Heavy Quark Baryons and Exotica

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The LHC as a Heavy Baryon Factory

Proton-Proton Collisions at $\sqrt{s} = 13 \text{ TeV} \sim 20\,000 \,\text{b}\bar{\text{b}}$ -pairs per second

A CONTRACTOR OF THE CONTRACTOR

High B-baryon production fraction

| – Mesons – | | | | Baryons | | |
|----------------|---|------------------|---|--------------|---|------------------|
| \mathbf{B}^+ | + | B^{0} | : | ${ m B_s^0}$ | : | $\Lambda_{ m b}$ |
| $(u\bar{b})$ | | $(d\bar{b})$ | | $(s\bar{b})$ | | (udb) |
| | 4 | | : | 1 | : | 2 |
| Unique dataset | | | | | | |

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CMS

SUISSE

Heavy Quark Baryons and Exotica

ATLAS



LICE

J = 1/2 Baryon Multiplets







Charmed Baryons





The Λ_c^+ excitation spectrum

- Well studied heavy-light-light system
 - Orbitally excited states
 - D-wave doublet predicted more states in other models
 - Missing state?
 - Indication by BaBar for structure in D⁰p at 2.84 GeV [PRL98(2007)012001]



heavy-quark + light-diquark



Predictions from [EPJ A51(2015)82]

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Amplitude analysis of $\Lambda_{ m b} o { m D}^0{ m p}\pi$ at LHCb [JHEP05(2017)030]

1800

1600

1400 1200

1000 800

> 600 400

200

5400

LHCb

5500 5600

 $\Lambda^0_{\mu} \rightarrow D^0 p \pi$

· Comb. bkg.

5700 5800

5900

Part. rec. bkg

Candidates / (5 MeV

- Data set 3 fb^{-1} (Run I) $\sim 11\,000 \ \Lambda_b$ decays
- 5D amplitude analysis in helicity formalism
 Investigating D⁰p resonances





A new Λ_c^* state

[JHEP05(2017)030]

S. N. Friday 10:05h Session: Baryons

Three Λ_{c}^{+} resonances

| State | M[MeV] | $\Gamma[\text{MeV}]$ | JP |
|-----------------------------|--------|----------------------|--------------|
| $\Lambda_{\rm c}^{+}(2860)$ | 2856 | 67 | $3/2^+$ |
| $\Lambda_{\rm c}^{+}(2880)$ | 2881 | 5.4 | $5/2^{+}$ |
| $\Lambda_{\rm c}^+(2940)$ | 2945 | 28 | $3/2^-$ fav. |

- First constraint on $\Lambda^+_{
 m c}(2940)$ spin
- $\Lambda_c^+(2880)$, $\Lambda_c^+(2940)$ in agreement with previous measurements





Only $3/2^+$ gives physical phase motion

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The Λ_c^+ at BES III: Hadronic Branchings [PRL116(2016)052001]

Exclusive $e^+e^- \to \Lambda_c^+\Lambda_c^-$ production at threshold

- **Data sample corresponding to** $567 \,\mathrm{pb}^{-1}$ at $\sqrt{\mathrm{s}} = 4.599 \,\mathrm{GeV}$
- Beam energy-momentum constraint



P. Yue Thu 17:10h Session: Hadron decays

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Beam energy-momentum constraint⁻

