Precision Calculations for FCC-ee EW Observables

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1. Introduction
2. Electroweak Precision Observables
3. Higgs Observables
4. Conclusions

1. Introduction

Experimental situation:

LHC/ILC/FCC-ee/CEPC/. . . will provide (high!) accuracy measurements!

Theory situation:

– Measurements are performed using theory predictions

– measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, . . .)

Full uncertainty is given by the (linear) sum of experimental and theoretical uncertainties!

Write-up in preparation:

> **Theoretical uncertainties for electroweak and Higgs-boson precision measurements at the FCC-ee**
>
> Conveners: A. Freitas[1], S. Heinemeyer[2],
> Contributors: M. Beneke[3], A. Blondel[4], A. Hoang[5], P. Janot[6], J. Reuter[7], C. Schwinn[8], and S. Weinzierl[9]

⇒ will go into CDR ?!

⇒ should be taken into account by other (exp) groups!

Where we need theory prediction:

1. Prediction of the measured quantity

   Example: M_W

   $\to$ at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity

   Example: $e^+e^- \to W^+W^-$

   $\to$ better than then "pure" experimental precision

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Two types of theory uncertainties:

1. intrinsic: missing higher orders

2. parametric: uncertainty due to exp. uncertainty in SM input parameters
   Example: m_t, m_b, α_s, $\Delta\alpha_{\text{had}}$, ...

Options for the evaluation of intrinsic uncertainties:

1. Determine all prefactors of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the loop is $\mathcal{O}(1)$

2. Take the known contribution at n-loop and $(n-1)$-loop and thus estimate the $n+1$-loop contribution:

$$\frac{(n+1)^{(\text{estimated})}}{n^{(\text{known})}} \approx \frac{n^{(\text{known})}}{(n-1)^{(\text{known})}}$$

   ⇒ simplified example! Has to be done
   "coupling constant by coupling constant"

3. Variation of $\mu^{\overline{\text{DR}}}$ (QCD!, EW?)

4. Compare different renormalizations

⇒ Mostly used here: 1 & 2

Our future estimates:

- assume to go substantially beyond what is known now
- assume that many theorists will put many^2 hours of work into it (motivation?)
- do not assume that magically new calculational methods are invented
- are overall optimistic

⇒ they should be taken seriously!

Saying "Ah, theorists will have to work a bit harder and solve this"
is not a realistic option!

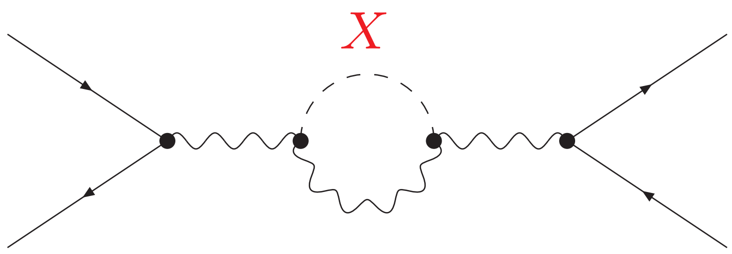
2. Electroweak Precision Observables

Comparison of observables with theory:

Precision data: $M_W, \sin^2\theta_{\rm eff}, a_\mu, \; M_h$	$\leftrightarrow$	Theory: SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. X



$\Downarrow$

SM: limits on M_H, BSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed
$\Rightarrow$ only models "ready" so far: SM, MSSM

The EWPO:

M_W

$$\sigma^0_{\text{had}} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}], \qquad \text{(from a fit to } \sigma_f(s) \text{ at various values of } s\text{)}$$

$$R_\ell = \left[\textstyle\sum_q \sigma_q(M_Z^2)\right] / \sigma_\ell(M_Z^2), \qquad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2) / \left[\textstyle\sum_q \sigma_q(M_Z^2)\right], \qquad (q = b, c)$$

$$A^f_{\text{FB}} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\text{LR}} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|$$

$\sin^2\theta^\ell_{\text{eff}}$ from A^f_{FB} and A^f_{LR}

Intrinsic uncertainties: $\Rightarrow$ always a limiting factor!

