Triangle Singularities in the $\Lambda_b \to J/\psi K^- p$ Reaction

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Based On: M. Bayar, F. Aceti, F.-K. Guo and E. Oset, Phys. Rev.D 94, 074039 (2016)

Outline

- Introduction
- Detailed analysis of the triangle singularity
- Results for the $\Lambda_b \to K^- J/\psi\, p$
- Conclusion

Introduction

- Triangle singularities in physical processes were introduced by Landau (L. D. Landau, Nucl. Phys. 13, 181 (1959))
- Some examples of effects of the triangle singularity :
  J. J. Wu, X. H. Liu, Q. Zhao and B. S. Zou, PRL108(2012)081803
  X. G. Wu, J. J. Wu, Q. Zhao and B. S. Zou, PRD87(2013)014023(2013)
  F. Aceti, W. H. Liang, E. Oset, J. J. Wu and B. S. Zou, Phys. Rev. D 86, 114007 (2012)
  M. Mikhasenko, B. Ketzer and A. Sarantsev, Phys. Rev. D 91, 094015 (2015)
  F. Aceti, L. R. Dai and E. Oset, Phys.Rev. D94 (2016) no.9, 096015
- LHCb pentaquark-like structures: in the $\Lambda_b \to J/\psi K^- p$ reaction in the $J/\psi p$ spectrum $\Rightarrow$ a narrow peak $\Rightarrow$ at 4450 MeV

  R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 072001 (2015), R. Aaij et al. [LHCb Collaboration], Chin. Phys. C 40, 011001 (2016)
- The possibility that this peak is due to triangle singularity is discussed:

  F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, Phys. Rev. D 92, 071502 (2015), X. H. Liu, Q. Wang and Q. Zhao, Phys. Lett. B 757, 231 (2016), F.-K. Guo, U.-G. Meißer, J. Nieves and Z. Yang Eur.Phys.J. A52 (2016) no.10, 318

Detailed analysis of the triangle singularity

The $\Lambda_b \to J/\psi K^- p$ Reaction

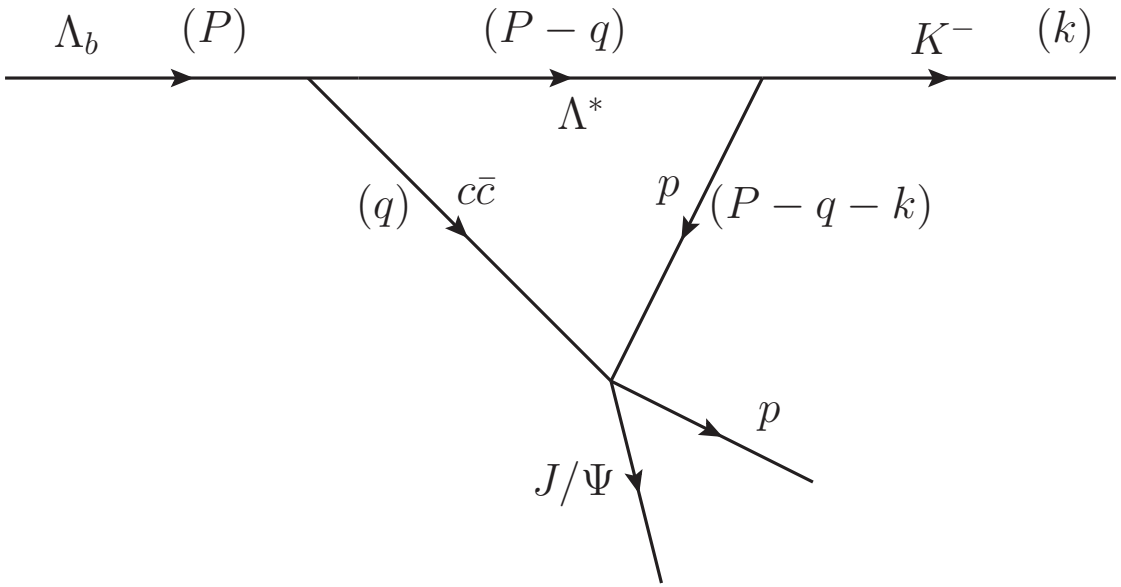


Figure: Triangle diagram for $\Lambda_b \to J/\psi K^- p$ decay, where Λ^* stands for the different Λ^* and $c\bar{c}$ stands for different charmonium states.