BESII T

| Mode | This work $(\%)$ | PDG (%) |
|-----------------------------|--------------------------|-----------------|
| pK_S^0 | $1.52 \pm 0.08 \pm 0.03$ | 1.15 ± 0.30 |
| $pK^{-}\pi^{+}$ | $5.84 \pm 0.27 \pm 0.23$ | 5.0 ± 1.3 |
| $pK_S^0\pi^0$ | $1.87 \pm 0.13 \pm 0.05$ | 1.65 ± 0.50 |
| $pK_S^0\pi^+\pi^-$ | $1.53 \pm 0.11 \pm 0.09$ | 1.30 ± 0.35 |
| $pK^-\pi^+\pi^0$ | $4.53 \pm 0.23 \pm 0.30$ | 3.4 ± 1.0 |
| $\Lambda \pi^+$ | $1.24 \pm 0.07 \pm 0.03$ | 1.07 ± 0.28 |
| $\Lambda \pi^+ \pi^0$ | $7.01 \pm 0.37 \pm 0.19$ | 3.6 ± 1.3 |
| $\Lambda \pi^+ \pi^- \pi^+$ | $3.81 \pm 0.24 \pm 0.18$ | 2.6 ± 0.7 |
| $\Sigma^0 \pi^+$ | $1.27 \pm 0.08 \pm 0.03$ | 1.05 ± 0.28 |
| $\Sigma^+ \pi^0$ | $1.18 \pm 0.10 \pm 0.03$ | 1.00 ± 0.34 |
| $\Sigma^+\pi^+\pi^-$ | $4.25 \pm 0.24 \pm 0.20$ | 3.6 ± 1.0 |
| $\Sigma^+ \omega$ | $1.56 \pm 0.20 \pm 0.07$ | 2.7 ± 1.0 |
| | | |

P. Yue Thu 17:10h Session: Hadron decays



Semileptonic Λ_c^+ decays

- \blacksquare Tag one $\Lambda_{\,\rm c}^+$ hadronically
- Lepton PID (dE/dx + TOF + EMC)
- Use beam constraint to infer missing energy

$$\begin{aligned} \mathcal{B}(\Lambda_{\rm c}^+ \to \Lambda e^+ \nu_{\rm e}) &= (3.63 \pm 0.38 \pm 0.20)\%\\ \mathcal{B}(\Lambda_{\rm c}^+ \to \Lambda \mu^+ \nu_{\mu}) &= (3.49 \pm 0.46 \pm 0.27)\%\\ \frac{\mathcal{B}(\Lambda_{\rm c}^+ \to \Lambda e^+ \nu_{\rm e})}{\mathcal{B}(\Lambda_{\rm c}^+ \to \Lambda \mu^+ \nu_{\mu})} &= 0.96 \pm 0.16 \pm 0.04 \end{aligned}$$

$$\begin{split} & \Lambda_{c}^{+} \rightarrow \Lambda e^{+} \nu_{e} \text{ [PRL115(2015)221805]} \\ & \vdots \\$$

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Strangely charming baryons: The Ω_c

[PRL118(2017)182001]

- The |css > system is a proving ground for HQET
- Popular model: heavy quark + light diquark
- Two S-wave ground states Ω^0_{c} and $\Omega^0_{c}(2770)$ observed
- 5 P-wave states predicted

Summary of theoretical predictions





[PRL118(2017)182001]

Reconstruct $\Xi_c \to p K^- \pi^+$



- Ξ_c detached from primary vertex
- PID of daughter tracks
- pointing to primary vertex

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- $\blacksquare \Xi_{c}$ detached from primary vertex
- PID of daughter tracks
- pointing to primary vertex

Adding another kaon:



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[PRL118(2017)182001]

| Resonance | Mass (MeV) | Γ (MeV) | Yield | N_{σ} |
|-------------------------------|--|------------------------------|------------------------|--------------|
| $\Omega_{c}(3000)^{0}$ | $3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$ | $4.5\pm0.6\pm0.3$ | $1300 \pm 100 \pm 80$ | 20.4 |
| $\Omega_{c}(3050)^{0}$ | $3050.2\pm0.1\pm0.1^{+0.3}_{-0.5}$ | $0.8\pm0.2\pm0.1$ | $970\pm60\pm20$ | 20.4 |
| | | $< 1.2\mathrm{MeV}, 95\%$ CL | | |
| $\Omega_{c}(3066)^{0}$ | $3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$ | $3.5\pm0.4\pm0.2$ | $1740 \pm 100 \pm 50$ | 23.9 |
| $\Omega_{c}(3090)^{0}$ | $3090.2\pm0.3\pm0.5^{+0.3}_{-0.5}$ | $8.7\pm1.0\pm0.8$ | $2000\pm140\pm130$ | 21.1 |
| $\Omega_{\rm c}(3119)^0$ | $3119.1\pm0.3\pm0.9^{+0.3}_{-0.5}$ | $1.1\pm0.8\pm0.4$ | $480\pm70\pm30$ | 10.4 |
| | | $<2.6{\rm MeV},95\%$ CL | | |
| $\Omega_{\mathbf{c}}(3188)^0$ | $3188 \pm 5 \pm 13$ | $60\pm~15\pm11$ | $1670 \pm 450 \pm 360$ | |

