Portorož, Slovenia

April 21th, 2017

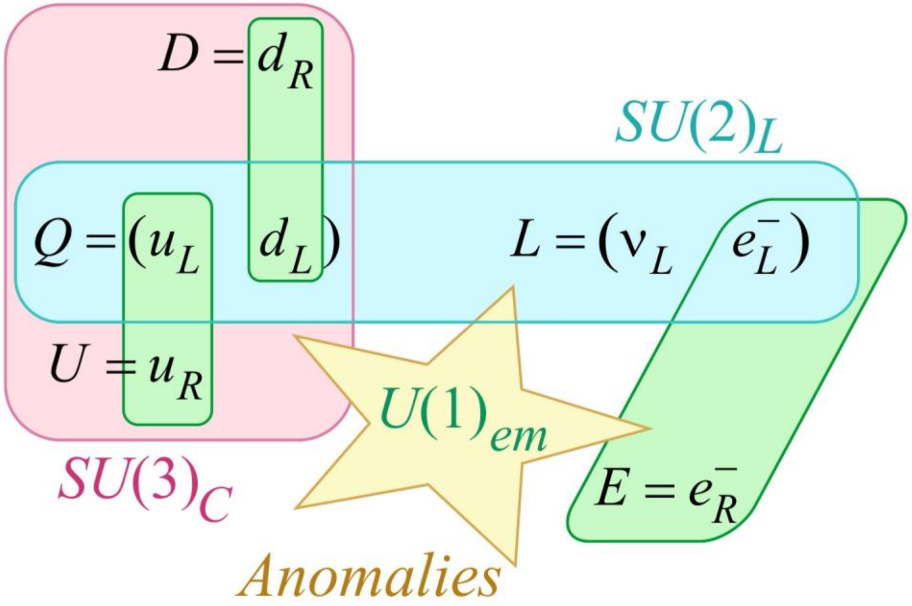
Towards a new paradigm for quark-lepton unification

Christopher Smith

Introduction

A. Flavors and unification

From the gauge point of view, fermions are well unified:



From the flavor point of view, the situation is far less satisfactory:

$$\mathcal{L}_{Yukawa} = \bar{U}\mathbf{Y}_u QH + \bar{D}\mathbf{Y}_d QH^\dagger + \bar{E}\mathbf{Y}_e LH^\dagger$$

Unrelated 3x3 complex matrices

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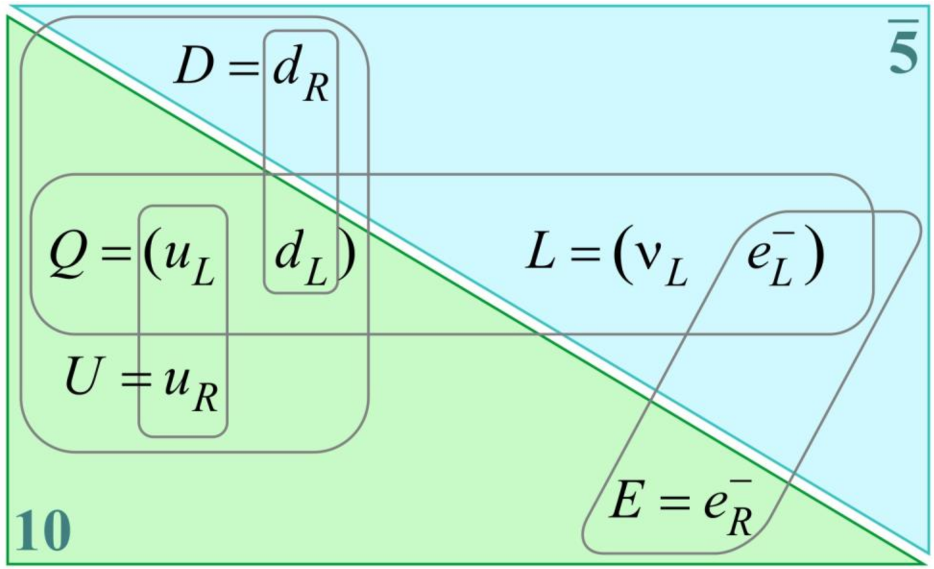

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$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\bar{\chi}_{10}^{C}\mathrm{Y}_{10}\chi_{10}h_5 + \sqrt{2}\bar{\psi}_{\bar{5}}^{C}\mathrm{Y}_5\chi_{10}h_5^{\dagger}$$

$$\mathrm{Y}_{10} = \mathrm{Y}_{10}^{T} = \mathrm{Y}_u \qquad \mathrm{Y}_5 = \mathrm{Y}_e^{T} = \mathrm{Y}_d \rightarrow \frac{m_d}{m_s} = \frac{m_e}{m_\mu} \text{ ????}$$

A. Flavors and unification

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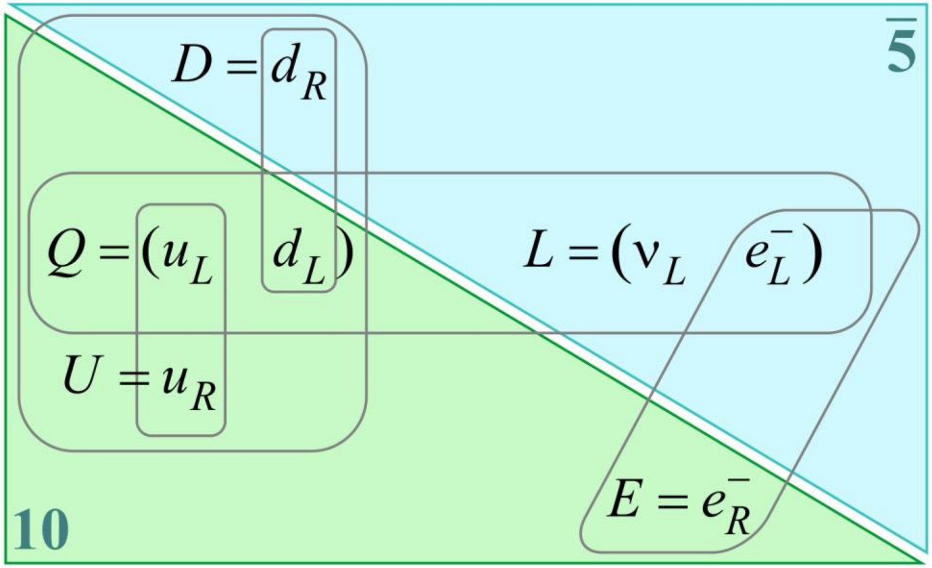


From the flavor point of view, the situation is far less satisfactory:

$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\bar{\chi}_{10}^{C} Y_{10} \chi_{10} h_5 + \sqrt{2}\bar{\psi}_{\bar{5}}^{C} Y_5 \chi_{10} h_5^{\dagger} - \sqrt{12}\bar{\psi}_{\bar{5}}^{C} Y_5' \chi_{10} h_{45}^{\dagger} + \dots$$

$$Y_{10} = Y_{10}^{T} = Y_u \qquad Y_5 - 3Y_5' = Y_e^{T}\ ,\ Y_5 + Y_5' = Y_d$$

B. Minimal Flavor Violation

The three generations of quarks/leptons have identical gauge interactions

→ flavor symmetry: $G_F = U(3)^5 = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

Chivukula, Georgi '87

- The only sources of breaking are the Yukawa couplings:

$$\mathcal{L}_{Yukawa} = U\mathrm{Y}_u QH + D\mathrm{Y}_d QH^\dagger + E\mathrm{Y}_e LH^\dagger$$

...but artificially invariant if $\mathrm{Y}_{u,d,e}$ transform under G_F (= spurions).

