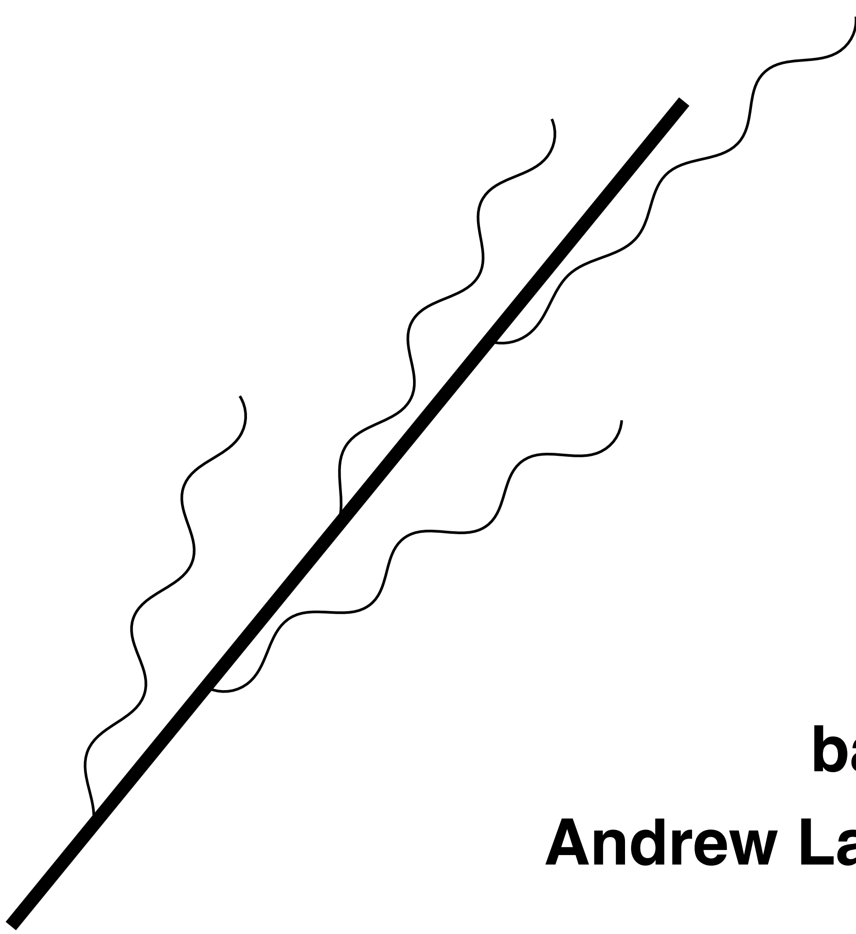
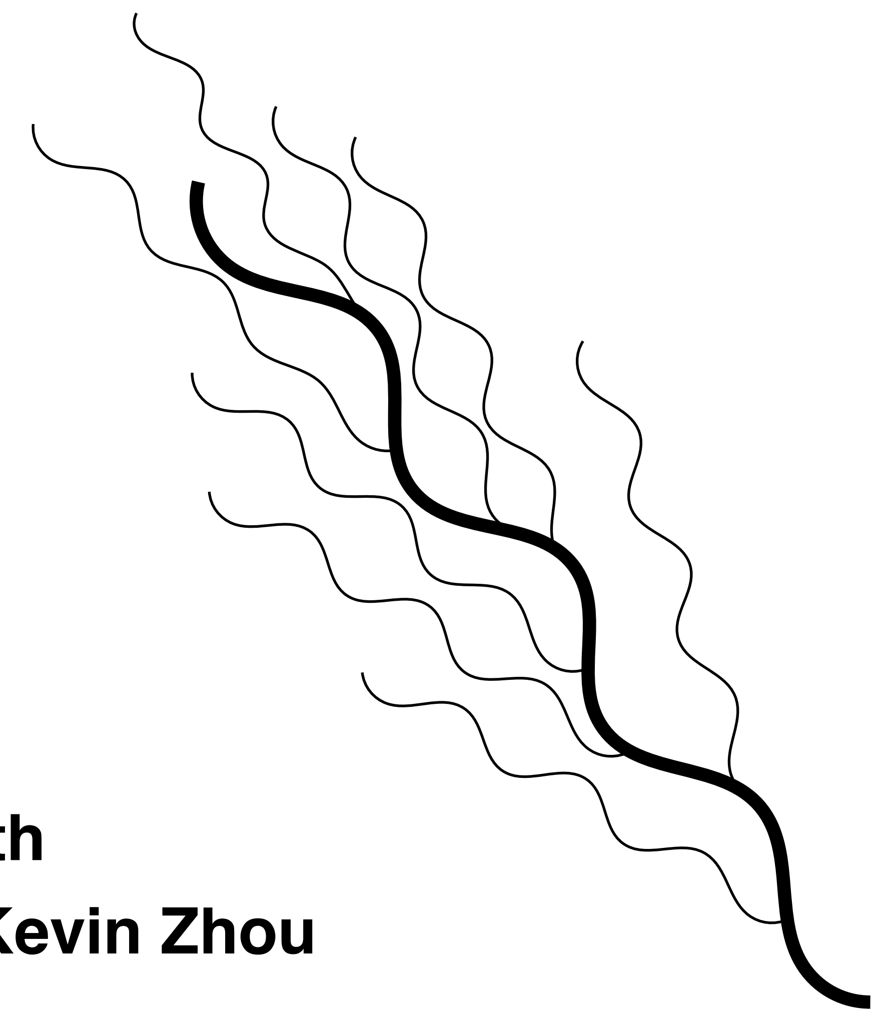
Casimir Meets Poisson:

analytic control over counting observables for quark-gluon discrimination



Christopher Frye

BOOST 2017



based on 1704.06266 with

Andrew Larkoski, Jesse Thaler, Kevin Zhou

Goals of this talk:

- To explain why counting observables perform better than Sudakov distributed observables in quark-gluon discrimination
- To demonstrate analytic control over a new counting observable: soft drop multiplicity

jet mass — soft drop multiplicity — track multiplicity

Calculability ⟷ Performance

Sudakov distributed observables

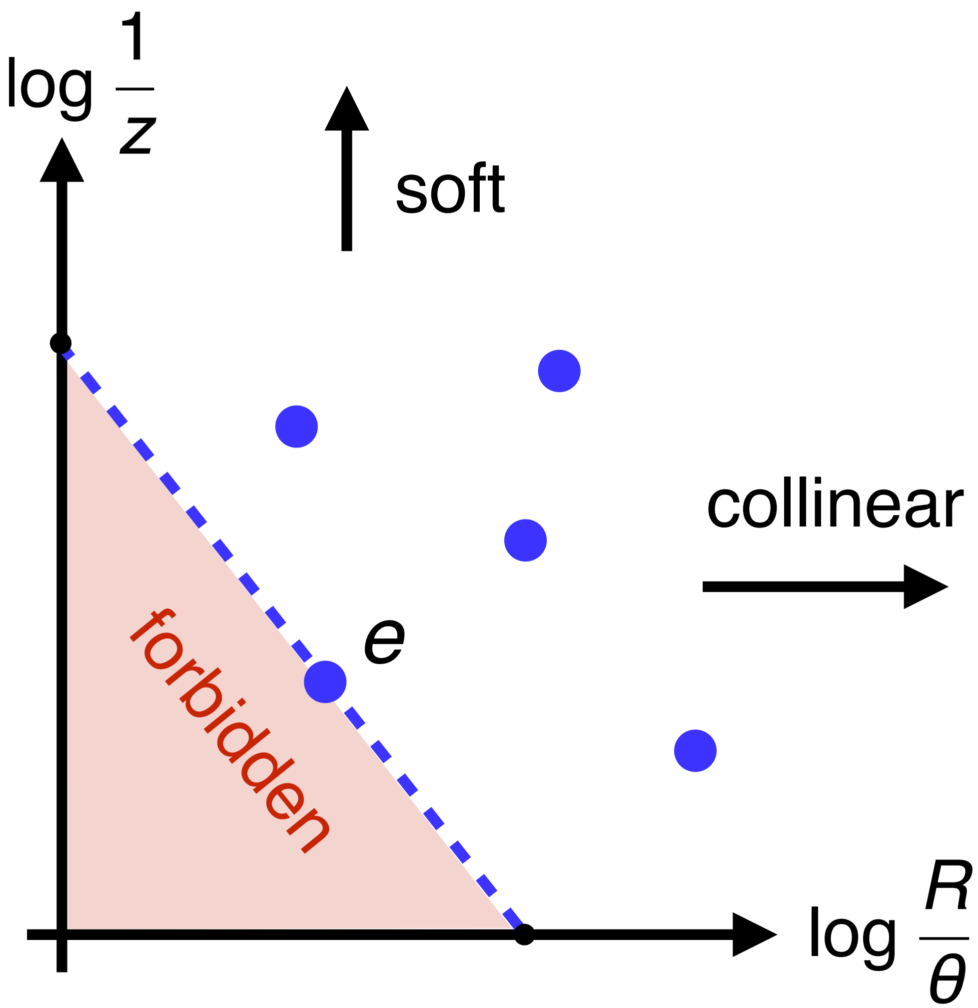
Sudakov distributed observables typically take form:

$$e = \sum_{i \in J} f(p_i)$$

in limit of soft and collinear emissions.

- infrared and collinear (IRC) safe provided f(p) is linear in p's energy
- e.g. jet mass: $m^2 = \sum_{i,j \in J} 2\, E_i\, E_j\, (1 - \cos\theta_{ij}) \to E_J \sum_{i \in J} E_i\, \theta_i^2$
- good analytic control but poor quark-gluon discrimination

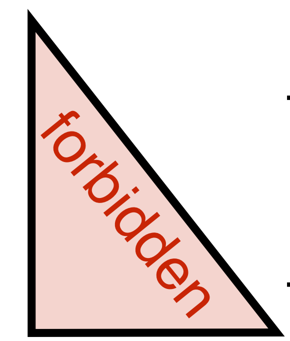
Sudakov suppression



emission probability at LL from parton of flavor i = q or g:

$$dP_i = \frac{2\,\alpha_s\,C_i}{\pi}\,\frac{dz}{z}\,\frac{d\theta}{\theta}$$

cumulative probability distribution:

$$\Sigma_i(e) = \exp\left[-\frac{2\,\alpha_s\,C_i}{\pi}\right.$$  $$\left.\vphantom{\frac{1}{1}}\right]$$

$$\Longrightarrow \quad \Sigma_g(e) = [\Sigma_q(e)]^{C_A/C_F}$$

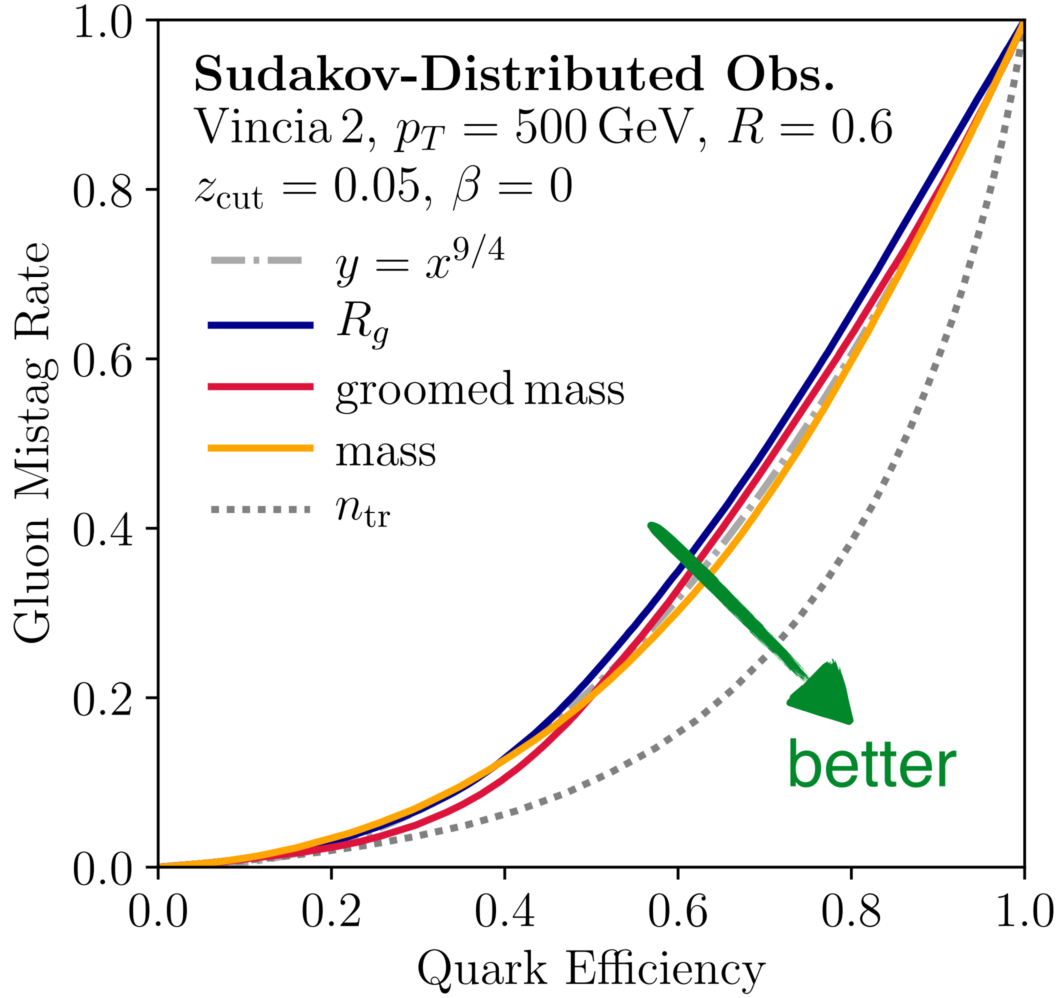
discrimination power limited by $\frac{C_A}{C_F} = \frac{9}{4}$

