge Trajectories in (n,M^2) and (J,M^2) planes





$$n = \beta M^2 + \beta_0$$
$$J = \alpha M^2 + \alpha_0$$

Magnetic Moments

- Magnetic moment of baryons is one of the most important quantities in investigation of their electromagnetic structure, and can provide essential information about the dynamics of the strong interaction at low energies.
- The magnetic moments of heavy flavor baryon computed based on the nonrelativistic hypercentral constituent quark model using the spin-flavour wave functions of the constituting quarks and their effective masses.
- As the combination of quarks in baryon changes, its binding interaction affects and m_i^{eff} differs. The effective mass for each of the constituting quark m_i^{eff} can be defined as

$$m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i}\right) \tag{9}$$

where, $\langle H \rangle$ = E + $\langle V_{spin} \rangle$. Thus, the magnetic moment of baryons with bound quarks are given as[1]

Magnetic Moments

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle) \tag{10}$$

where

$$\mu_i = \frac{e_i \sigma_i}{2 m_i^{eff}} \tag{11}$$

e_i is a charge and σ_i is the spin of the respective constituent quark corresponds to the spin flavor wave function of the baryonic state.

Table : Magnetic Moment(in nuclear magnetons)

Baryons	wave function [30]	Our	[31]	[32]	[33]
Ω_{bbb}^{*-}	$3\mu_b$	-0.191	-0.194	-0.198	-0.196

Conclusion I

- The present work studied the properties of the triply beauty Ω_{bbb} baryon using the potential model approach. The Hypercentral constituent quark model is used for non-reletivistic approach extended by adding first order correction.
- No experimental evidence is observed yet for triply heavy baryons. Thus, we determine the mass spectra for radial excited states (2S-5S) and orbital excited states(1P-5P, 1D-4D, 1F-2F) with first order correction(B) and without correction(A).
- The ground state is determined for $J^P = \frac{3}{2}^+$. It's mass is 73 MeV lower than Ref. [22] and 98 MeV higher than Ref.[13]. In all other cases our 1S state mass prediction show difference of more than 100 MeV from all other predictions such that the experimental outcome will decide the actual range of the existence.
- Such mass study will definitely help to identify the nearly available resonances belong to Ω_{bbb} baryon. As we can see very few theoretical study has presented the excited states of the baryon.

.

Conclusion II

- The regge Trajectories are drawn in (n, M^2) and (J, M^2) planes and it would be helpful to identify the unknown J^P values for all baryons. We observe that the trajectories are linear, parallel and equidistant.
- Ground state magnetic moments are also calculated and compared. It is observed that it is in good agreement with others.

Acknowledgment

The organizing committee of Hadron 2017 !!

Financial Assistance is provided by Science and Engineering Research Board (SERB) to attend the conference under Internatonal Travel Grant Scheme(ITS) governed by Government of India.

The financial support extended by DST, India under SERB fast track scheme SR/FTP/PS-152/2012.

Group Members:

- Dr. Ajay Kumar Rai (Supervisor)
- Dr. Naynesh Devlani
- Mr. Virendrasinh Kher
- Mr. Dharmesh Rathaud
- Mr. Raghav Chaturvedi
- Mr. Keval Gandhi

Collaborators: Prof. P. C. Vinodkumar & Dr. Kaushal Thakkar

References I

[1] C. Patrignani et al. (Particle Data Group), Chin. Phys. **C40**(10), 100001 (2016)
[2] R. Aaij et al. (LHCb), JHEP **05**, 030 (2017), 1701.07873
[3] R. Aaij et al. (LHCb) (2017), 1703.04639
[4] R. Aaij et al. (LHCb) (2017), 1707.01621
[5] M.M. Giannini, E. Santopinto, Chin. J. Phys. **53**, 020301 (2015), 1501.03722
[6] E. Santopinto, F. Iachello, M.M. Giannini, Eur. Phys. J. **A1**, 307 (1998)
[7] R. Bijker, F. Iachello, A. Leviatan, Annals Phys. **284**, 89 (2000), nucl-th/0004034
[8] Y. Koma, M. Koma, H. Wittig, Phys. Rev. Lett. **97**, 122003 (2006), hep-lat/0607009
[9] A.K. Rai, D.P. Rathaud, Eur. Phys. J. **C75**(9), 462 (2015), 1507.01409
[10] M.B. Voloshin, Prog.Part.Nucl.Phys. **51**, 455 (2008)
[11] W. Roberts, M. Pervin, Int. J. Mod. Phys. **A23**, 2817 (2008), 0711.2492
[12] A. Bernotas, V. Simonis, Lith. J. Phys. **49**, 19 (2009), 0808.1220
[13] J.M. Flynn, E. Hernandez, J. Nieves, Phys. Rev. **D85**, 014012 (2012), 1110.2962
[14] K.W. Wei, B. Chen, X.H. Guo, Phys. Rev. **D92**(7), 076008 (2015), 1503.05184
[15] K.W. Wei, B. Chen, N. Liu, Q.Q. Wang, X.H. Guo (2016), 1609.02512
[16] J.R. Zhang, M.Q. Huang, Phys. Lett. **B674**, 28 (2009), 0902.3297
[17] P. Hasenfratz, R.R. Horgan, J. Kuti, J.M. Richard, Phys. Lett. **B94**, 401 (1980)
[18] K. Thakkar, A. Majethiya, P.C. Vinodkumar, Eur. Phys. J. Plus **131**(9), 339 (2016), 1609.05444
[19] T.M. Aliev, K. Azizi, M. Savc, J. Phys. **G41**, 065003 (2014), 1404.2091
[20] T.M. Aliev, K. Azizi, M. Savci, JHEP **04**, 042 (2013), 1212.6065
[21] Z.F. Sun, Z.W. Liu, X. Liu, S.L. Zhu, Phys. Rev. **D91**(9), 094030 (2015), 1411.2117

References II

[22] A.P. Martynenko, Phys. Lett. **B663**, 317 (2008), 0708.2033

[23] Z.S. Brown, W. Detmold, S. Meinel, K. Orginos, Phys. Rev. **D90**(9), 094507 (2014), 1409.0497

[24] S. Meinel, Phys. Rev. **D85**, 114510 (2012), 1202.1312

[25] H. Sanchis-Alepuz, R. Alkofer, G. Eichmann, R. Williams, PoS **QCD-TNT-II**, 041 (2011), 1112.3214

[26] F.J. Llanes-Estrada, O.I. Pavlova, R. Williams, The European Physical Journal C **72**(6), 2019 (2012), ISSN 1434-6052

[27] Z.G. Wang, Commun. Theor. Phys. **58**, 723 (2012), 1112.2274

[28] J. Vijande, A. Valcarce, H. Garcilazo, Phys. Rev. **D91**(5), 054011 (2015), 1507.03735

[29] T. Regge, Nuovo Cim. **14**, 951 (1959)

[30] Fayyazuddin, Riazuddin, A Modern introduction to particle physics (1994)

[31] A. Bernotas, V. imonis, Lith. J. Phys. **53**, 84 (2013), 1209.2900

[32] R. Dhir, R.C. Verma, Eur. Phys. J. **A42**, 243 (2009), 0904.2124

[33] B. Patel, A.K. Rai, P.C. Vinodkumar, Pramana **70**, 797 (2008), 0802.4408

Thank You