Spontaneous symmetry breaking in three-Higgs-doublet S3-symmetric models

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Motivation for three Higgs doublets

Three fermion generations may suggest three doublets

Interesting scenario for dark matter

Possibility of having a discrete symmetry and still having spontaneous CP violation

Rich phenomenology

Motivation for imposing discrete symmetries

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help to control HFCNC

Example: NFC, no HFCNC due to Z_2 symmetry(ies)

Example: MFV suppression of HFCNC, BGL models

Symmetries are needed to stabilise dark matter

Three Higgs doublet models with S_3 Symmetry

(extended to flavour)

Despite

many works aiming at explaining neutrino masses and leptonic mixing

Ma, Koide, Kubo, Mondragon, Rodriguez-Jauregui, Chen, Wolfenstein, Mohapatra, Nasri, Yu, Harrison, Scott, Frigerio, Grimus, Lavoura, Branco, Silva-Marcos…

several works addressing masses and mixing in the quark sector

Harari, Haut, Weyers, Meloni, Teshima, Melic, Canales, S Salazar, Velasco-Sevilla ,…

a lot of work already done analysing the Higgs potential

Derman, Tsao, Pakvasa, Sugawra, Wyler, Branco, Gerard, Grimus, Das, Dey, Bhattacharyya, Leser, Pas, Ivanov, Nishi…

inert dark matter candidates from S_3 3HDM considered

Fortes, Machado, Montano, Pleitez…

Interesting open questions still remain!

The Scalar potential

S_3 is the permutation group involving three objects, ϕ_1, ϕ_2, ϕ_3

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2}\gamma \sum_{i<j} [\phi_i^\dagger \phi_j + \mathrm{hc}]$$

$$V_4 = A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i<j} \{ C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \mathrm{hc}]\}$$
$$+ \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \mathrm{hc}] + \sum_{i \neq j \neq k \neq i, j<k} \{ \frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \mathrm{hc}]$$
$$+ \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \mathrm{hc}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \mathrm{hc}]\}$$

Derman, 1979

here all fields appear on equal footing

this representation is not irreducible, for instance, the combination

$$\phi_1 + \phi_2 + \phi_3$$

remains invariant, it splits into two irreducible representations,

doublet and singlet: $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad h_S$

Decomposition into these two irreducible representations

$$\begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

This definition does not treat equally ϕ_1, ϕ_2, ϕ_3 **, they could be interchanged**

Notice similarity with tribimaximal mixing: Harrison, Perkins and Scott, 1999

$$(F =) \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The matrix F diagonalizes the democratic matrix , Δ

$$F'^T \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} F' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad \Delta = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The democratic mass matrix can be obtained from S_3 flavour symmetries

S_{3L} x S_{3R}: $M_l = \lambda' \ \Delta \quad ; \quad M_D = \lambda \ \Delta \quad ; \quad M_R = \mu \ (\Delta + a \ \mathbb{1})$

Very interesting alternative, democratic with phases (USY)

The scalar potential in terms of fields from irreducible representations

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2(h_1^\dagger h_1 + h_2^\dagger h_2),$$

$$\begin{aligned}V_4 = &\lambda_8(h_S^\dagger h_S)^2 + \lambda_5(h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) + \lambda_1(h_1^\dagger h_1 + h_2^\dagger h_2)^2 \\ &+ \lambda_2(h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3[(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ &+ \lambda_6[(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] \\ &+ \lambda_7[(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ &+ \lambda_4[(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}]\end{aligned}$$

Das and Dey

no symmetry under the interchange of h_1 and h_2

however there is symmetry for $h_1 \to -h_1$

equivalent doublet representation $\begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ -i & 1\end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

now there is symmetry for $\chi_1 \leftrightarrow \chi_2$

In the special case $\lambda_4 = 0$ **the potential has SO(2) symmetry:**

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Danger: massless scalar!

