

Electroweak Splitting Functions and High Energy Showering

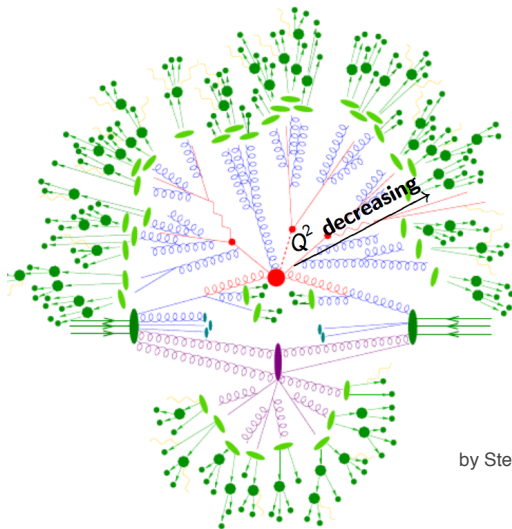
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arXiv:1611.00788 J.C. T. Han B. Tweedie

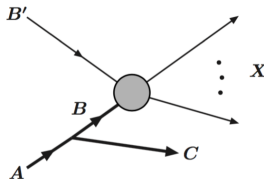
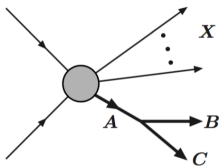
July 18, 2017

Collisions in Colliders



by Steffen Schumann

Collinear Splitting Functions



$\frac{d\mathcal{P}_{A \rightarrow B+C}}{dzdk_T^2}$ is “collinear splitting function”, iteration of “splittings” leads to parton shower:

- ISR: DGLAP evolution of PDFs
- FSR: jets

Assumption for splitting formalism: Interferences are small

Facing Electroweak Shower

- For Strong Interactions, when hard scattering scales ~ 50 GeV

$$\frac{50 \text{ GeV}}{\Lambda_{\text{QCD}} \sim 500 \text{ MeV}} \gg 1$$

Parton shower emerges

- In a 100 TeV Collider, hard scattering could scale ~ 10 TeV

$$\frac{10 \text{ TeV}}{m_W \sim 100 \text{ GeV}} \sim \frac{50 \text{ GeV}}{\Lambda_{\text{QCD}} \sim 500 \text{ MeV}} \gg 1$$

- We are facing a new phenomenon: electroweak parton shower.

Novelties of Electroweak Showering

Novelties of electroweak showering:

- Spontaneous symmetry breaking:
 - ① massive particle: perturbative shut-off
 - ② additional degrees of freedom: longitudinal and higgs
- Chirality: Left-handed fermions and right-handed fermions couple differently.
- Neutral bosons interference: γ/Z_T ; h/Z_L
- Weak isospin self-averaging.

Focus today:

- Neutral interference.
- VEV corrections, i.e. the effects of EW symmetry breaking.

Splitting Functions in Unbroken Limit (showering at high energy scale $\gg v$).

$$d\sigma_{X,B+C} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A \rightarrow B+C}$$

Splittings analogue to QCD



$$P_{f_L \rightarrow f_L + W_T / Z_T} \sim \alpha_W \frac{1+(1+z)^2}{z} \cdot \frac{1}{p_T^2}$$



$$P_{W_T \rightarrow W_T Z_T / \gamma} \sim \alpha_W \frac{(1-z\bar{z})^2}{z(1-z)} \cdot \frac{1}{p_T^2}$$

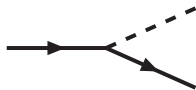


$$P_{W_T / Z_T \rightarrow f_L \bar{f}_R} \sim \alpha_W \frac{1}{p_T^2}$$

Captures leading log in EW Shower

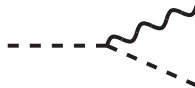
Splitting Functions in Unbroken Limit (Showering at high energy scale $\gg v$).

