

# Controlled FCNC in the Two Higgs Doublet Models

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based on work done with various  
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# Two Higgs Doublet Models

## Several motivations

- New sources of CP violation

SM cannot account for BAU

- Possibility of having spontaneous CP violation

EW symmetry breaking and CP violation same footing

T. D. Lee 1973, Kobayashi and Maskawa 1973

- Strong CP Problem, Peccei-Quinn

- Supersymmetry

**LHC important role**

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## ***In general two Higgs doublet models have FCNC***

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

**EPS Prize in 2009 to Gargamelle, CERN**

In the Standard Model Flavour changing neutral currents (FCNC) are forbidden at tree level

- **in the gauge sector, no ZFCNC**
- **in the Higgs sector, no HFCNC**

**Models with two or more Higgs doublets have potentially large HFCNC**

***Strict limits on FCNC processes!***

3/ In two Higgs Doublet Models (2HDM) Flavour Changing Neutral Currents (FCNC) have to be eliminated at tree level or naturally suppressed, in order to conform to experiment.

- $Z_2$  symmetry leading to Natural Flavour Conservation

Glashow and Weinberg (1977)

Paschos (1977)

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Can one have a framework where there are FCNC at tree level, but naturally suppressed?

Is it possible to have a framework where the FCNC exist, but are only functions of  $V^{CKM}$  and the ratio  $v_2/v_1$ ?

The suppression of FCNC could be related to the smallness of some of the  $V^{CKM}$  elements.

- Naturally suppressed FCNC as a result of a symmetry of the Lagrangian. The suppression is due to small  $V^{CKM}$  elements

G.C.B, Grimus, Lavoura (1996) (BGL)

- Extension to the leptonic sector

F. Botella, GCB, MN Rebelo  
F. Botella, GCB., MN Rebelo, M. Nebot

- Thorough Phenomenological Analysis

F. Botella, GCB, A. Carmoza, M. Nebot, L. Pedro, M.N. Rebelo

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## Notation

### Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q_L^0} \Gamma_1 \Phi_1 d_R^0 - \overline{Q_L^0} \Gamma_2 \Phi_2 d_R^0 - \overline{Q_L^0} \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q_L^0} \Delta_2 \tilde{\Phi}_2 u_R^0 + \text{h.c.}$$

$$\tilde{\Phi}_i = -i\tau_2 \Phi_i^*$$

### Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}(v_1 \Gamma_1 + v_2 e^{i\theta} \Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1 \Delta_1 + v_2 e^{-i\theta} \Delta_2),$$

### Diagonalised by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag} (m_d, m_s, m_b),$$

$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag} (m_u, m_c, m_t).$$



# Leptonic Sector

Charged  
leptons

$$\left( -\bar{L}_L^{\circ} \pi_1 \Phi_1 \ell_R^{\circ} - \bar{L}_L^{\circ} \pi_2 \Phi_2 \ell_R^{\circ} + \text{h.c.} \right)$$

$$+ \left( -\bar{L}_L^{\circ} \Sigma_1 \tilde{\Phi}_1 \nu_R^{\circ} - \bar{L}_L^{\circ} \Sigma_2 \tilde{\Phi}_2 \nu_R^{\circ} + \text{h.c.} \right) +$$

$$+ \left( \frac{1}{2} \nu_R^{\circ T} \bar{C}^{-1} M_R \nu_R^{\circ} + \text{h.c.} \right)$$

neutrino  
Dirac

neutrino Majorana

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Expansion around the v.e.v.:

$$\Phi_j = \begin{bmatrix} \phi_j^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}} (v_j + \rho_j + i\eta_j) \end{bmatrix}$$

It is convenient to define:

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = U \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}; \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = U \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = U \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$U = \frac{1}{v} \begin{bmatrix} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{bmatrix}; \quad v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}$$

