B Decays into multi-hadron final states

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Introduction

Why Multi-Hadron Final States?

+ Due to the large mass of the B these modes are abundant
+ Many single hadron modes are "pseudo-one particle": $B \to \rho\ell\bar{\nu} \to \pi\pi\ell\bar{\nu}$, $B \to K^*\ell\ell \to K\pi\ell\ell$ etc.
- Theoretically quite difficult: Already $B \to \pi\pi$ is difficult
- New, not yet explored non-perturbative inputs needed

$$\boxed{\text{Make Use of the fact that } \alpha_s(m_b) \ll 1}$$

Is there a QCDF framework?

- Effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) O_i(\mu)$$

- Decay amplitudes:

$$\mathcal{A}(B \to f) = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) \langle f | O_i(\mu) | B \rangle$$

- $\langle f | O_i(\mu) | B \rangle$ still contains the large scale m_b
- ... which can be calculated perturbatively
- Factorization of these perturbative contributions
- Suitable definition of non-perturbative quantities

Part 1:

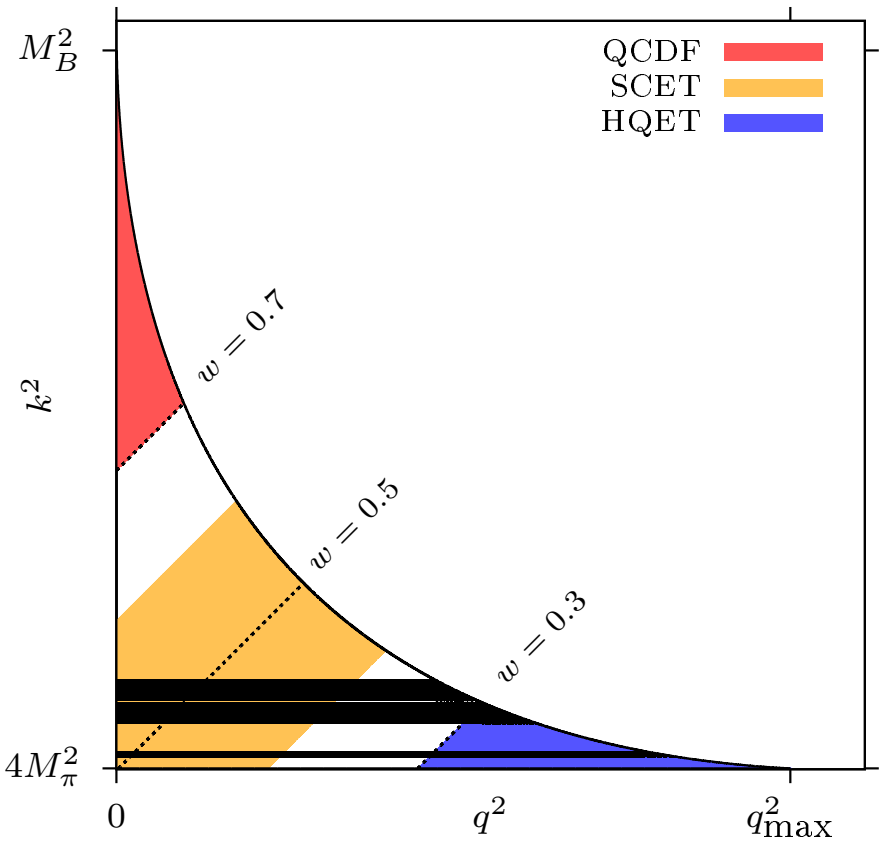
Semileptonic Decays: $B \to \pi\pi\ell\bar{\nu}$

Motivation

- Precise determination of V_{ub} with more channels
- Full phase space distribution for $B \to \rho\ell\bar{\nu} \to \pi\pi\ell\bar{\nu}$
- Needed as an input for factorized formulae for $B \to \pi\pi\pi$
- Methods also relevant for $B \to K^*\ell\ell \to K\pi\ell\ell$

(Faller, Feldmann, Khodjamirian, ThM, van Dyk 1310.6660)

Phase Space for $B \to \pi\pi\ell\bar{\nu}$



- k^2: Invariant mass of the two pions
- q^2 Leptonic invariant mass
- $M_b\omega = v \cdot k$
- dark areas: ρ resonances

- LCSR at small k^2 and small q^2
- HHχPT at small k^2 and large q^2
- QCDF at large k^2 and small q^2

Hadronic Matrix Elements

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu b|B^-(p)\rangle = iF_\perp \frac{1}{k^2\sqrt{\lambda}}\epsilon^{\mu\alpha\beta\gamma} q_\alpha k_{1\beta} k_{2\gamma}$$

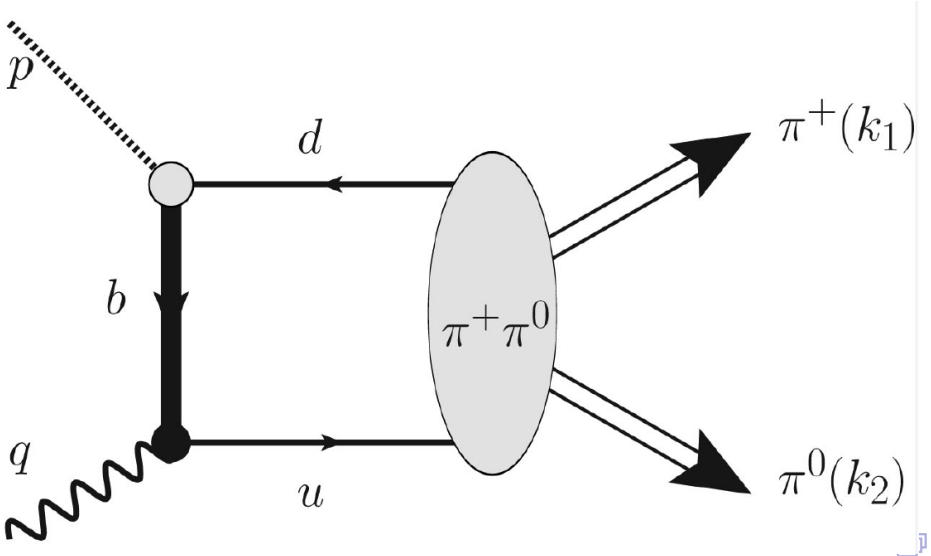
$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma^\mu\gamma_5 b|B^-(p)\rangle = ...F_t,\ ...F_0,\ ...F_{||}$$

- Form factors depend on three variables: q^2, k^2 and $q(k_1 - k_2)$
- Partial wave analysis for the two pions

$B \to \pi\pi\ell\bar{\nu}$ at small k^2 and large q^2

Khodjamirian, Hambrock 1511.02509

- Similar kinematics as in $B \to \pi\ell\bar{\nu}$:
  Light Cone Sum Rule
- Interpolate the B meson
- Use a light cone distribution for the final state:
  Two Pion Light Cone Distribution (Polyakov, Diehl, Grozin)



Definition of the 2πLCDA:

$$\zeta = \frac{k_1^+}{k^+} \quad \text{(light-cone components)}$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du e^{iu(k\cdot x)}\Phi_\perp^{I=1}(u,\zeta,k^2)\,,$$

Normalization in terms of time-like pion form factors:

$$\int_0^1 du\,\Phi_\parallel^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{em}(k^2)\,, \qquad \int_0^1 du\,\Phi_\perp^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{t}(k^2)$$

- Use the measured (electromagnetic) pion from factor
- Extraction of the ρ contribution by cuts and angular analyses

