Triangle Singularities in the $\Lambda_b \rightarrow J/\psi K^- p$ Reaction

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Outline

- Introduction
- Detailed analysis of the triangle singularity
- Results for the $\Lambda_b \rightarrow K^- J/\psi \ p$
- Conclusion
Triangle singularities in physical processes were introduced by Landau (L. D. Landau, Nucl. Phys. 13, 181 (1959))

Some examples of effects of the triangle singularity:
F. Aceti, L. R. Dai and E. Oset, Phys.Rev. D94 (2016) no.9, 096015

LHCb pentaquark-like structures: in the $\Lambda_b \rightarrow J/\psi K^- p$ reaction in the $J/\psi p$ spectrum ⇒ a narrow peak ⇒ at 4450 MeV

The possibility that this peak is due to triangle singularity is discussed:
Detailed analysis of the triangle singularity
The $\Lambda_b \rightarrow J/\psi K^- p$ Reaction

![Triangle diagram for $\Lambda_b \rightarrow J/\psi K^- p$ decay](image)

**Figure:** Triangle diagram for $\Lambda_b \rightarrow J/\psi K^- p$ decay, where $\Lambda^*$ stands for the different $\Lambda^*$ and $c\bar{c}$ stands for different charmonium states.

The scalar three-point loop integral

$$I_1 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left(q^2 - m_{c\bar{c}}^2 + i\epsilon\right) \left((P - q)^2 - m_{\Lambda^*}^2 + i\epsilon\right) \left((P - q - k)^2 - m_p^2 + i\epsilon\right)}.$$  (1)
Detailed analysis of the triangle singularity

![Diagram of a triangle with labels and dashed lines]

**Figure:** A triangle diagram. The two dashed vertical lines correspond to the two relevant cuts.

The scalar three-point loop integral

\[
l_1 = i \int \frac{d^4 q}{(2\pi)^4} \left( \frac{1}{(q^2 - m_2^2 + i \epsilon)} \right) \left[ \frac{1}{(P - q)^2 - m_1^2 + i \epsilon} \right] \left[ \frac{1}{(p_{23} - q)^2 - m_3^2 + i \epsilon} \right]. \tag{2}\]

Rewriting a propagator into two poles

\[
\frac{1}{(q^2 - m_2^2 + i \epsilon)} = \frac{1}{(q^0 - \omega_2 + i \epsilon)(q^0 + \omega_2 + i \epsilon)}, \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^2} \tag{3}\]

focus on the positive-energy poles

\[
l_1 \approx \frac{i}{8m_1 m_2 m_3} \int \frac{dq^0 d^3 \vec{q}}{(2\pi)^4} \left( \frac{1}{(q^0 - \omega_2 + i \epsilon)} \right) \left[ \frac{1}{(P^0 - q^0 - \omega_1 + i \epsilon)} \right] \left[ \frac{1}{(p_{23}^0 - q^0 - \omega_3 + i \epsilon)} \right]. \tag{4}\]
\[ I(m_{23}) = \int d^3q \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i \epsilon][E_{23} - \omega_2(q) - \omega_3(k + q) + i \epsilon]} \]

\[ = 2\pi \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i \epsilon} f(q) \]

\[ f(q) = \int_{-1}^1 dz \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2qkz + i \epsilon}} , \]

\[ \rightarrow \text{singularity of integrand does not necessarily give a singularity of integral} \]
Relation between singularities of integrand and integral

\[ \text{Singularity of integrand does not necessarily give a singularity of integral:} \]

- integral contour can be deformed to avoid the singularity

\[ \Rightarrow \text{Two cases that a singularity cannot be avoided:} \]

- end point singularity
- pinch singularity
\[ I \sim \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i \epsilon} f(q), \quad f(q) \sim \int_{-1}^1 dz \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 + 2qp_{23}z + i \epsilon}} \]