Alternative choice of irreducible representations

S_3 has three irreducible representations, doublet, singlet and pseudo singlet, h_A

Take S_3 doublet and h_A

No direct translation into initial fields Φ_1, Φ_2, Φ_3

New potential (only term in λ_4 changes):

$$V_2 = \mu_0^2 h_A^\dagger h_A + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \tag{2.75a}$$

$$\begin{aligned} V_4 = {} & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_A^\dagger h_2)(h_1^\dagger h_2 + h_2^\dagger h_1) - (h_A^\dagger h_1)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_A^\dagger h_A)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_A^\dagger h_1)(h_1^\dagger h_A) + (h_A^\dagger h_2)(h_2^\dagger h_A)] + \lambda_7 [(h_A^\dagger h_1)(h_A^\dagger h_1) + (h_A^\dagger h_2)(h_A^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_A^\dagger h_A)^2. \end{aligned} \tag{2.75b}$$

reduces to the same potential we had before with h_1 and h_2 interchanged, no new physics!

Constraining the potential by the vevs

Possibility of SCPV - real parameters

Let us start with real vacua (no CP violation)

Three minimisation conditions:

can be solved to give μ_0^2 and μ_1^2 in terms of the quartic coefficients:

$$\mu_0^2 = \frac{1}{2w_S}\left[\lambda_4(w_2^2-3w_1^2)w_2-(\lambda_5+\lambda_6+2\lambda_7)(w_1^2+w_2^2)w_S-2\lambda_8 w_S^3\right], \tag{4.2a}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1+\lambda_3)(w_1^2+w_2^2)+6\lambda_4 w_2 w_S+(\lambda_5+\lambda_6+2\lambda_7)w_S^2\right], \tag{4.2b}$$

$$\mu_1^2 = -\frac{1}{2}\left[2(\lambda_1+\lambda_3)(w_1^2+w_2^2)-3\lambda_4(w_2^2-w_1^2)\frac{w_S}{w_2}+(\lambda_5+\lambda_6+2\lambda_7)w_S^2\right]. \tag{4.2c}$$

Eqs (4.2b) and (4.2c) obtained dividing by w_1 and w_2 respectively

$$\lambda_4 = 4A-2(C+\bar{C}+D)-E_1-E_2+E_4=0$$

Consistency requires:

- for $w_1=0$ the corresponding derivative is zero - no clash
- or else $\lambda_4(3w_2^2-w_1^2)w_S=0$ i. e., $\lambda_4=0$ or $w_1=\pm\sqrt{3}w_2$ or $w_S=0.$
- for $w_S=0.$ special condition: $\lambda_4 w_2(3w_1^2-w_2^2)=0,$ i. e., in addition:

$$\lambda_4=0 \text{ or } w_2=\pm\sqrt{3}w_1, \text{ or else } w_2=0.$$

SSB, real vacua, residual symmetries

Derman, Tsao Phys. Rev. D20 (1979) 1207:

(x, x, x) S_3 ; (x, x, y) S_2 ; (x, y, z) = (x, -x, 0) S_2 $\lambda_4 \neq 0$

Translation into doublet singlet notation

(x, x, x) $\longrightarrow$ (0,0, ω_S) $w_1 = 0$ (also verifies $w_1 = \pm\sqrt{3}w_2$)

(x, -x, 0) $\longrightarrow$ $(\omega_1, 0, 0)$ $w_S = 0$ together with $w_2 = 0.$

(x, 0, -x) $\longrightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together $w_2 = \sqrt{3}w_1$

(0,x, -x) $\longrightarrow$ $(\omega_1, \omega_2, 0)$ $w_S = 0$ together with $w_2 = -\sqrt{3}w_1$

(x, x, y) translates into $(0, w_2, w_S)$; consistency condition: $w_1 = 0.$

(x, y, x) translates into $(w_1, -\frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = -\sqrt{3}w_2$

(y, x, x) translates into $(w_1, \frac{1}{\sqrt{3}}w_1, w_S)$; consistency condition: $w_1 = \sqrt{3}w_2$