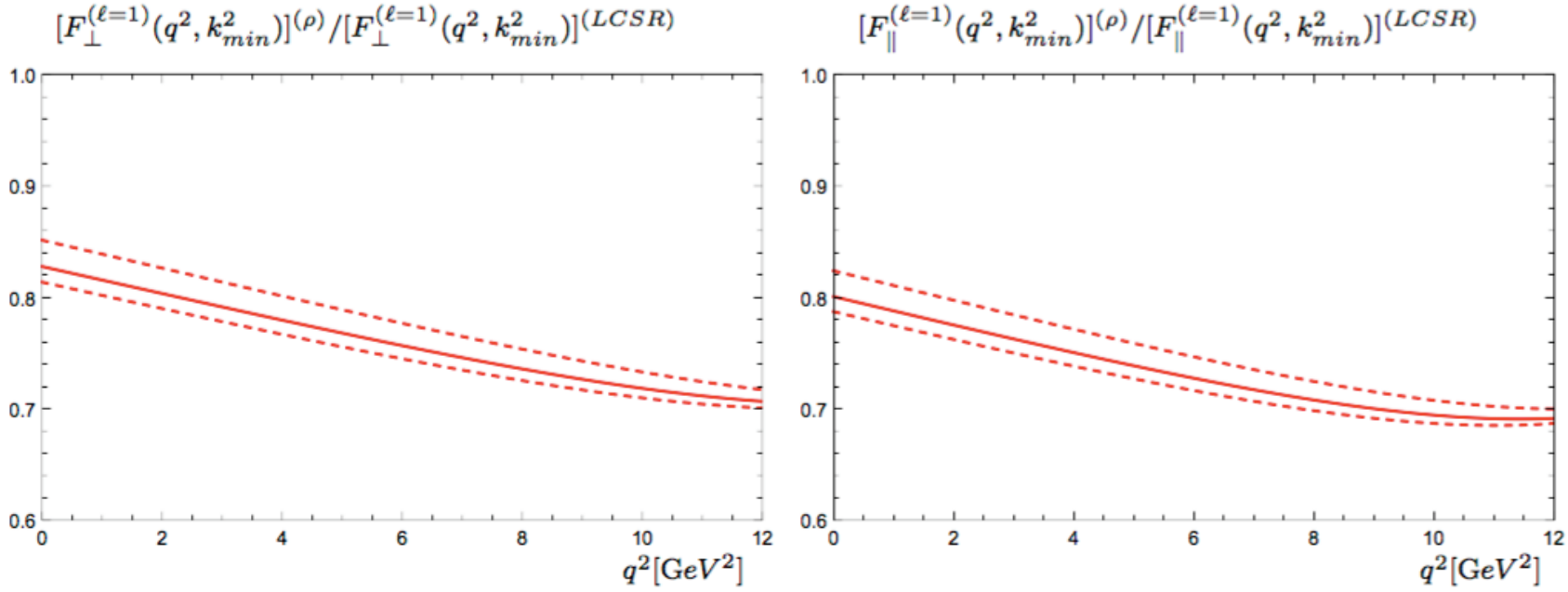


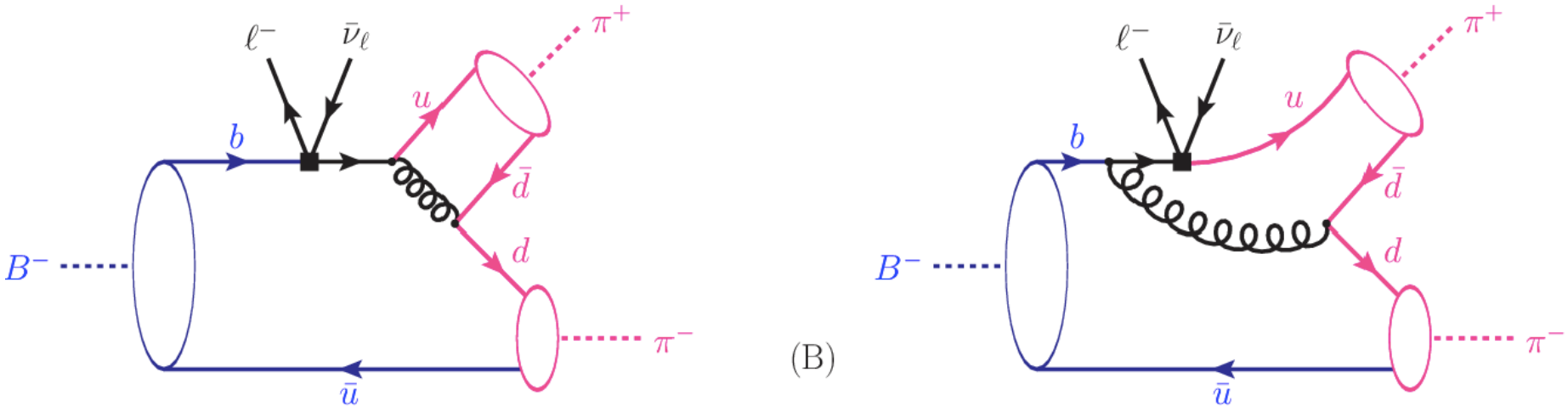
Figure 4: *The relative contribution of ρ-meson to the P-wave* $B \to \pi^+\pi^0$ *form factors* $F_\perp^{(\ell=1)}(q^2, k^2_{min})$ *(left panel) and* $F_\parallel^{(\ell=1)}(q^2, k^2_{min})$ *(right panel) calculated from LCSRs at central values of the input. Dashed lines indicate the uncertainty due to the variation of the Borel parameter.*

(from Khodjamirian, Hambrock 1511.02509)

$B \to \pi\pi\ell\bar{\nu}$ at large k^2

Make us of QCDF: (Faldmann, Böer, van Dyk 1608.07127)

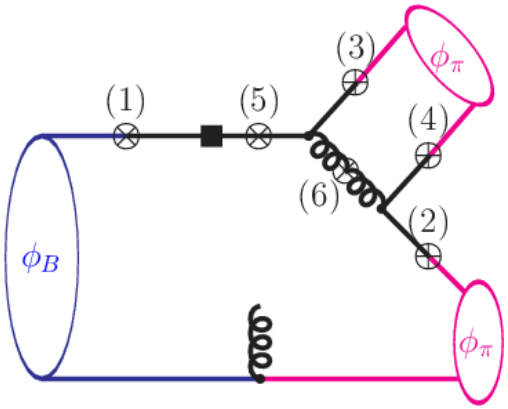
- Large Di-pion mass: $k^2 \gg \Lambda^2_{\rm QCD}$
- Large Energies of the pions in the rest frame of the B meson
  $vk_1 \gg \Lambda_{\rm QCD} \quad vk_2 \gg \Lambda_{\rm QCD}$



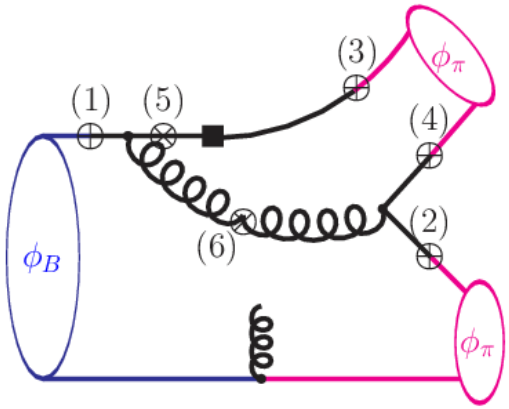
Show that QCD corrections can be computed:

Spectator scattering:

(A)



(B)



QCDF works, however ...