- Are these the 5 P-wave states? [PRD95(2017)114012]
- Why two very narrow states? [PRD96(2017)014009]
- Next steps: quantum numbers and Isospin multiplet

V. Belyaev Friday 09:45h Session: Baryons

LHCb

12 / 24

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A Doubly Charmed Baryon Ξ_{cc}^{++} at LHCb [PRL119(2017)112001]



with $\Lambda_c^+ \to p K^- \pi^+$







A Doubly Charmed Baryon Ξ_{cc}^{++} at LHCb [PRL119(2017)112001]



Adding $K^{-}\pi^{+}\pi^{+}$, background suppression using neural network



2016 Dataset: 1.7 fb⁻¹, 13 TeV
 \(\mathcal{E}_{cc}^{++}\) reconstructed entirely online in the trigger



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Heavy Quark Baryons and Exotica

Hadron2017, Salamanca

13 / 24

Comparison with Selected Theory Predictions

| Method | $m_{\Xi_{cc}^{++}}[MeV/c^2]$ | Reference |
|--------------------------|--------------------------------------|-------------------------------------|
| Experiment | $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ | [PRL119(2017)112001] |
| Effective potential | 3627 ± 12 | [PRD90(2014)094007] |
| Relativized Quark Model | 3613 | arXiv:1708.04468 |
| Relativistic Quark Model | 3620 | [PRD66(2002)014008] |
| Lattice QCD | $3610\pm23\pm22$ | [PRD90(2014)094507] |
| HQ effective theory | 3610 | [Pr. Part. Nucl. Phys. 33(1994)787] |

- Excellent agreement in several theoretical approaches!
- Comment on Selex observation:
 - Observation of Ξ_{cc}^+ at m = $3519 \pm 2 \,\text{MeV/c}^2$ can't be Isospin partner [PRL89(2002)112001][PLB628(2005)18]
 - Low statistics (yields 15.9 and 5.62 events)
 - Short lifetime 33 fs and too large production xsection

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Heavy Baryons with Hidden Charm





6D Amplitude analysis allows to measure resonance parameters



| State | Mass [MeV] | Width [MeV] | JP |
|-------------------|--------------------------|-----------------|-----------|
| $P_{c}(4380)^{+}$ | $4380\pm8\pm29$ | $205\pm18\pm86$ | $3/2^{-}$ |
| $P_{c}(4450)^{+}$ | $4449.8 \pm 1.7 \pm 2.5$ | $39\pm5\pm19$ | $5/2^{+}$ |

Spin parity assignment not unique

Excluded: same parity solution

Results confirmed in two subsequent analyses

- $\label{eq:Ab} \Lambda_{\rm b} \to J/\psi p K \text{ moments analysis} \\ \text{[PRL117(2016)082002]}$
- $\Lambda_{\rm b} \rightarrow {\rm J}/\psi_{\rm P}\pi$ amplitude analysis [PRL117(2016)082003]



Pentaquark Models

Many contributions Session: Exotics

Proximity of thresholds suggests two-body contributions



| Closeby thresholds | | |
|---|-------------------|--------------------------|
| [MeV] | $P_{c}(4380)^{+}$ | $P_{c}(4450)^{+}$ |
| Mass | $4380\pm8\pm29$ | $4449.8 \pm 1.7 \pm 2.5$ |
| $\Sigma_{c}^{*+}\overline{D}^{0}$ | 4382.3 ± 2.4 | |
| $\chi_{c1}(1P)p$ | | 4448.93 ± 0.07 |
| $\Lambda_{c}^{+*}\overline{D}^{0}$ | | 4457.09 ± 0.35 |
| $\Sigma_{c}\overline{D}^{0*}$ | | 4459.9 ± 0.5 |
| $\Sigma_{\rm c}\overline{\rm D}^0\pi^0$ | | 4452.7 ± 0.5 |
| [EPJ A51(2015)11,152] | | |

| Rescattering | Hadronic molecules | Tightly bound states |
|------------------|---|----------------------------|
| kinematic effect | loosely bound system | constituents |
| | of color-singlets | carrying color (di-quarks) |
| above threshold | below threshold | no association |
| - | S-wave binding restricts J ^P | large multiplets |