The scalar three-point loop integral

$$I_1 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left(q^2 - m^2_{c\bar{c}} + i\,\epsilon\right)\left[(P-q)^2 - m^2_{\Lambda^*} + i\,\epsilon\right]\left[(P-q-k)^2 - m^2_p + i\,\epsilon\right]} \,. \tag{1}$$

Detailed analysis of the triangle singularity

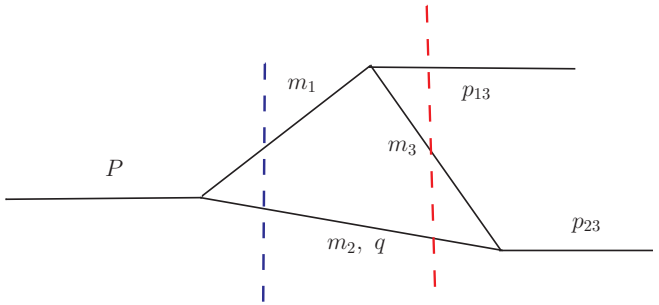


Figure: A triangle diagram. The two dashed vertical lines correspond to the two relevant cuts.

The scalar three-point loop integral

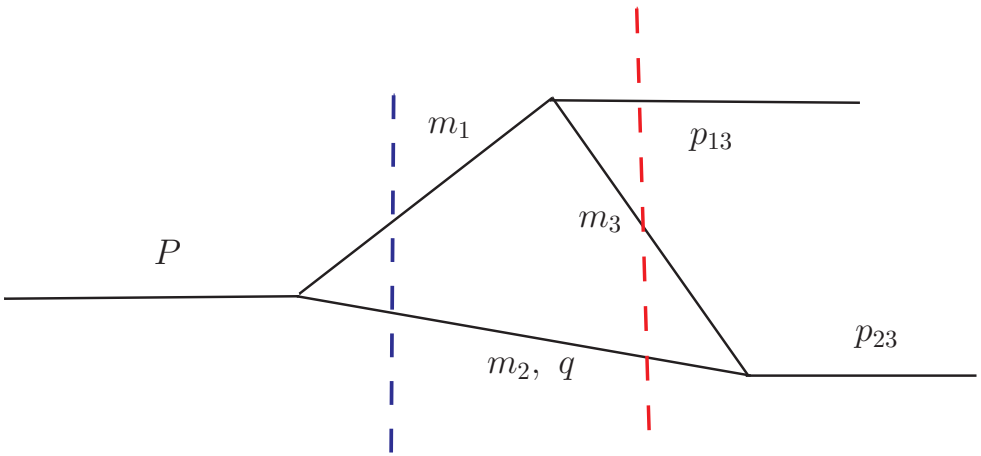
$$I_1 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{\left(q^2 - m_2^2 + i\,\epsilon\right)\left[(P-q)^2 - m_1^2 + i\,\epsilon\right]\left[(p_{23}-q)^2 - m_3^2 + i\,\epsilon\right]} \,. \tag{2}$$

Rewriting a propagator into two poles

$$\frac{1}{\left(q^2 - m_2^2 + i\,\epsilon\right)} = \frac{1}{\left(q^0 - \omega_2 + i\,\epsilon\right)\left(q^0 + \omega_2 + i\,\epsilon\right)}, \qquad \omega_2 = \sqrt{m_2^2 + \vec{q}^{\,2}} \tag{3}$$

focus on the positive-energy poles

$$I_1 \simeq \frac{i}{8 m_1 m_2 m_3} \int \frac{dq^0 d^3\vec{q}}{(2\pi)^4} \frac{1}{\left(q^0 - \omega_2 + i\,\epsilon\right)\left[P^0 - q^0 - \omega_1 + i\,\epsilon\right]\left[p_{23}^0 - q^0 - \omega_3 + i\,\epsilon\right]} \,. \tag{4}$$