- New physics couplings/operators are assumed to be also invariant:

Example: $d_L^I \to d_L^J \gamma^*$ from $\mathcal{O}_\gamma \sim c^{IJ} \bar{Q}^I \gamma_\nu Q^J D_\mu F^{\mu\nu}$

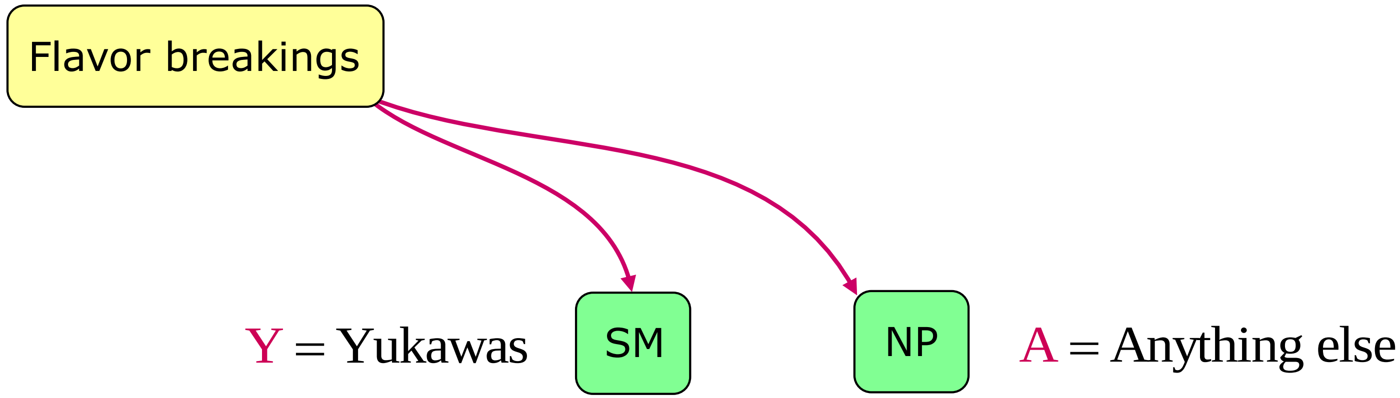
MFV expansion: $c^{IJ} \sim (\alpha 1 + \beta \mathrm{Y}_u^\dagger \mathrm{Y}_u + \gamma \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots)^{IJ}$

Background values: $v\mathrm{Y}_u = m_u V_{CKM}$, $v\mathrm{Y}_{d,e} = m_{d,e}$.

D'Ambrosio, et al. '02

C. The redundancy interpretation

Nikolidakis,CS ‘07
Colangelo,Nikolidakis,CS ‘08
CS ‘11



Some NP mechanism is at the origin of all the flavor structures.

C. The redundancy interpretation

Nikolidakis,CS '07
Colangelo,Nikolidakis,CS '08
CS '11

Flavor breakings

Imagine there is a single fundamental spurion X at the high energy scale ← minimality

SM

$$\mathrm{Y} = x_0 1 + x_1 \mathrm{X} + x_2 \mathrm{X}^2$$

$$x_i \sim \mathcal{O}(1)$$

NP

$$\mathrm{A} = x'_0 1 + x'_1 \mathrm{X} + x'_2 \mathrm{X}^2$$

$$x'_i \sim \mathcal{O}(1)$$ ← naturality

Some NP mechanism is at the origin of all the flavor structures.

Remark: finite expansions thanks to Cayley-Hamilton identities:

$$\mathrm{X}^3 - \langle \mathrm{X} \rangle \mathrm{X}^2 + \tfrac{1}{2}\mathrm{X}(\langle \mathrm{X} \rangle^2 - \langle \mathrm{X}^2 \rangle) = \tfrac{1}{3}\langle \mathrm{X}^3 \rangle - \tfrac{1}{2}\langle \mathrm{X} \rangle \langle \mathrm{X}^2 \rangle + \tfrac{1}{6}\langle \mathrm{X} \rangle^3$$

C. The redundancy interpretation

Nikolidakis,CS ‘07
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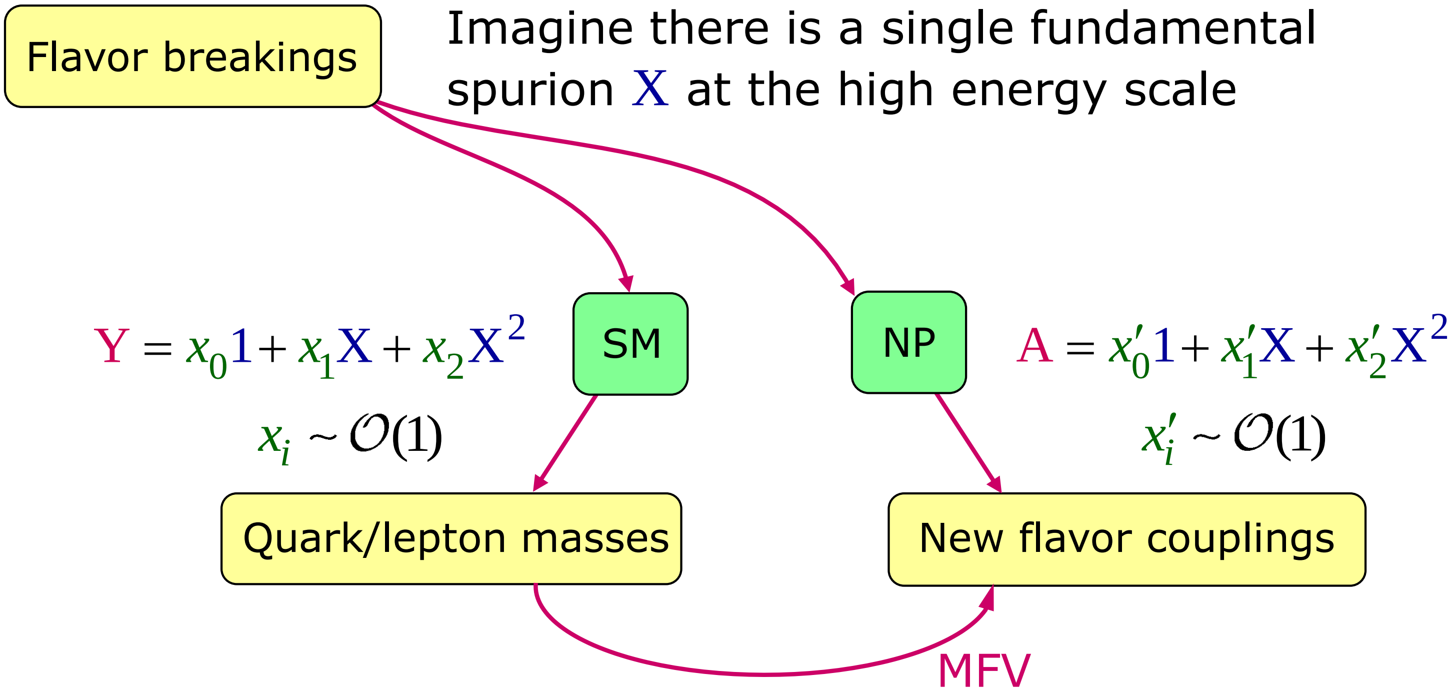
Imagine there is a single fundamental spurion X at the high energy scale ← minimality



← naturality

Then, all that is observable are MFV relations among couplings:

$\mathrm{A} = a_0 1 + a_1 \mathrm{Y} + a_2 \mathrm{Y}^2$ or $\mathrm{Y} = b_0 1 + b_1 \mathrm{A} + b_2 \mathrm{A}^2$ with $a_i, b_i \sim \mathcal{O}(1)$

C. The redundancy interpretation

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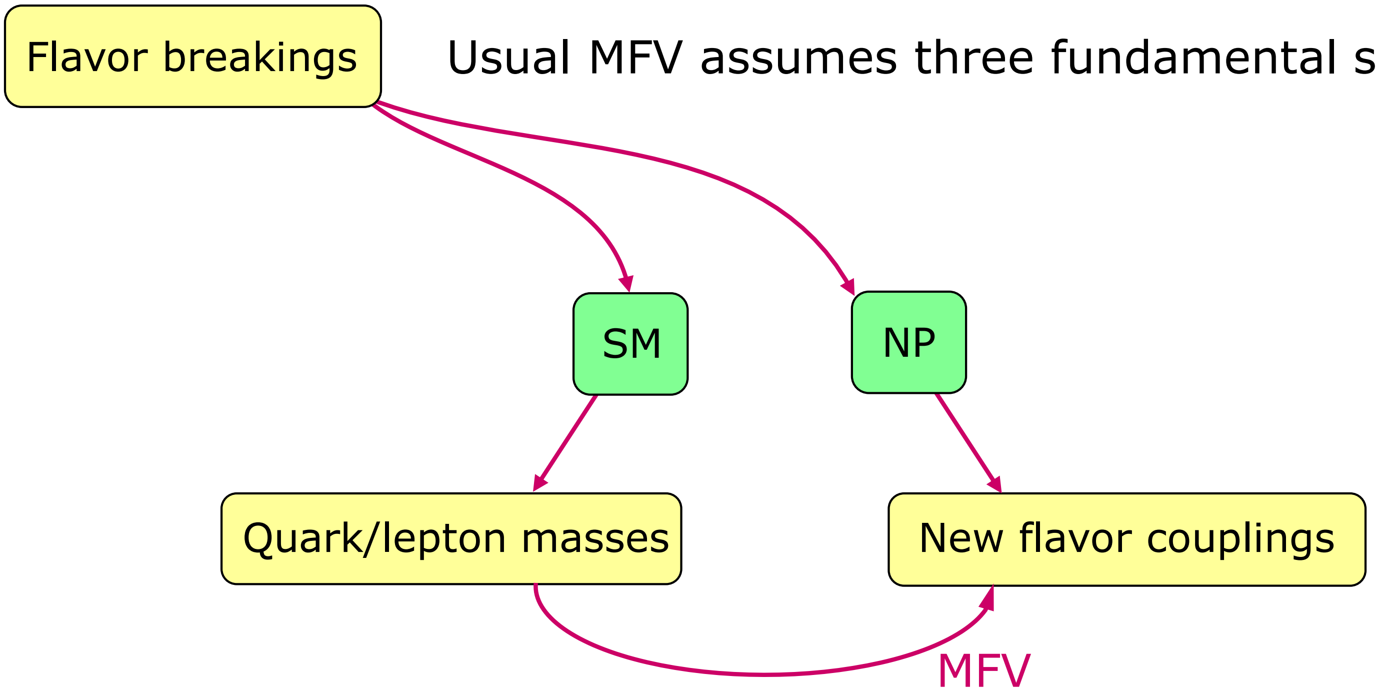