$H^0 \rightarrow$  couplings proportional to mass matrices

$G^0 \rightarrow$  neutral pseudo-Goldstone boson

$G^\pm \rightarrow$  Charged pseudo-Goldstone bosons

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## Neutral and charged Higgs Interactions for the quark sector

$$\begin{aligned}
 \mathcal{L}_Y(\text{quark, Higgs}) = & -\overline{d}_L^0 \frac{1}{v} [M_d H^0 + N_d^0 R + i N_d^0 I] d_R^0 \\
 & -\overline{u}_L^0 \frac{1}{v} [M_u H^0 + N_u^0 R + i N_u^0 I] u_R^0 \\
 & -\frac{\sqrt{2} H^+}{v} (\overline{u}_L^0 N_d^0 d_R^0 - \overline{u}_R^0 N_u^{0\dagger} d_L^0) + \text{h.c.}
 \end{aligned}$$

$$N_d^0 = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\theta} \Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}} (v_2 \Delta_1 - v_1 e^{-i\theta} \Delta_2).$$

**Flavour structure of quark sector of 2HDM characterised by:**

four matrices  $M_d$ ,  $M_u$ ,  $N_d^0$ ,  $N_u^0$ .

**Likewise for Leptonic sector, Dirac neutrinos:**

$$M_\ell, M_\nu, N_\ell^0, N_\nu^0.$$

# Yukawa Couplings in terms of quark mass eigenstates

for  $H^+, H^0, R, I$

$\mathcal{L}_Y(\text{quark, Higgs}) =$

$$\begin{aligned}
 & - \frac{\sqrt{2}H^+}{v} \bar{u} \left( V N_d \gamma_R - N_u^\dagger V \gamma_L \right) d + \text{h.c.} - \frac{H^0}{v} (\bar{u} D_u u + \bar{d} D_d d) - \\
 & - \frac{R}{v} \left[ \bar{u} (N_u \gamma_R + N_u^\dagger \gamma_L) u + \bar{d} (N_d \gamma_R + N_d^\dagger \gamma_L) d \right] + \\
 & + i \frac{I}{v} \left[ \bar{u} (N_u \gamma_R - N_u^\dagger \gamma_L) u - \bar{d} (N_d \gamma_R - N_d^\dagger \gamma_L) d \right]
 \end{aligned}$$

$$\gamma_L = (1 - \gamma_5)/2$$

$$\gamma_R = (1 + \gamma_5)/2$$

$$V = V_{CKM}$$

FCNC controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger \left( v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger \left( v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{uR}$$

For general 2 HDM,  $N_d, N_u$  are arbitrary complex  $3 \times 3$  matrices.

One can rewrite  $N_d$  as:

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left( t + \frac{1}{t} \right) \underbrace{U_{dL}^\dagger e^{i\alpha} \Gamma_2 U_{dR}}_{\text{leads to FCNC}}$$

$$t \equiv \tan \beta = \frac{v_2}{v_1} \quad \text{leads to FCNC}$$

The general 2HDM has a **HUGE** number of New Flavour Parameters encoded by the two matrices  $N_d^0$  and  $N_u^0$  which are arbitrary complex matrices

$18 + 18 = 36$  new physics parameters

Questions :

- How to parametrize  $N_d, N_u$  in a sensible way?
- Can one use a parametrization which resembles the one used in VCKM?

Answer: Yes!

Any arbitrary complex matrix can be written:

$$N_d^0 = X_L^\dagger D^{Nd} X_R$$

$$N_d^0 = K_L \hat{V}_L^{Nd} D^{Nd} \bar{K} (\hat{V}_R^{Nd})^\dagger K_R^\dagger$$

$$K_{L,R} = \text{diag.} [1, \exp(i\varphi_{1L,R}), \exp(i\varphi_{2L,R})]$$

$$\bar{K} = \text{diag.} [e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}]$$

$$\text{phases: } 2(K_L) + 2K_R + 1 \hat{V}_L^{Nd} + 1 \hat{V}_R^{Nd} + 3\bar{K} = 9$$

$$\text{real parameters: } 3 \hat{V}_L^{Nd} + 3 \hat{V}_R^{Nd} + 3 D^{Nd} = 9$$

There are many **new sources of flavour changing and of CP violation**

$$I_1 \equiv \text{tr} (M_d N_d^{\circ\dagger}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_d^*)_{33}$$

where  $N_d$  is the matrix  $N_d^{\circ}$  in the basis where it couples to quark mass eigenstates.