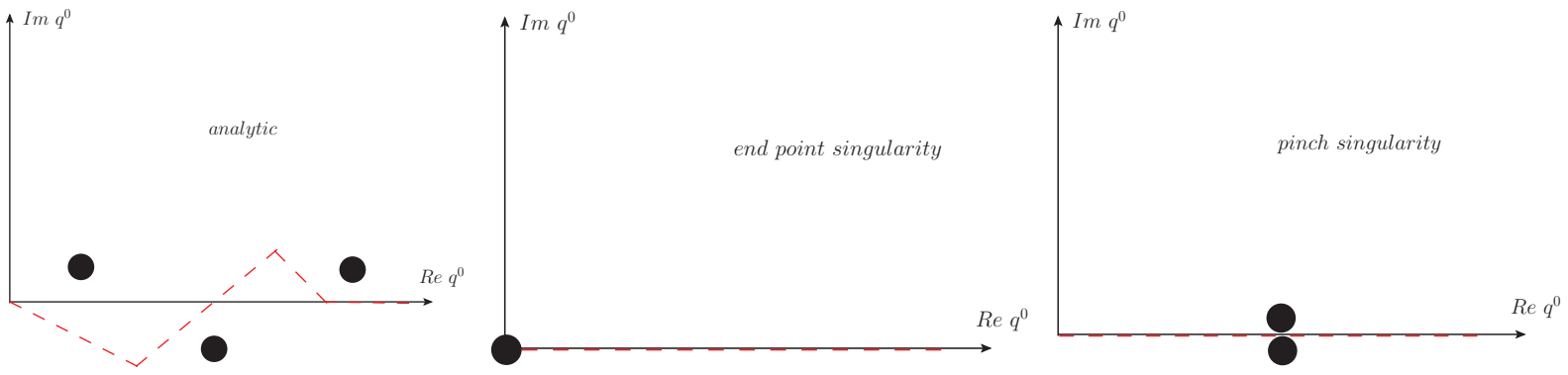


$$
\begin{aligned}
I(m_{23}) &= \int d^3q \, \frac{1}{\left[P^0 - \omega_1(\vec{q}\,) - \omega_2(\vec{q}\,) + i\,\epsilon\right]\left[E_{23} - \omega_2(\vec{q}\,) - \omega_3(\vec{k} + \vec{q}\,) + i\,\epsilon\right]} \\
&= 2\,\pi \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q) \qquad (5)
\end{aligned}
$$

$$
f(q) = \int_{-1}^{1} dz \, \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2\,q\,k\,z} + i\,\epsilon}\,, \qquad (6)
$$

$\Longrightarrow$ singularity of integrand does not necessarily give a singularity of integral

Relation between singularities of integrand and integral

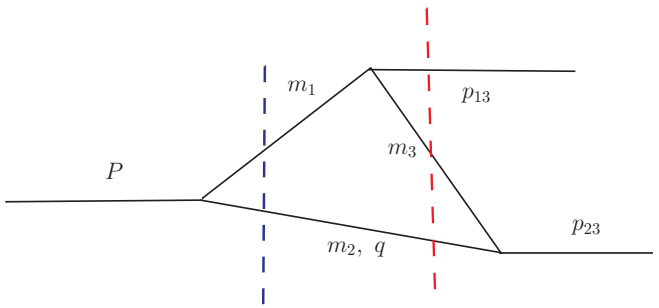


$\Longrightarrow$ Singularity of integrand does not necessarily give a singularity of integral:

- integral contour can be deformed to avoid the singularity

$\Longrightarrow$ Two cases that a singularity cannot be avoided:

- end point singularity
- pinch singularity



$$I \sim \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q), \quad f(q) \sim \int_{-1}^{1} dz \, \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 + 2\,q\,p_{23}\,z} + i\,\epsilon}$$

Singularities of the integrand of I in the rest frame of initial particle:

- cut 1: $P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon = 0$

$\Longrightarrow \quad q_{\text{on}+} = q_{\text{on}} + i\,\epsilon$ with $q_{\text{on}} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}$.

- cut 2: $E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 + 2\,q\,p_{23}\,z} + i\,\epsilon = 0$, end point singularities

