Jet Fragmentation and Fractal Observables

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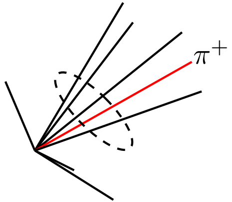
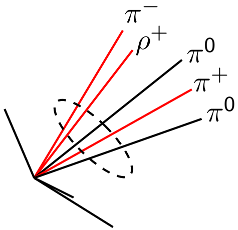
Based on work with: Massimiliano Procura, Jesse Thaler, Wouter Waalewijn, and Kevin Zhou

Fragmentation Functions

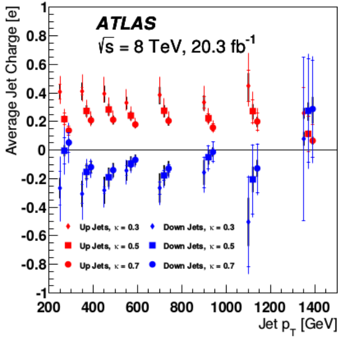
- Fragmentation function (FF) $D_i^h(x, \mu)$:
  - Probability of hadron h resulting from parton i, carrying momentum fraction x
  - Non-perturbative (must be extracted from data)
  - Process independent
  - Perturbative RG evolution
- Jet substructure: typically don't care about individual identified hadron
- Today's talk: subsets of jet particles $\rightarrow$ generalized fragmentation functions (GFFs)
- GFF $\mathcal{F}_i(x, \mu)$ describes distribution of observable x among some subset $\mathcal{S}$ of jet particles



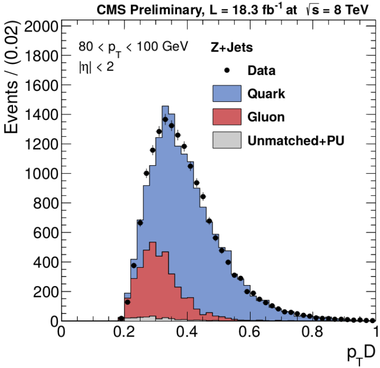


Collinear Unsafe Observables

$$\text{Jet Charge} = \sum_{i \in \text{jet}} Q_i z_i^{\kappa} \qquad p_T^D = \sum_{i \in \text{jet}} z_i^2 \qquad z_i = \frac{p_{T,i}}{p_{T,\text{jet}}}$$



Phys.Rev. D93 (2016) no.5, 052003 following Krohn, Shwartz, Lin, Waalewijn:1209.2421

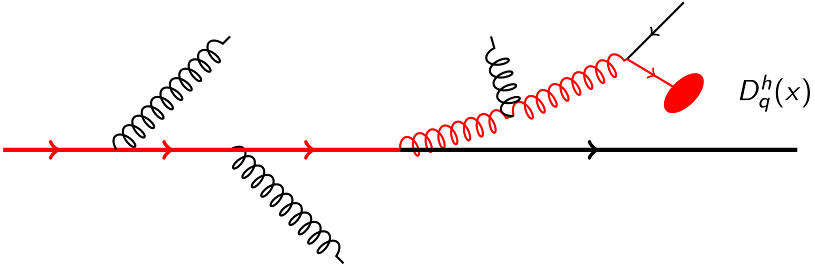


CMS Collab.-CMS-PAS-JME-13-002

RG Evolution: Standard Fragmentation Functions

- Leading order evolution $\to$ DGLAP equations
- Follow evolution on one path $\to$ linear

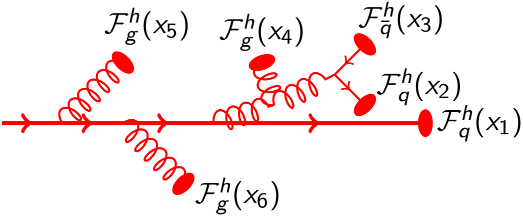
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} D_i^h(x,\mu) = \frac{1}{2} \sum_{j,k} \int_x^1 \frac{\mathrm{d}z}{z} \frac{\alpha_s(\mu)}{\pi} P_{i\to j,k}(z,\alpha_s) D_j^h(x/z,\mu)$$



RG Evolution: Generalized Fragmentation Functions

- $\mathcal{F}_i(x, \mu)$ carries information about all particles in $\mathcal{S}$
- Leading order evolution follows evolution along all paths $\rightarrow$ nonlinear
- NLO evolution involves $1 \rightarrow 3$ splittings