For $\lambda_4 = 0$ SO(2) symmetry implies (x, y, z) possible solution

Vacuum	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	Comment
R-0	$0, 0, 0$	$0, 0, 0$	Not interesting
R-I-1	x, x, x	$0, 0, w_S$	$\mu_0^2 = -\lambda_8 w_S^2$
R-I-2a	$x, -x, 0$	$w, 0, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_1^2$
R-I-2b	$x, 0, -x$	$w, \sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-I-2c	$0, x, -x$	$w, -\sqrt{3}w, 0$	$\mu_1^2 = -\frac{4}{3}\left(\lambda_1 + \lambda_3\right) w_2^2$
R-II-1a	x, x, y	$0, w, w_S$	$\mu_0^2 = \frac{1}{2}\lambda_4 \frac{w_2^3}{w_S} - \frac{1}{2}\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2 + \frac{3}{2}\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1b	x, y, x	$w, -w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-1c	y, x, x	$w, w/\sqrt{3}, w_S$	$\mu_0^2 = -4\lambda_4 \frac{w_2^3}{w_S} - 2\lambda_a w_2^2 - \lambda_8 w_S^2$, $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right) w_2^2 - 3\lambda_4 w_2 w_S - \frac{1}{2}\lambda_a w_S^2$
R-II-2	$x, x, -2x$	$0, w, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right) w_2^2$, $\lambda_4 = 0$
R-II-3	$x, y, -x-y$	$w_1, w_2, 0$	$\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right), \lambda_4 = 0$
R-III	ρ_1, ρ_2, ρ_3	w_1, w_2, w_S	$\mu_0^2 = -\frac{1}{2}\lambda_a(w_1^2 + w_2^2) - \lambda_8 w_S^2$, $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(w_1^2 + w_2^2\right) - \frac{1}{2}\lambda_a w_S^2$, $\lambda_4 = 0$

$$\lambda_a = \lambda_5 + \lambda_6 + 2\lambda_7,$$
$$\lambda_b = \lambda_5 + \lambda_6 - 2\lambda_7.$$

Complex vacua

Table 2: Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3\sin 2\rho_1/\sin 2\rho_2}$, $\psi = \sqrt{[3+3\cos(\rho_2 - 2\rho_1)]/(2\cos\rho_2)}$. With the constraints of Table 4 the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm\frac{2\pi i}{3}}, xe^{\mp\frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x+iy, x-iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon\hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2\sigma_1)}{1+9\tan^2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\pm\hat{w}_2 e^{-i\arctan(3\tan\sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$
C-IV-c	$\sqrt{1+2\cos^2\sigma_2}\hat{w}_2$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1+2\cos^2\rho)} + x$, $re^{i\rho} - r\sqrt{3(1+2\cos^2\rho)} + x, -2re^{i\rho} + x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1}\xi + x, re^{i\rho_2} - re^{i\rho_1}\xi + x$, $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos\sigma_1}}\hat{w}_2 e^{i\sigma_1}$, $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2}\psi + x$, $re^{i\rho_1} - re^{i\rho_2}\psi + x, -2re^{i\rho_1} + x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

Constraints

Vacuum	Constraints
C-I-a	$\mu_1^2 = -2\left(\lambda_1 - \lambda_2\right)\hat{w}_1^2$
C-III-a	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \frac{4\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-b	$\mu_0^2 = -\frac{1}{2}\lambda_b \hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\lambda_b \hat{w}_S^2,$ $\lambda_4 = 0$
C-III-c	$\mu_1^2 = -(\lambda_1 + \lambda_3)(\hat{w}_1^2 + \hat{w}_2^2),$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0$
C-III-d,e	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{(\hat{w}_1^2 - \hat{w}_2^2)^2}{\hat{w}_S^2} - \epsilon\lambda_4 \frac{(\hat{w}_1^2 - \hat{w}_2^2)(\hat{w}_1^2 - 3\hat{w}_2^2)}{4\hat{w}_2 \hat{w}_S}$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \epsilon\lambda_4 \frac{\hat{w}_S(\hat{w}_1^2 - \hat{w}_2^2)}{4\hat{w}_2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_7 = \frac{\hat{w}_1^2 - \hat{w}_2^2}{\hat{w}_S^2}(\lambda_2 + \lambda_3) - \epsilon\frac{(\hat{w}_1^2 - 5\hat{w}_2^2)}{4\hat{w}_2 \hat{w}_S}\lambda_4$
C-III-f,g	$\mu_0^2 = -\frac{1}{2}\lambda_b\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\lambda_b \hat{w}_S^2, \lambda_4 = 0$
C-III-h	$\mu_0^2 = -2\lambda_b \hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -4\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_b - 8\cos^2\sigma_2\lambda_7\right)\hat{w}_S^2,$ $\lambda_4 = \mp\frac{2\cos\sigma_2 \hat{w}_S}{\hat{w}_2}\lambda_7$
C-III-i	$\mu_0^2 = \frac{16\left(1-3\tan^2\sigma_1\right)^2}{(1+9\tan^2\sigma_1)^2}(\lambda_2 + \lambda_3)\frac{\hat{w}_2^4}{\hat{w}_S^2} \pm \frac{6\left(1-\tan^2\sigma_1\right)(1-3\tan^2\sigma_1)}{(1+9\tan^2\sigma_1)^{\frac{3}{2}}}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{2(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_5 + \lambda_6)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\frac{4(1+3\tan^2\sigma_1)}{1+9\tan^2\sigma_1}(\lambda_1 - \lambda_2)\hat{w}_2^2 \mp \frac{\left(1-3\tan^2\sigma_1\right)}{2\sqrt{1+9\tan^2\sigma_1}}\lambda_4 \hat{w}_2 \hat{w}_S$ $-\frac{1}{2}(\lambda_5 + \lambda_6)\hat{w}_S^2,$ $\lambda_7 = -\frac{4\left(1-3\tan^2\sigma_1\right)\hat{w}_2^2}{(1+9\tan^2\sigma_1)\hat{w}_S^2}(\lambda_2 + \lambda_3) \mp \frac{\left(5-3\tan^2\sigma_1\right)\hat{w}_2}{2\sqrt{1+9\tan^2\sigma_1}\hat{w}_S}\lambda_4$