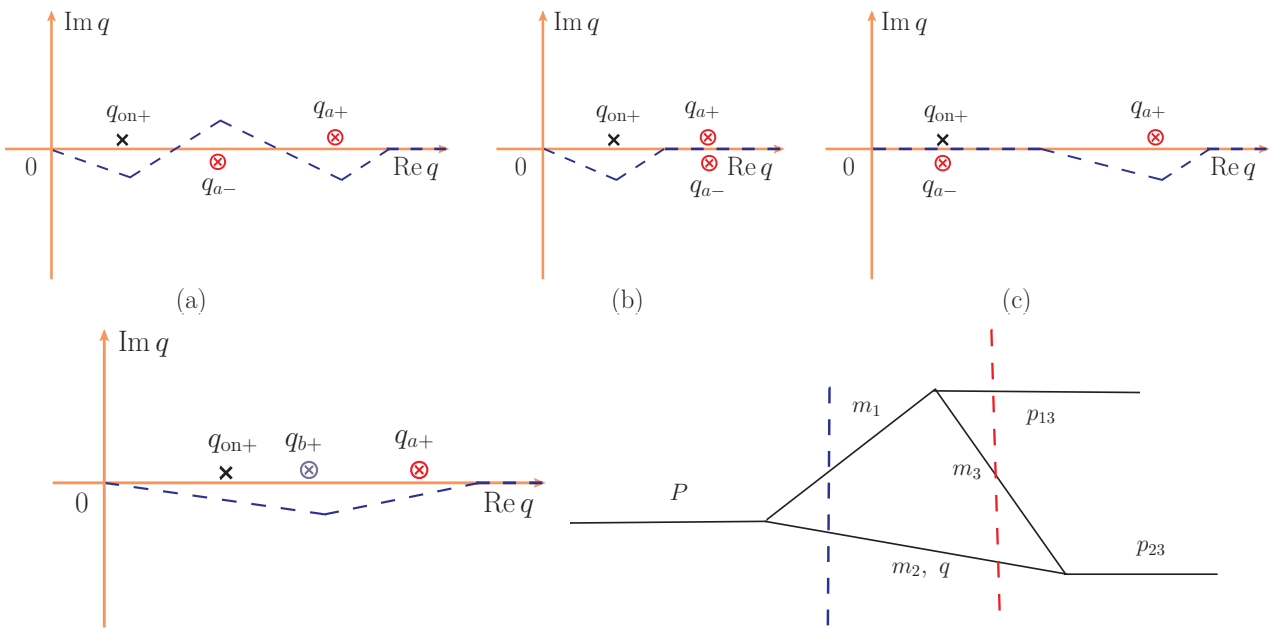
$z = -1$: $\quad q_{a+} = \gamma\left(v\,E_2^* + p_2^*\right) + i\,\epsilon \qquad q_{a-} = \gamma\left(v\,E_2^* - p_2^*\right) - i\,\epsilon$

$z = +1$: $\quad q_{b+} = \gamma\left(-v\,E_2^* + p_2^*\right) + i\,\epsilon \qquad q_{b-} = -\gamma\left(v\,E_2^* + p_2^*\right) - i\,\epsilon$

$$v = \frac{k}{E_{23}}, \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}}$$

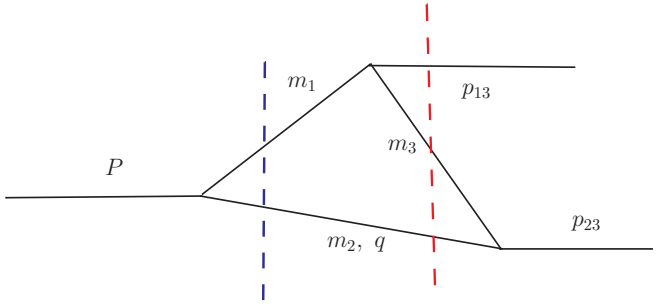
E_2^* (p_2^*) are the energy (momentum) of particle-2 in the center-of-mass frame of the (2,3) system

All singularities of the integrand:



- q_{on+}, $q_{a+} = \gamma\left(v\,E_2^* + p_2^*\right) + i\,\epsilon$ $\quad q_{a-} = \gamma\left(v\,E_2^* - p_2^*\right) - i\,\epsilon$
- threshold singularity $\Longrightarrow q_{a+} = q_{a-}$
- triangle singularity $\Longrightarrow q_{on+} = q_{a-}$
- $q_{a-} < 0, \quad q_{b+} = -q_{a-}$

The Coleman Norton Theorem $\rightarrow$ The singularity is in the physical region only when the process can happen classically: all the intermediate states are on shell, and the particle 3 emitted from the decay of the particle 1 moves along the same direction as the particle 2 with a large speed than the particle 2 and can catch up with it to rescatter.

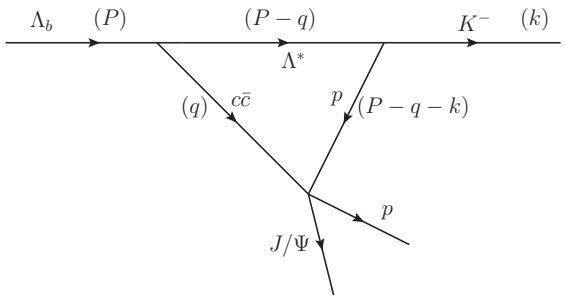


For $q_{\rm on}$ and q_{a-} ($\epsilon = 0$), physical regions $\Rightarrow m_1 \leq M - m_2$ and $m_{23} \geq m_2 + m_3$
using $q_{on+} = q_{a-}$

$$m_1^2 \in \left[\frac{M^2 m_3 + m_{13}^2 m_2}{m_2 + m_3} - m_2 m_3\,,\ (M - m_2)^2\right], \tag{8}$$

$$m_{23}^2 \in \left[(m_2 + m_3)^2,\ \frac{M m_3^2 - m_{13}^2 m_2}{M - m_2} + M m_2\right]. \tag{9}$$