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{F}_i(x, \mu) = \frac{1}{2} \sum_{j,k} \int \mathrm{d}z \frac{\alpha_s(\mu)}{\pi} P_{i \to j,k}(z, \alpha_s) \int \mathrm{d}x_1 \mathrm{d}x_2 \, \mathcal{F}_j(x_1, \mu) \mathcal{F}_k(x_2, \mu) \times \delta(x - \hat{x}(z, x_1, x_2))$$



$\nearrow$

Jet Charge: $\hat{x} = z^\kappa x_1 + (1-z)^\kappa x_2$

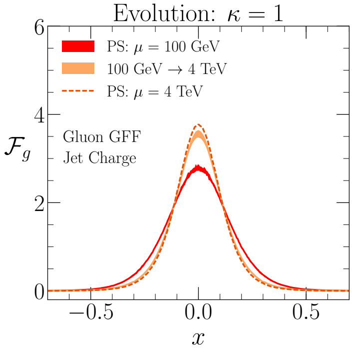
p_T^D : $\hat{x} = z^2 x_1 + (1-z)^2 x_2$

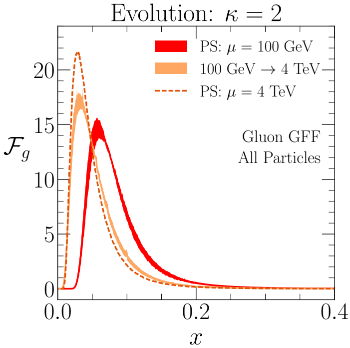
RG Evolution: Comparison to Parton Showers

Gluon GFF, Weighted Jet Charge and p_T^D

$$pp:\ \mu = p_T R,\ z_i = \frac{p_{T,i}}{p_{T,\mathrm{jet}}};\quad e^+e^-:\ \mu = ER,\ z_i = \frac{E_i}{E_{\mathrm{jet}}}$$

envelopes: PYTHIA PS, VINCIA PS, DIRE PS; E, R combinations



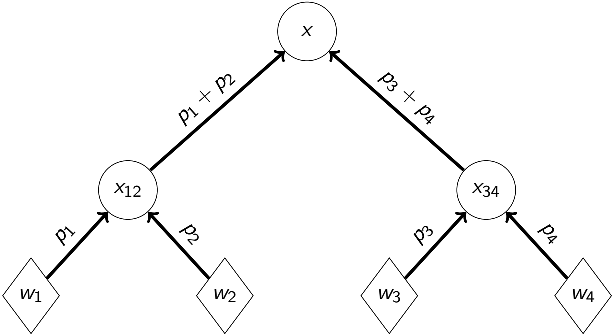


$$\text{Jet Charge} = \sum_{i \in jet} Q_i z_i^\kappa \qquad p_T^D = \sum_{i \in jet} z_i^2$$

Fractal Observables

- Construct observable with structure of (leading order) evolution equation
- Define clustering tree, final state jet particles = leaves of tree
- Assign weights to jet constituents (non-kinematic quantum numbers)
- Recursively combine from bottom to top of tree using recursion relation $\hat{x}(z, x_1, x_2)$