Counting observables

$n_{\rm tr}$ = (number of charged particles in a jet)

- good quark-gluon discrimination; a useful benchmark for our study
- not IRC safe
- cannot calculate discrimination power, but at asymptotically high energies:

$$\langle n_{\rm tr}\rangle_g \sim \frac{C_A}{C_F}\langle n_{\rm tr}\rangle_q$$



Ellis, Stirling, Webber
QCD and Collider Physics (1996)

Iterated soft drop

see Frederic Dreyer's talk for "recursive soft drop"

algorithm's parameters: $z_{\rm cut}\,,\,\beta\,,\,\theta_{\rm cut}$
used to define variables: $z_n\,,\,\theta_n$

- begin at trunk of C/A clustering tree with $n = 1$
- at branching into subjets i,j require

$$\theta_{ij} > \theta_{\rm cut}$$

otherwise terminate algorithm

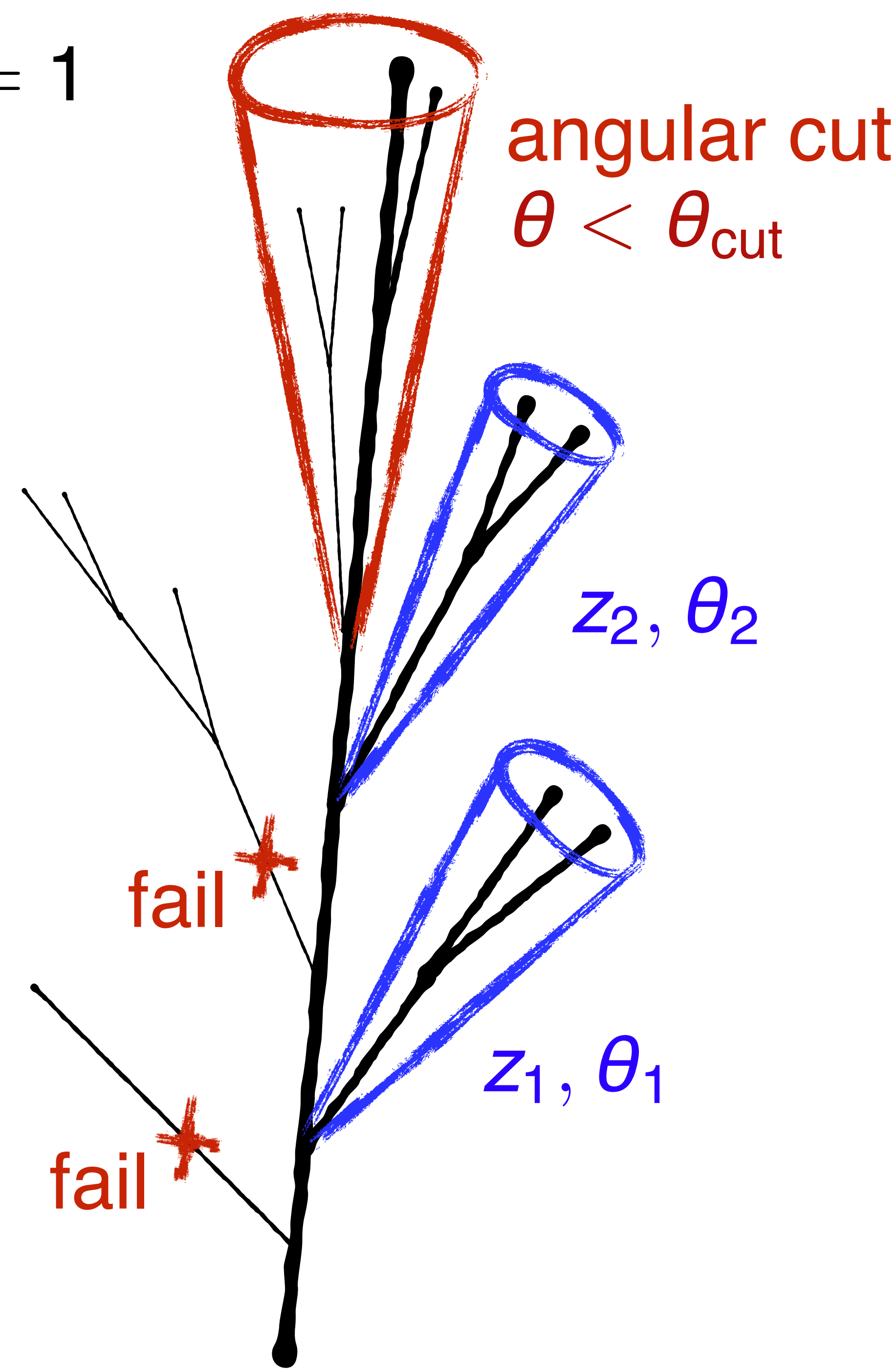
- if soft drop criterion is satisfied

$$z_{ij} > z_{\rm cut}\left(\frac{\theta_{ij}}{R}\right)^{\beta}$$

then

$$z_n = z_{ij}$$
$$\theta_n = \theta_{ij}$$
$$n \to n+1$$

- follow harder subjet i or j and recurse

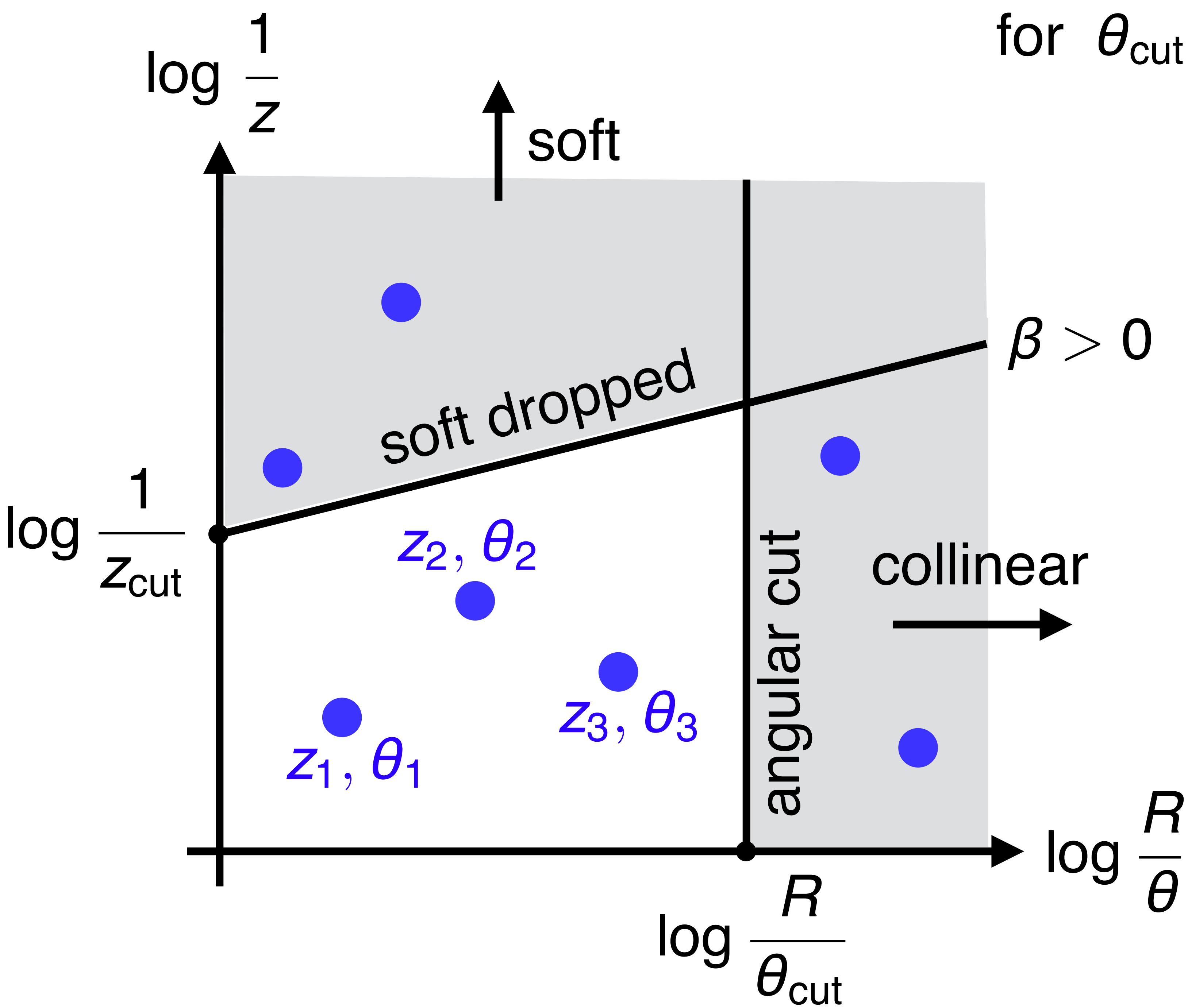


Larkoski, Marzani, Soyez, Thaler
JHEP 1405 (2014) 146

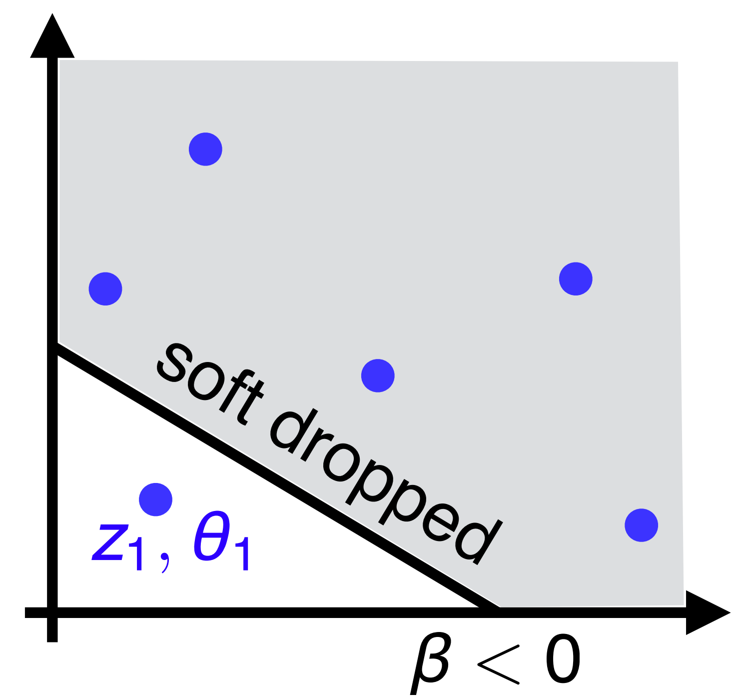
ISD phase space

IRC safety of $\{z_n\,,\ \theta_n\}$ set:

- IR: soft emissions at finite angle fail soft drop for $z_{\rm cut} > 0$
- C: collinear splittings fail soft drop for $\theta_{\rm cut} > 0$ or $\beta < 0$



see Benjamin Elder's talk for $\beta = 0,\ \theta_{\rm cut} = 0$ with GFF's



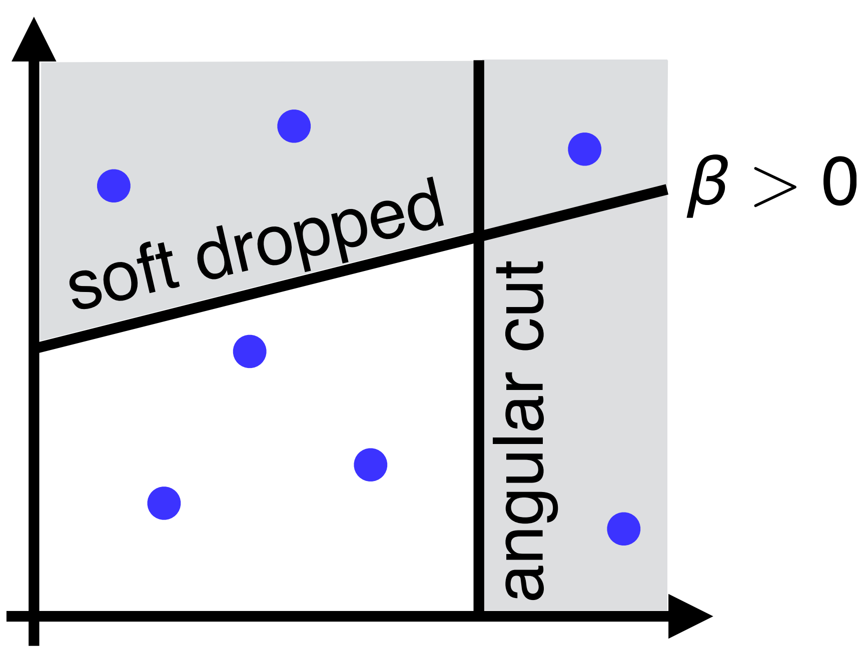
soft drop: $z > z_{\rm cut} \left(\frac{\theta}{R}\right)^{\beta}$