**$\text{Im } I_1$  is very important** since it probes the phases in  $(N_d)_{jj}$  which contribute to the electric dipole moment of down quarks.

Recall the CP odd invariant of the SM:

$$\underline{I}^{\text{CP}} \equiv \text{tr} [H_u, H_d]^3 = 6i (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \text{Im } Q_{uscb}$$



In the SM  $V_{CKM}$  reflects a misalignment in flavour space between the two Hermitian matrices  $H_d = M_d M_d^\dagger$  and  $H_u = M_u M_u^\dagger$ .

In an analogous way, in 2HDM, one has New mixing matrices which reflect misalignment of various Hermitian matrices.

Examples:  $I_2^{CP} \equiv \text{tr}[H_u, H_{N_d}]^3 = 6i \Delta_u \Delta_{N_d} \text{Im} Q_2$

where  $Q_2$  is a rephasing invariant quartet of

$$V_2 \equiv U_{uL}^\dagger U_{N_d L} \quad \text{and} \quad \Delta_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$\Delta_{N_d}$  defined by an analogous expression for the eigenvalues of  $N_d^0 N_d^{0\dagger}$

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 One may also construct WB invariants sensitive to right-handed mixings

$$I_7^{\text{CP}} \equiv \text{tr} [H'_d, H'_{N_d^0}]^3 = 6i \Delta_d \Delta_{N_d} \text{Im} Q_7$$

where  $H'_d \equiv M_d^\dagger M_d$  ;  $H'_{N_d^0} = N_d^{\text{ot}} N_d^0$

$Q_7$  is a rephasing quartet of  $U_{dR} U_{N_R}^\dagger$ .

One may have also CP even WB invariants:

$$I_2 \equiv \text{tr} [M_d N_d^0, M_d M_d^\dagger]^2 = -2 m_d m_s (m_s^2 - m_d^2) N_{d12}^* N_{d21}^* - 2 m_d m_b (m_b^2 - m_d^2)^2 (N_d^*)_{13} (N_d^*)_{31} - 2 m_s m_b (m_b^2 - m_s^2) (N_d^*)_{23} (N_d^*)_{32}$$

$I_2$  can probe the strength of FCNC

One of the reasons why in the SM one cannot generate sufficient BAU is due to the smallness of CP violation :

$$\frac{\text{tr} [H_u, H_d]^3}{v^{12}} \approx 10^{-20}$$

Situation is much better in 2HDM

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In the SM the invariant which controls the strength of CP violation is of order  $M^2$ .

Even in constrained 2HDM like BGL models, the lowest order invariant sensitive to CP violation is of much lower order, namely:

$$I_g^{CP} \equiv \text{Im tr} [M_d N_d^{\circ\dagger} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger]$$

For one of the most interesting BGL models (top model)

$$I_g^{CP} = - \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (m_b^2 - m_s^2) (m_b^2 - m_d^2) (m_s^2 - m_d^2) (m_c^2 - m_u^2) \times$$

$$\text{Im} (V_{22}^* V_{32} V_{33}^* V_{23})$$

$$V \rightarrow V^{CKM} !!$$

Example of a **BGL-type model**: Impose the following discrete symmetry:

$$Q_{Lj}^{\circ} \rightarrow \exp(i\tau) Q_{Lj}^{\circ}; \quad u_{Rj}^{\circ} \rightarrow \exp(2i\tau) u_{Rj}^{\circ};$$

$$\phi_2 \rightarrow \exp(i\tau) \phi_2$$

$\Gamma_j, \Delta_j$  have the form:

$$\tau \neq 0, \pi$$

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}; \quad \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