Pentaquark Models

Many contributions Session: Exotics

 $P_{a}(4450)^{+}$

Closeby thresholds

 $P_{-}(4380)^{+}$

Proximity of thresholds suggests two-body contributions

| | [] | - (() | - (|
|-------------------|--|-----------------|--------------------------|
| | Mass | $4380\pm8\pm29$ | $4449.8 \pm 1.7 \pm 2.5$ |
| | $\Sigma_{c}^{*+}\overline{D}^{0}$ | 4382.3 ± 2.4 | |
| \longrightarrow | $\chi_{c1}(1P)p$ | | 4448.93 ± 0.07 |
| | $\Lambda_{\rm c}^{+*}\overline{\rm D}^0$ | | 4457.09 ± 0.35 |
| | $\Sigma_{c}\overline{D}^{0*}$ | | 4459.9 ± 0.5 |
| | $\Sigma_{\rm c}\overline{\rm D}^0\pi^0$ | | 4452.7 ± 0.5 |
| | | [EPJ A51(2015) | 11,152] |
| V | | | |
| 1 | | | |

[MeV]

| Rescattering | Hadronic molecules | Tightly bound states |
|------------------|---|----------------------------|
| kinematic effect | loosely bound system | constituents |
| | of color-singlets | carrying color (di-quarks) |
| above threshold | below threshold | no association |
| _ | S-wave binding restricts J ^P | large multiplets |





- Even in amplitude analyses cusps are difficult to distinguish from real resonances [PRD92(2015)071502]
- phase motion: resonance vs cusp



Add complementary data:
 Rescattering can be ruled out if there is a narrow enhancement in the elastic channel \(\chi_{c1}(1P))\) p

First observation of $\Lambda_b \to \chi_{\textbf{c}1(2)} p K$



- Even in amplitude analyses cusps are difficult to distinguish from real resonances
 [PRD92(2015)071502]
- phase motion: resonance vs cusp



Add complementary data: Rescattering can be ruled out if there is a narrow enhancement in the elastic channel \(\chi_{c1}(1P))\) p First observation of $\Lambda_{\rm b} \to \chi_{{\rm c1}(2)} p K$



[PRL119(2017)062001]

Next step: amplitude analysis

Both final states provide access to strange pentaquarks $usdc\bar{c}$

 $\Lambda_{b}^{0} \begin{pmatrix} \mathbf{b} \\ \mathbf{u} \\ \mathbf{d} \\ \mathbf{$

- **J**/ $\psi \phi$ system $\rightarrow c \bar{c} s \bar{s}$ Tetraquarks
- LHCb Analyse $B \rightarrow J/\psi \phi K$: [PRL118(2017)022003] [PRD95(2017)012002]



- Less tracks reconstruct
- Lower Ξ_{b} production cross section
- Expect comparable statistics



First Observation of $\Xi_{\mathbf{b}}^{-} \rightarrow \mathrm{J}/\psi \Lambda \mathrm{K}^{-}$

$$= (4.19 \pm 0.29 \pm 0.14) \times 10^{-2}$$

$$m(\Xi_b^-) - m(\Lambda_b)$$

= 177.08 ± 0.47 ± 0.16 MeV/c²

Use Run II data set to study $J/\psi \Lambda K^-$ amplitudes

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[PLB772(2017)265]