Soft drop multiplicity

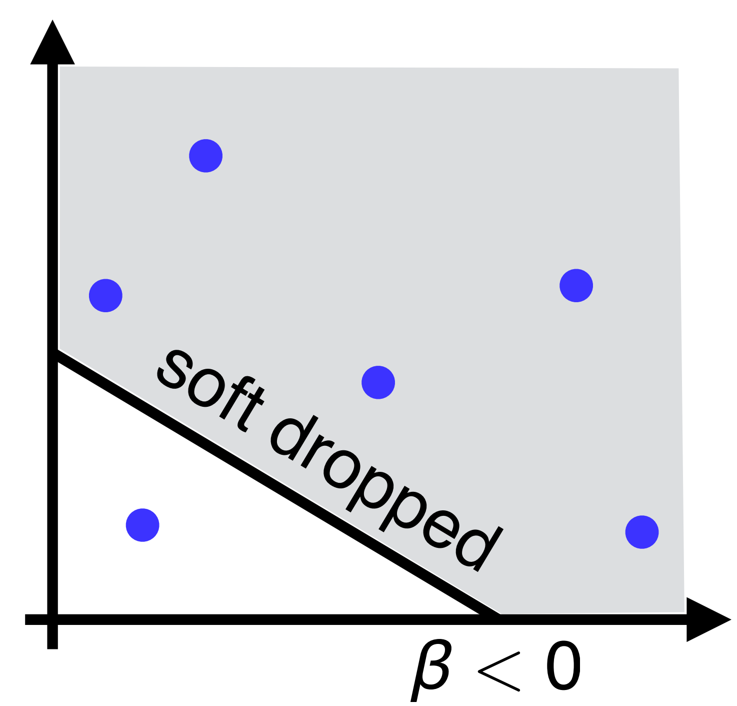
$$n_{\text{SD}} = (\text{number of emissions counted by ISD})$$

- implicit dependence on ISD parameters: $z_{\text{cut}}\,,\ \beta\,,\ \theta_{\text{cut}}$
- simply an emission count: an IRC safe counting observable amenable to pQCD calculation

$n_{\text{SD}} = 3$:

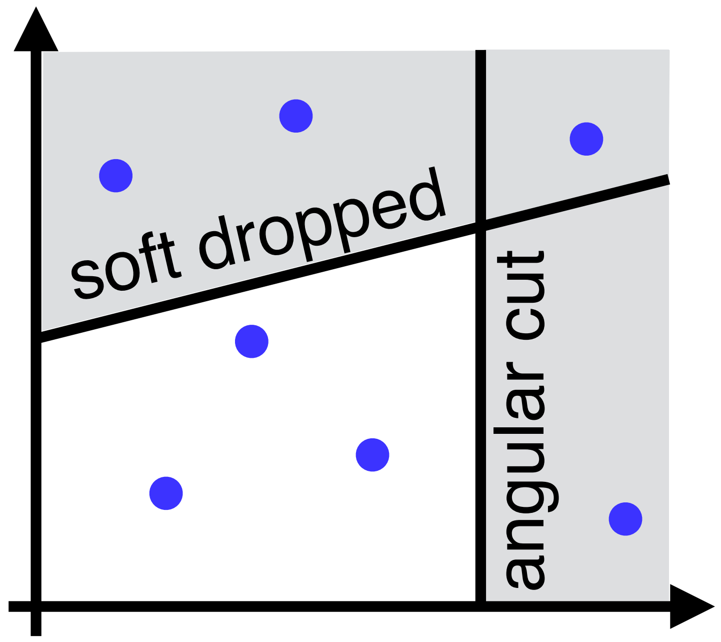


$n_{\text{SD}} = 1$:



n_{SD} as a discriminator

- could define other observables on the $\{z_n, \theta_n\}$ set
  - — e.g. weighted multiplicity $\sum_n (z_n)^\kappa$ which we have studied
- but n_{SD} contains all discriminatory info in $\{z_n, \theta_n\}$ set at LL
  - — distribution of counted emissions in (z, θ) plane has identical prediction for quarks and gluons



$$dP_i = \frac{2\,\alpha_S\, C_i}{\pi}\,\frac{dz}{z}\,\frac{d\theta}{\theta}$$

LL evolution equations

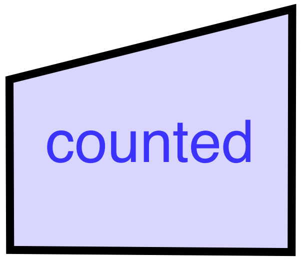
define $P_n(\theta) = (\text{probability of } n \text{ counts by ISD with } \theta_{\text{cut}} = \theta)$

evolution equations

$$P_n(\theta - \delta\theta) = P_{n-1}(\theta) \left[\frac{\delta\theta}{\theta} \int_{\substack{\text{counted} \\ \text{by ISD}}} \frac{dz}{z} \frac{2\,\alpha_S\, C_i}{\pi} \right] + P_n(\theta) \left[1 - \frac{\delta\theta}{\theta} \int_{\substack{\text{counted} \\ \text{by ISD}}} \frac{dz}{z} \frac{2\,\alpha_S\, C_i}{\pi} \right]$$

$\Longrightarrow$ solution

$$P_n = \frac{1}{n!} A^n e^{-A}, \quad \text{where} \quad A = \frac{2\,\alpha_S\, C_i}{\pi} \cdot$$

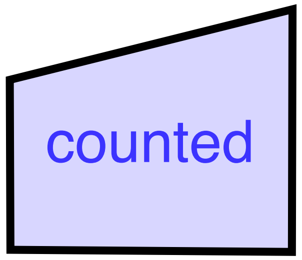


$$A = \frac{2\,\alpha_S\, C_i}{\pi} \left(\log \frac{R}{\theta_{\text{cut}}} \right) \left(\log \frac{1}{2 z_{\text{cut}}} + \frac{\beta}{2} \log \frac{R}{\theta_{\text{cut}}} \right)$$

fixed coupling

Casimir Meets Poisson

$$P_n \overset{\text{LL}}{=} \frac{1}{n!} A^n e^{-A}, \quad \text{where} \quad A = \frac{2\,\alpha_S\, C_i}{\pi} \cdot$$



mean and relative width:

$$\langle n_{\text{SD}} \rangle_g = \frac{C_A}{C_F} \langle n_{\text{SD}} \rangle_q$$

just like n_{tr}

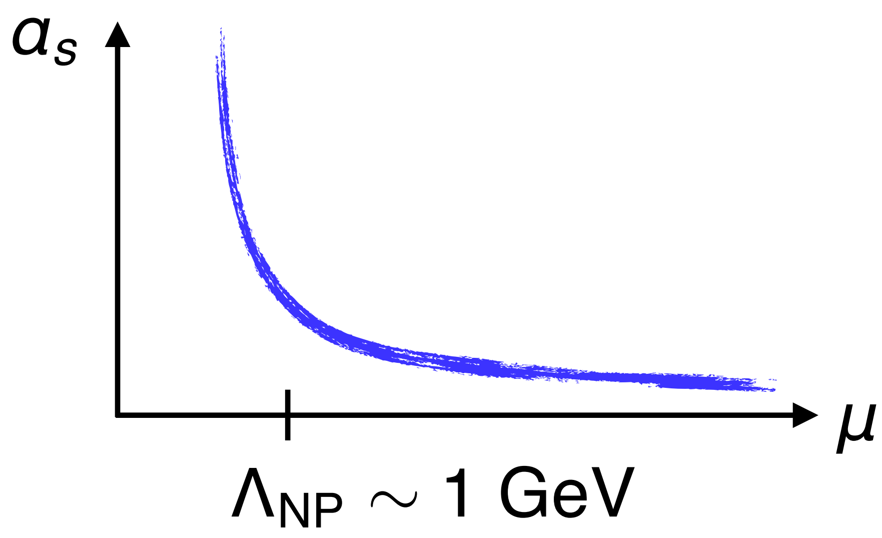
$$\frac{\Delta n_{\text{SD}}}{n_{\text{SD}}} = \frac{1}{\sqrt{\langle n_{\text{SD}} \rangle}}$$

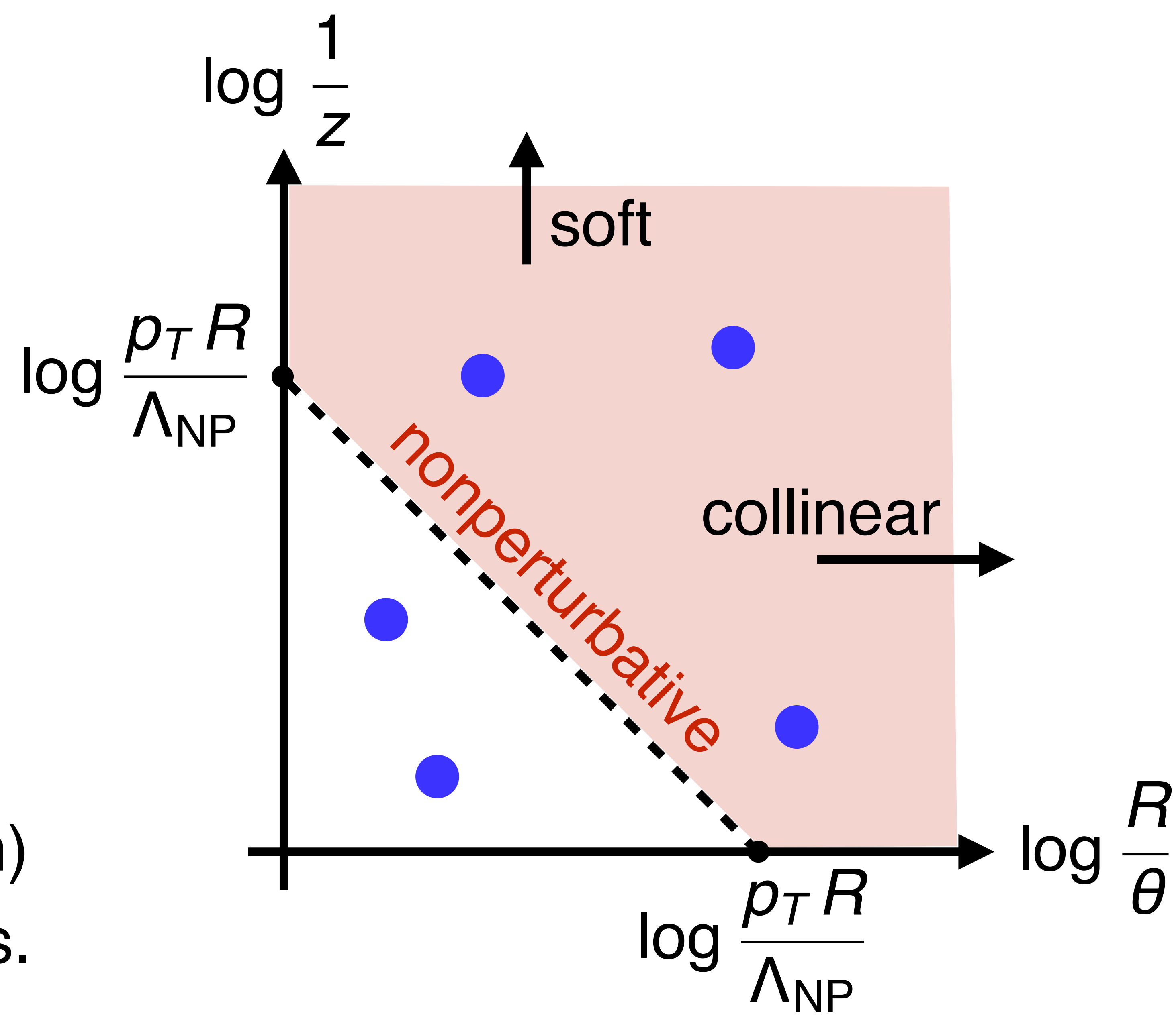
as means increase
(e.g. with ISD parameters)
relative widths decrease
$\Longrightarrow$ improved discrimination

What stops us from achieving arbitrarily good discrimination power?