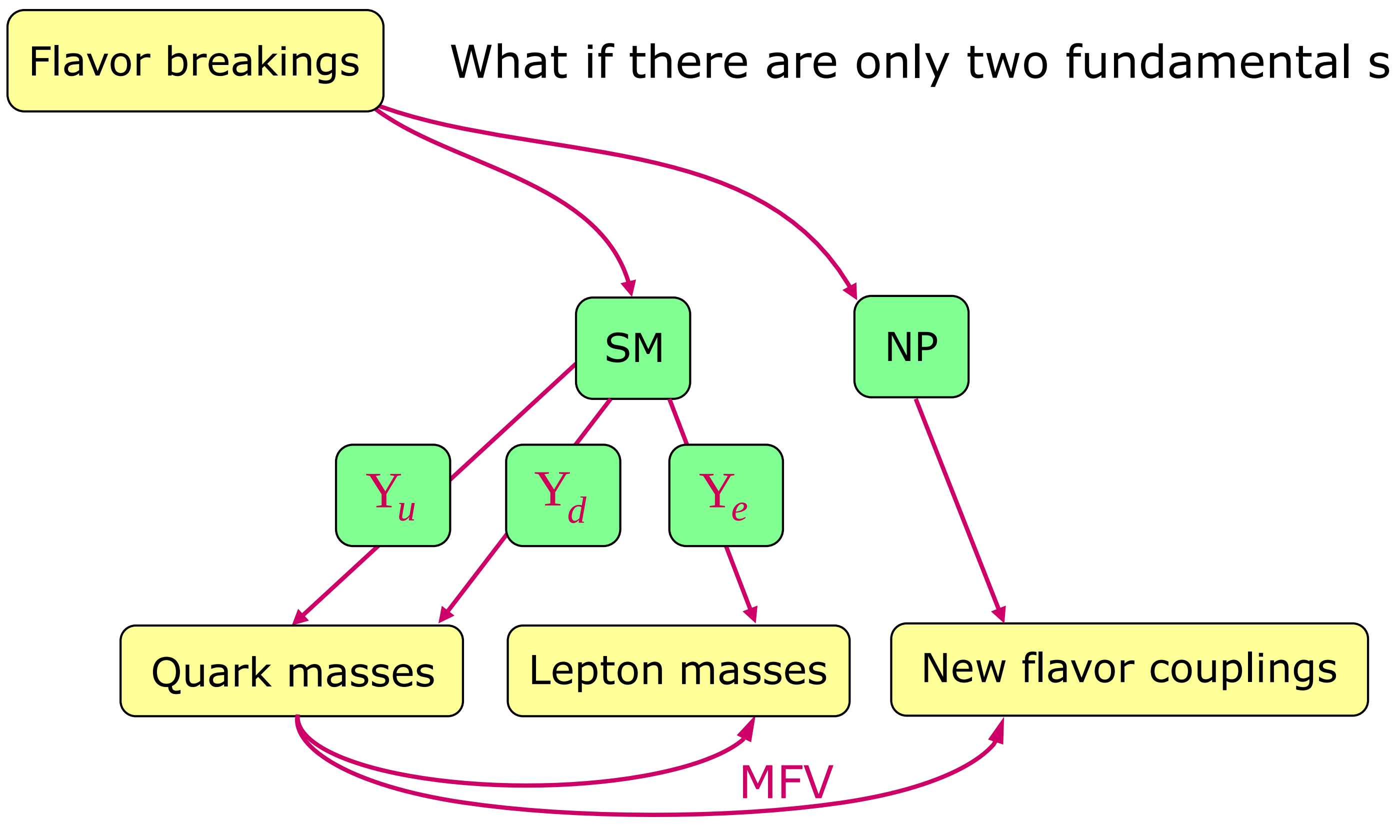
Usual MFV assumes three fundamental spurions X, Y, Z.

Trade X, Y, Z for Y_u, Y_d, Y_e and express NP couplings in terms of them.

D. Going beyond...

CS '16

What if there are only two fundamental spurions $\mathbf{X},\mathbf{Y}$?



At low energy, the SM couplings must satisfy $\mathbf{Y}_e = F(\mathbf{Y}_u, \mathbf{Y}_d)$.

[Neutrinos kept massless here]

- Outline

  I. Flavor perspective on Yukawa unification

  II. Geometric MFV

  III. Application to the MSSM

  IV. Application to minimal SU(5)

Based on arXiv:1612.03825

(to appear in JHEP)

I. Flavor perspective on Yukawa unification

A. Setting up MFV for Yukawas

To proceed, a choice must be made about the fundamental spurions.

Flavor symmetry: $G'_F = U(3)^3 = U(3)_{Q=L} \times U(3)_U \times U(3)_{D=E}$

Spurions: $\mathrm{Y}_u, \mathrm{Y}_d$ Known in the same gauge basis:

$\xrightarrow{G'_F}$ $v\mathrm{Y}_u = m_u V_{CKM}\ ,\ v\mathrm{Y}_d = m_d\ .$

Expansion: $$\mathrm{Y}_e = x_0 \mathrm{Y}_d \cdot \left(1 + x_1 \mathrm{A} + x_2 \mathrm{B} + x_3 \mathrm{B}^2 + x_4 \{\mathrm{A},\mathrm{B}\} + x_5 \mathrm{BAB} + x_6 i[\mathrm{A},\mathrm{B}] + x_7 i[\mathrm{A},\mathrm{B}^2] + x_8 i(\mathrm{BAB}^2 - \mathrm{B}^2\mathrm{AB})\right)$$

$$(\mathrm{A} \equiv \mathrm{Y}_d^\dagger \mathrm{Y}_d, \mathrm{B} \equiv \mathrm{Y}_u^\dagger \mathrm{Y}_u)$$

These choices match SU(5), up to an irrelevant transposition.

A. Setting up MFV for Yukawas

The MFV basis is nearly singular → Very large coefficients in general.

Mercoli,CS '09

Assuming alignment of the lepton and down-quark mass basis:

$$Y_e \equiv 0.2 Y_d \cdot \left(1 + 10^8 Y_d^\dagger Y_d - 10^{11} (Y_d^\dagger Y_d)^2\right) \quad \text{for } \tan\beta = 50.$$

Allowing for subsequent SVD, keeping only three terms ($\tan\beta = 50$):