Vacuum	Constraints
C-IV-a*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_1^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\hat{w}_1^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-b	$\mu_0^2 = \left(\lambda_2 + \lambda_3\right)\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)^2}{\hat{w}_S^2} - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 - \lambda_2\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\left(\hat{w}_1^2 - \hat{w}_2^2\right)}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-c	$\mu_0^2 = 2\cos^2\sigma_2\left(1 + \cos^2\sigma_2\right)\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\left(1 + \cos^2\sigma_2\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left[2\left(1 + \cos^2\sigma_2\right)\lambda_1 - \left(2 + 3\cos^2\sigma_2\right)\lambda_2 - \cos^2\sigma_2\lambda_3\right]\hat{w}_2^2$ $-\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = -\frac{2\cos\sigma_2 \hat{w}_2}{\hat{w}_S}\left(\lambda_2 + \lambda_3\right), \lambda_7 = \frac{\cos^2\sigma_2 \hat{w}_2^2}{\hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-d*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = 0$
C-IV-e	$\mu_0^2 = \frac{\sin^2(2(\sigma_1 - \sigma_2))}{\sin^2(2\sigma_1)}\left(\lambda_2 + \lambda_3\right)\frac{\hat{w}_2^4}{\hat{w}_S^2}$ $-\frac{1}{2}\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(1 - \frac{\sin 2\sigma_2}{\sin 2\sigma_1}\right)\left(\lambda_1 - \lambda_2\right)\hat{w}_2^2 - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_4 = 0, \lambda_7 = -\frac{\sin(2(\sigma_1 - \sigma_2))\hat{w}_2^2}{\sin 2\sigma_1 \hat{w}_S^2}\left(\lambda_2 + \lambda_3\right)$
C-IV-f	$\mu_0^2 = -\frac{(\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1)\cos(\sigma_2 - \sigma_1)}{2\cos^2\sigma_1}\lambda_4 \frac{\hat{w}_2^3}{\hat{w}_S}$ $-\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{2\cos\sigma_1}\left(\lambda_5 + \lambda_6\right)\hat{w}_2^2 - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\frac{\cos(\sigma_1 - 2\sigma_2) + 3\cos\sigma_1}{\cos\sigma_1}\left(\lambda_1 + \lambda_3\right)\hat{w}_2^2$ $-\frac{3\cos 2\sigma_1 + 2\cos(2(\sigma_1 - \sigma_2)) + \cos 2\sigma_2 + 4}{4\cos(\sigma_1 - \sigma_2)\cos\sigma_1}\lambda_4 \hat{w}_2 \hat{w}_S - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = -\frac{\cos\sigma_1 \hat{w}_S}{2\cos(\sigma_2 - \sigma_1)\hat{w}_2}\lambda_4, \lambda_7 = -\frac{\cos(\sigma_2 - \sigma_1)\hat{w}_2}{2\cos\sigma_1 \hat{w}_S}\lambda_4$
C-V*	$\mu_0^2 = -\frac{1}{2}\left(\lambda_5 + \lambda_6\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \lambda_8 \hat{w}_S^2,$ $\mu_1^2 = -\left(\lambda_1 + \lambda_3\right)\left(\hat{w}_1^2 + \hat{w}_2^2\right) - \frac{1}{2}\left(\lambda_5 + \lambda_6\right)\hat{w}_S^2,$ $\lambda_2 + \lambda_3 = 0, \lambda_4 = 0, \lambda_7 = 0$