nonperturbative and higher order effects…

Nonperturbative effects





Strong coupling evaluated at

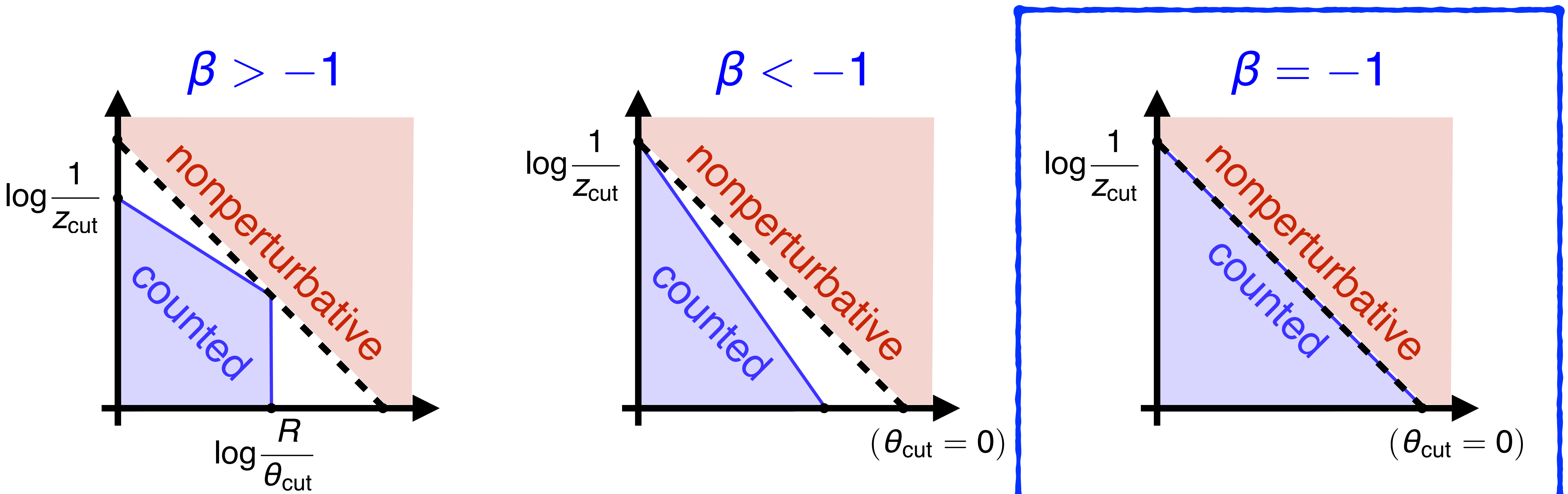
$$\mu = z\,\theta\,p_T$$

(the relative k_t of the emission) in evolution eq's to resum logs.

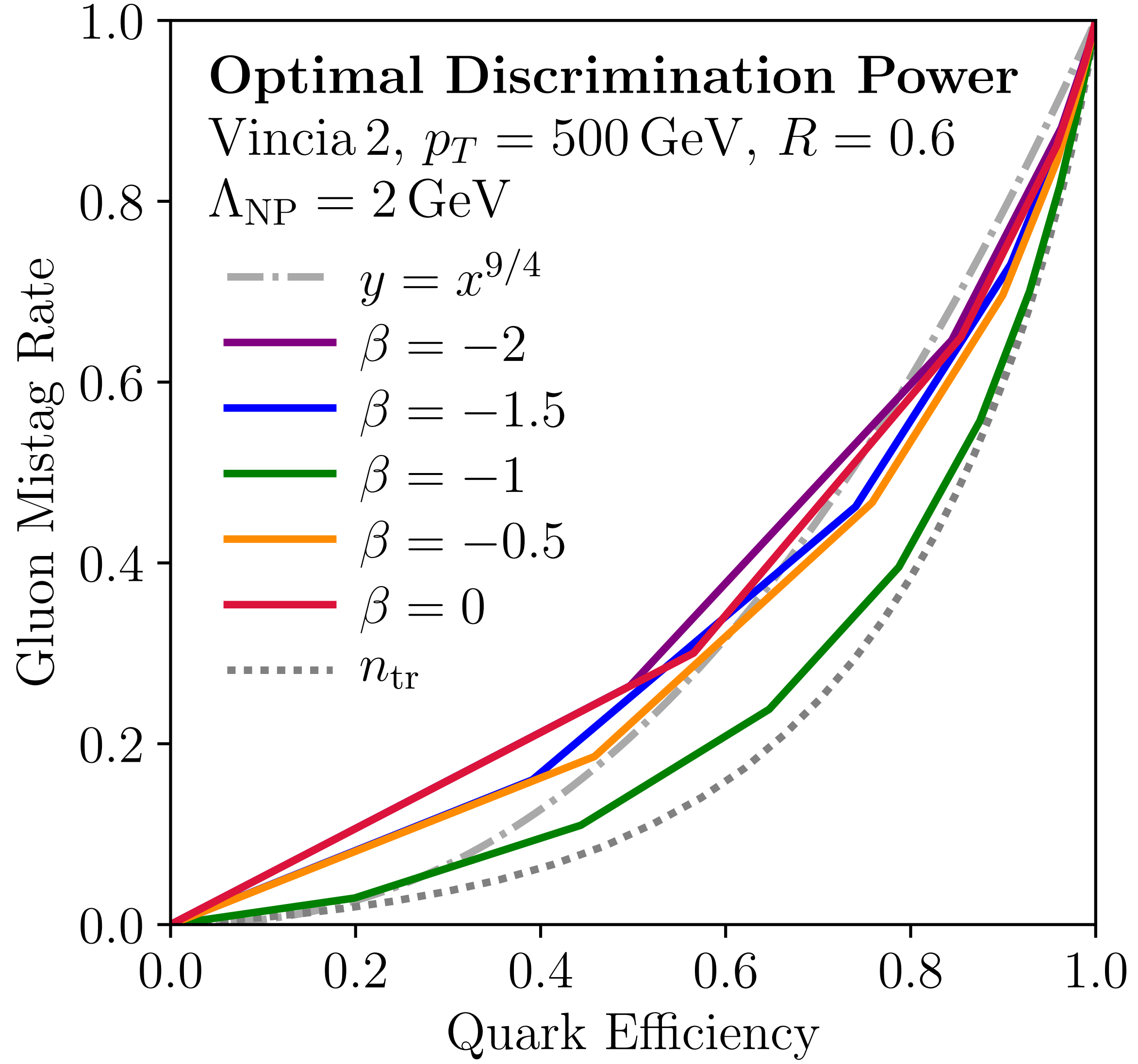
For a reliable analytic calculation, choose $z_{\rm cut}, \beta, \theta_{\rm cut}$ to avoid the nonperturbative regime.

Optimizing discrimination power

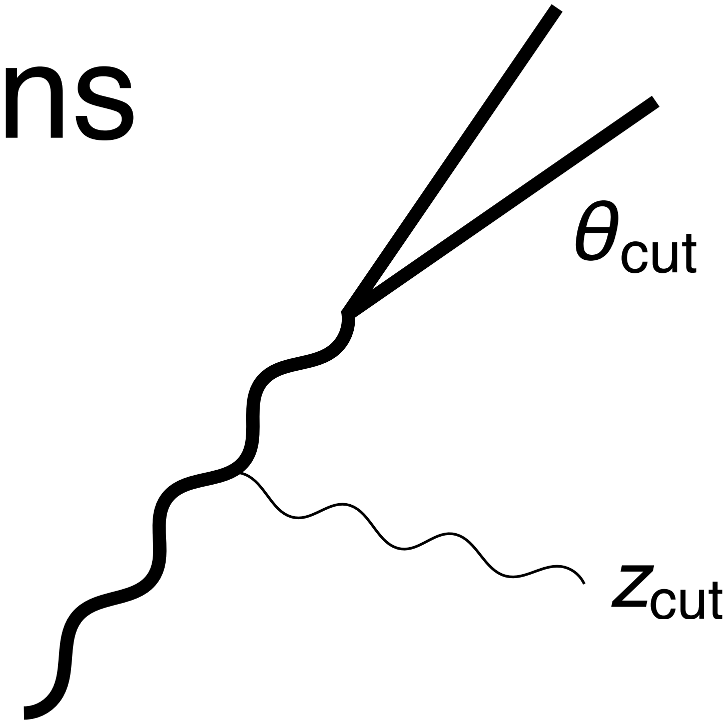
- Larger area of measured phase space leads to better discrimination power, due to: $\frac{\Delta n_{\mathrm{SD}}}{n_{\mathrm{SD}}} = \frac{1}{\sqrt{\langle n_{\mathrm{SD}} \rangle}}$
- With $\beta =$ fixed, choose $z_{\mathrm{cut}}, \theta_{\mathrm{cut}}$ to maximize perturbative area:



Optimized discrimination power: $\beta = -1$



Next-to-leading log corrections



(LL) (NLL)

$$P_n \sim \exp\left[\alpha_S^n L^{n+1} + \alpha_S^n L^n + \cdots\right]$$

- At NLL, emissions not necessarily soft,
  can be either quarks or gluons,
  must keep track of energy losses and flavor changes:

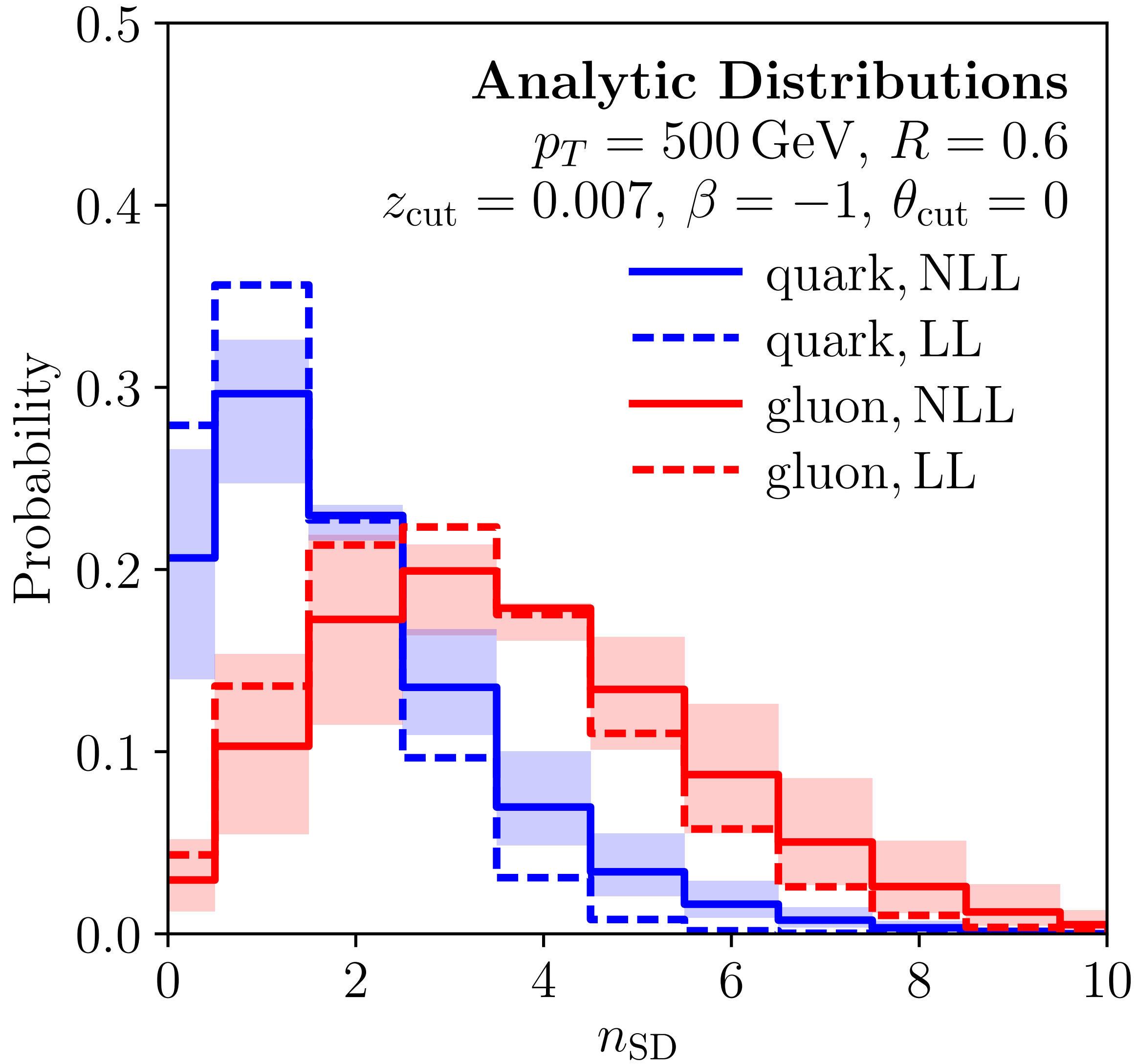
$$P_n^{(i)}(\theta_{\rm cut}) = \sum_{j=q,g} \int dZ \;\; P_n^{i\to j(Z)}(\theta_{\rm cut})$$