Results for the $\Lambda_b \to K^- J/\psi\, p$



$\Lambda^* \Longrightarrow \Lambda(1405), \Lambda(1520), \Lambda(1600), \Lambda(1670), \Lambda(1690), \Lambda(1800), \Lambda(1810), \Lambda(1820), \Lambda(1830), \Lambda(1890), \Lambda(2100), \Lambda(2110), \Lambda(2350), \Lambda(2585)$

$c\bar{c}$	Most relevant range of M_{Λ^*} (MeV)	Range of triangle singularity (MeV)
η_c	[2226, 2639]	[3919, 4283]
J/ψ	[2151, 2523]	[4035, 4366]
χ_{c0}	[1949, 2205]	[4353, 4588]
χ_{c1}	[1887, 2109]	[4449, 4654]
χ_{c2}	[1858, 2063]	[4494, 4686]
h_{c1}	[1878, 2094]	[4464, 4664]
$\eta_c(2S)$	[1806, 1983]	[4575, 4741]
$\psi(2S)$	[1774, 1933]	[4624, 4775]

The χ_{c1} intermediate charmonium state

Assume the $\chi_{c1}\, p \Rightarrow$ S wave in the $\chi_{c1}\, p \rightarrow J/\psi\, p$

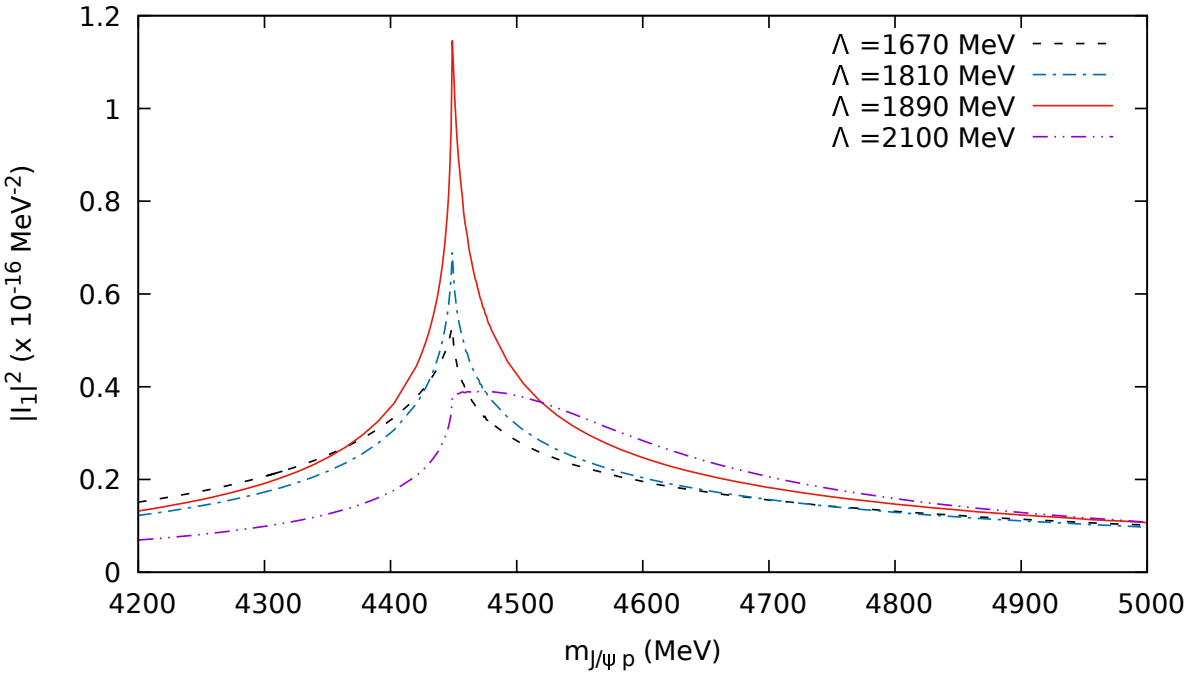
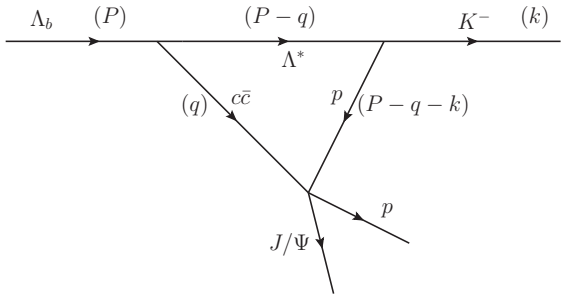


Figure: The value of $|I_1|$ for $\Lambda^* \chi_{c1}$ with a width $\Gamma = 100$ MeV for the hyperon.