$$z = \frac{E_L}{E_L + E_R}$$

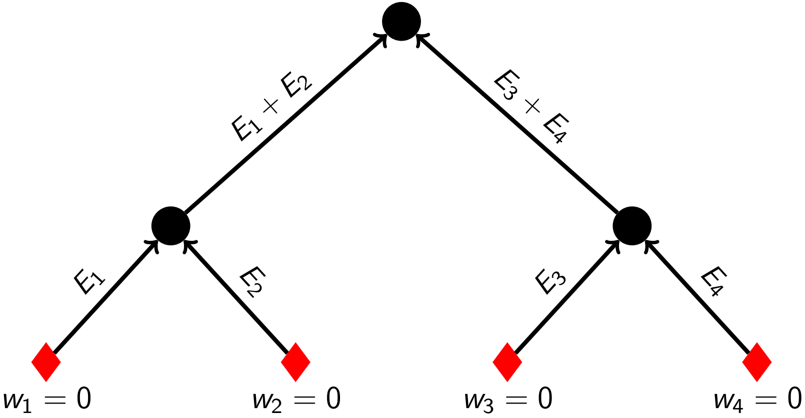


Fractal Observables: Familiar Examples

- Weighted jet charge:
  - $w_i = Q_i$
  - $\hat{x} = z^\kappa x_1 + (1-z)^\kappa x_2$
  - $x = \sum_{i \in \mathrm{jet}} Q_i z_i^\kappa$
- p_T^D:
  - $w_i = 1$
  - $\hat{x} = z^2 x_1 + (1-z)^2 x_2$
  - $x = \sum_{i \in \mathrm{jet}} z_i^2$
- These recursion relations are associative $\rightarrow$ independent of clustering tree

Fractal Observables: Non-Associative Example

- $\hat{x} = z^{\kappa} x_1 + (1-z)^{\kappa} x_2 + (4z(1-z))^{\kappa/2}$
- $w_i = 0$



Fractal Observables: Non-Associative Example

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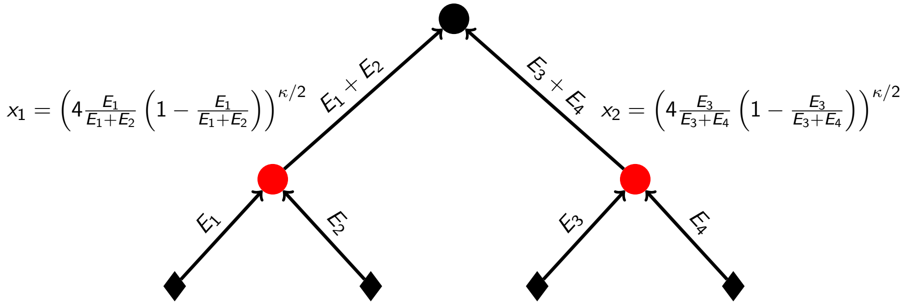
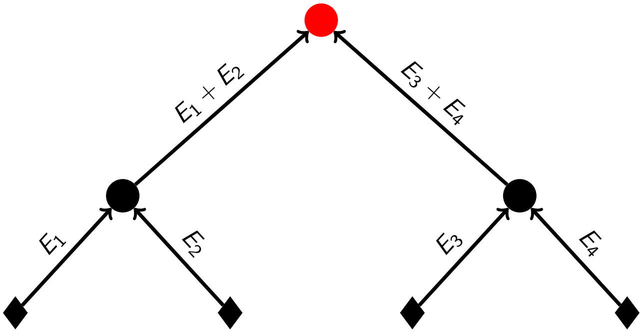


Fractal Observables: Non-Associative Example

- $\hat{x} = z^\kappa x_1 + (1-z)^\kappa x_2 + (4z(1-z))^{\kappa/2}$
- $w_i = 0$

$$x = \left(\frac{E_1+E_2}{E_{\text{jet}}}\right)^\kappa x_1 + \left(1-\frac{E_1+E_2}{E_{\text{jet}}}\right)^\kappa x_2 + \left(4\frac{E_1+E_2}{E_{\text{jet}}}\left(1-\frac{E_1+E_2}{E_{\text{jet}}}\right)\right)^{\kappa/2}$$



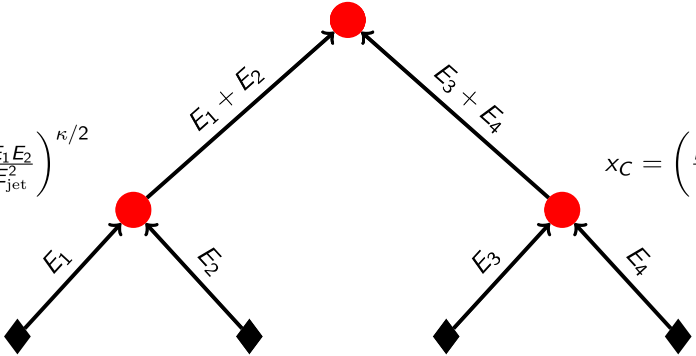
Fractal Observables: Node Definition

$$x = x_A + x_B + x_C = \sum_{\text{nodes}} \left(\frac{E_L E_R}{E_{\text{jet}}^2} \right)^{\kappa/2} \qquad \kappa = 2 \implies x = 2(1 - p_T^D)$$