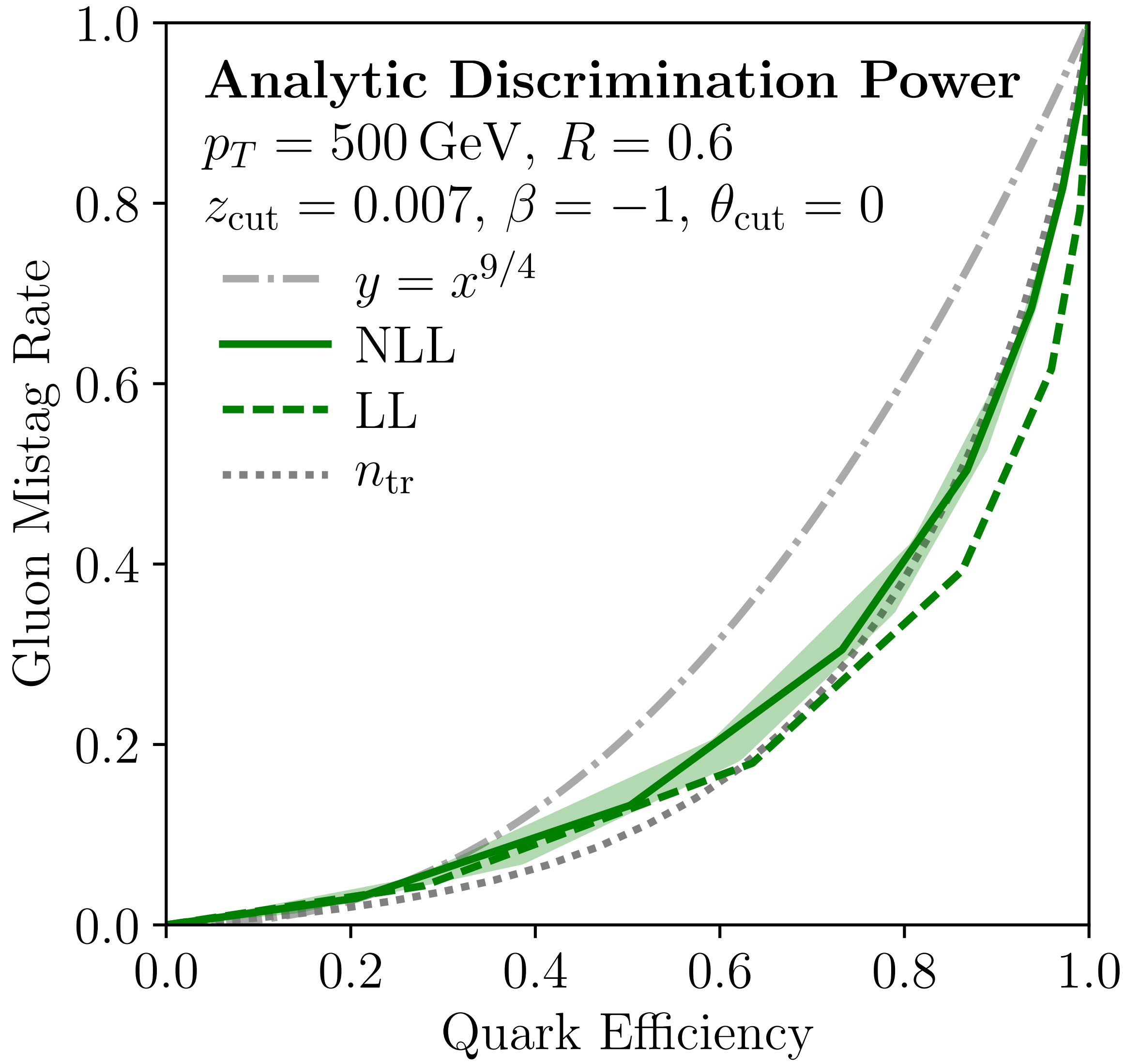
- More complicated evolution equations: solution not Poissonian.

- Discrimination power reduced at NLL due to flavor mixing:

  a quark may convert into a collinear gluon

Analytic results: LL, NLL

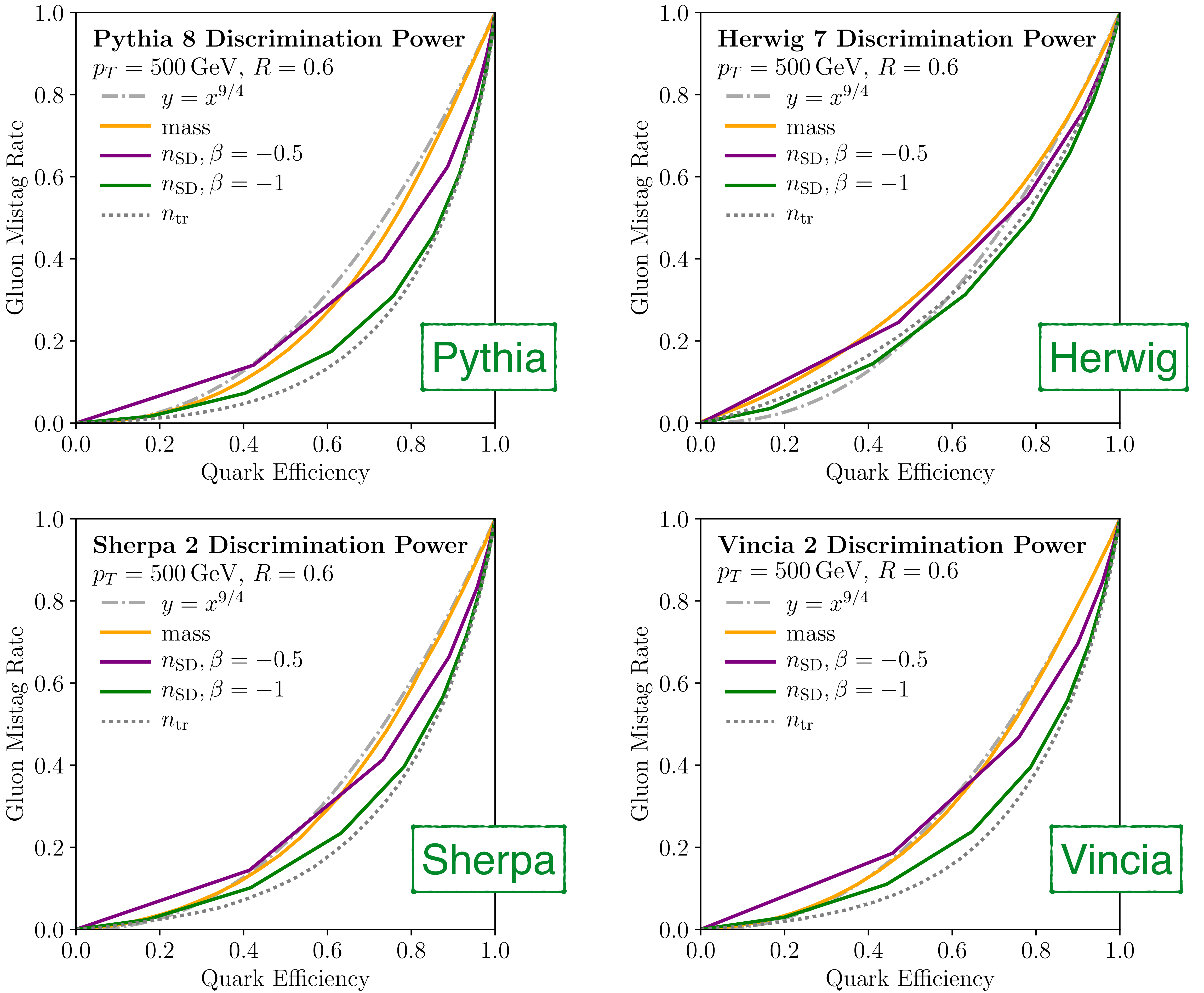




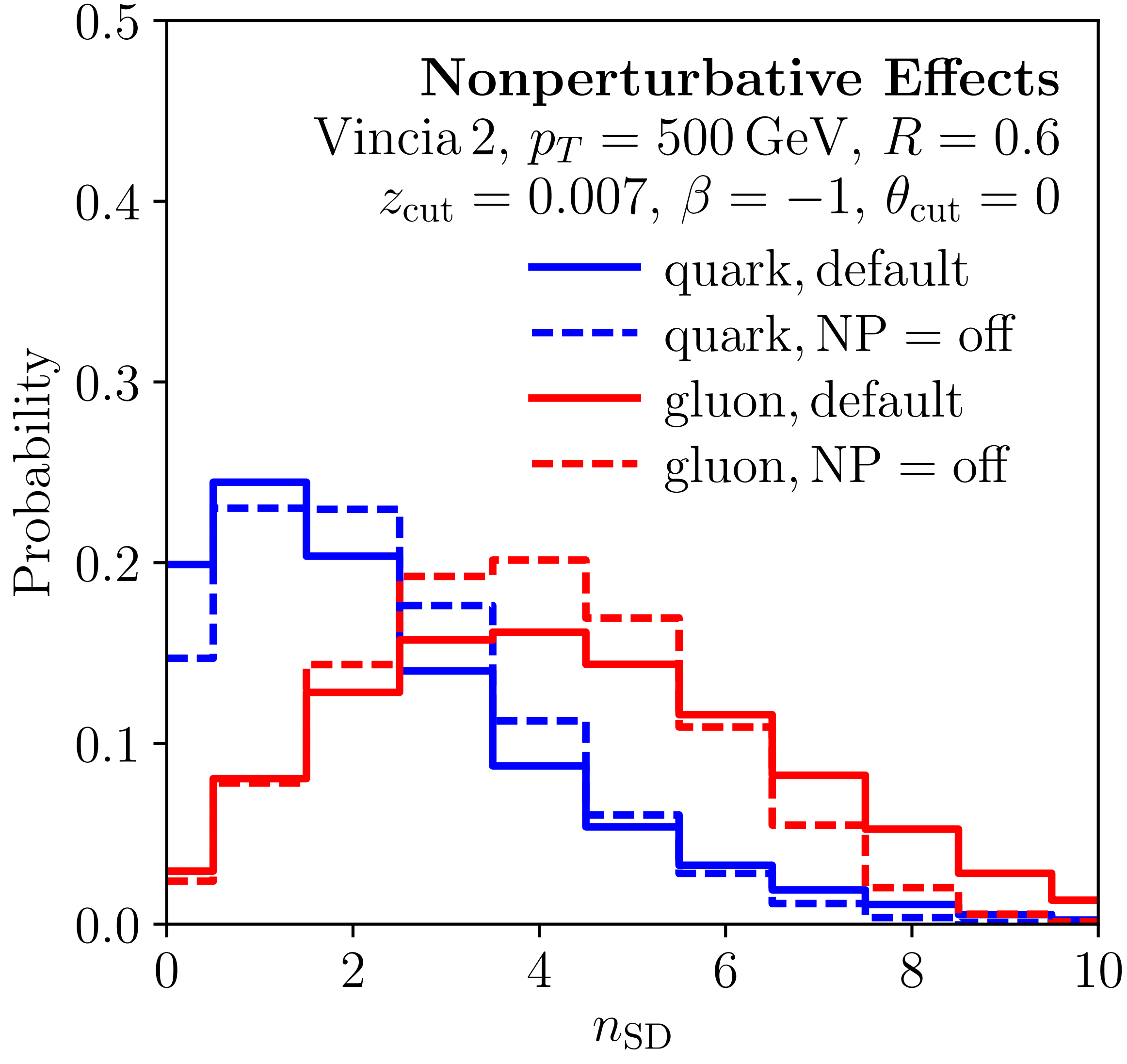
MONTE CARLO RESULTS

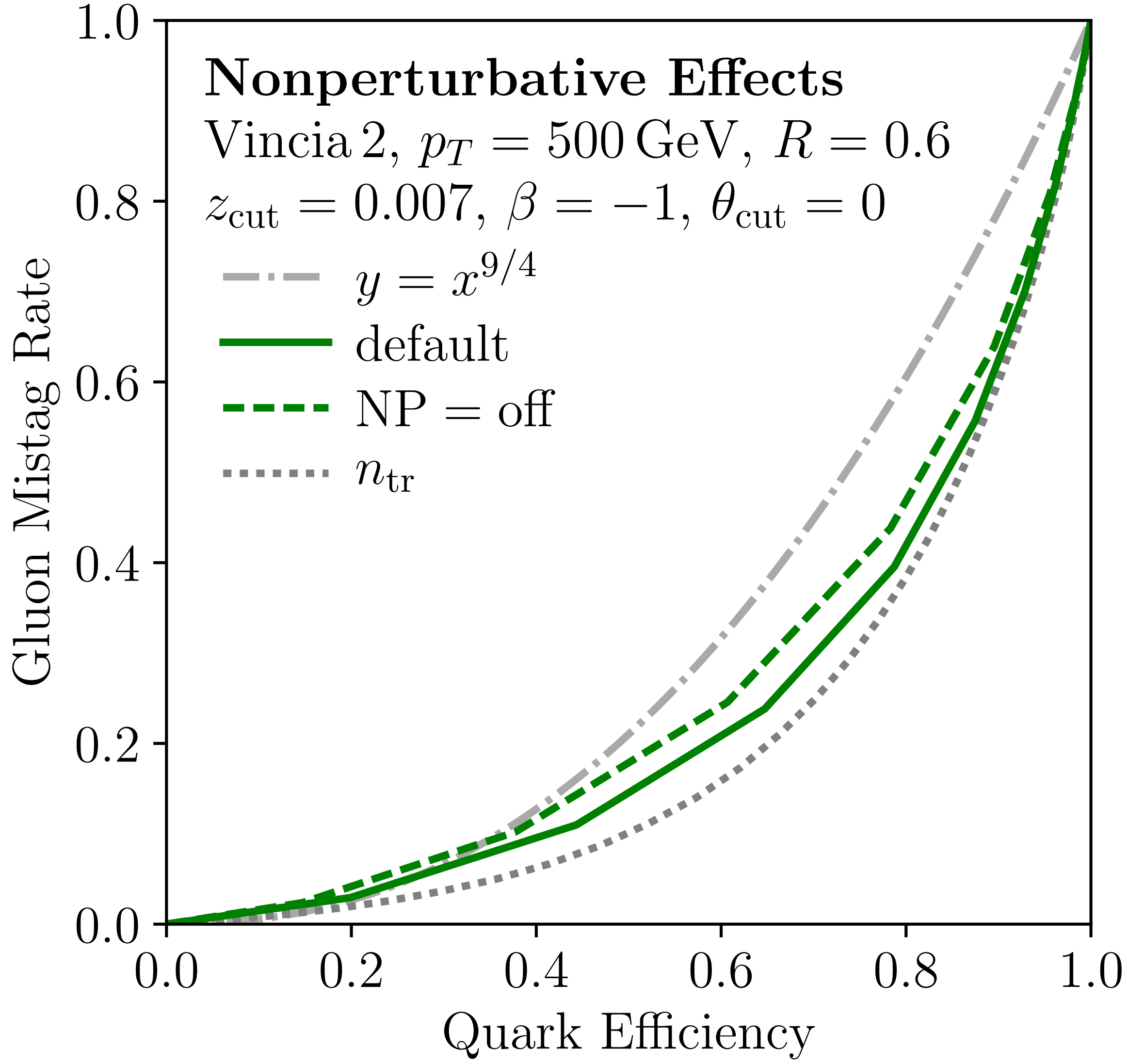
- Pythia 8.219
- Herwig 7.0.1
- Sherpa 2.2.0
- Vincia 2.0.01

Sjöstrand, Mrenna, Skands, JHEP05 (2006) 026
Bellm et al, Eur. Phys. J. C 76 (2016) 196
Gleisberg et al, JHEP 0902 (2009) 007
Giele, Kosower, Skands, Phys. Rev. D78 (2008) 014026

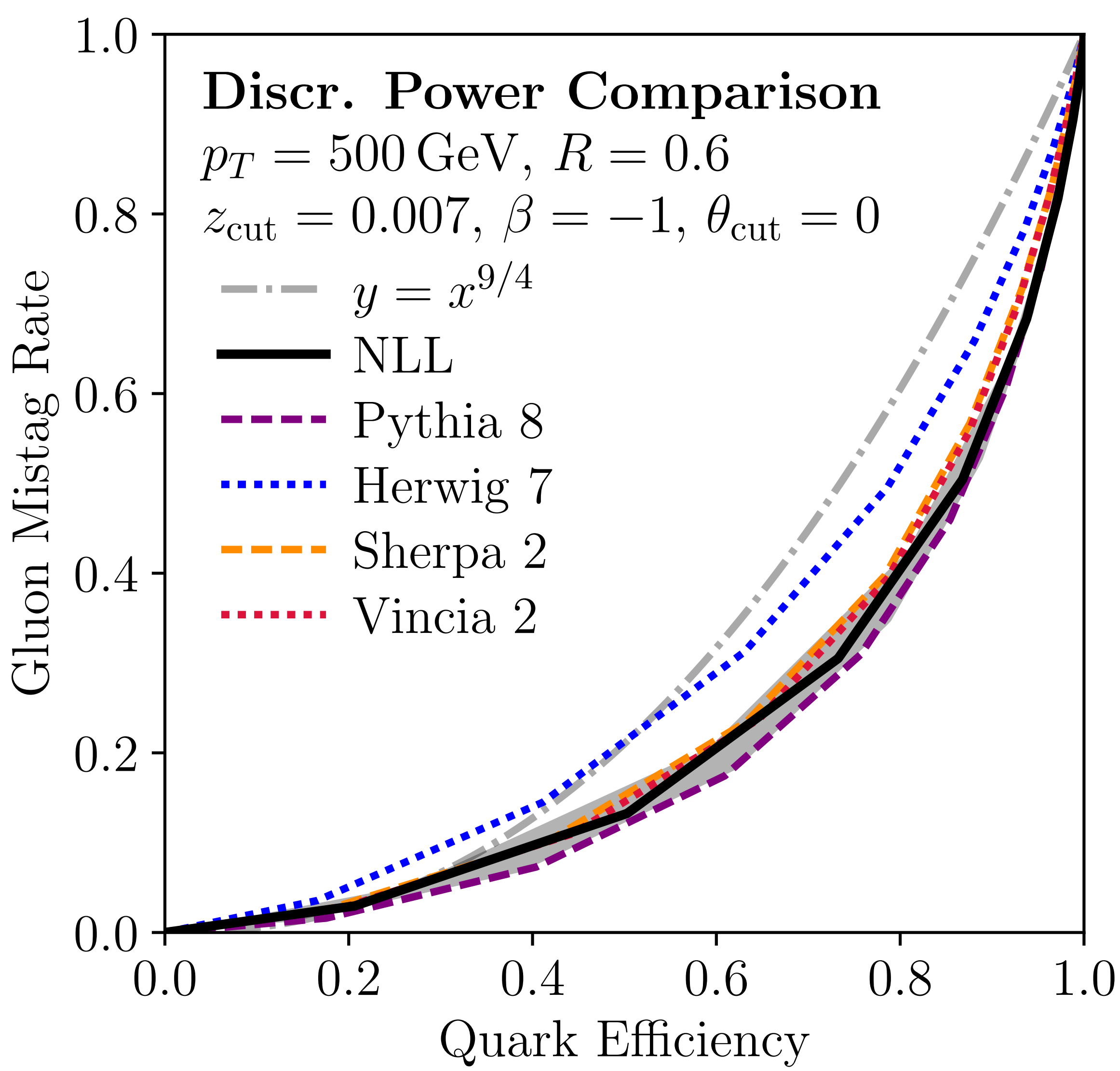

Pythia 8 Discrimination Power
$p_T = 500\,\mathrm{GeV},\ R = 0.6$
$y = x^{9/4}$
mass
$n_{\mathrm{SD}}, \beta = -0.5$
$n_{\mathrm{SD}}, \beta = -1$
n_{tr}
Pythia
Herwig 7 Discrimination Power
$p_T = 500\,\mathrm{GeV},\ R = 0.6$
$y = x^{9/4}$
mass
$n_{\mathrm{SD}}, \beta = -0.5$
$n_{\mathrm{SD}}, \beta = -1$
n_{tr}
Herwig
Sherpa 2 Discrimination Power
$p_T = 500\,\mathrm{GeV},\ R = 0.6$
$y = x^{9/4}$
mass
$n_{\mathrm{SD}}, \beta = -0.5$
$n_{\mathrm{SD}}, \beta = -1$
n_{tr}
Sherpa
Vincia 2 Discrimination Power
$p_T = 500\,\mathrm{GeV},\ R = 0.6$
$y = x^{9/4}$
mass
$n_{\mathrm{SD}}, \beta = -0.5$
$n_{\mathrm{SD}}, \beta = -1$
n_{tr}
Vincia
Gluon Mistag Rate
Quark Efficiency