Beauty Baryons





$\Xi_{\mathbf{b}}$ Excitations: The $\Xi_{\mathbf{b}}^{0*}(5945)$

- Discovered by CMS [PRL108(2012)252002]
- in $\Xi_{\mathbf{b}}^{0*} \to \Xi_{\mathbf{b}}^{-} \pi^{+}$ with $\Xi_{\mathbf{b}}^{-} \to J/\psi \Xi^{-}$ and $\Xi^{-} \to \Lambda^{0} \pi^{-}$
- Compatible with J⁺ = 3/2⁺ state No other states seen in Ξ⁻_bπ⁺
- Precise measurement of mass and width at LHCb [JHEP05(2016)161]

■ in
$$\Xi_{\mathbf{b}}^{0*} \to \Xi_{\mathbf{b}}^{-} \pi^{+}$$
 with $\Xi_{\mathbf{b}}^{-} \to \Xi_{\mathbf{c}}^{0} \pi^{-}$
and $\Xi_{\mathbf{c}}^{0} \to \mathrm{pK}^{-} \mathrm{K}^{-} \pi^{+}$

$$\mathbf{m}(\Xi_{\mathbf{b}}^{0*}) - \mathbf{m}(\Xi_{\mathbf{b}}^{-}) - \mathbf{m}(\pi^{+}) = 15.727 \pm 0.068 \pm 0.023 \, \mathbf{MeV}$$

$$\Gamma(\Xi_{\mathbf{b}}^{0*}) = 0.90 \pm 0.16 \pm 0.08 \, \mathbf{MeV}$$





Isospin partners: Two Ξ_b^- Excitations [PRL114(2015)062004]

- Two excited Ξ_{b}^{-} states found at LHCb
- in $\Xi_{\mathbf{b}}^{-*} \to \Xi_{\mathbf{b}}^{0} \pi^{-}$ with $\Xi_{\mathbf{b}}^{0} \to \Xi_{\mathbf{c}}^{+} \pi^{-}$ and $\Xi_{\mathbf{c}}^{+} \to \mathrm{pK}^{-} \pi^{+}$
- decay angle distributions compatible with quark-model spin assignments
- Ξ_{b}^{-*} and Ξ_{b}^{0*} isospin partners Isospin-splitting $\delta m_{iso} \approx 2.3 \text{ MeV}$
- Isospin partner to Ξ^{-'}_b below Ξ⁻_bπ⁺ threshold?



| | $\delta {f M}[{f MeV}]$ | $\Gamma[MeV]$ | JP |
|---------------------|-----------------------------|--------------------------|-----------|
| $\Xi_{b}^{-\prime}$ | $3.653 \pm 0.018 \pm 0.006$ | < 0.0895%C.L. | $1/2^{+}$ |
| Ξ_{b}^{-*} | $23.96 \pm 0.12 \pm 0.06$ | $1.65 \pm 0.31 \pm 0.10$ | $3/2^{+}$ |





- LHC is a heavy quark baryon factory
- **Discovery of 5 new** Ω_{c} states and
- **Doubly charmed** Ξ_{cc}^{++} this year's highlight
- Impressive success for theory
- More puzzles to solve which role do multiquark states play in the baryon spectrum?





Backup





More Theory Predictions

| Reference | Value (MeV) | Method |
|--------------------------------------|----------------------|---|
| [Karliner and Rosner, 2014] | 3627 ± 12 | |
| [De Rujula et al., 1975] | 3550 - 3760 | QCD-motivated quark model |
| J. Bjorken (unpublished draft, 1986) | 3668 ± 62 | QCD-motivated quark model |
| [Anikeev et al., 2001] | 3651 | QCD-motivated quark model |
| [Fleck and Richard, 1989] | 3613 | Potential and bag models |
| [Richard, 1994] | 3630 | Potential model |
| [Korner et al., 1994] | 3610 | Heavy quark effective theory |
| [Roncaglia et al., 1995] | 3660 ± 70 | Feynman-Hellmann + semi-empirical |
| [Lichtenberg et al., 1996] | 3676 | Mass sum rules |
| [Ebert et al., 1997] | 3660 | Relativistic quasipotential quark model |
| [Silvestre-Brac, 1996] | 3607 | Three-body Faddeev equations. |
| [Gerasyuta and Ivanov, 1999] | 3527 | Bootstrap quark model + Faddeev eqs. |
| [Itoh et al., 2000] | ucc: 3649 ± 12 , | |
| | $dcc: 3644 \pm 12$ | Quark model |
| [Kiselev and Likhoded, 2002a] | 3480 ± 50 | Potential approach + QCD sum rules |
| [Narodetskii and Trusov, 2002] | 3690 | Nonperturbative string |
| [Ebert et al., 2002] | 3620 | Relativistic quark-diquark |
| | | • |