$$x_B = \left(\frac{(E_1+E_2)(E_3+E_4)}{E_{\text{jet}}^2} \right)^{\kappa/2}$$

$$x_A = \left(\frac{E_1 E_2}{E_{\text{jet}}^2} \right)^{\kappa/2}$$

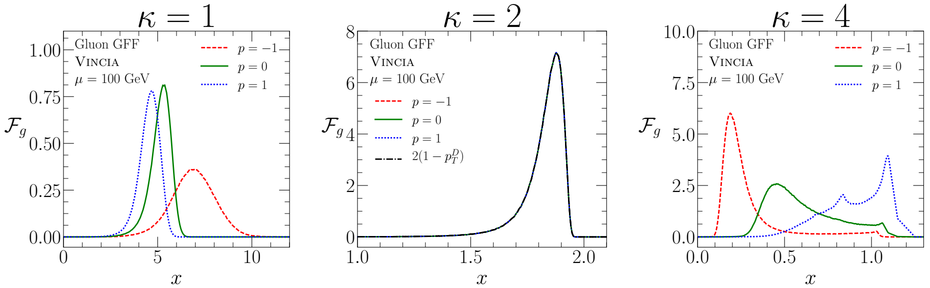
$$x_C = \left(\frac{E_3 E_4}{E_{\text{jet}}^2} \right)^{\kappa/2}$$



Fractal Observables: Tree Dependence

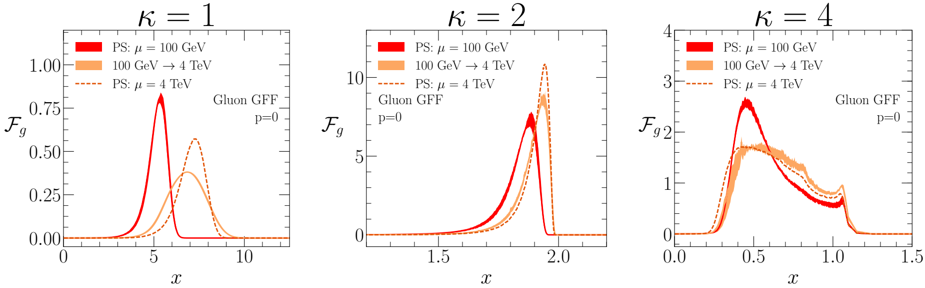
- Compute fractal observable on jet ensemble from VINCIA
- anti-k_T jets with $R = 0.6$
- Recluster into fractal observable tree
- Non-associative recursion relation $\rightarrow$ tree dependence

$$p = -1 \rightarrow \text{anti} - k_T$$
$$p = 0 \rightarrow \text{C/A}$$
$$p = 1 \rightarrow k_T$$



Fractal Observables: RG Evolution

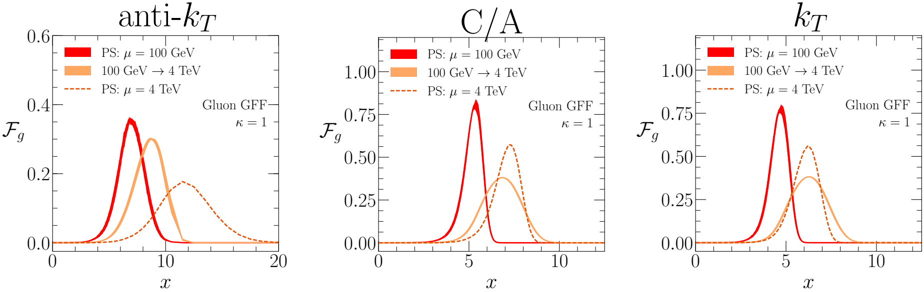
Gluon GFF, C/A Trees



$$\hat{x}(z, x_1, x_2) = z^\kappa x_1 + (1-z)^\kappa x_2 + (4z(1-z))^{\kappa/2}$$