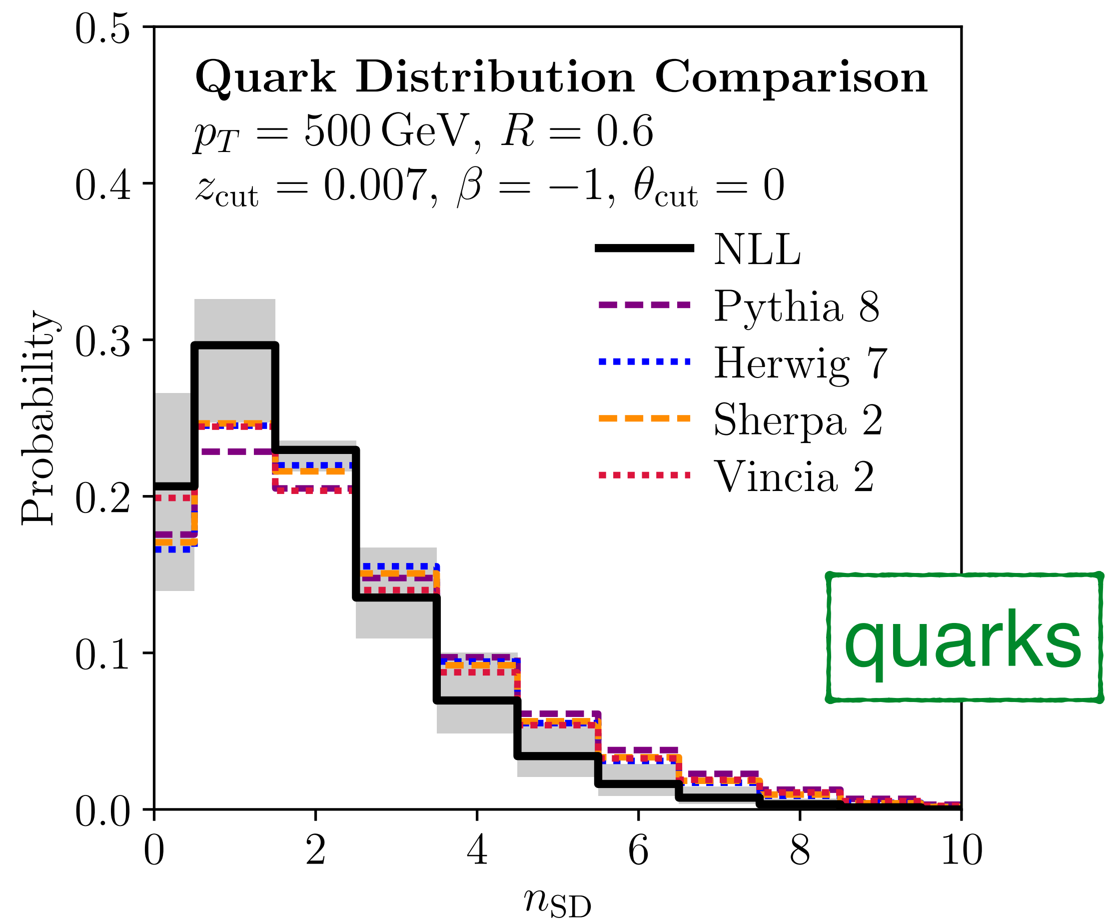
Nonperturbative effects in MC

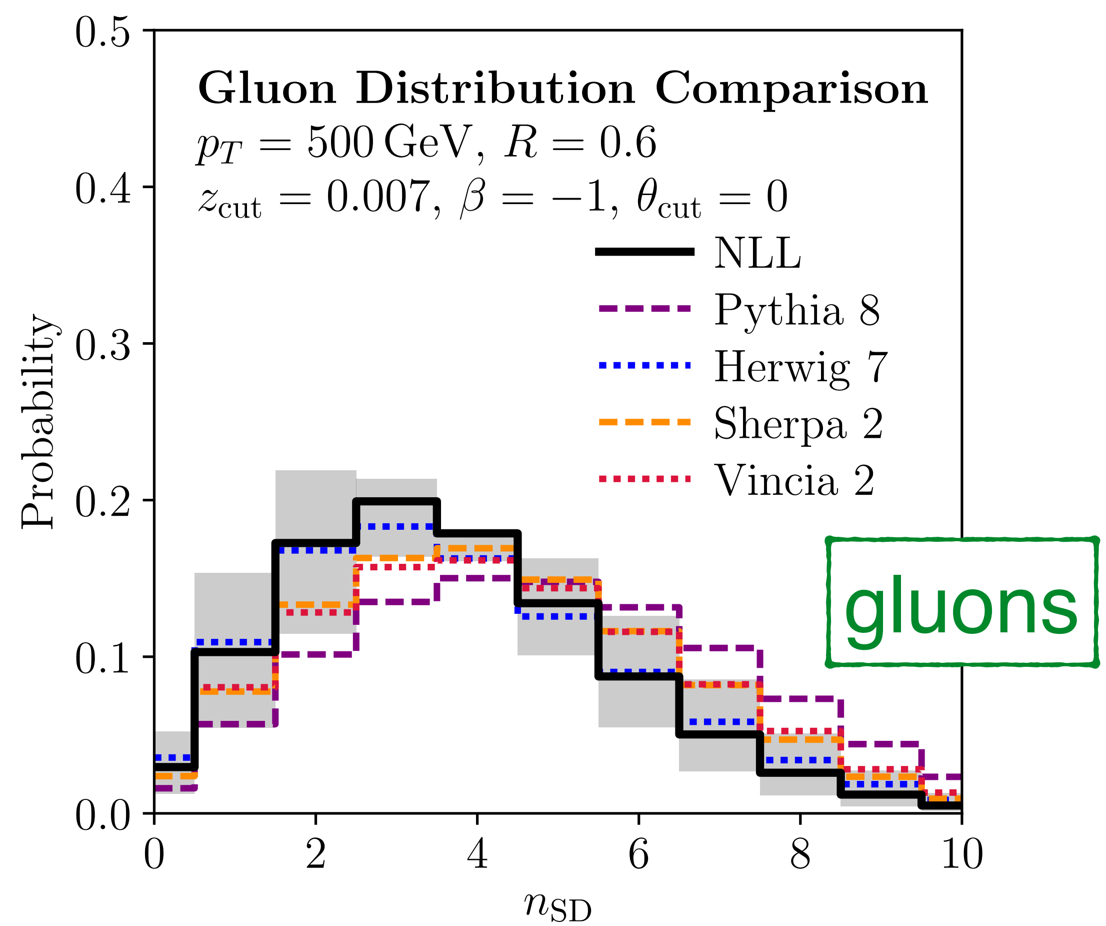




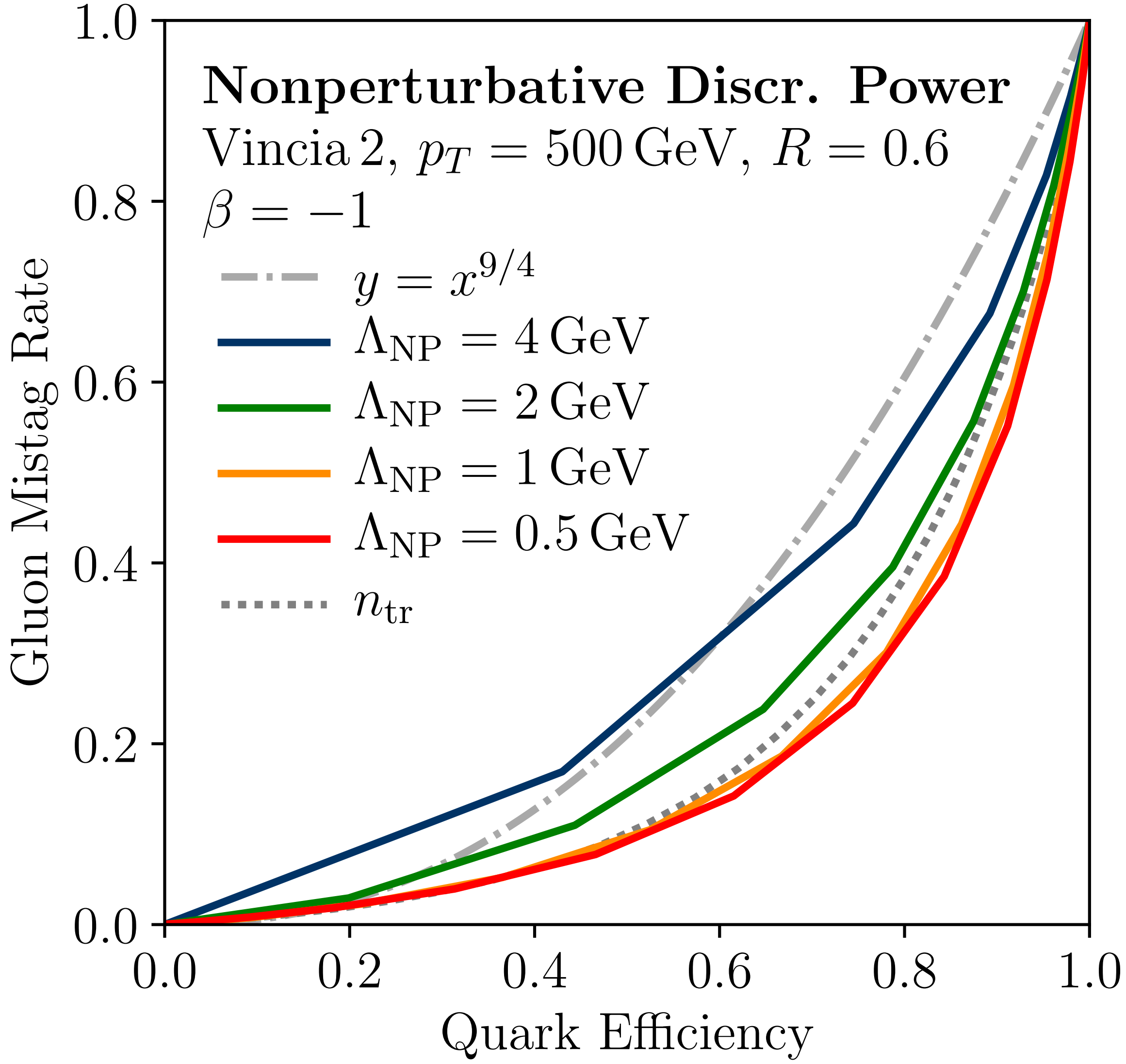
Comparison: NLL, MC







If we do allow significant nonperturbative sensitivity…



Summary and outlook

- Due to their Poisson-like nature, counting observables outperform Sudakov distributed observables in quark-gluon discrimination.

- A new IRC safe counting observable:
  — soft drop multiplicity n_{SD}.

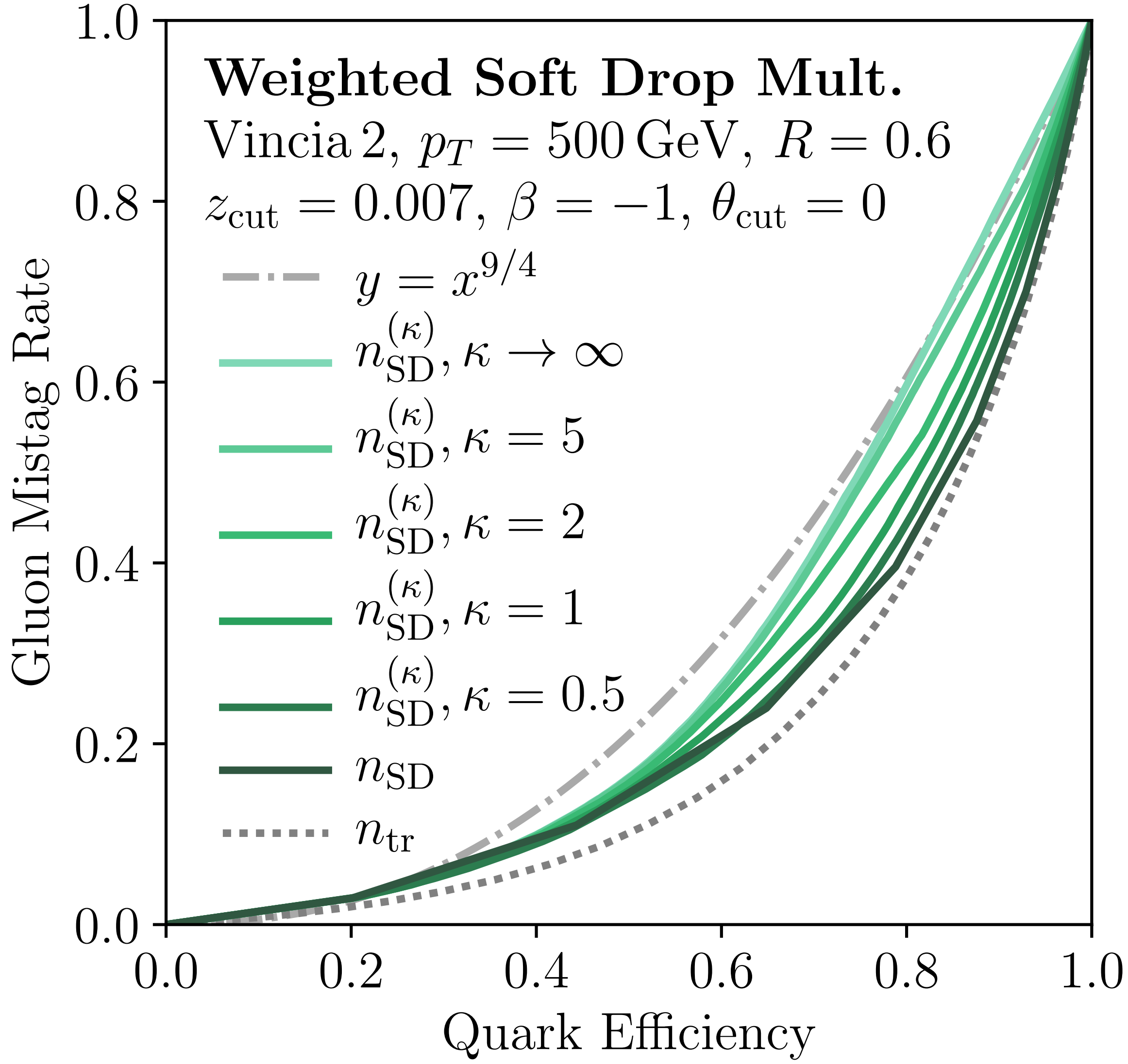
- Used NLL evolution equations to reliably compute n_{SD} distribution and compared to MC's.

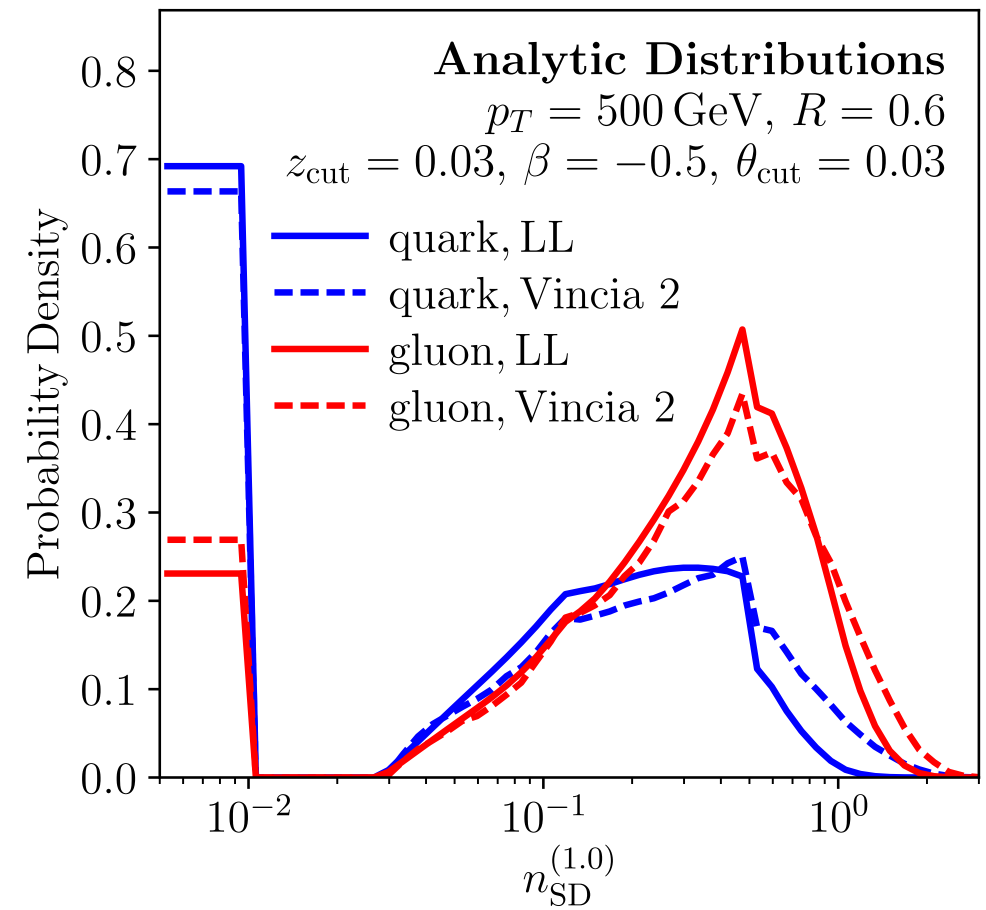
- What next:
  — compare LHC measurement, analytic calculation, and MC's
  — other counting observables, e.g. trimmed subjet multiplicity