The case of $\lambda_4 = 0$

Potential has additional continuous SO(2) symmetry

$$\lambda_4 = 4A - 2(C + \bar{C} + D) - E_1 - E_2 + E_4 = 0$$

Derman (1979), “unnatural”

Spontaneous breaking of this SO(2) symmetry leads to massless particles

Possible solution: break the symmetry softly, the most general quadratic potential can be written:

$$V = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2) + \mu_2^2 (h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}\nu^2 (h_2^\dagger h_1 + h_1^\dagger h_2)$$
$$+ \mu_3^2 (h_S^\dagger h_1 + h_1^\dagger h_S) + \mu_4^2 (h_S^\dagger h_2 + h_2^\dagger h_S)$$

Complex vacua, Spontaneous CP Violation

Table 1: Spontaneous CP violation

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

Next we present a few illustrative examples. Important tool:

most general CP transformation

$$\Phi_i \xrightarrow{\text{CP}} U_{ij}\Phi_j^*$$

together with assumption that vacuum is invariant

$$\text{CP}|0\rangle = |0\rangle$$

leads to the following condition

$\mathcal{L}(U\phi) = \mathcal{L}(\phi)$

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

G. C. Branco, J. M. Gerard and W. Grimus (1984)

Vacuum C-I-a

$$U_{ij}\langle 0|\Phi_j|0\rangle^* = \langle 0|\Phi_i|0\rangle$$

$$x,\ xe^{\frac{2\pi i}{3}},\ xe^{-\frac{2\pi i}{3}}$$

geometrical phases

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

CP is conserved

calculable non-trivial phases, fixed by symmetry of V, no explicit dependence on parameters of the potential.

G. C. Branco, J. M. Gerard and W. Grimus (1984)

Vacuum C-III-c

$$\hat{w}_1 e^{\imath\sigma_1}, \hat{w}_2 e^{\imath\sigma_2}, 0$$

$\lambda_4 = 0$ SO(2) rotation

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$(ae^{i\delta_1}, ae^{i\delta_2}, 0)$$

$$\tan 2\theta = \frac{\hat{w}_1^2 - \hat{w}_2^2}{2\hat{w}_1\hat{w}_2\cos\sigma}.$$

followed by overall phase rotation

$$(ae^{i\delta}, ae^{-i\delta}, 0)$$

symmetry for interchange: $h_1' \leftrightarrow h_2'$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}^* = \begin{pmatrix} ae^{i\delta} \\ ae^{-i\delta} \\ 0 \end{pmatrix}$$

CP is conserved

Next: Two possible complex vacua discussed previously in the literature

Two possible complex vacua discussed previously in the literature

$\hat{w}e^{i\sigma}, \quad \hat{w}e^{-i\sigma}, \quad \hat{w}_S$ Pakwasa and Sugawara, 1978 (PS)

$\hat{w}e^{i\sigma}, \quad \hat{w}e^{i\sigma}, \quad \hat{w}_S$ Ivanov and Nishi, 2014 (IN)

$\hat{w} \neq 0$ and $\hat{w}_S \neq 0$.

Both solutions require $\lambda_4 = 0$ for consistency

The PS vacuum is requires $\lambda_4 = 0$

therefore there is symmetry under the interchange of the components of the S_3 doublet. As a result the Branco, Gerard, Grimus condition is verified

For the IN vacuum it is also possible to show that the allowed region of parameter space where this solution minimises the potential is such that no spontaneous CP violation occurs

Final Remarks

Models with three Higgs doublets have rich phenomenology

Aims and challenges

Exploit possible dark matter candidates in this context, beyond cases where the singlet plays the role of the SM Higgs doublet

Study how to generate realistic fermion masses and mixing with the fermions transforming non trivially under S_3

Look for viable models in the context of spontaneous CP violation

Look for interesting scenarios with the potential of being tested at the LHC