Splittings with higgs and longitudinal, with $W_L^\pm \sim \phi_L^\pm$ and $Z_L \sim \phi_0$

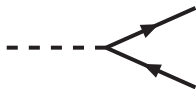


$$P_{f_L \rightarrow f_R^{(\prime)} + h/W_L^\pm} \sim y_{f^{(\prime)}}^2 \frac{1}{p_T^2}$$

(helicity flipped)



$$P_{W_L^\pm \rightarrow W_T^\pm h/Z_L} \sim \alpha_W \frac{1}{z} \frac{1}{p_T^2}$$



$$P_{h/W_L^\pm \rightarrow f_L \bar{f}_L^{(\prime)}} \sim y_{f^{(\prime)}}^2 \frac{1}{p_T^2}$$





$$P_{V_T \rightarrow V_L(/h) V_L/h} \sim \alpha_W \frac{1}{p_T^2}$$

Captures leading log in EW shower

Neutral Interference

- Splitting formalism assumes interference to be suppressed.
- Interference can be significant for γ/Z intermediated splittings.
- Solution: matrix. $\frac{d\hat{P}_{ij}}{dk_T^2 dz}$.
 - 1 When $i = j$, "single state splitting functions".
 - 2 When $i \neq j$, interference terms.

		
	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left(\frac{(1-z\bar{z})^2}{z\bar{z}} \right)$	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left(\frac{z^2 + \bar{z}^2}{2} \right)$
	$\rightarrow W_T W_T$	$f_s \bar{f}_s^{(f)}$
V_T	$2g_2^2 (V=W^{0,\pm})$	$N_f g_V^2 (Q_{f_s}^V)^2$
$[BW]_T^0$	0	$N_f g_1 g_2 Y_{f_s} T_{f_s}^3$

: *

Longitudinal in Broken(Real) Theory: Huge Interference

Longitudinal vector boson $\epsilon_L \sim \frac{k^\mu}{m_W}$, bad energy behavior and huge interference!

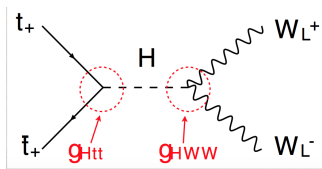
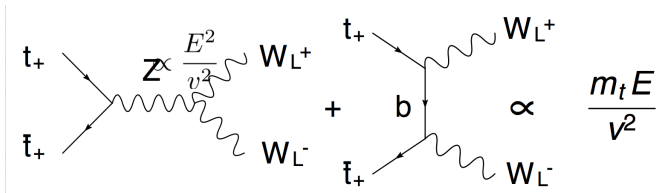


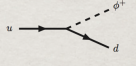
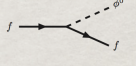
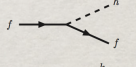
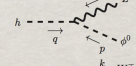
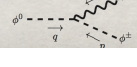


Figure: higgs contribution $\propto \frac{m_t m_h}{v^2}$ fixes the problem.

Goldstone Equivalence Gauge

- Hint to the solution: $\epsilon_L \sim \frac{k^\mu}{m_W} \rightarrow \phi_W$ in high energy — goldstone equivalence theorem.
- Write $\epsilon_L^\mu(k) = \frac{k^\mu}{m_W} - \frac{m_W}{n \cdot k} n^\mu$, with $n^\mu = (-1, \hat{k})$. So impose gauge-fixing $\frac{1}{2\xi} (n^\mu W_\mu)^2$.
- Consequences:
 - ① $n \cdot k \neq 0$: k^μ terms are eliminated – power counting is good
 - ② $n \cdot n = 0$, $\epsilon_n \simeq \frac{m_W}{2E}$, VEV corrections to unbroken limit.
 - ③ $|W_L \rangle = |W_n \rangle + i|\phi \rangle$ — manifest goldstone equivalence theorem
 - ④ Broken theory continuously goes to unbroken theory when $v \rightarrow 0$

$W_\phi^\pm \rightsquigarrow W_\phi^\pm$	$= \frac{i}{k^2 - m_W^2} \text{sign}(k^2)$		$= i \frac{g_2}{\sqrt{2}} f_{T,n} P_L$
$Z_\phi \rightsquigarrow Z_\phi$	$= \frac{i}{k^2 - m_Z^2} \text{sign}(k^2)$		$= ig_Z f_{T,n} ((T_3^f - Q_f^{\text{EM}} s_W^2) P_L - Q_f^{\text{EM}} s_W^2 P_R)$
$W_\phi^\pm \rightsquigarrow \phi^\pm$	$= \frac{i}{k^2 - m_W^2} \frac{m_W}{\sqrt{ k^2 }}$		$= i(-y_d P_L + y_u P_R)$
$Z_\phi \rightsquigarrow \phi^0$	$= \frac{i}{k^2 - m_Z^2} \frac{m_Z}{\sqrt{ k^2 }}$		$= i(\delta_{f_u} - \delta_{f_d}) \frac{y_f}{\sqrt{2}} i\gamma_5$
			$= -i \frac{y_f}{\sqrt{2}}$
			$= \frac{g_2}{2c_W} (q-p) \cdot \epsilon(k)$
			$= \frac{g_2}{2} (q-p) \cdot \epsilon(k)$

$\sim k^\mu k^\nu / m_W^2$ term removed;
 Gauge – Goldstone mixing exists.