Fractal Observables: RG Evolution

Gluon GFF, $\kappa = 1$

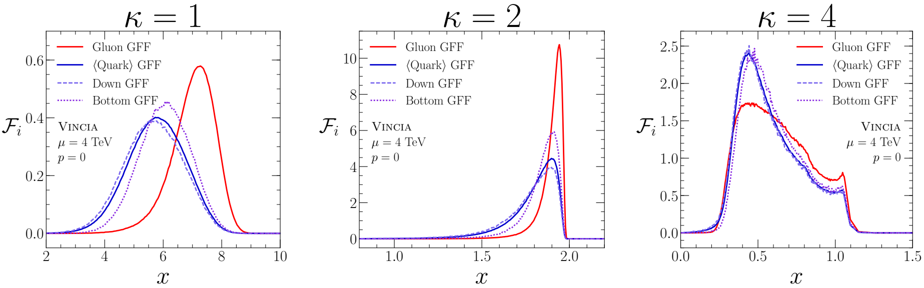


$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{F}_i(x, \mu) = \frac{1}{2} \sum_{j,k} \int \mathrm{d}z \frac{\alpha_s(\mu)}{\pi} P_{i \to j,k}(z, \alpha_s) \int \mathrm{d}x_1 \mathrm{d}x_2\, \mathcal{F}_j(x_1, \mu) \mathcal{F}_k(x_2, \mu)$$

$$\times\, \delta(x - zx_1 - (1-z)x_2 - (4z(1-z))^{1/2})$$

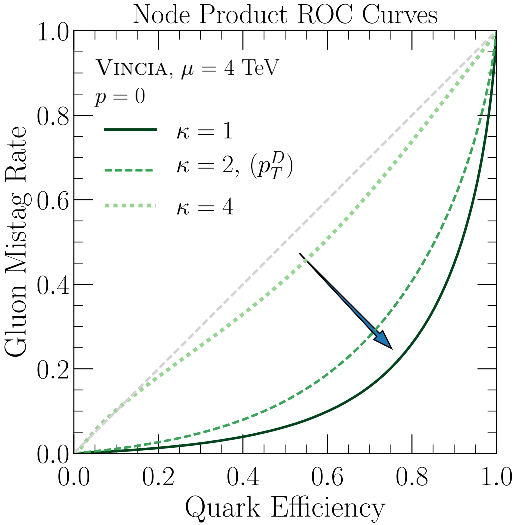
Quark/Gluon Discrimination: Distributions

Multiple Partons, C/A Trees



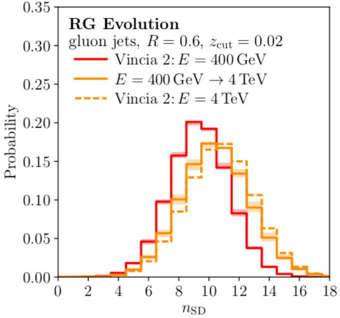
better (more like multiplicity) $\leftarrow$ p_T^D $\rightarrow$ worse (less like multiplicity)

Quark/Gluon Discrimination: ROC Curves



Soft-Drop Multiplicity

More about this in Chris Frye's talk tomorrow at 4pm.



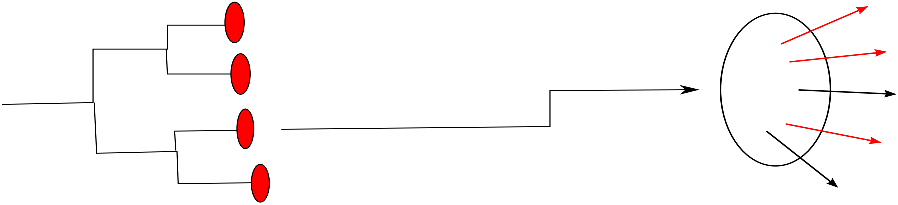
- C-unsafe in $\beta \to 0$ limit
- Another application of GFFs
- Recursion relation:

$$\hat{x}(z, x_1, x_2) = \begin{cases} x_2 & 0 \leq z < z_{\rm cut} \\ x_2 + f(z) & z_{\rm cut} \leq z \leq 1/2 \\ x_1 + f(z) & 1/2 \leq z \leq 1 - z_{\rm cut} \\ x_1 & 1 - z_{\rm cut} < z \leq 1 \end{cases}$$

Frye, Larkoski, Thaler, Zhou:
ArXiv:1704.06266

Conclusion

- FFs $\rightarrow$ cross sections with single identified hadron
- GFFs $\rightarrow$ cross sections of fractal observables with subsets of final state particles
- GFFs are non-perturbative
- Nonlinear, DGLAP-like perturbative evolution
- Fractal observables at the LHC: jet charge and p_T^D
- Non-associative generalizations show promise for quark/gluon discrimination



Thank You