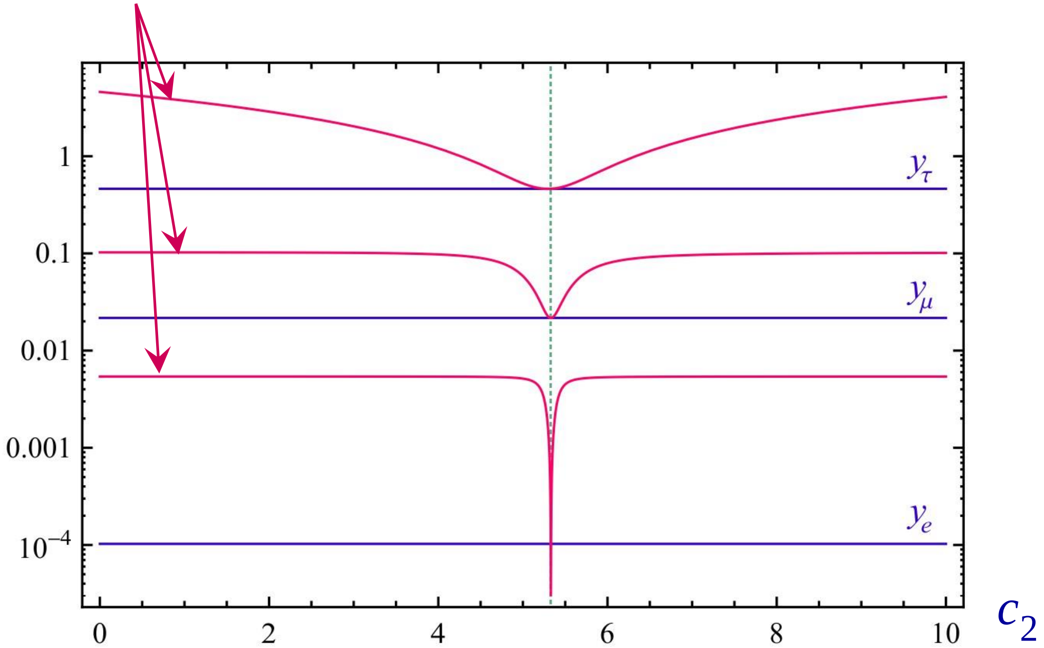
$$Y_e = c_0 Y_d \cdot \left(1 + c_1 Y_u^\dagger Y_u + c_2 Y_d^\dagger Y_d\right)$$

	c_0	c_1	c_2
Masses at M_Z	8.6	-1.8	1.2
SM at M_{GUT}	22	6	-20
MSSM at M_{GUT}	20	-7.9	5.3

Remarkable that reasonable coefficients are possible at all!!!

B. On the anatomy of a fine-tuning

If we plot the SVD of $Y_e = c_0 Y_d \cdot X\,,\ \ X = 1 + c_1 Y_u^\dagger Y_u + c_2 Y_d^\dagger Y_d$:

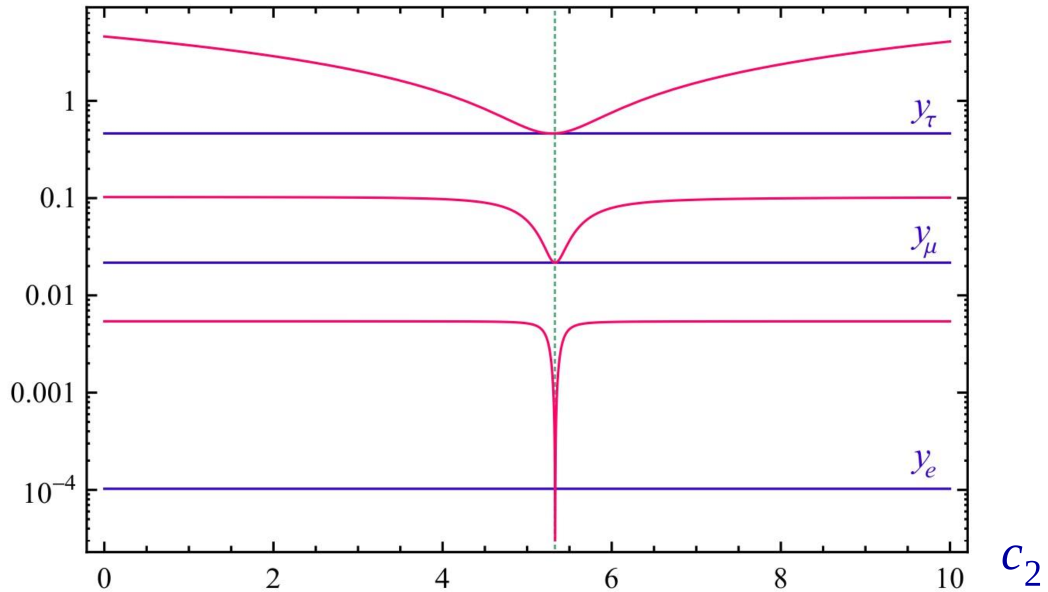


At the physical point: $|X| \approx \begin{pmatrix} 1 & 0.0005 & 0.01 \\ 0.0005 & 1 & 0.06 \\ 0.01 & 0.06 & 0.004 \end{pmatrix}$

Very delicate cancellation $1 + c_1 y_t^2 + c_2 y_b^2 \approx 0$!

B. On the anatomy of a fine-tuning

If we plot the SVD of $Y_e = c_0 Y_d \cdot X$, $X = 1 + c_1 Y_u^\dagger Y_u + c_2 Y_d^\dagger Y_d$:



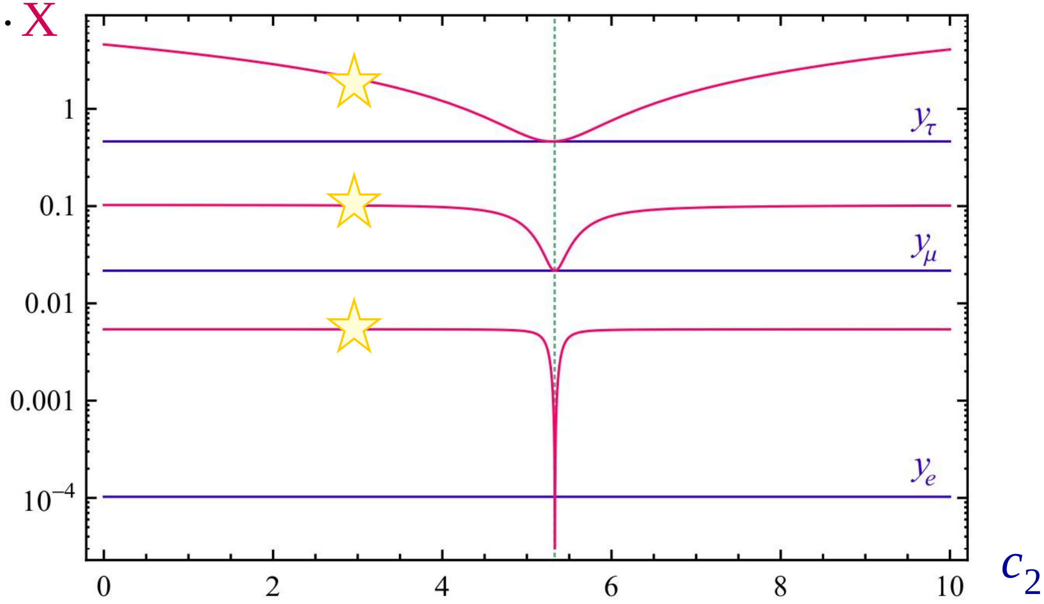
Mathematically, there is a singularity within the natural $c_{1,2}$ ranges:

$$\det Y_e = c_0^3 \times \det Y_d \times \det X \quad \Rightarrow \quad \det X \approx 1 + c_1 \langle Y_u^\dagger Y_u \rangle + c_2 \langle Y_d^\dagger Y_d \rangle \approx 0$$

No finite polynomial relation between $Y_{e,u,d}$ will ever be natural!

C. The twisted persona of the leptons

$$Y_e = c_0 Y_d \cdot X$$

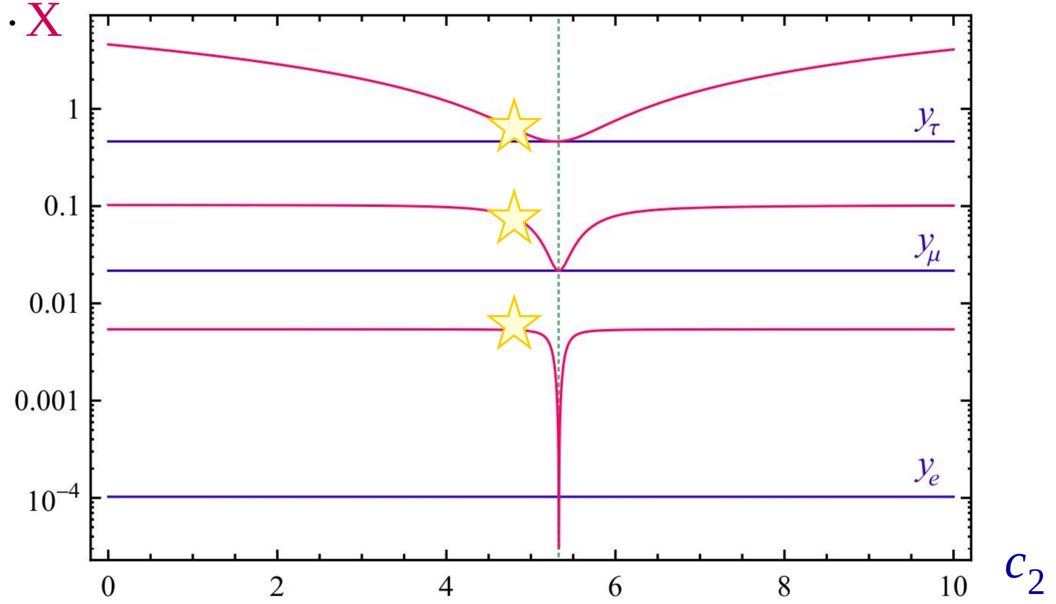


$v g_E Y_e g_L^\dagger = m_e$, $c_2 = 2.5$: Small CKM-like mixings

$$|g_E| \approx \begin{pmatrix} 1.000 & 0.0002 & 0.00005 \\ 0.0002 & 1.000 & 0.005 \\ 0.00006 & 0.005 & 1.000 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 1.00 & 0.005 & 0.04 \\ 0.002 & 0.98 & 0.18 \\ 0.04 & 0.18 & 0.98 \end{pmatrix}$$