Quantity	FCC-ee	Current intrinsic unc.		Projected unc.
M_W [MeV]	1	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.2
R_b [10^{-5}]	6	15	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$	7
R_l [10^{-3}]	1	5	$(\alpha^2_{\rm bos}, \alpha^3, \alpha^2\alpha_s)$	1.5

These calculations are required for the projection:

– complete $\mathcal{O}\left(\alpha\alpha_s^2\right)$ corrections
– fermionic $\mathcal{O}\left(\alpha^2\alpha_s\right)$ corrections
– double-fermionic $\mathcal{O}\left(\alpha^3\right)$ corrections
– leading four-loop corrections enhanced by the top Yukawa coupling

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

Parametric uncertainties:

1. M_H: better than 50 MeV $\Rightarrow$ negligible

2. M_Z: $\sim$ 0.1 MeV with negligible theory uncertainties
   $\Rightarrow$ negligible

3. $\alpha_s(M_Z)$: from (mainly) R_ℓ
   $\delta\alpha_s^{\text{exp}} \sim 10^{-4}$, $\delta\alpha_s^{\text{theo}} \sim 1.5 \times 10^{-4}$

4. m_t: from threshold scan
   $\delta m_t^{\text{exp}} \sim \mathcal{O}(10\ \text{MeV})$
   $\delta m_t^{\text{theo}} \sim 50$ MeV (NNNLO/NNLL $\oplus$ $1S \to \overline{\text{MS}}$ $\oplus$ $\delta\alpha_s$)

5. m_b: from lattice calculations
   $\delta m_b \sim 10$ MeV (still under discussion, too optimistic?)

6. $\Delta\alpha_{\text{had}}$: BES III and Belle II: $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$
   better from measurements "around the Z pole?

SM input: $\Delta\alpha_{\text{had}}$ $\Rightarrow$ could be limiting factor!

From $e^+e^- \to$ had. using dispersion relation

today: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
possible improvement in the future: $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$

Direct determination at FCC-ee from $e^+e^- \to f\bar{f}$ off the Z peak

[*P. Janot '15*]
possible improvement in the future: $\delta(\Delta\alpha_{\text{had}}) \sim 2 \times 10^{-5}$ $\Rightarrow$ TU neglected

Calculation of $e^+e^- \to f\bar{f}$ needed at 3-loop and beyond: [*A. Freitas '16*]
current techniques (2L/3L): corrections of $\sim 10^{-3}$
new calculation methods (2L/3L): corrections of $\sim 10^{-4}$
unknown methods 3L: $\lesssim 10^{-5}$
unknown methods 4L: $\sim 10^{-5}$
(+ higher-orders in real photon emission)
$\Rightarrow$ improvement unclear $\Rightarrow \delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$

Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \to W^{(*)}W^{(*)}$, but $e^+e^- \to WW \to$ 4f needed

Current status:

full one-loop for $2 \to 4$ process

[*A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02*]

$\Rightarrow$ extraction of M_W at the level of $\sim$ 6 MeV

Most recent improvement:

leading 2L corrections from EFT

[*Actis, Beneke, Falgari, Schwinn '08*]

$\Rightarrow$ impact on M_W at the level of $\sim$ 3 MeV

$\Rightarrow$ full 2L for $2 \to 4$ process not foreseeable

Potentially possible:

2L resummed higher-order terms for $e^+e^- \to WW$ and $W \to ff'$

$\Rightarrow$ extraction of M_W at $\sim$ 1 MeV??

Summary of future parametric uncertainties:

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$1 - 1.5$	1 (0.6)	$\delta(\Delta\alpha_{\rm had})$
$\sin^2\theta^{\ell}_{\rm eff}$ [10^{-5}]	0.6	2 (1)	$\delta(\Delta\alpha_{\rm had})$
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$

$\delta(\Delta\alpha_{\rm had}) = 5(3) \times 10^{-5}$

$\Rightarrow$ add quadratic to experimental uncertainties!

$\Rightarrow$ add linearly to intrinsic uncertainties!