$\epsilon_{s=\pm,n}^\mu$ are “stripped” out of propagators, and inserted into vertices.

Systematic VEV(mass) corrections

“Scalarizing” k^μ/m_W enables systematic incorporation of all splittings (up to log) proportional to VEV(masses)

- Massless $f \rightarrow f + V_L$ corresponds to $f \rightarrow f\phi$ according to goldstone equivalence

$$y_f = 0 \rightarrow \text{gives } 0$$

- Contribution comes from VEV correction $V_n \sim \frac{m_V}{E}$: “Effective W approximation”.

$$\frac{d\mathcal{P}_{f \rightarrow f+V_L}}{dz dk_T^2} \sim \frac{\alpha_2}{2\pi} \frac{1-z}{z} \frac{m_V^2}{k_T^4} \quad (1)$$

- Has been known for years: vector boson fusion observed in LHC

S. Dawson (1985); G. Kane et al. (1984); Chanowitz & Gailard (1984)

Ultra-collinear splitting

There are characteristically new channels
 in the broken phase:

“Ultra collinear”:

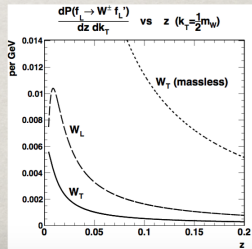
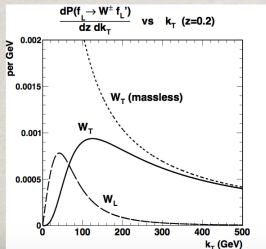
$$\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$$

$k_T^2 > m_W^2$, it shuts off;

$k_T^2 < m_W^2$, flattens out!

The DPFs for W_L thus don't run at leading log:

“Bjorken scaling” restored (higher-twist effects)!



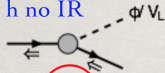
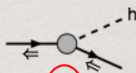
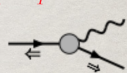
New Broken Splittings: ultra-collinear behavior (11 new types)

A whole new set of broken splittings

SPLITTING IN THE BROKEN GAUGE

New fermion splitting: $\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$

V_L is of IR, h no IR

			
	$\frac{1}{16\pi^2} \frac{v^2}{k_T^4} \left(\frac{1}{z} \right)$	$\frac{1}{16\pi^2} \frac{v^2}{k_T^4}$	$\frac{1}{16\pi^2} \frac{v^2}{k_T^4}$
	$\rightarrow V_L f_s^{(\prime)}$ ($V = \gamma$)	$h \xi$	$V_T f_s^{(\prime)}$
$f_{s=L}$	$(l_f^Y (y_f^2 \bar{z} - y_{f(\prime)}^2) z - Q_{f_L}^Y g_V^2 \bar{z})^2$	$\frac{1}{4} y_f^4 z (1 + \bar{z})^2$	$g_V^2 z (Q_{f_L}^Y y_f \bar{z} - Q_{f_L}^Y y_{f(\prime)})^2$
$f_{s=R}$	$(l_f^Y y_f y_{f(\prime)} z^2 - Q_{f_R}^Y g_V^2 \bar{z})^2$	$\frac{1}{4} y_f^4 z (1 + \bar{z})^2$	$g_V^2 z (Q_{f_R}^Y y_f \bar{z} - Q_{f_R}^Y y_{f(\prime)})^2$

Chirality conserving:
 Non-zero for massless f

Chirality flipping: $\sim m_f$

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SPLITTING IN THE BROKEN GAUGE

New gauge boson splitting in $3\text{-}W_L$

Vector boson V_L is of IR.

$$\frac{1}{16\pi^2} \frac{v^2}{k_T^4} \left(\frac{1}{z\bar{z}} \right) \quad \frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$$