- peak around 4450 MeV !!
- The largest strength $\Rightarrow$ $\Lambda(1890)$; the threshold and the triangle singularities merge
- $\Lambda(1670)$ and $\Lambda(1810)$: the cusp structure comes from the threshold singularity
- $\Lambda(2100)$: inside the range of the triangle singularities

Detailed analysis of the S and P-wave amplitudes for χ_{c1} and $\psi(2S)$ and $\Lambda(1890)$



In the $c\bar{c}\ p \to J/\psi\ p$ we have several situations:

$\Lambda(1890)$ $[I(J^P) = 0(\frac{3}{2}^+)]$

1. The quantum numbers of the $J/\psi\, p \Rightarrow J^P = 3/2^-$ ($P_c(4450)$) :
   - a) $c\bar{c} = \chi_{c1}$: requires a P-wave in the $\chi_{c1}\ p$ system
   - b) $c\bar{c} = \psi(2S)$: requires an S-wave in both the $\psi(2S)\ p$ and $J/\psi\ p$
2. The quantum numbers of the $J/\psi\, p$ are $J^P = 1/2^+$ or $3/2^+$:
   - a) $c\bar{c} = \chi_{c1}$: the $\chi_{c1} p$ system (S-wave)and the $J/\psi p$ (P-wave)
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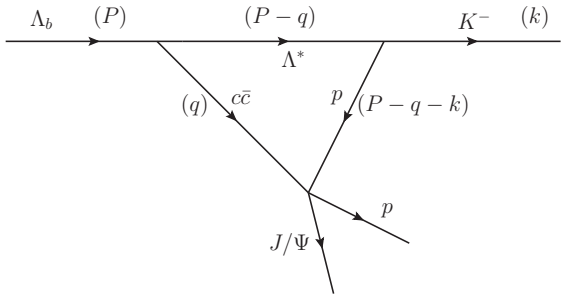

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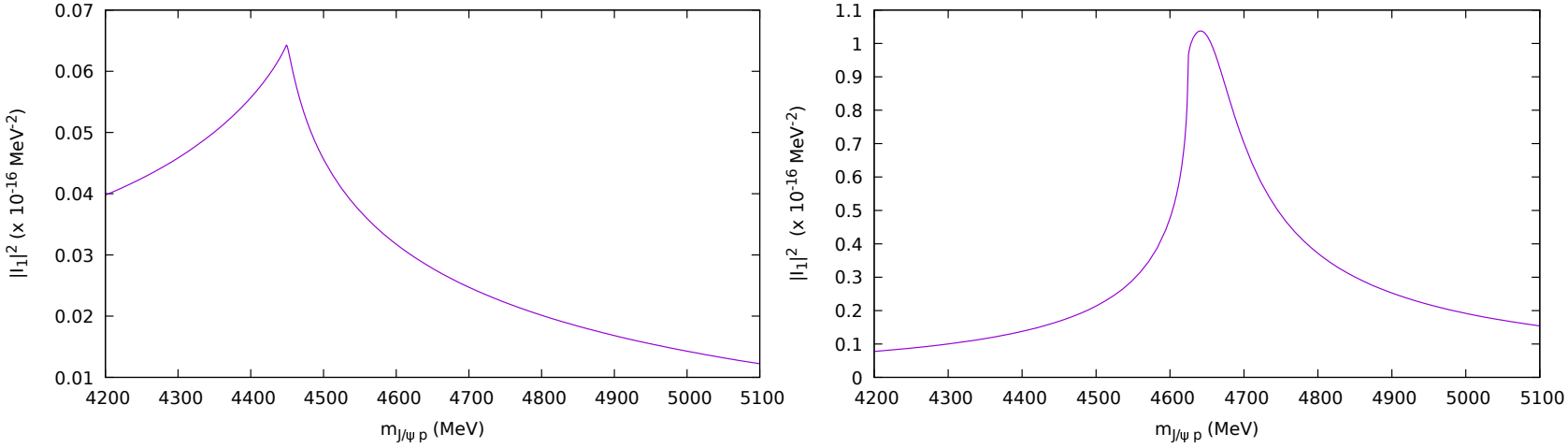
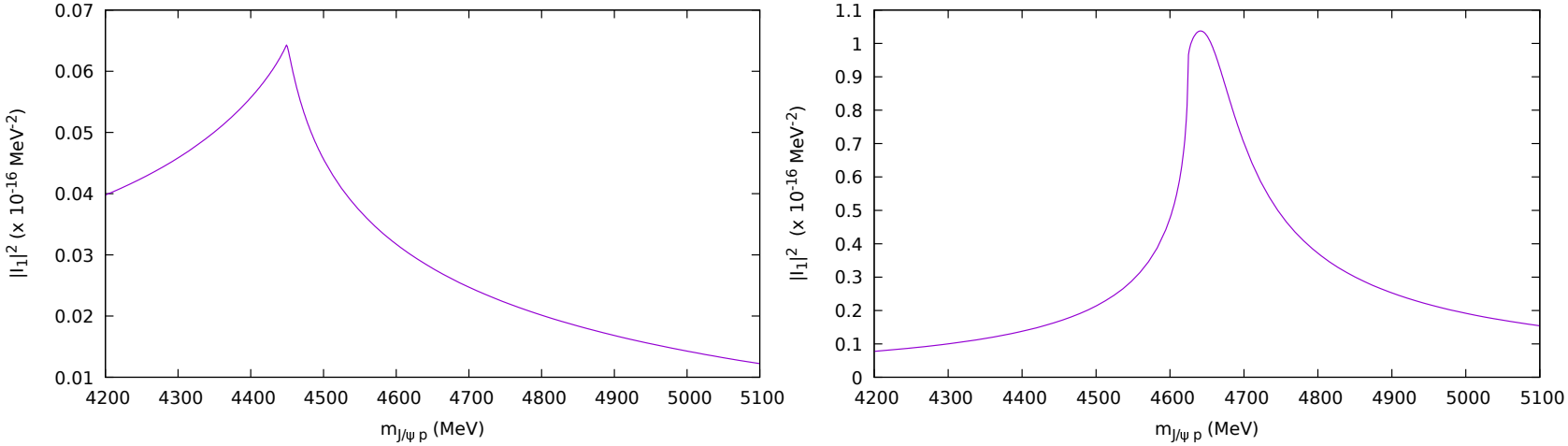