C. The twisted persona of the leptons

$\mathrm{Y}_e = c_0 \mathrm{Y}_d \cdot \mathrm{X}$



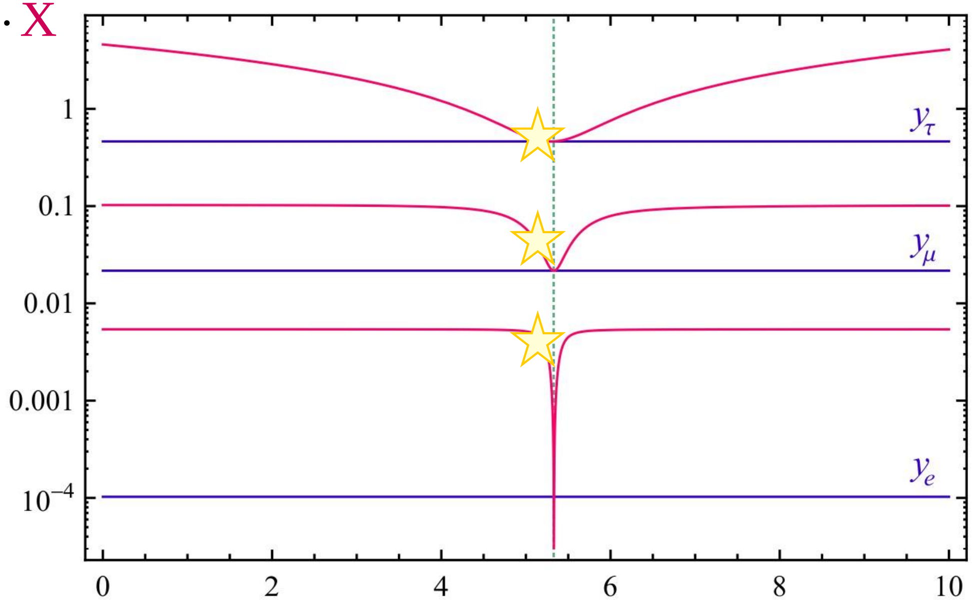
$v g_E \mathrm{Y}_e g_L^\dagger = m_e$, $c_2 = 4.8$: Large mixing close to the SVD reordering

$$|g_E| \approx \begin{pmatrix} 1.000 & 0.008 & 0.0001 \\ 0.008 & 0.99 & 0.11 \\ 0.002 & 0.11 & 0.99 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.98 & 0.13 & 0.14 \\ 0.01 & 0.71 & 0.70 \\ 0.20 & 0.69 & 0.70 \end{pmatrix}$$

$$g_E \cdot \mathrm{Y}_d^\dagger \cdot \mathrm{X}^2 \cdot \mathrm{Y}_d \cdot g_E^\dagger \qquad g_L \cdot \mathrm{X} \cdot \mathrm{Y}_d^\dagger \mathrm{Y}_d \cdot \mathrm{X} \cdot g_L^\dagger$$

C. The twisted persona of the leptons

$$Y_e = c_0 Y_d \cdot X$$

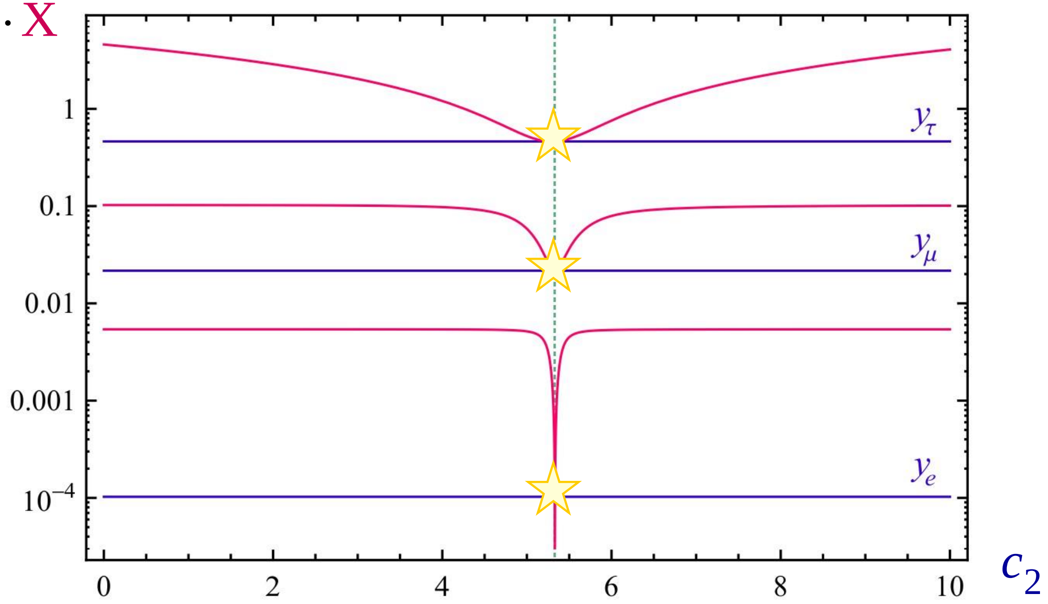


$v g_E Y_e g_L^\dagger = m_e$, $c_2 = 5.2$: Towards a second SVD reordering

$$|g_E| \approx \begin{pmatrix} 0.97 & 0.09 & 0.002 \\ 0.08 & 0.98 & 0.20 \\ 0.02 & 0.20 & 0.98 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.80 & 0.56 & 0.20 \\ 0.03 & 0.28 & 0.96 \\ 0.59 & 0.77 & 0.21 \end{pmatrix}$$

C. The twisted persona of the leptons

$Y_e = c_0 Y_d \cdot X$

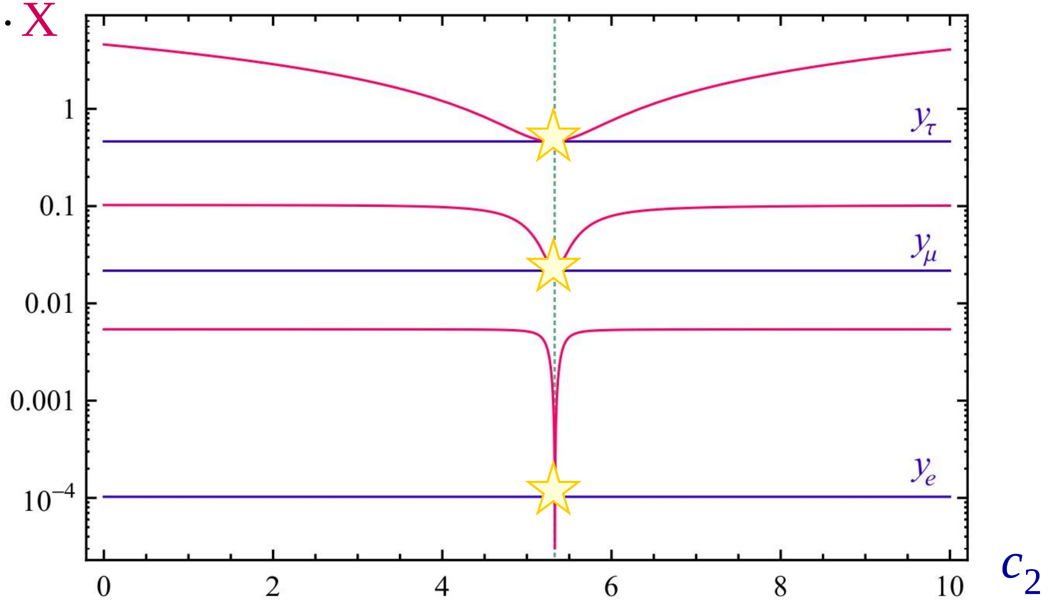


$v g_E Y_e g_L^\dagger = m_e$, $c_2 = 5.3$: At the physical point, CKM-like mixings

$$|g_E| \approx \begin{pmatrix} 0.97 & 0.24 & 0.002 \\ 0.24 & 0.95 & 0.22 \\ 0.06 & 0.21 & 0.98 \end{pmatrix} \qquad |g_L| \approx \begin{pmatrix} 0.03 & 0.98 & 0.20 \\ 0.06 & 0.20 & 0.98 \\ 1.00 & 0.02 & 0.07 \end{pmatrix}$$