	$\rightarrow W_L^+ W_L^-$	$Z_L W_L^\pm / Z_L$
W_L^\pm	0	$\frac{1}{16} g_2^4 ((z-z)(2+z\bar{z}) - t_W^2 z(1+z))^2$
h	$\frac{1}{4} (g_2^2(1-z\bar{z}) - \lambda_h z\bar{z})^2$	$\frac{1}{8} (g_2^2(1-z\bar{z}) - \lambda_h z\bar{z})^2$
Z_L	$\frac{1}{16} g_2^4 ((z-z)(2+z\bar{z}) - t_W^2 z\bar{z})^2$	0
$[hZ_L]$	$\frac{1}{8} g_2^2 (g_2^2(1-z\bar{z}) - \lambda_h z\bar{z}) (z-z)(2+z\bar{z}) - t_W^2 z\bar{z}$	0

h has no IR.

$$\frac{1}{16\pi^2} \frac{v^2}{k_T^4} \left(\frac{1}{\bar{z}} \right) \quad \frac{1}{16\pi^2} \frac{v^2}{k_T^4}$$

	$\rightarrow h W_L^\pm / Z_L$	$h h$
W_L^\pm	$\frac{1}{4} z (g_2^2(1-z\bar{z}) + \lambda_h z\bar{z})^2$	0

Estimations of Splitting Rates at 1 TeV and 10 TeV

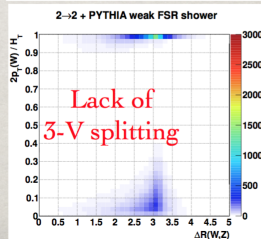
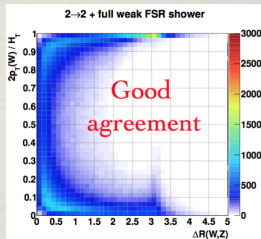
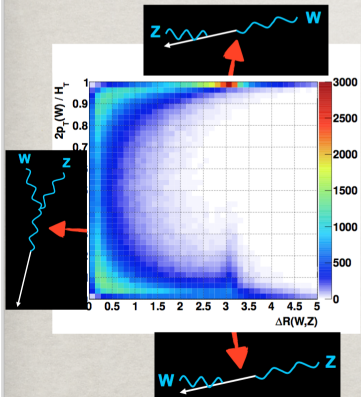
Process	$\approx \mathcal{P}(E)$ (leading-log term)	$\mathcal{P}(1 \text{ TeV})$	$\mathcal{P}(10 \text{ TeV})$
$q \rightarrow V_T q^{(\prime)}$ (CL+IR)	$(3 \times 10^{-3}) \left[\log \frac{E}{m_W} \right]^2$	1.6%	7%
$q \rightarrow V_L q^{(\prime)}$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.4%	1.1%
$t_R \rightarrow W_L^+ b_L$ (CL)	$(8 \times 10^{-3}) \log \frac{E}{m_W}$	2.5%	4%
$t_R \rightarrow W_T^+ b_L$ (UC)	(6×10^{-3})	0.6%	0.6%
$V_T \rightarrow V_T V_T$ (CL+IR)	$(0.015) \left[\log \frac{E}{m_W} \right]^2$	7%	34%
$V_T \rightarrow V_L V_T$ (UC+IR)	$(0.014) \log \frac{E}{m_W}$	2.7%	7%
$V_T \rightarrow f \bar{f}$ (CL)	$(0.02) \log \frac{E}{m_W}$	5%	10%
$V_L \rightarrow V_T h$ (CL+IR)	$(2 \times 10^{-3}) \left[\log \frac{E}{m_W} \right]^2$	0.8%	4%
$V_L \rightarrow V_L h$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.5%	1%

- Double log $>$ single log.
- “Color” factor $C_A(\text{adjoint}) > C_F(\text{fundamental})$: biggest from gauge-triple splitting

“Weak jets”: EW Showering with Multiple Gauge Bosons

AN EXAMPLE: $WZ+J$ @ 100 TEV

MadGraph $2 \rightarrow 3$ fixed order



EW Showering with Multiple Gauge Bosons

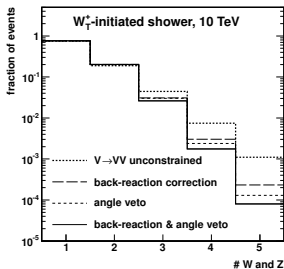
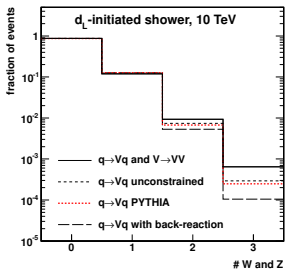


Figure: $O(10\%)$ suppression for every additional boson radiation;
 Back reaction & angular veto: more precisely (comparing with Pythia 8) taking care of phase space.