scenario	$\mathcal{B}(B^- \to \pi^+\pi^-\mu^-\bar{\nu}_\mu) \,/\, \lvert V_{ub}\rvert^2$
(A)	$(2.9^{+1.0}_{-0.5}) \cdot 10^{-8}$
(B)	$(9.6^{+3.4}_{-1.5}) \cdot 10^{-7}$
(C)	$(3.2^{+0.8}_{-0.7}) \cdot 10^{-5}$

Part 2:

Nonleptonic Decays: $B \to \pi\pi\pi$

Three-body non-leptonics

Kinematics: $p_B \to p_1 + p_2 + p_3$

- Two independent kinematical variables $p_i^2 = 0$

$$s_{ij}^2 = (p_i + p_j)^2 \quad s_{12} + s_{13} + s_{23} = M_B^2$$

Historically:

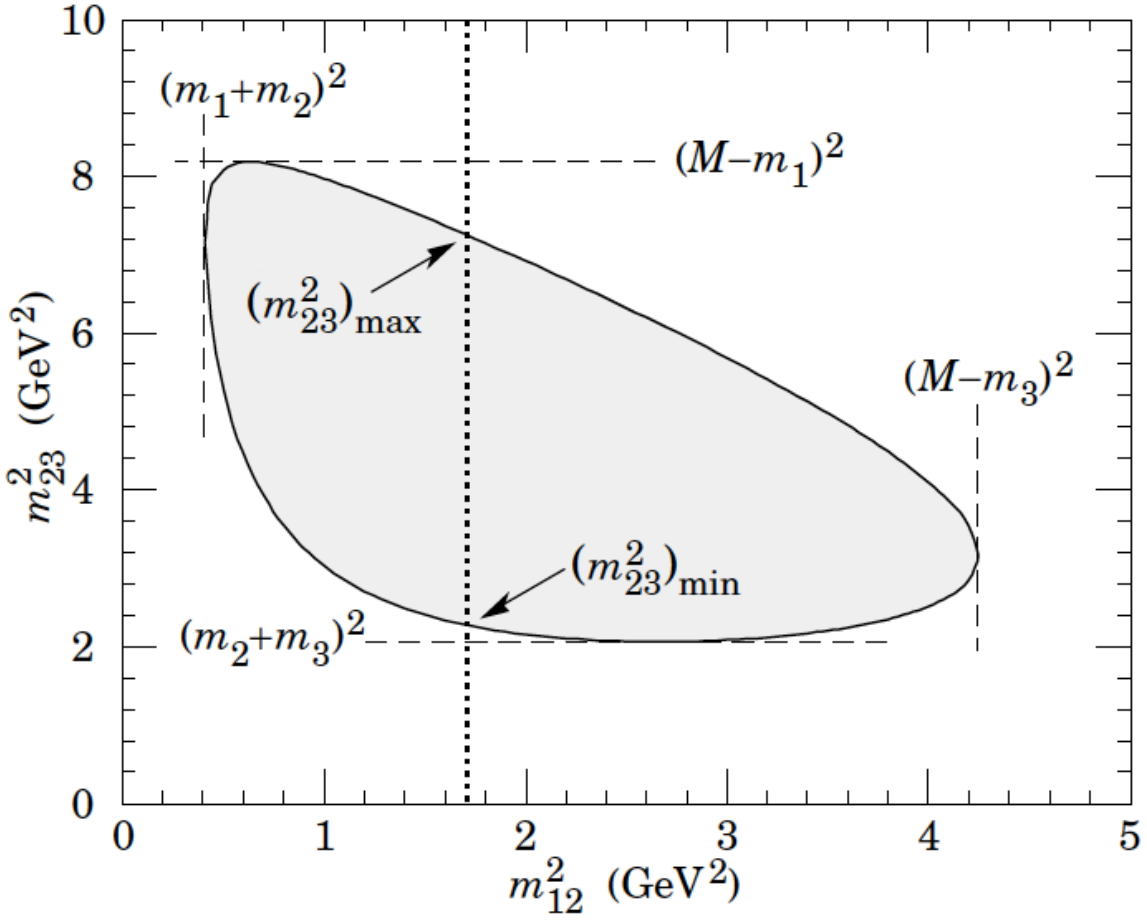
- "Isobar" Model: (Bediaga et al. ...)
- Description via pseudo two-particle decays:

$$(B \to M\, M_1 M_2) = (B \to M^* M \quad \text{and} \quad M^* \to M_1 M_2)$$

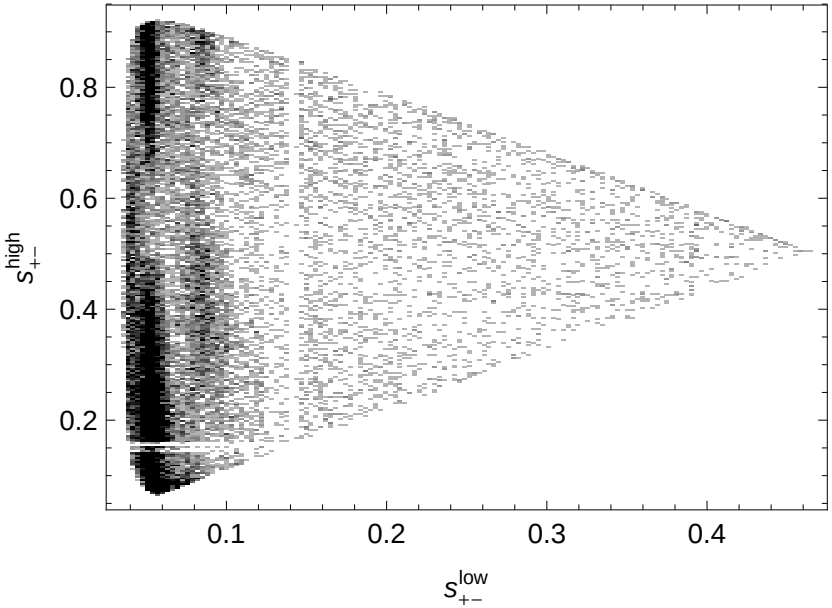
- sum over all possibilities for M^*, including $\Gamma(M^*)$
- possibly add a flat "non-resonant" backgound!

$B \to \pi\pi\pi$

Study the Dalitz Distribution:



Specifically for $B^+ \to \pi^+\pi^-\pi^+$



(Plot form LHCb arXiv:1408.5373)

Dalitz Plot is symmetric:

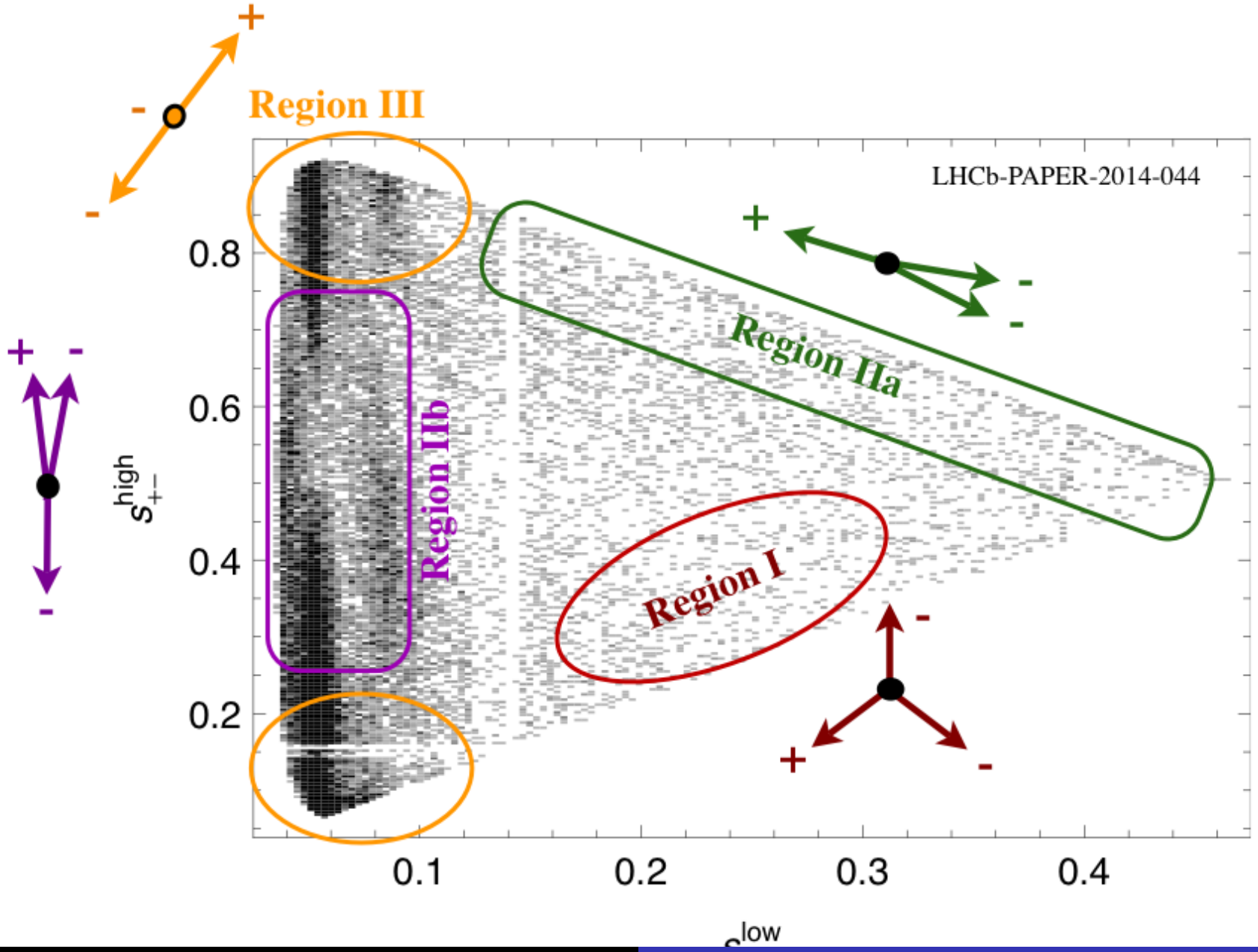
$$s_{12} = s_{+-}^{\rm low} \quad s_{23} = s_{+-}^{\rm high}$$

$$s_{12} = s_{++}$$

Regions

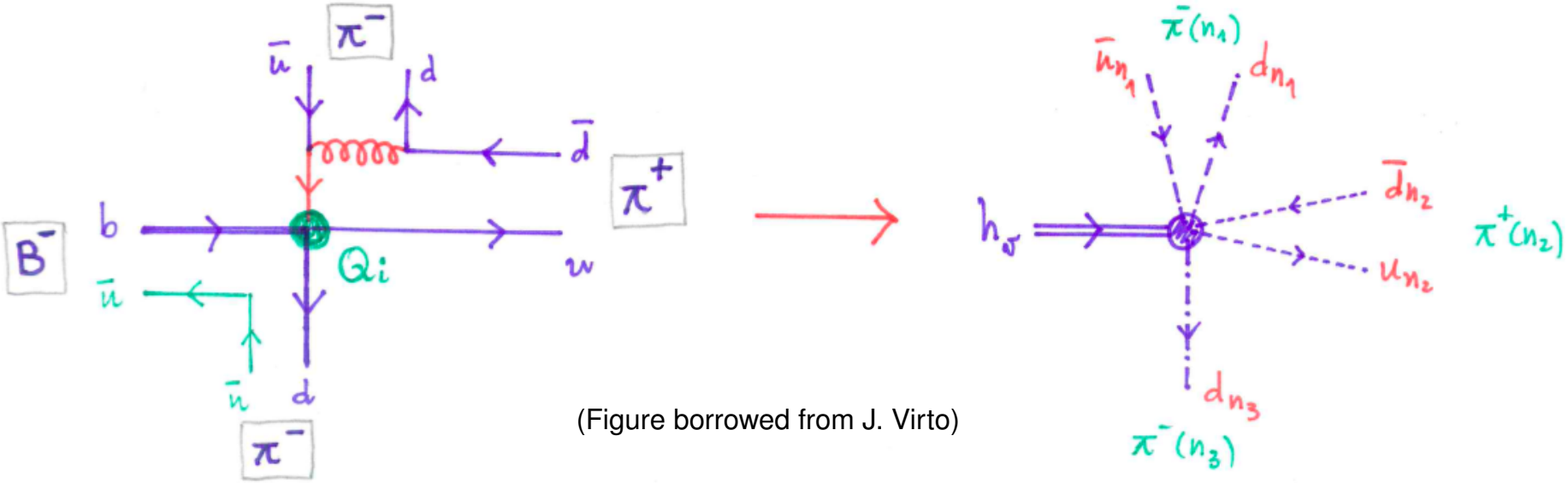
Split the Dalitz Plot into Regions: (S. Kränkl, ThM, J. Virto 1505.04111)

- **Region 1:** "Mercedes Star"
  $s_{++} \sim s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/3$
- **Region 2:** Collinear Decay Products
  - **Region 2a:** $(\pi^+\pi^+)_{\text{coll}}$ recoil against π^-
    $s_{++} \sim 0, \quad s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/2$
  - **Region 2b:** $(\pi^+\pi^-)_{\text{coll}}$ recoil against π^+
    $s_{+-}^{\text{low}} \sim 0, \quad s_{++} \sim s_{+-}^{\text{high}} \sim 1/2$
- **Region 3:** Soft Decay Products
  - **Region 3a:** Soft π^+
    $s_{++} \sim s_{+-}^{\text{low}} \sim 0 \quad s_{+-}^{\text{high}} \sim 1$
  - **Region 3b:** Soft π^-
    $s_{+-}^{\text{low}} \sim {}_{+-}^{\text{high}} \sim 0, \quad s_{++} \sim 1$


Region III
LHCb-PAPER-2014-044
Region IIa
Region IIb
Region I
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
s_{+-}^{high}
s^{low}

Region 1: The Center

- Three "disconnected" collinear directions: n_1 n_2 n_3



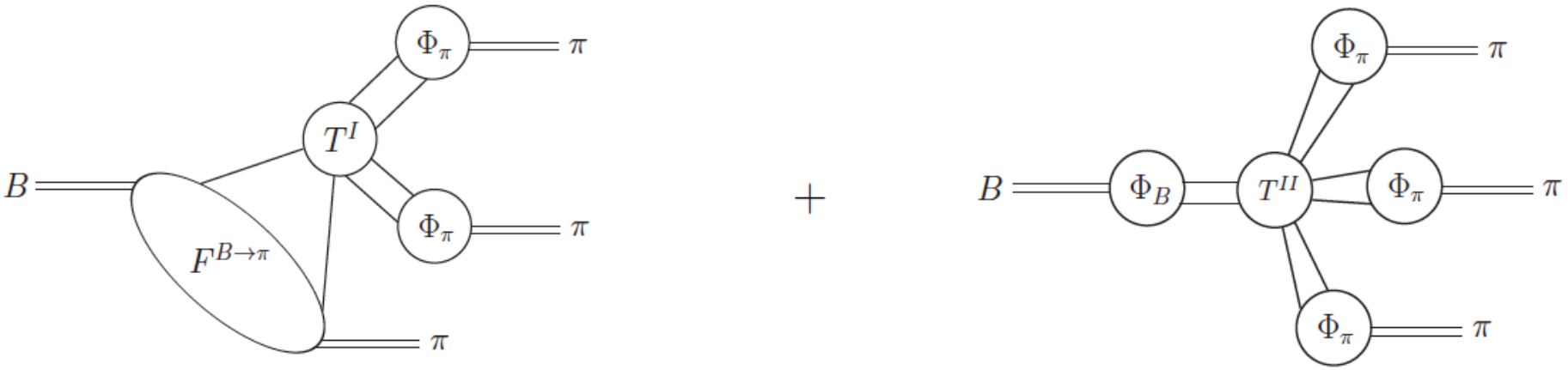
(Figure borrowed from J. Virto)