Figure: (1.a) The value of $|I_1|^2$ for P-wave $\Lambda(1890)\chi_{c1}$ (1.b) The value of $|I_1|^2$ for S-wave $\Lambda(1890)\psi(2S)$.

$\Gamma_{\Lambda(1890)} = 100$ MeV

- the amplitude (1.a) is very much suppressed than the S-wave case (1.b)
  This is natural!! the singularity appears $\Rightarrow$ the χ_{c1} p on shell and at threshold $\Rightarrow$ the P-wave factor vanishes.
- (1.a) the P-wave has a "background" below the peak
  the shape is too broad!!
- (1.b) the S-wave structure is very much peaked and narrow
  a narrow peak around 4624 MeV
  the experimental data $\Rightarrow$ the $J/\psi p$ invariant mass distribution in this region is flat.
- Conclusion: if the narrow $J/\psi\, p$ has $3/2^-$ $\Rightarrow$ the triangle singularities due to Λ^* $c\bar{c}\, p$ intermediate states cannot play an important role in the decay $\Lambda_b \to K^- J/\psi\, p$



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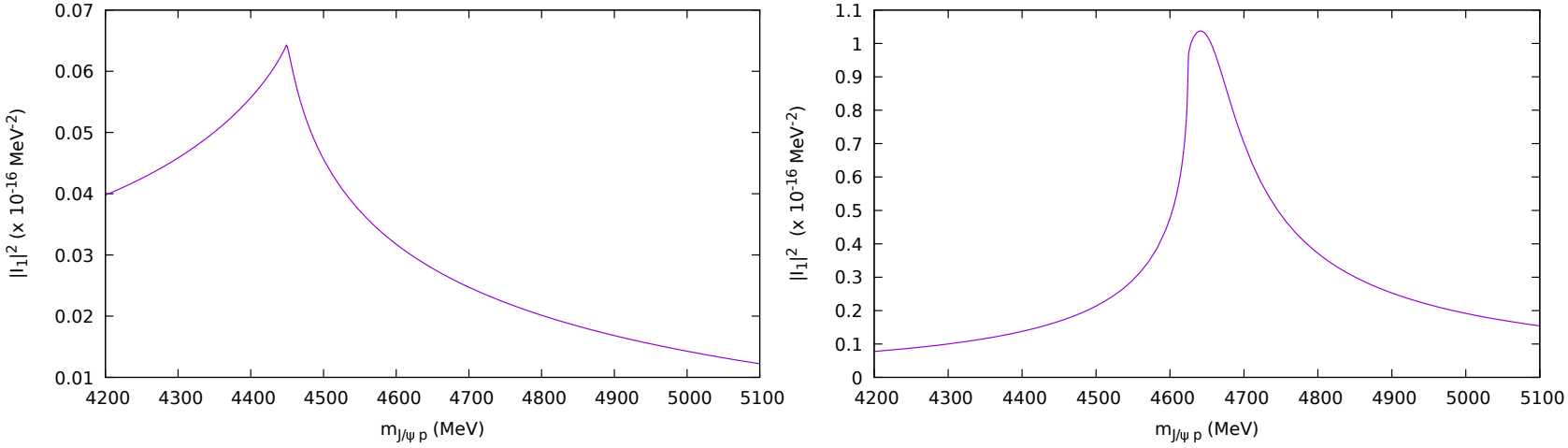