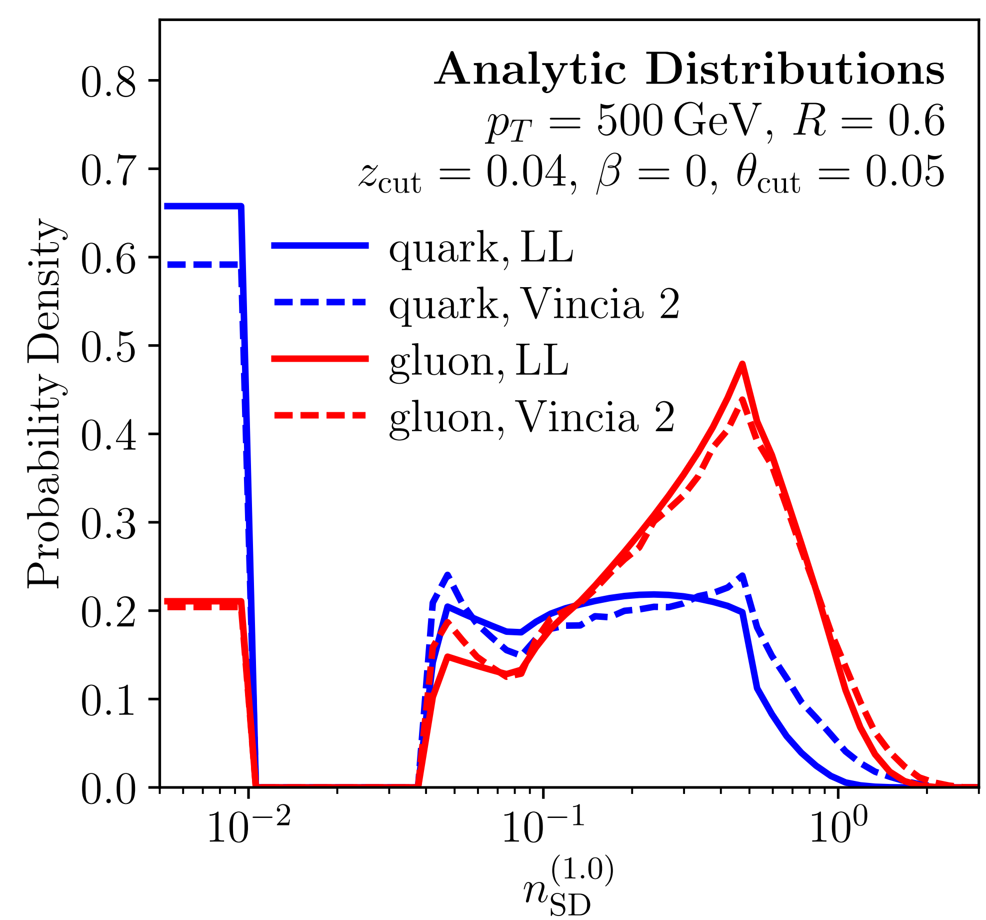
BACKUP SLIDES

Weighted multiplicity

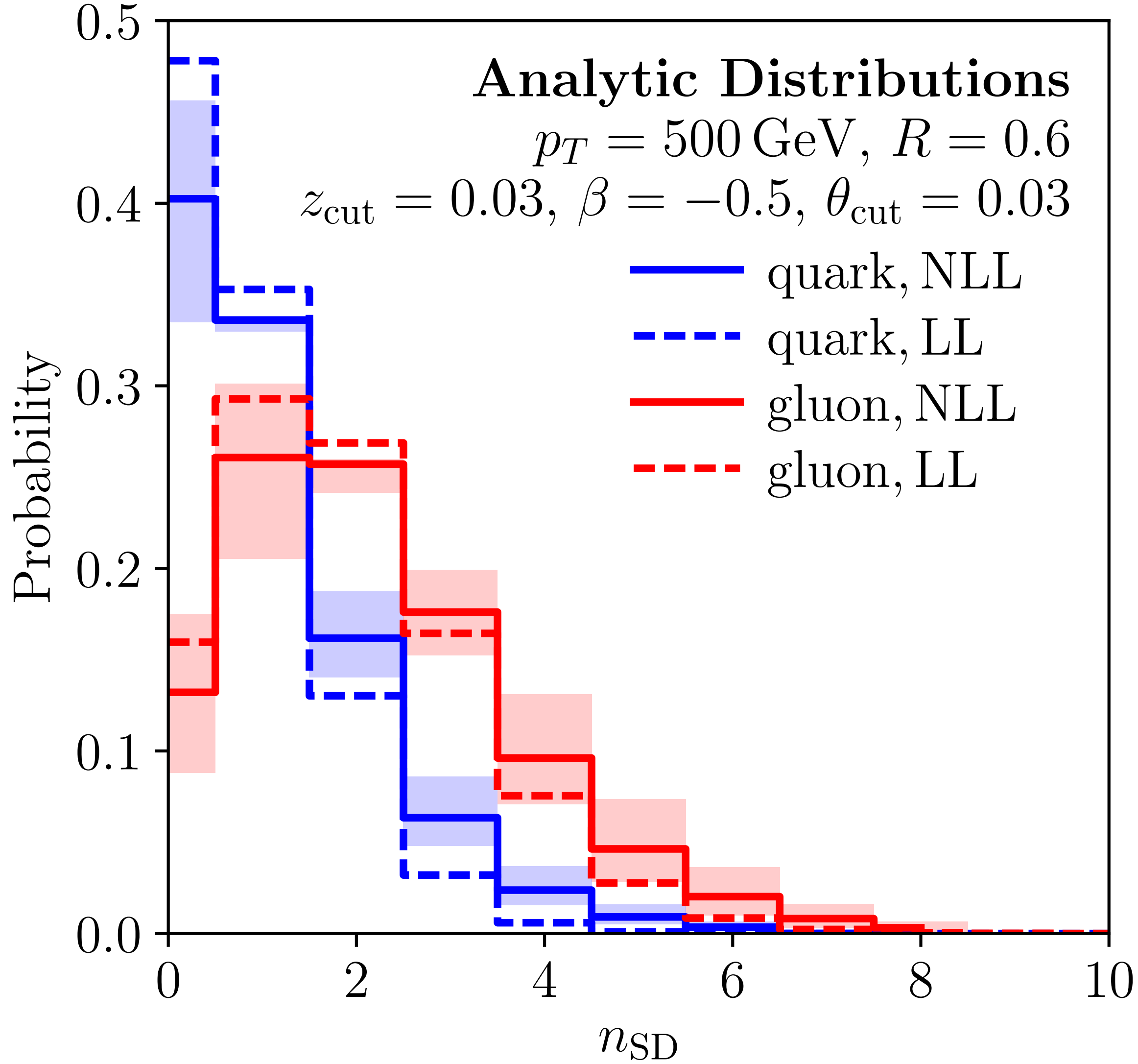
$$n_{\mathrm{SD}}^{(\kappa)} = \sum_n (z_n)^{\kappa}$$

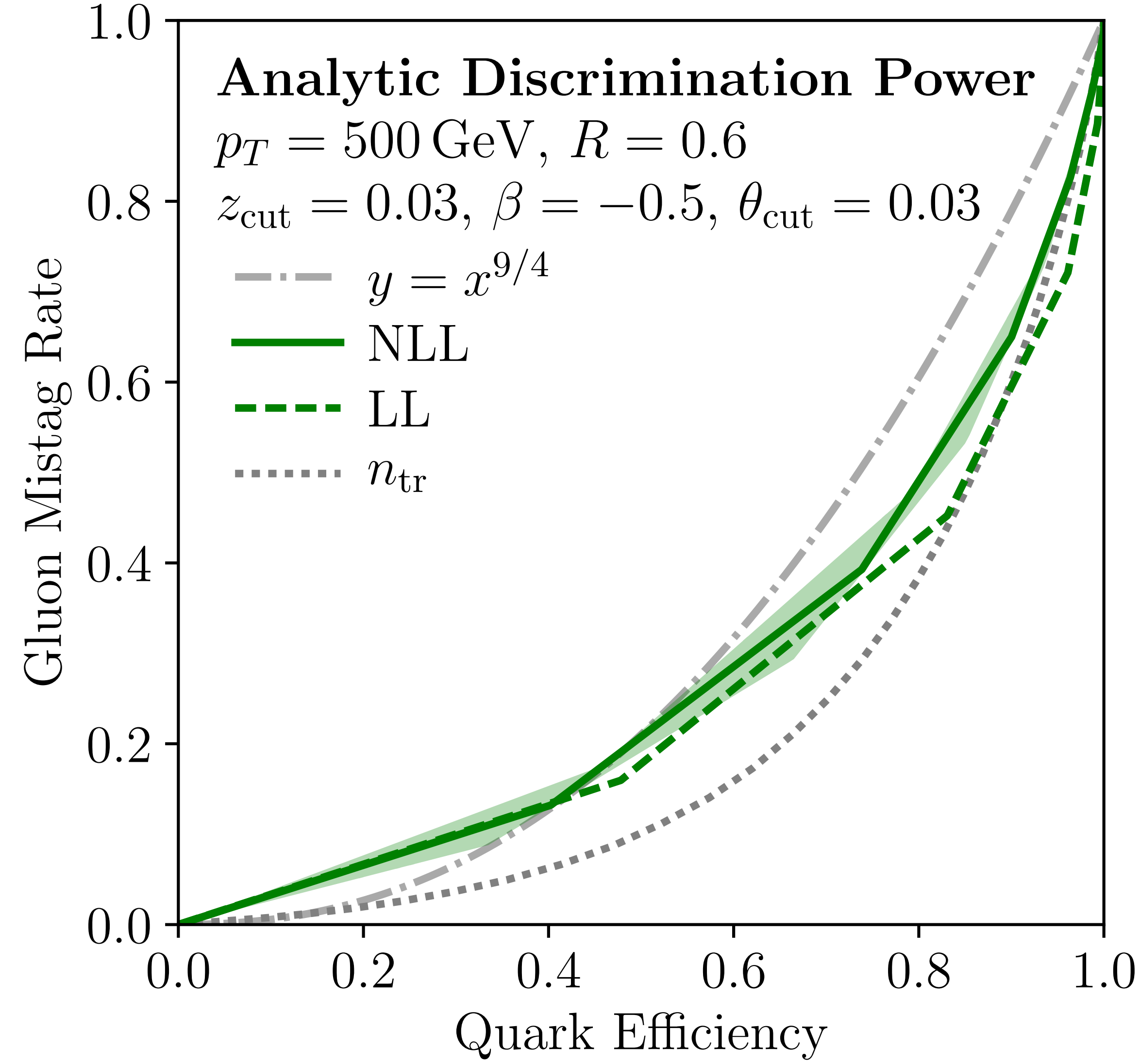




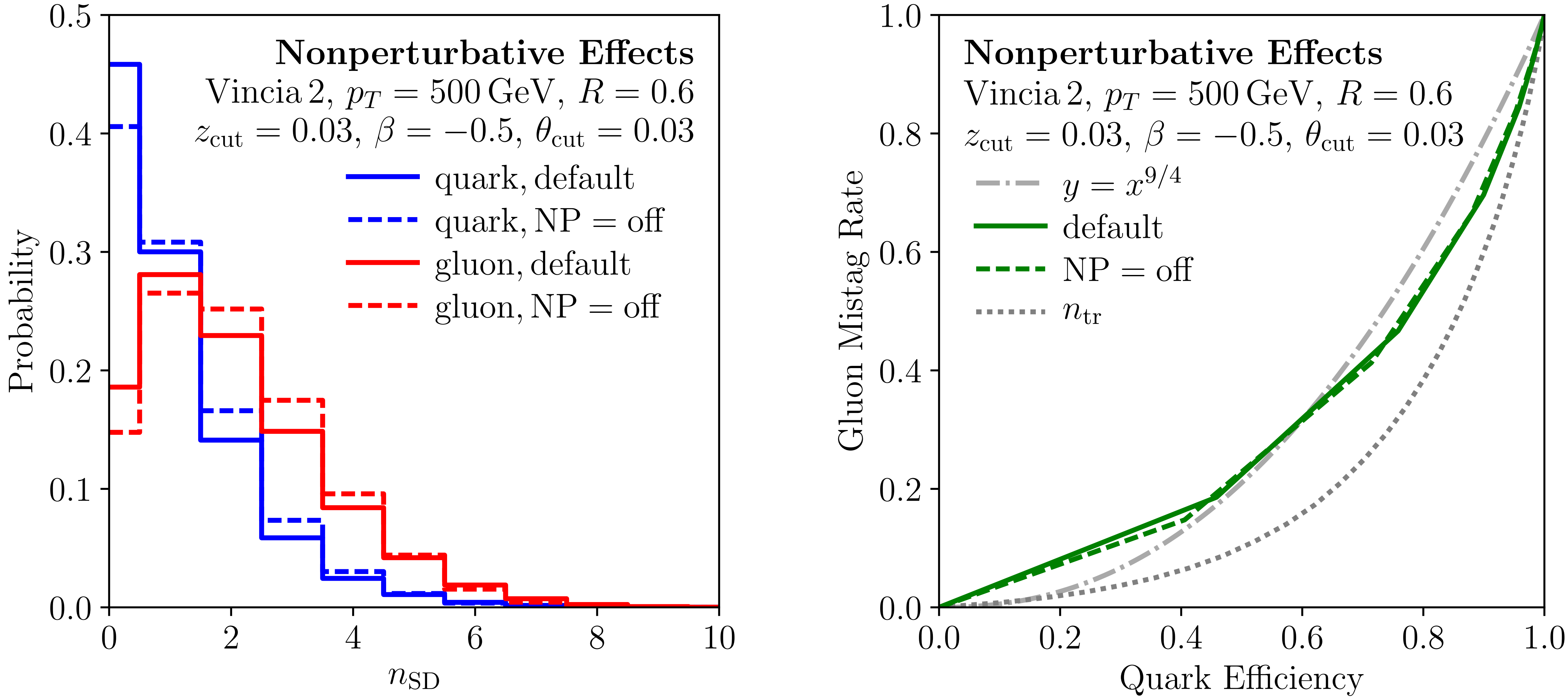


Analytic results: $\beta = -0.5$





Nonperturbative effects: $\beta = -0.5$



Comparison: $\beta = -0.5$