$$\langle \pi^-_{n_1} \pi^+_{n_2} \pi^-_{n_3} | O_i | B \rangle = \langle \pi^-_{n_3} | \bar{d}_{n_3} \Gamma_3 h_v | B \rangle$$

$$\times \int du dv \, T_i(u,v) \langle \pi^-_{n_1} | \bar{d}_{n_1}(\bar{u}) \Gamma_1 u_{n_1}(u) | 0 \rangle \langle \pi^+_{n_2} | \bar{u}_{n_2}(\bar{v}) \Gamma_2 d_{n_2}(v) | 0 \rangle$$

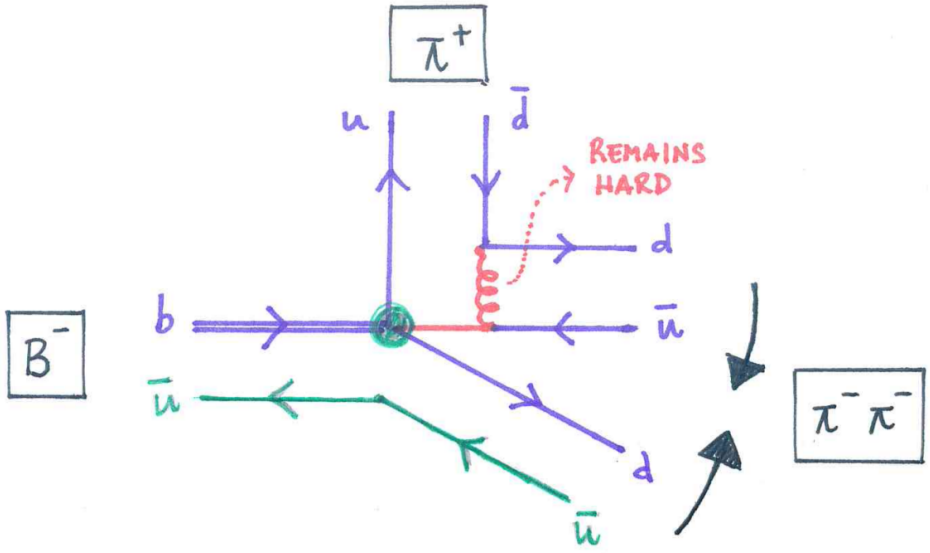
$$\sim F^{B\to\pi} \, T_i \otimes \phi_\pi \otimes \phi_\pi$$

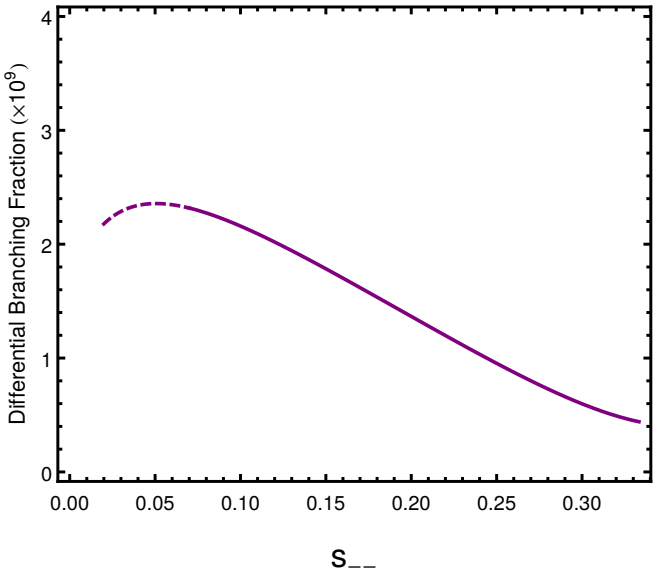
- $1/m_b^2$ and α_s supressed with respect to a two body decay
- At leading order / leading power / leading twist all convolutions are finite
  $\to$ factorization:



Extrapolation to collinear $\pi^-\pi^-$

- There are no resonances in this channel
- No infrared / collinear problems expected
- Perturbative result expected to be regular: No "soft" propagators

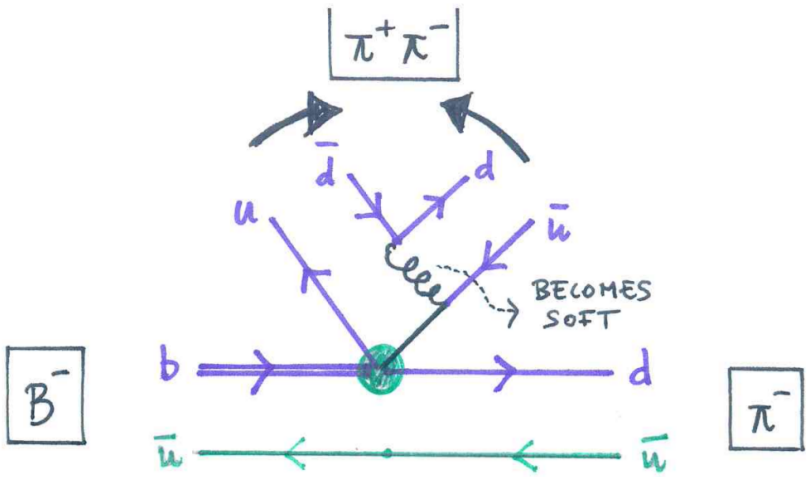


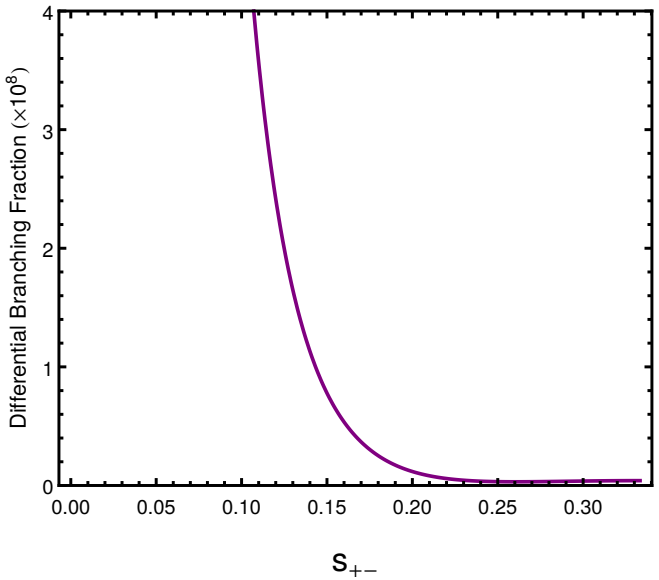


$$\frac{d\Gamma}{ds_{--}\,ds_{+-}} \simeq 0.84\,\Gamma_0\, f_+(m_B^2/2)^2 + \mathcal{O}(s_{--})$$