Top decay & shower

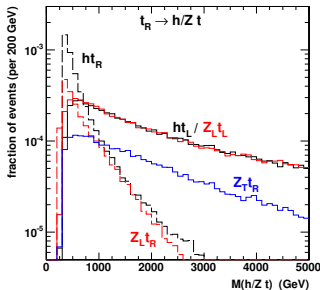
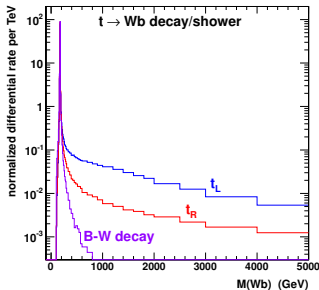


Figure: a) Splitting corrections to “tail” of top decay; b) interplay of multiple elements of top showering: yukawa (ϕ) & gauge coupling (W_n); helicity conserving & flipping.

Neutral boson interference: γ/Z_T initiated splittings

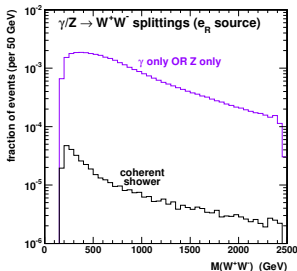
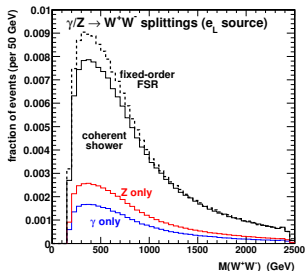


Figure: Left: LH electron source, constructive interference;
 Right: RH electron source, mainly B_0 intermediated, destructive. More transparent in B_0/W_0 basis.

Collimated higgses

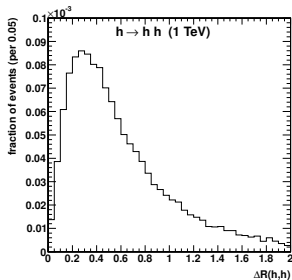


Figure: $h \rightarrow hh$, rare but interesting.

Summary

- With the increase of energy scale in colliders, we are facing a new phenomenon of EW shower.
- EW sector presents rich physics
- With the help of a new gauge (GEG), we are able to calculate all the EW (collinear log) splitting functions.
 - ① Unbroken limit: analogue to QCD
 - ② Broken: systematic VEV corrections – ultra-collinear splittings
- We incorporate EW showering into a Monte Carlo program
 - ① More complete
 - ② More accurate in implementation
- Looking forward to seeing them in real world! (and in your programs).

Thank you!

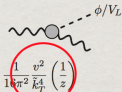
New Broken Splittings: ultra-collinear behavior

A whole new set of broken splittings

SPLITTING IN THE BROKEN GAUGE

New gauge boson splitting to $W_L W_T$

Vector boson V_L is of IR.



$$\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$$

	$\rightarrow W_L^\pm \gamma_T$	$W_L^\pm Z_T$	$Z_L W_T^\pm$	$W_L^+ W_T^-$ or $W_L^- W_T^+$
W_T^\pm	$e^2 g_2^2 \bar{z}^3$	$\frac{1}{4} c_W^2 g_2^4 \bar{z} ((1 + \bar{z}) + t_W^2 z)^2$	$\frac{1}{4} g_2^4 \bar{z} (1 + \bar{z})^2$	0
γ_T	0	0	0	$e^2 g_2^2 \bar{z}$
Z_T	0	0	0	$\frac{1}{4} c_W^2 g_2^4 \bar{z} ((1 + \bar{z}) - t_W^2 z)^2$
$[\gamma Z]_T$	0	0	0	$\frac{1}{2} c_W e g_2^3 \bar{z} ((1 + \bar{z}) - t_W^2 z)$

h & f have no IR.