C. The twisted persona of the leptons

$$\mathrm{Y}_e = c_0 \mathrm{Y}_d \cdot \mathrm{X}$$



$v g_E \mathrm{Y}_e g_L^\dagger = m_e$, $c_2 = 5.3$: At the physical point, twisted leptons!

$$|\mathrm{X}| \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{gauge} \approx \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{phys}$$

Only singularity at natural $c_{1,2}$ values

Top partner is the lightest

II. Geometric MFV

A. Pseudo fine tuning thanks to the geometric expansion

Polynomial expansions are fine-tuned: What about infinite series?

$$\mathrm{X} = 1 + \eta \mathrm{Y}_u^\dagger \mathrm{Y}_u + \eta^2 (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^2 + \eta^3 (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^3 + \ldots = \frac{1}{1 - \eta \mathrm{Y}_u^\dagger \mathrm{Y}_u}$$

If η is large enough: $\mathrm{X} = \begin{pmatrix} \frac{1}{1-\eta y_u^2} \approx 1 & & \\ & \frac{1}{1-\eta y_c^2} \approx 1 & \\ & & \frac{1}{1-\eta y_t^2} \approx 0 \end{pmatrix}$.

Actually, the large top mass ensures: $(\mathrm{Y}_u^\dagger \mathrm{Y}_u)^n \approx \langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle^{n-1} \mathrm{Y}_u^\dagger \mathrm{Y}_u$

$$\mathrm{X} = \sum_{n=0}^{\infty} \eta^n (\mathrm{Y}_u^\dagger \mathrm{Y}_u)^n = 1 + \frac{\eta}{1 - \eta \langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle} \mathrm{Y}_u^\dagger \mathrm{Y}_u \xrightarrow{\eta \to \infty} 1 - \frac{1}{\langle \mathrm{Y}_u^\dagger \mathrm{Y}_u \rangle} \mathrm{Y}_u^\dagger \mathrm{Y}_u$$

Precisely the fictitious fine-tuning we need!

B. Toy model with vector-like lepton doublets

Toy model to sum the MFV series outside its radius of convergence.

Add – Weak doublet of vector-like flavor triplet leptons $X_{L,R}$.

– Scalar singlet H_s with vev v_s.

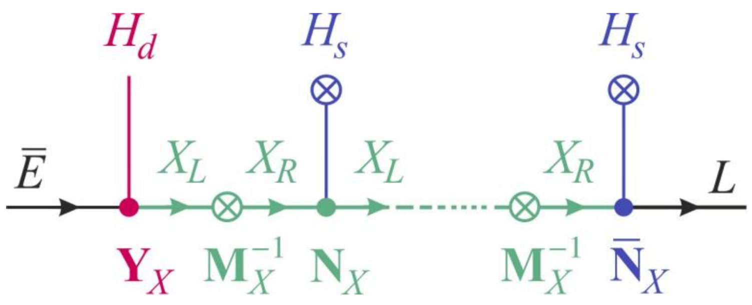
$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{X}_{L,R} \not{D} X_{L,R} + \bar{X}_{L,R} \mathrm{M}_X X_{R,L} + (\bar{X}_L \mathrm{N}_X X_L H_s + \bar{X}_R \bar{\mathrm{N}}_X L H_s + E \mathrm{Y}_X X_L H^* + h.c.)$$

Impose MFV under $G_F = U(3)^3 = U(3)_{Q=L=X_{L,R}} \times U(3)_U \times U(3)_{D=E}$

$$\begin{cases} \mathrm{M}_X = M_X \mathbf{1} \\ \mathrm{Y}_e = \mathrm{Y}_X = \gamma \mathrm{Y}_d \\ \mathrm{N}_X = \bar{\mathrm{N}}_X = \alpha \mathrm{Y}_u^\dagger \mathrm{Y}_u + \beta \mathrm{Y}_d^\dagger \mathrm{Y}_d \end{cases}$$

B. Toy model with vector-like lepton doublets

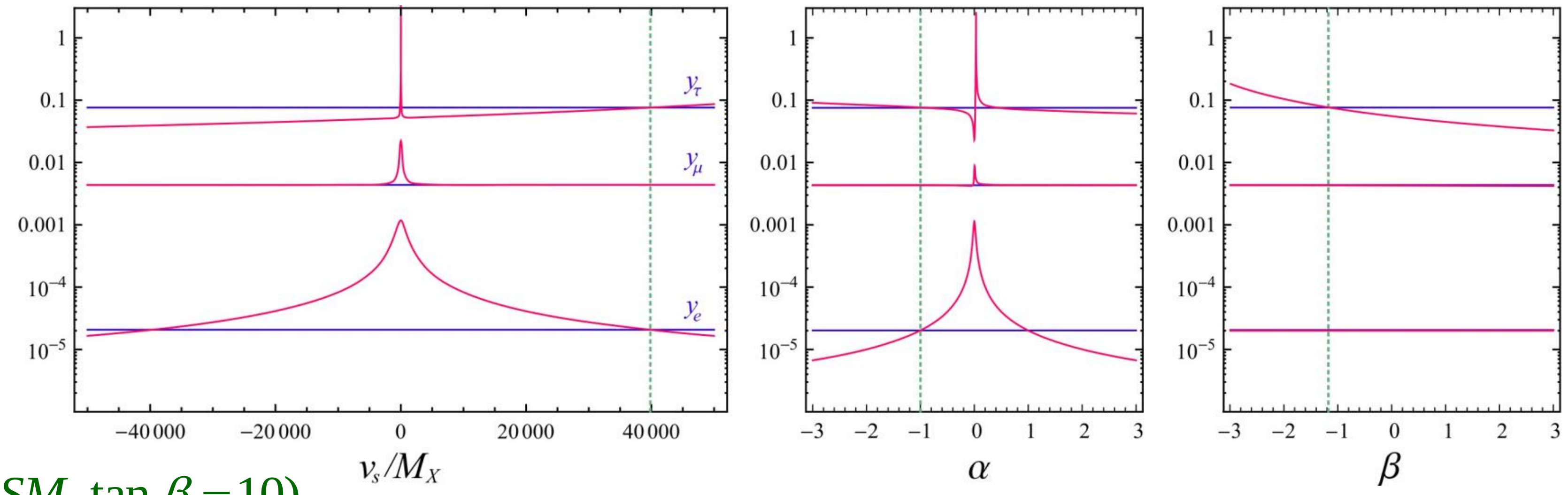
Integrating out the vector leptons:



Effective lepton Yukawa coupling:

$$\mathrm{Y}_e^{eff} = \mathrm{Y}_e - \mathrm{Y}_X \frac{1}{\mathrm{M}_X + \mathrm{N}_X H_s} \bar{\mathrm{N}}_X H_s = \gamma \mathrm{Y}_d \frac{1}{1 + (v_s / M_X)(\alpha \mathrm{Y}_u^\dagger \mathrm{Y}_u + \beta \mathrm{Y}_d^\dagger \mathrm{Y}_d)}$$

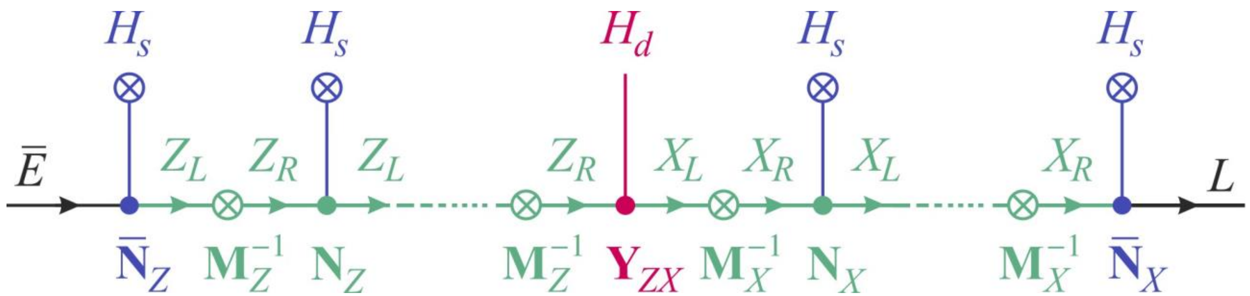
No more fine-tuning, and all parameters natural:



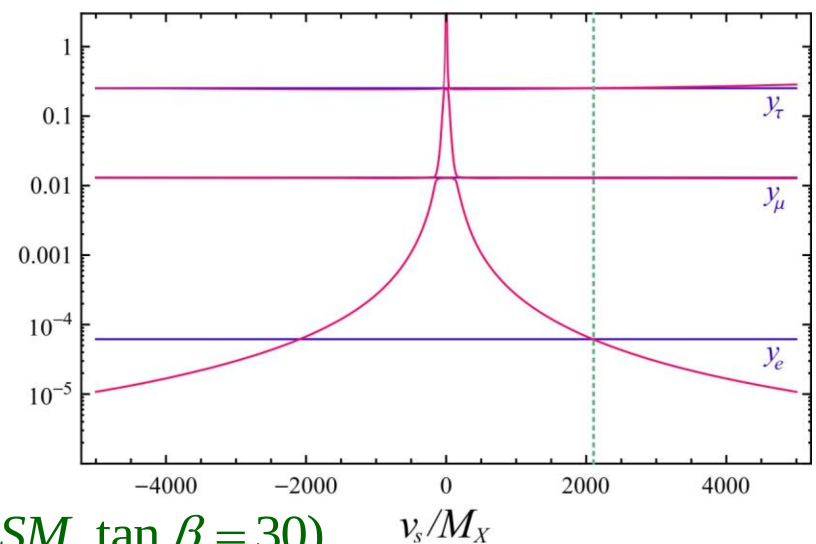
$(MSSM, \tan\beta = 10)$

C. Adding vector-like lepton singlets

Further adding weak singlet vector-like leptons $Z_{L,R}$:



$$\mathbf{Y}_e^{eff} = \frac{1}{1+(v_s/M_Z)(\varepsilon \mathbf{Y}_d \mathbf{Y}_d^\dagger)} \gamma \mathbf{Y}_d \frac{1}{1+(v_s/M_X)(\alpha \mathbf{Y}_u^\dagger \mathbf{Y}_u + \beta \mathbf{Y}_d^\dagger \mathbf{Y}_d)}$$



$(MSSM, \tan\beta = 30)$

Full twisting of the leptons:

$$|g_{E,L}| \approx \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(0.01)$$

III. Application to the MSSM

A. Geometric MFV for squark soft breaking terms

Assume a simplified structure: $\begin{cases} m^2_{Q,U,D} = m_0^2 X_{Q,U,D} \\ A_{u,d} = A_0 X_{U,D} \cdot Y_{u,d} \cdot X_Q \end{cases}$

where $X_{Q,U,D}$ are geometric:

$$X_Q = \frac{1}{1-\eta_q(\alpha^q Y_u^\dagger Y_u + \beta^q Y_d^\dagger Y_d + \ldots)}, \quad X_{U,D} = \frac{1}{1-\eta_{u,d}(\alpha^{u,d} Y_{u,d} Y_{u,d}^\dagger + \ldots)}$$

When $\eta_{q,u,d}$ are large, third-generation squarks tend to be much lighter:

For example, $X_U \to 1 - \langle Y_u^\dagger Y_u \rangle^{-1} Y_u Y_u^\dagger \approx \mathrm{diag}(1,1,0)$.

After RGE down, NSUSY-like squark spectrum:

Brummer,Kraml, Kulkarni,CS, '14

- $A_{u,d}$ geometric behavior washed out,
- stop quark(s) and possibly left sbottom remain much lighter,
- MFV remains at all scales → No problem with FCNC!

B. Geometric MFV for slepton soft breaking terms

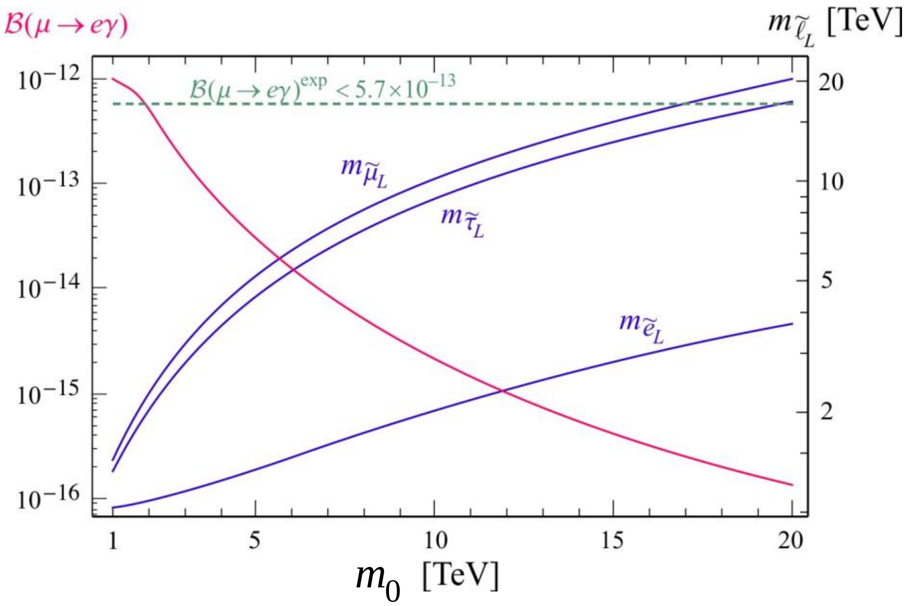
Assume a simplified structure:
$$\begin{cases} \mathbf{Y}_e = \gamma \mathbf{X}_D \cdot \mathbf{Y}_d \cdot \mathbf{X}_Q \\ \mathbf{m}^2_{L,E} = m_0^2 \mathbf{X}_{Q,D} \\ \mathbf{A}_e = A_0 \mathbf{X}_D \cdot \mathbf{Y}_d \cdot \mathbf{X}_Q \end{cases}$$

1. Lepton-slepton are not aligned, but all mixing matrices computable.

2. Much lighter $\tilde{e}_{R,L}$, corresponding to the $\tilde{\tau}_{R,L}$ gauge state.

3. LFV induced entirely by the CKM mixing

→ lower bound on m_0:



Caution: PMNS is still absent since $m_\nu = 0$!

C. What about R-parity violation?

Nikolidakis, CS '07

The flavor symmetry $U(3)_{Q=L} \times U(3)_U \times U(3)_{D=E}$ with only $\mathbf{Y}_{u,d}$ as spurions:

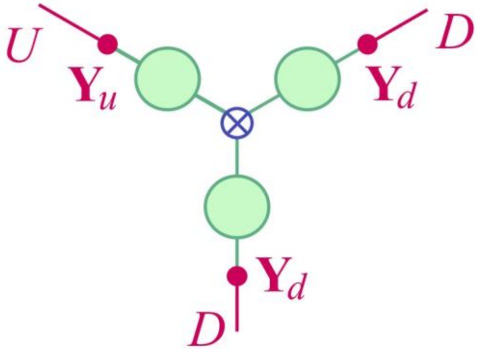
Forbids $\mathcal{L}$ violation but allows for $\mathcal{B}$ violation: $\mathcal{W}_{RPV} \supset \lambda''^{IJK} U^I D^J D^K$

Holomorphy ? → $\lambda''^{IJK} = \lambda \varepsilon^{LMN} \mathbf{Y}_u^{IL} \mathbf{Y}_d^{JM} \mathbf{Y}_d^{KN}$

Csaki, Grossman, Heidenreich '11

With a geometric behavior, holomorphy in $\mathbf{Y}_{u,d}$ is lost, but:

$$\lambda''^{IJK} = \lambda \varepsilon^{LMN} (X_U \cdot \gamma \mathbf{Y}_u \cdot X_Q)^{IL} (X_D \cdot \gamma \mathbf{Y}_d \cdot X_Q)^{JM} (X_D \cdot \gamma \mathbf{Y}_d \cdot X_Q)^{KN}$$



Numerically: holomorphy comes back after the RG evolution starting from this MFV input.

Holomorphy is a very strong attractor!