Extrapolation to collinear $\pi^+\pi^-$

- There are resonances in this channel: ρ. ω, ...
- Perturbative result expected to be IR singular
- "soft" propagators

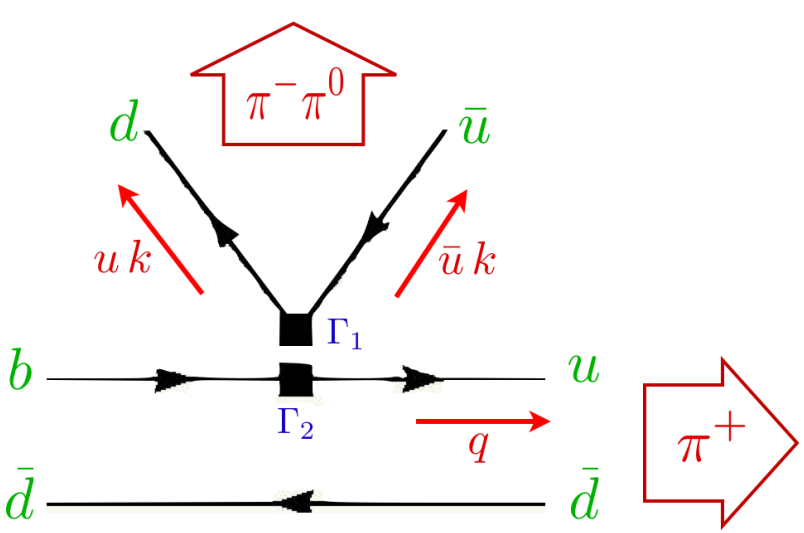


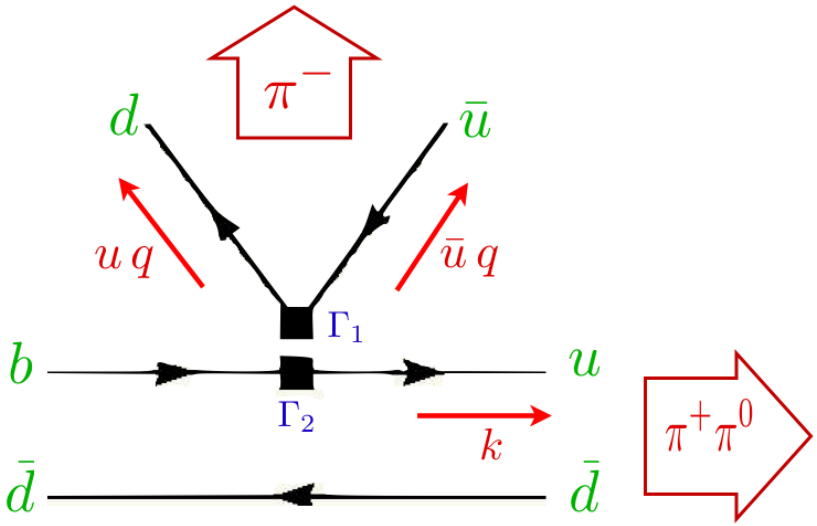


$$\frac{d\Gamma}{ds_{+-}\,ds_{--}} \simeq \frac{0.38}{s_{+-}}\,\Gamma_0\, f_+(0)^2 + \text{regular}$$

Region 2b: new non-perturbative input

- Factorization breaks down in the resonance regions
- New, nonperturbative input is needed
- Three-body decay resembles two-body decay





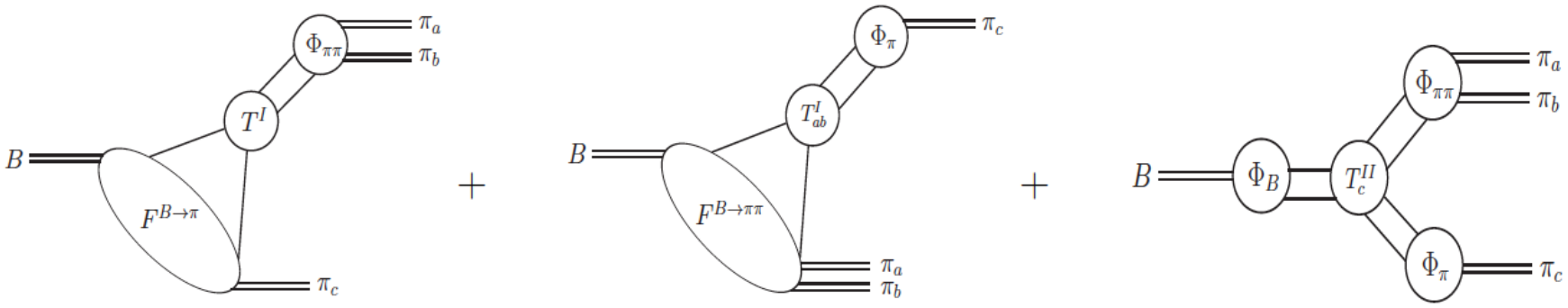
- Operators are the same as in two-body decays ...

- ... but the final states are different

$$\langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} \pi^-_n | O_i | B \rangle =$$
$$\langle \pi^-_n | \bar{h}_v \Gamma \xi_n | B \rangle \times \int dz\, T_1(z) \langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} | \bar{\chi}_{\bar{n}}(z\bar{n}) \Gamma' \chi_{\bar{n}}(0) | 0 \rangle$$
$$+ \langle \pi^-_{\bar{n}} \pi^+_{\bar{n}} | \bar{h}_v \Gamma \xi_{\bar{n}} | B \rangle \times \int dz\, T_2(z) \langle \pi^-_n | \bar{\chi}_{\bar{n}}(zn) \Gamma' \chi_n(0) | 0 \rangle$$
$$\sim F^{B\to\pi}\, T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi}\, T_2 \otimes \phi_\pi$$