**Singularities of the integrand of \( I \) in the rest frame of initial particle:**

- **cut 1:** \( P^0 - \omega_1(q) - \omega_2(q) + i \epsilon = 0 \)
  \[ \Rightarrow q_{on}^+ = q_{on} + i \epsilon \quad \text{with} \quad q_{on} = \frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)}. \]

- **cut 2:** \( E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 + 2qp_{23}z + i \epsilon} = 0 \), end point singularities
  
  \[ \begin{align*}
  z = -1: & \quad q_{a+} = \gamma (v E_2^* + p_2^*) + i \epsilon \quad q_{a-} = \gamma (v E_2^* - p_2^*) - i \epsilon \\
  z = +1: & \quad q_{b+} = \gamma (-v E_2^* + p_2^*) + i \epsilon \quad q_{b-} = -\gamma (v E_2^* + p_2^*) - i \epsilon \\
  \end{align*} \]

\[ v = \frac{k}{E_{23}}, \quad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}} \]

\( E_2^* (p_2^*) \) are the energy (momentum) of particle-2 in the center-of-mass frame of the (2,3) system.
All singularities of the integrand:

- $q_{on+}, \quad q_{a+} = \gamma (v E^*_2 + p^*_2) + i \epsilon \quad q_{a-} = \gamma (v E^*_2 - p^*_2) - i \epsilon$
- Threshold singularity $\implies q_{a+} = q_{a-}$
- Triangle singularity $\implies q_{on+} = q_{a-}$
- $q_{a-} < 0, \quad q_{b+} = -q_{a-}$
The Coleman Norton Theorem \( \rightarrow \) The singularity is in the physical region only when the process can happen classically: all the intermediate states are on shell, and the particle 3 emitted from the decay of the particle 1 moves along the same direction as the particle 2 with a large speed than the particle 2 and can catch up with it to rescatter.

For \( q_{on} \) and \( q_{a-} \) (\( \epsilon = 0 \)), physical regions \( \Rightarrow m_1 \leq M - m_2 \) and \( m_{23} \geq m_2 + m_3 \) using \( q_{on+} = q_{a-} \)

\[
m^2_1 \in \left[ \frac{M^2 m_3 + m_{13}^2 m_2}{m_2 + m_3} - m_2 m_3, (M - m_2)^2 \right],
\]

\[
m^2_{23} \in \left[ (m_2 + m_3)^2, \frac{Mm_3^2 - m_{13}^2 m_2}{M - m_2} + Mm_2 \right].
\]
Results for the $\Lambda_b \rightarrow K^- J/\psi p$

$\Lambda^* \rightarrow \Lambda(1405), \Lambda(1520), \Lambda(1600), \Lambda(1670), \Lambda(1690), \Lambda(1800), \Lambda(1810), \Lambda(1820), \Lambda(1830), \Lambda(1890), \Lambda(2100), \Lambda(2110), \Lambda(2350), \Lambda(2585)$

<table>
<thead>
<tr>
<th>$c\bar{c}$</th>
<th>Most relevant range of $M_{\Lambda^*}$ (MeV)</th>
<th>Range of triangle singularity (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c$</td>
<td>[2226, 2639]</td>
<td>[3919, 4283]</td>
</tr>
<tr>
<td>$J/\psi$</td>
<td>[2151, 2523]</td>
<td>[4035, 4366]</td>
</tr>
<tr>
<td>$\chi_{c0}$</td>
<td>[1949, 2205]</td>
<td>[4353, 4588]</td>
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<tr>
<td>$\chi_{c1}$</td>
<td>[1887, 2109]</td>
<td>[4449, 4654]</td>
</tr>
<tr>
<td>$\chi_{c2}$</td>
<td>[1858, 2063]</td>
<td>[4494, 4686]</td>
</tr>
<tr>
<td>$h_{c1}$</td>
<td>[1878, 2094]</td>
<td>[4464, 4664]</td>
</tr>
<tr>
<td>$\eta_c(2S)$</td>
<td>[1806, 1983]</td>
<td>[4575, 4741]</td>
</tr>
<tr>
<td>$\psi(2S)$</td>
<td>[1774, 1933]</td>
<td>[4624, 4775]</td>
</tr>
</tbody>
</table>
Assume the $\chi_{c1} p \Rightarrow S$ wave in the $\chi_{c1} p \rightarrow J/\psi p$