3. Higgs observables: Higgs couplings

Initial measurement: $\sigma \times$ BR

recoil method: $e^+e^- \to ZH$, $Z \to e^+e^-, \mu^+\mu^-$

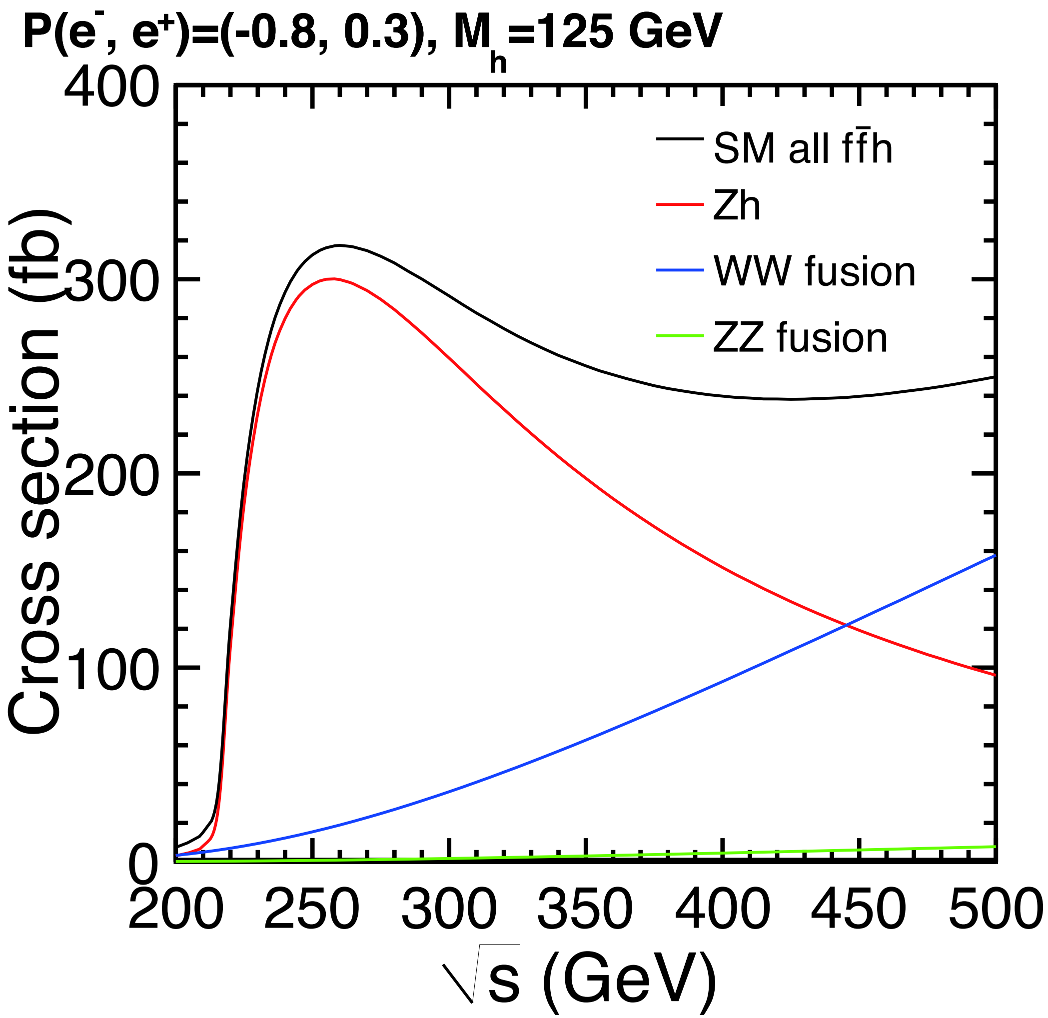
$\Rightarrow$ measurement of the Higgs production cross section

$\Rightarrow$ NO additional theoretical assumptions needed for absolute determination of partial widths

$\Rightarrow$ indirect measurement of total width

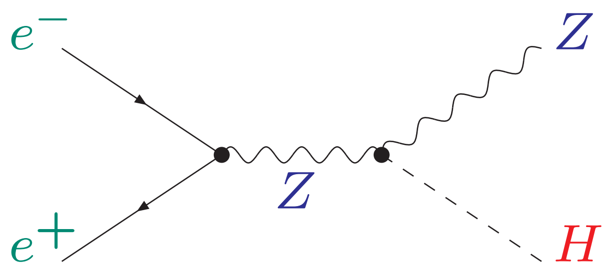
$\Rightarrow$ direct extraction of partial widths (couplings)

Higgs production cross sections:



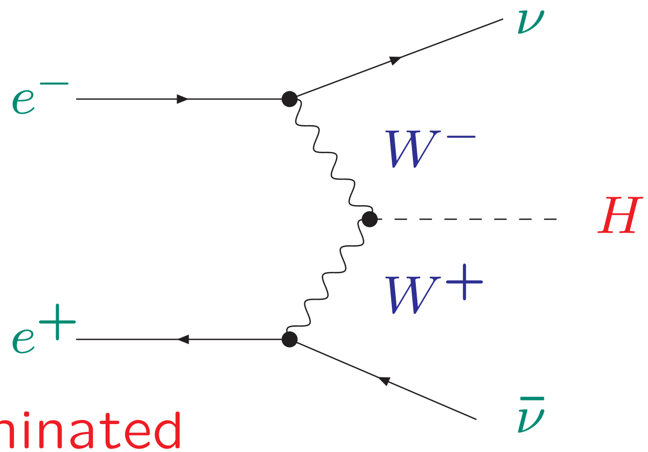
Higgs-strahlung:

$e^+e^- \to Z^* \to ZH$



weak boson fusion (WBF):

$e + e- \to \nu\bar{\nu}H$



FCC-ee: $\sqrt{s} \sim 250$ GeV, Higgs-strahlung dominated

$\underline{e^+e^- \to ZH}$:

$\delta\sigma_{HZ}^{\text{exp}} \sim 0.4\%$

full one-loop available, corrections of 5-10%

rough estimate: $\delta\sigma_{HZ}^{\text{theo}} \sim 1\%$ from missing two-loop corrections

Two-loop corrections for $2 \to 2$ can in principle be done . . .

$\Rightarrow$ theory uncertainties sufficiently small

$\underline{e^+e^- \to \nu\bar{\nu}H}$:

small contribution . . .

Partial two-loop calculation (with closed fermion loops)
can in principle be done . . .

$\Rightarrow$ theory uncertainties sufficiently small

Decay width theoretical uncertainties: General recipe:

[LHCHXSWG BR group '15]

1. Parametric Uncertainties: $p \pm \Delta p$
   - Evaluate partial widths and BRs with p, $p + \Delta p$, $p - \Delta p$ and take the differences w.r.t. central values
   - Upper $(p + \Delta p)$ and lower $(p - \Delta p)$ uncertainties summed in quadrature to obtain the Combined Parametric Uncertainty

2. Theoretical Uncertainties:
   - Calculate uncertainty for partial widths and corresponding BRs for each theoretical uncertainty
   - Combine the individual theoretical uncertainties linearly to obtain the Total Theoretical Uncertainty

   $\Rightarrow$ estimate based on "what is included in the codes"!

3. Total Uncertainty:

   Linear sum of the Combined Parametric Uncertainty and the Total Theoretical Uncertainties

Intrinsic uncertainties for decay widths:

"FCC-ee" = expected precision on g^2_{Hxx}

Partial width	QCD	electroweak	total	future	FCC-ee
$H \to b\bar{b}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	$\sim 1.0\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$	$\sim 0.2\%$	$\sim 1.7\%$
$H \to \tau^+\tau^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	$\sim 1.3\%$
$H \to \mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$	$< 0.1\%$	$\sim 15\%$
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$	$\sim 1\%$	$\sim 2\%$
$H \to \gamma\gamma$	$< 0.1\%$	$< 1\%$	$<1\%$	$< 1\%$	$\sim 3.6\%$
$H \to Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$	$\sim 1\%$	
$H \to WW \to 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.4\%$	$\sim 0.5\%$
$H \to ZZ \to 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$	$\lesssim 0.3\%$	$\sim 0.4\%$
Γ_{tot}				$\sim 0.3\%$	$\sim 1\%$

$\Rightarrow$ non-negligible for $H \to WW/ZZ \to 4f$

Future parametric uncertainties for decay widths:

decay	fut. intr.	fut. para. m_q	para. α_s	para. M_H	FCC-ee
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 1.0\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.7\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.3\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 15\%$
$H \to gg$	$\sim 1\%$		0.5%	–	$\sim 2\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.6\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.4\%$	–	–	$\sim 0.1\%$	$\sim 0.5\%$
$H \to ZZ$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

Γ_{tot} applies "to all" (partial cancelations . . .)
$\Rightarrow$ non-negligible in particular for $H \to WW/ZZ \to 4f$ (δm_b optimistic?)

One more word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO/Γ_H can be obtained.

We know that BSM physics must exist! (DM, gravity, . . .)

As soon as BSM physics will be discovered, an evaluation of the EWPO/Γ_H in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

4. Conclusions

- The full uncertainty of a measured quantity is given by the (linear) sum of experimental and theoretical uncertainties!

- We give (realistic/optimistic) estimates for future intrinsic and parametric uncertainties

- EWPO: intrinsic unc. larger than anticipated experimental unc.
  parametric unc. often larger than experimental uncertainties
  $\Rightarrow$ particularly true for M_W and $\sin^2\theta_{\text{eff}}$

- Higgs: cross section calculations can be under control
  intrinsic unc. can be relevant for $H \to WW/ZZ \to$ 4f
  parametric unc. can be relevant, in particular for $H \to WW/ZZ \to$ 4f

- Write-up is in preparation, will go into CDR
  Uncertainties should be taken into account by other (exp) groups!