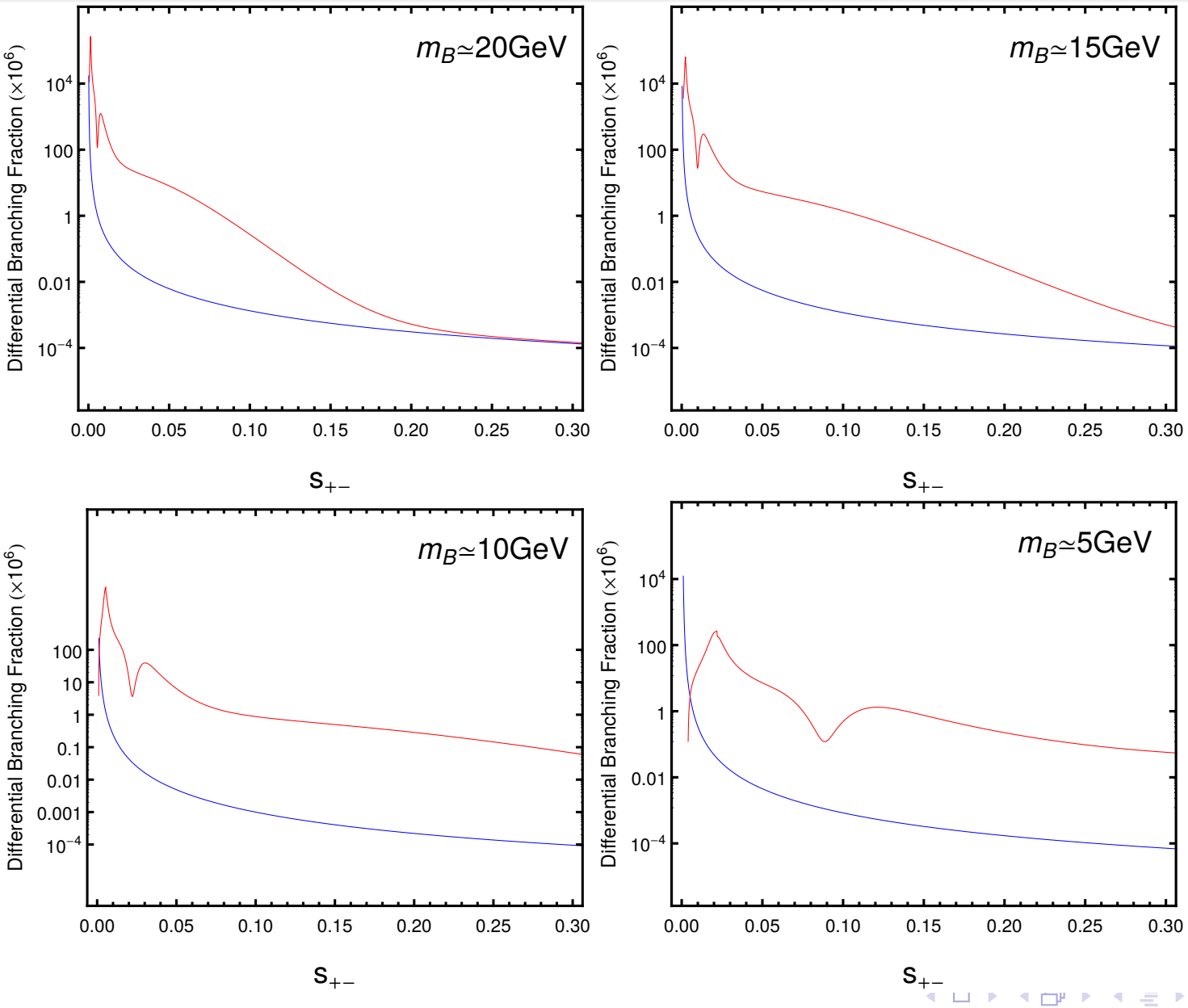
- Two-Pion light-cone distribution (Polyakov, Diehl, Gousset, ...)
- Generalized (soft) Form factor (Feldmann, Khofjamirian, van Dyk, ThM ...)

- Factorization formula similar to the two-body case



Merging the Regions ...

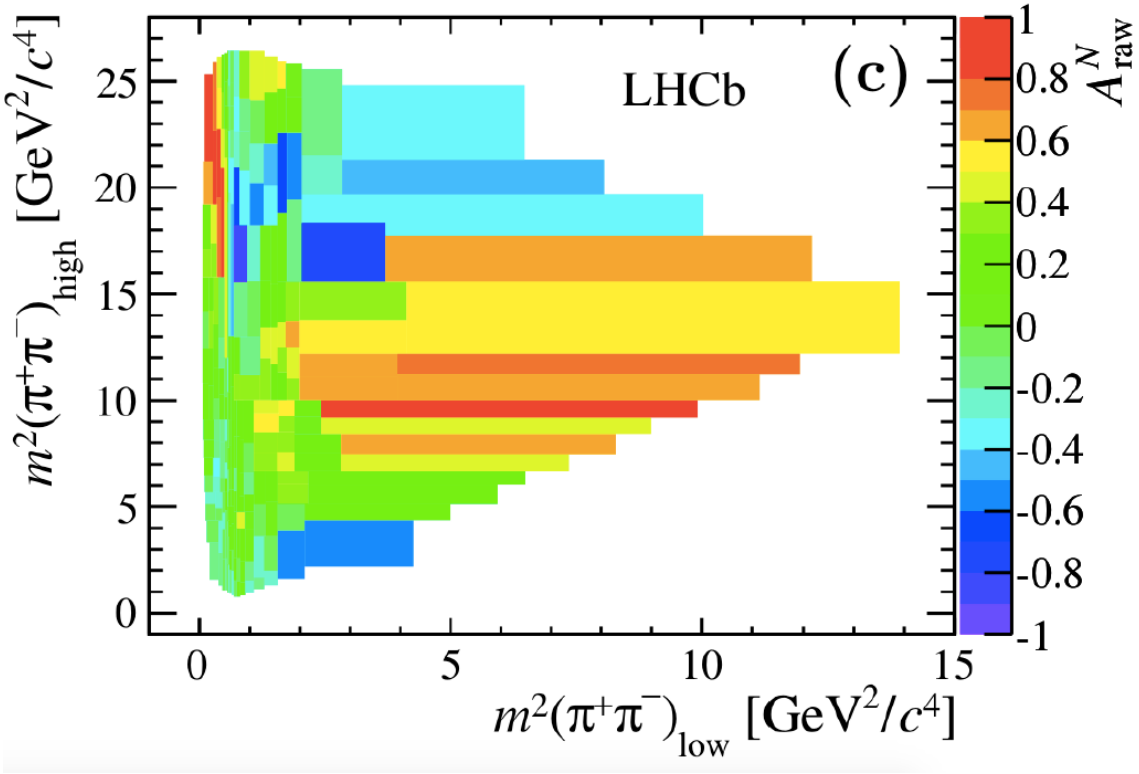
- The starting point is the large-m_b limit
- Do the regions match properly?
- Is m_b large enough?
- Is there a central region for $m_b \sim 5$ GeV?


$m_B \simeq 20$GeV
$m_B \simeq 15$GeV
$m_B \simeq 10$GeV
$m_B \simeq 5$GeV
Differential Branching Fraction ($\times 10^6$)
s_{+-}
0.00
0.05
0.10
0.15
0.20
0.25
0.30
10^4
100
1
0.01
10^{-4}
10
0.1
0.001

- Probably there is no perturbatively calculable central region for realistic m_b
- For realistic m_b the Dalitz plot consists only of edges
- Three-body decays become quasi two-body
- The factorzation formula is the one knwon form the two-body decays with new non-perturbative input
- These are just first results, many more checks need to be performed.

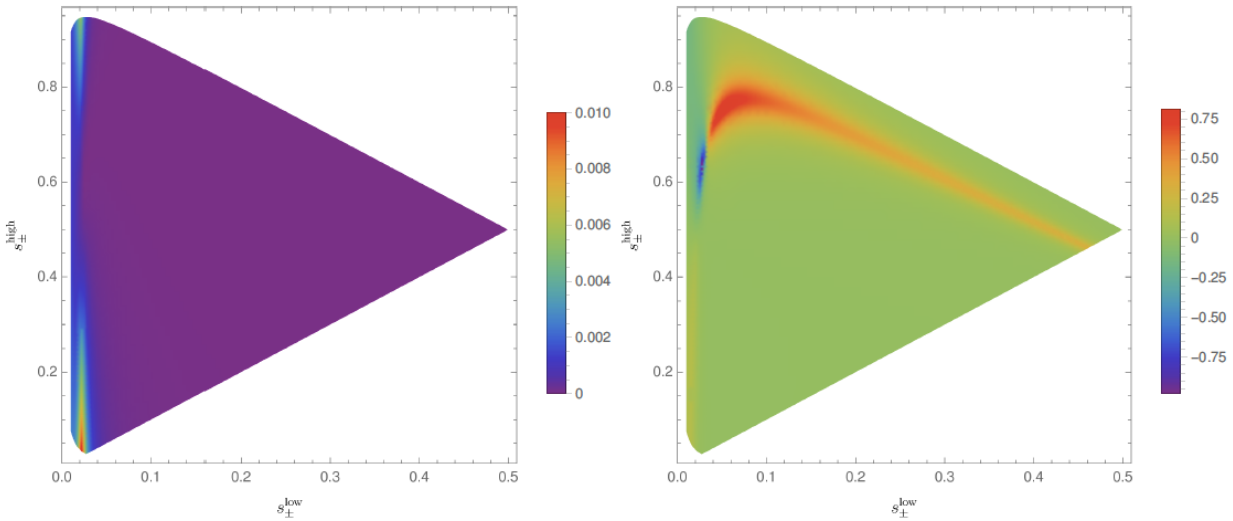
CP Violation Studies

Motivation: Local CP asymmetries from LHCb:

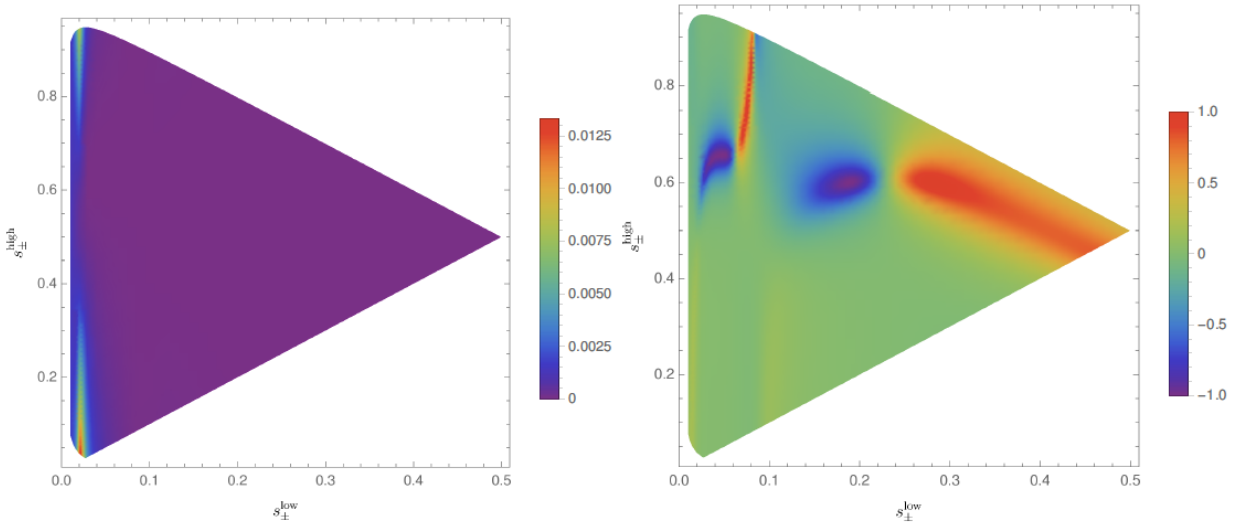


$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\lambda_u (\bar{b}u)(\bar{u}d) + \lambda_c (\bar{b}c)(\bar{u}c) \right)$$

- CPV is induced by the (local) non-perturbative phases in the matrix elements $\langle \pi\pi\pi | (\bar{b}u)(\bar{u}d) | B \rangle$ and $\langle \pi\pi\pi | (\bar{b}c)(\bar{c}d) | B \rangle$
- After QCDF the phases originate from:
  - the $B \to \pi\pi$ form factor ($I = 0$ and $I = 1$ of the two pions)
  - the 2π-LCDA: time-like pion form factor ($I = 1$ only)
- Needs to be modelled:
  BW-shapes for the $I = 0$ and $I = 1$ channel
  Unknown relative strength and phase of the two contributions
  What is the role of charm resonances?
- Some preliminary studies indicate a strong model dependence (Klein, TM, Virto, Vos, in progress)



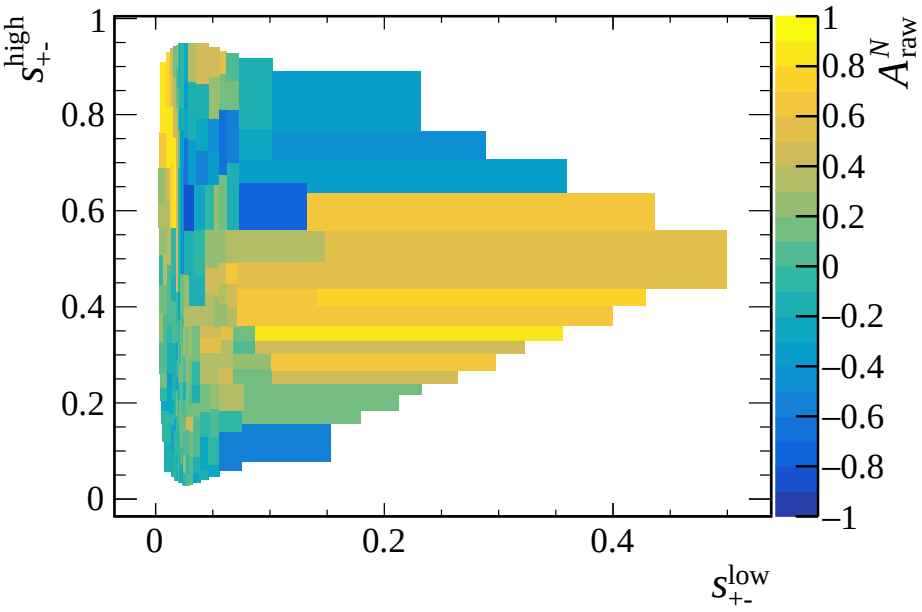
(a) Dalitz distribution (b) CP distribution



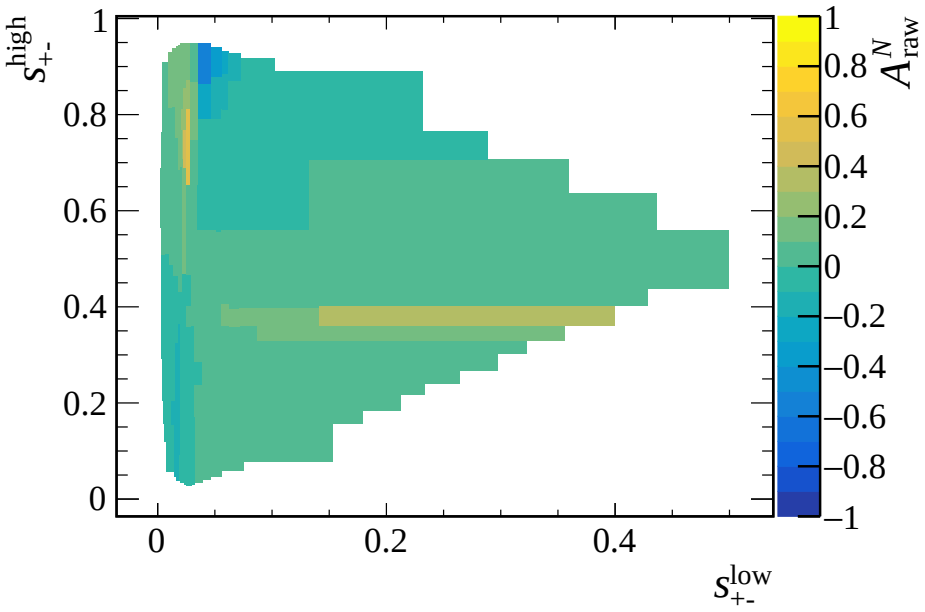
(a) Dalitz (b) CP distribution

(preliminary!!)

Do some modelling, including the charm region:



LHCb



(even more preliminary!!)

Summary

- Multi-body decays ...
  - ... are abundant
  - ... contain important information in their kinematic distributions
  - ... are theoretically most complex
- QCD based Ansatz: QCD factorization
- No perturbative central region, even for B decays
- Quasi two-body with new non-perturbative input
- Can we make use of the local CP asymmetry measurements?