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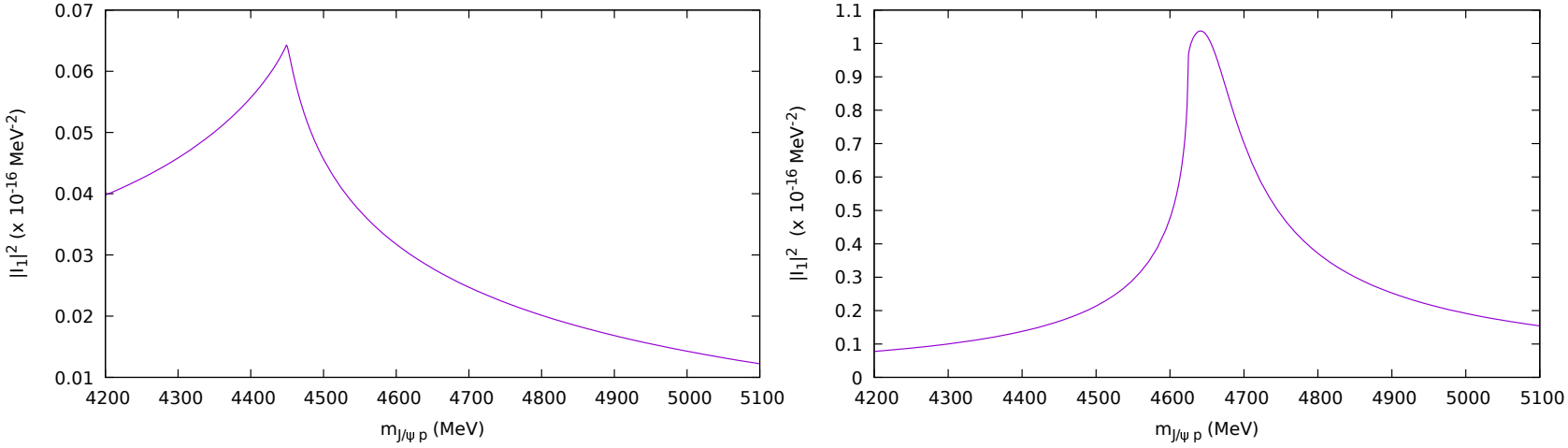

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Conclusion

- Analyzed the singularities of a triangle loop integral in detail
- Derived a formula for an easy evaluation of the triangle singularity on the physical boundary
- applied to the $\Lambda_b \to J/\psi K^- p$ process via Λ^*-charmonium-proton intermediate states
- most relevant states $\Rightarrow$ the χ_{c1} and the $\psi(2S)$ among all possible charmonia up to the $\psi(2S)$
  In many of the other cases $\Rightarrow$ not develop a triangle singularity, but the threshold cusp
- The $\Lambda(1890)\,\chi_{c1}\,p$ loop is very special $\Rightarrow$ normal threshold and triangle singularities merge at about 4450 MeV
- in the case of $J^P = \frac{3}{2}^-,\ \frac{5}{2}^+$ for the narrow P_c, the $\chi_{c1}\,p \to J/\psi\,p$ requires P- and D-waves in $\chi_{c1}\,p \Rightarrow$ drastically reduce the strength of the contribution
- In the case of $1/2^+, 3/2^+$: the $\chi_{c1}\,p$ in the $\chi_{c1}\,p \to J/\psi\,p$ amplitude be in an S-wave the $\Lambda(1890)\,\chi_{c1}\,p$ triangle diagram could play an important role