**Figure:** The value of $|I_1|$ for $\Lambda^* \chi_{c1}$ with a width $\Gamma = 100$ MeV for the hyperon.

- peak around 4450 MeV !!
- The largest strength $\Rightarrow \Lambda(1890)$; the threshold and the triangle singularities merge
- $\Lambda(1670)$ and $\Lambda(1810)$: the cusp structure comes from the threshold singularity
- $\Lambda(2100)$: inside the range of the triangle singularities
Detailed analysis of the $S$ and $P$-wave amplitudes for $\chi_{c1}$ and $\psi(2S)$ and $\Lambda(1890)$

In the $c\bar{c} \ p \rightarrow J/\psi \ p$ we have several situations:

1. The quantum numbers of the $J/\psi \ p \Rightarrow J^P = 3/2^-$ ($P_c(4450)$):
   - a) $c\bar{c} = \chi_{c1}$: requires a $P$-wave in the $\chi_{c1} \ p$ system
   - b) $c\bar{c} = \psi(2S)$: requires an $S$-wave in both the $\psi(2S) \ p$ and $J/\psi \ p$

2. The quantum numbers of the $J/\psi \ p$ are $J^P = 1/2^+$ or $3/2^+$:
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Figure: (1.a) The value of $|I_1|^2$ for $P$-wave $\Lambda(1890)\chi_{c1}$ (1.b) The value of $|I_1|^2$ for $S$-wave $\Lambda(1890)\psi(2S)$. $\Gamma_{\Lambda(1890)} = 100$ MeV

- The amplitude (1.a) is very much suppressed than the $S$-wave case (1.b).
  - This is natural!! the singularity appears $\Rightarrow$ the $\chi_{c1} p$ on shell and at threshold $\Rightarrow$ the $P$-wave factor vanishes.
- (1.a) the $P$-wave has a “background" below the peak the shape is too broad!!
- (1.b) the $S$-wave structure is very much peaked and narrow a narrow peak around 4624 MeV the experimental data $\Rightarrow$ the $J/\psi p$ invariant mass distribution in this region is flat.

Conclusion: if the narrow $J/\psi p$ has $3/2^-$ $\Rightarrow$ the triangle singularities due to $\Lambda^* c\bar{c} p$ intermediate states cannot play an important role in the decay $\Lambda_b \rightarrow K^- J/\psi p$
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\[ \Gamma_{\Lambda(1890)} = 100 \text{ MeV} \]

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Conclusion

- Analyzed the singularities of a triangle loop integral in detail
- Derived a formula for an easy evaluation of the triangle singularity on the physical boundary
- Applied to the $\Lambda_b \rightarrow J/\psi K^- p$ process via $\Lambda^*$-charmonium-proton intermediate states
- Most relevant states $\Rightarrow$ the $\chi_{c1}$ and the $\psi(2S)$ among all possible charmonia up to the $\psi(2S)$
  In many of the other cases $\Rightarrow$ not develop a triangle singularity, but the threshold cusp
- The $\Lambda(1890) \chi_{c1} p$ loop is very special $\Rightarrow$ normal threshold and triangle singularities merge at about 4450 MeV
- In the case of $J^P = \frac{3}{2}^-, \frac{5}{2}^+$ for the narrow $P_c$, the $\chi_{c1} p \rightarrow J/\psi p$
  requires $P$- and $D$-waves in $\chi_{c1} p \Rightarrow$ drastically reduce the strength of the contribution
- In the case of $1/2^+, 3/2^+$: the $\chi_{c1} p$ in the $\chi_{c1} p \rightarrow J/\psi p$
  amplitude be in an $S$-wave the $\Lambda(1890) \chi_{c1} p$ triangle diagram
  could play an important role