Bernon, CS '14

IV. Application to minimal SU(5)

A. Flavor troubles in minimal SU(5)

The minimal flavor content is not compatible with observed masses:

$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\bar{\chi}_{10}^{C}\mathbf{Y}_{10}\chi_{10}h_5 + \sqrt{2}\bar{\psi}_{\bar{5}}^{C}\mathbf{Y}_5\chi_{10}h_5^{\dagger} \longrightarrow \mathbf{Y}_e^T = \mathbf{Y}_d \Leftrightarrow \frac{m_d}{m_s} = \frac{m_e}{m_\mu}$$

This can be cured by adding a third Yukawa coupling:

$$\mathcal{L}_{Yukawa} = -\frac{1}{4}\bar{\chi}_{10}^{C}\mathbf{Y}_{10}\chi_{10}h_5 + \underbrace{\sqrt{2}\bar{\psi}_{\bar{5}}^{C}\mathbf{Y}_5\chi_{10}h_5^{\dagger} + \frac{\sqrt{2}}{\Lambda}\bar{\psi}_{\bar{5}}^{C}\mathbf{Y}_5'\chi_{10}H_{24}h_5^{\dagger}} + \ldots$$

$$\mathbf{Y}_5 - \frac{3v_{24}}{2\Lambda}\mathbf{Y}_5' = \mathbf{Y}_e^T \;,\quad \mathbf{Y}_5 + \frac{v_{24}}{\Lambda}\mathbf{Y}_5' = \mathbf{Y}_d$$

But then, MFV fails in SU(5) because $U(3)_{Q=U=E} \times U(3)_{D=L}$ is too small:

$$\mathbf{m}_{10}^2 = m_0^2(c_0 1 + c_1\mathbf{Y}_{10}^{\dagger}\mathbf{Y}_{10} + \underbrace{c_2\mathbf{Y}_5^{\dagger}\mathbf{Y}_5 + c_3\mathbf{Y}_5^{\dagger}\mathbf{Y}_5'} + \ldots)$$

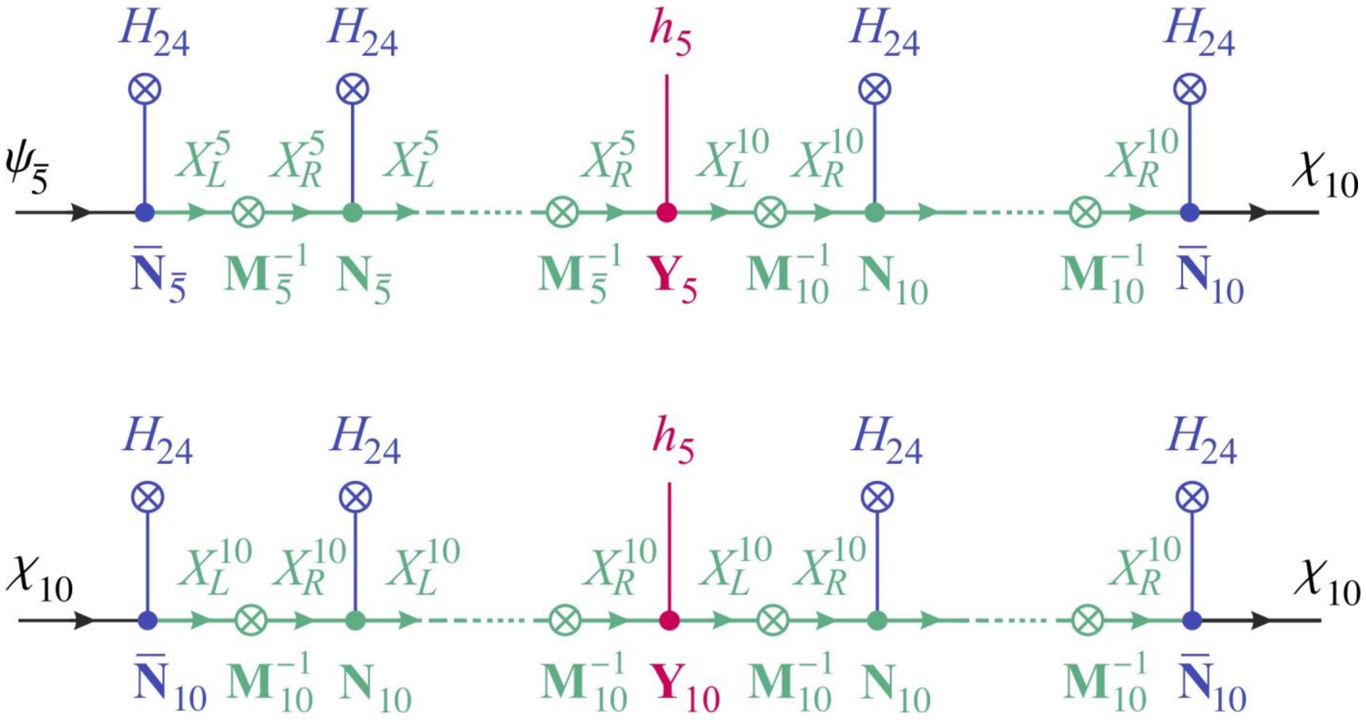
Not fixed in terms of fermion masses & CKM

Unknown and a priori generic mixing matrices threaten FCNC.

B. Towards dynamical flavor unification

We know: $Y_5' = F(Y_{10}, Y_5)$ is possible if F is not a finite polynomial.

Let us try a vector-like model, with new $X^{5}_{L,R}, X^{10}_{L,R}$ fermions:



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Let us try a vector-like model, with new $X_{L,R}^{5}, X_{L,R}^{10}$ fermions:

$$\begin{cases} \mathrm{Y}_u = F_{10}^{1} \cdot \mathrm{Y}_{10} \cdot F_{10}^{-1/4,T} \\ \mathrm{Y}_d = F_{5}^{1} \cdot \mathrm{Y}_{5} \cdot F_{10}^{-1/4,T} \\ \mathrm{Y}_e^T = F_{5}^{-3/2} \cdot \mathrm{Y}_{5} \cdot F_{10}^{-3/2,T} \end{cases} \qquad F_R^{\alpha} = \frac{1}{1+\alpha \dfrac{v_{24}}{M_R} \mathrm{N}_R}$$

Simple system, but incredibly difficult to solve:

Unknowns: $\mathrm{Y}_{5,10}$, $M_{5,10}$, and the MFV parameters in $\mathrm{N}_{5,10}$.

Constraints: SVD of $\mathrm{Y}_{u,d,e}$, and CKM mismatch between $\mathrm{Y}_{u,d}$.

Requirement: Natural solution + absence of fine-tuning.

Solutions found only in the no-mixing limit (not very illuminating).

Conclusion

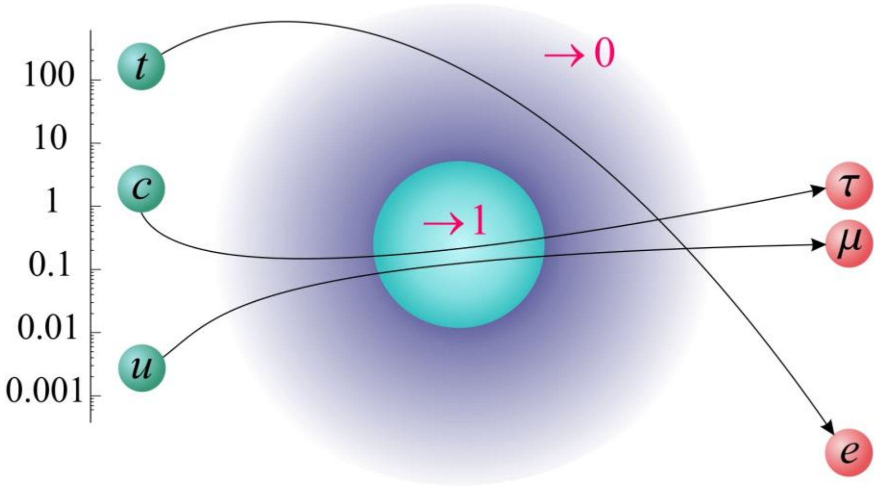
1. There could be only two fundamental flavor structures (+neutrinos)

The three $\Upsilon_{e,u,d}$ are redundant: they can be related!

Finite polynomial relationship necessarily fine-tuned.

Geometric MFV to achieve this automatically.

2. Third-generation partners of the top are the lightest



$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{phys} \approx \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}^{gauge}$$

Light stops: $M_{\tilde{Q}}^2 \approx m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

3. Perspectives: Real and complete dynamical implementation(s).

Consequences for models of neutrino masses.

Resolving the situation in minimal SU(5).