	$\rightarrow h V_T (V \neq \gamma)$	$f_s \bar{f}_s^{(f)}$
V_T	$\frac{1}{4} z \bar{z} g_V^4$	$\frac{1}{2} g_V^4 (Q_{f_s}^V y_{f(s)} z + Q_{f_s}^V y_{f \bar{s}} z)^2$
$[\gamma Z]_T$	0	$\frac{1}{2} e g z v^2 Q_f^2 (Q_{f_s}^Z z + Q_{f_s}^Z \bar{z})$

Implementation and New Phenomenon: EW PDF

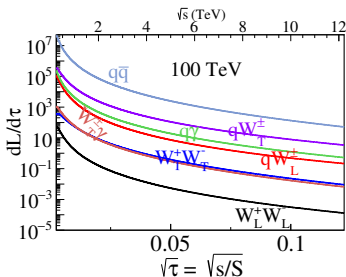
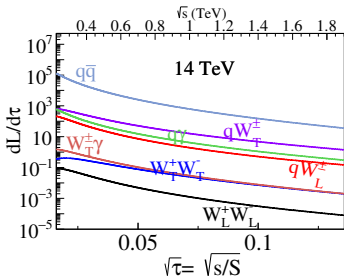
- At high energy, EW particles “show up” in the initial beams, i.e. EW PDFs.
- A useful intermediate concept is luminosities, with which the total cross section could be written as,

$$\sigma_{PP}(V_1 V_2 \rightarrow X) = \int_{\tau_{\text{low}}}^{\tau_{\text{high}}} d\tau \frac{d\mathcal{L}_{V_1 V_2}}{d\tau} \hat{\sigma}(V_1 V_2 \rightarrow \hat{X}_\tau), \quad (2)$$

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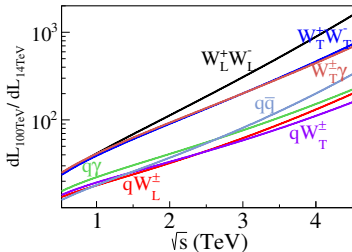
$$\frac{d\mathcal{L}_{V_1 V_2}}{d\tau} \simeq \frac{2}{(\delta_{V_1 V_2} + 1)} \int_\tau^1 \frac{d\xi}{\xi} \int_{\tau/\xi}^1 \frac{dz_1}{z_1} \int_{\tau/\xi/z_1}^1 \frac{dz_2}{z_2} \times$$

$$\sum_{q_1, q_2} f_{V_1 \in q_1}(z_1) f_{V_2 \in q_2}(z_2) f_{q_1 \in P}(\xi) f_{q_2 \in P}\left(\frac{\tau}{\xi z_1 z_2}\right) \quad (3)$$

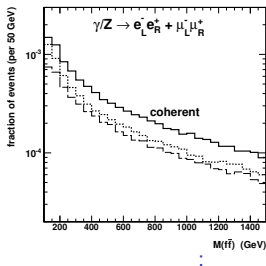
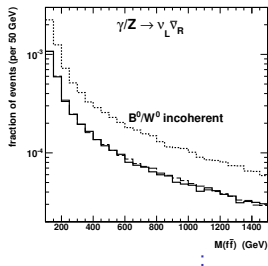


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More on neutral boson interference



Implementation and New Phenomenon: h/Z_L and higgs

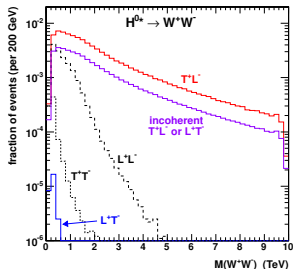


Figure: (a) h/Z_L interference, usually maximal. More transparent in H_0/H_0^* basis (b) $h \rightarrow hh$, rare but interesting.

EW Showering Effects on New Physics

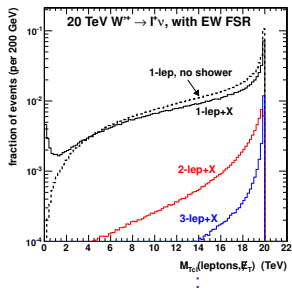


Figure: Showered events from 20 TeV W'^{+} decays. Invariant mass more sharply peaked for multiple splittings.

Isospin self-averaging & Combining QCD effects

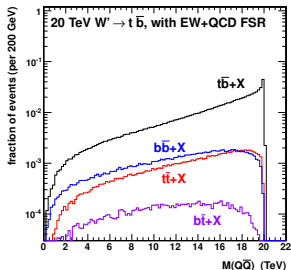
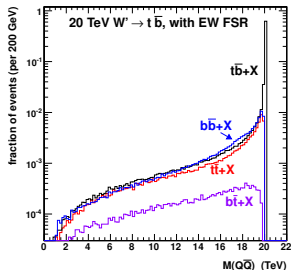


Figure: Showered events from 20 TeV W^{++} decays. (a) $W^{++} \rightarrow t_L \bar{b}_R$, full EW shower, and (b) combining EW and QCD showering.