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| Reference | Value (MeV) | Method |
|----------------------------|---------------------------------|---|
| [He et al., 2004] | 3520 | Bag model |
| [Richard and Stancu, 2005] | 3643 | Potential model |
| [Migura et al., 2006] | 3642 | Relativistic quark model + Bethe-Salpeter |
| [Albertus et al., 2007b] | 3612^{+17} | Variational |
| [Roberts and Pervin, 2008] | 3678 | Quark model |
| [Weng et al., 2011] | 3540 ± 20 | Instantaneous approx. + Bethe-Salpeter |
| [Zhang and Huang, 2008] | 4260 ± 190 | QCD sum rules |
| [Lewis et al., 2001] | $3608(15)({13\atop 35})$, | |
| | $3595(12)\binom{21}{22}$ | Quenched lattice |
| [Flynn et al., 2003] | 3549(13)(19)(92) | Quenched lattice |
| [Liu et al., 2010] | $3665 \pm 17 \pm 14^{+0}_{-78}$ | Lattice, domain-wall + KS fermions |
| [Namekawa, 2012] | 3603(15)(16) | Lattice, $N_f = 2 + 1$ |
| [Alexandrou et al., 2012] | 3513(23)(14) | LGT, twisted mass ferm., m_{π} =260 MeV |
| [Briceno et al., 2012] | 3595(39)(20)(6) | LGT, $N_f=2+1,\ m_\pi=200$ MeV |
| [Alexandrou et al., 2014] | 3568(14)(19)(1) | LGT, ${N_f}=2+1$, $m_{\pi}=210$ MeV |

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Rescattering: hadronic loops

[PRD92(2015)071502]





Nonrelativistic loop integral:

$$\mathbf{G}_{\Lambda}(\mathbf{E}) = \int \frac{\mathbf{d}^{3}\mathbf{q}}{\left(2\pi\right)^{3}} \frac{\vec{\mathbf{q}}^{2} \, \mathbf{f}_{\Lambda}(\vec{\mathbf{q}}^{2})}{\mathbf{E} - \mathbf{m}_{1} - \mathbf{m}_{2} - \vec{\mathbf{q}}^{2}/2\mu}$$

with a form factor $f_{\Lambda}(\vec{q}^2)$.

Triangle Singularity given by Landau-equation

$$1 + 2\mathbf{y}_{12}\mathbf{y}_{23}\mathbf{y}_{13} = \mathbf{y}_{12}^2 + \mathbf{y}_{23}^2 + \mathbf{y}_{13}^2$$

$$\textbf{y}_{\textbf{i}\textbf{j}} = \left(\textbf{m}_{\textbf{i}}^2 + \textbf{m}_{\textbf{j}}^2 - \left(\textbf{p}_{\textbf{i}} + \textbf{p}_{\textbf{j}}\right)^2\right) / 2\textbf{m}_{\textbf{i}}\textbf{m}_{\textbf{j}}$$





Rescattering: hadronic loops

[PRD92(2015)071502]



Nonrelativistic loop integral:

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with a form factor $f_{\Lambda}(\vec{q}^2)$.

Triangle Singularity given by Landau-equation

$$1 + 2\mathbf{y}_{12}\mathbf{y}_{23}\mathbf{y}_{13} = \mathbf{y}_{12}^2 + \mathbf{y}_{23}^2 + \mathbf{y}_{13}^2$$

$$\mathbf{y}_{ij} = \left(\mathbf{m}_{i}^{2} + \mathbf{m}_{j}^{2} - \left(\mathbf{p}_{i} + \mathbf{p}_{j}\right)^{2}\right)/2\mathbf{m}_{i}\mathbf{m}_{j}$$



LHCb