**FCNC only in the down sector**

If one imposes  $d_{Rj}^{\circ} \rightarrow \exp(2i\tau) d_{Rj}^{\circ}$  instead of

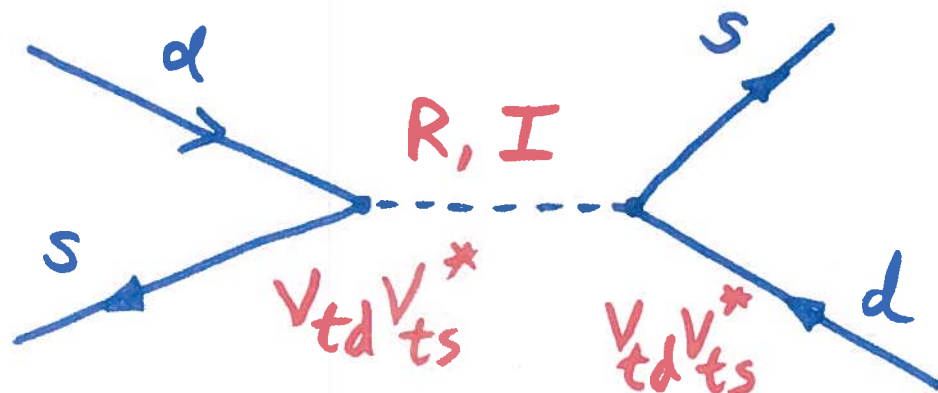
$u_{Rj}^{\circ} \rightarrow \exp(2i\tau) u_{Rj}^{\circ}$  **only FCNC in up sector**

Considering only the **Quark sector** there are **6 different BGL** type models. In the example considered, one has:

$$(N_d)_{rs} = \frac{\sqrt{2}}{\sqrt{1}} (D_d)_{rs} - \left( \frac{\sqrt{2}}{\sqrt{1}} + \frac{\sqrt{1}}{\sqrt{2}} \right) (V_{CKM})_{r3}^+ (V_{CKM})_{3s} (D_d)_{ss}$$

$$N_u = -\frac{\sqrt{1}}{\sqrt{2}} \text{diag}(0, 0, m_t) + \frac{\sqrt{2}}{\sqrt{1}} \text{diag.}(m_u, m_c, 0)$$

**Strong and natural suppression of  $K^0 - \bar{K}^0$  transitions**



$\rightarrow \lambda^{10}$  suppression

# Neutral couplings in BGL models

$$N_u = -\frac{v_1}{v_2} \text{diag}(0, 0, m_t) + \frac{v_2}{v_1} \text{diag}(m_u, m_c, 0)$$

Explicitly

$$N_d = \frac{v_2}{v_1} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$\rightarrow \left( \frac{v_1}{v_2} + \frac{v_2}{v_1} \right) \begin{pmatrix} m_d |V_{31}|^2 & m_s V_{31}^* V_{32} & m_b V_{31}^* V_{33} \\ m_d V_{32}^* V_{31} & m_s |V_{32}|^2 & m_b V_{32}^* V_{33} \\ m_d V_{33}^* V_{31} & m_s V_{33}^* V_{32} & m_b |V_{33}|^2 \end{pmatrix}$$

It all comes from the symmetry

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BGL models have some features in common with the Minimal Flavour Violation Framework

Buras, Gambino, Gorbahn, Jäger, Silvestrini (2001)

D'Ambrosio, Giudice, Isidori, Strumia (2002)

Namely, Flavour dependence of New Physics

is completely controlled by  $V^{CKM}$ , with no other flavour parameters.

Note: MFV is an "Hypothesis" not a model!!!



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## An important question:

Can one introduce other discrete symmetries leading to other models with FCNC, completely controlled by  $V_{CKM}$ ?

Answer: In the framework of 2HDM with Abelian symmetries and the constraint that FCNC only depend on  $V_{CKM}$ , BGL models are unique!

Ferreira and Silva 2010.

The explicit example of a BGL model was written in a weak-basis chosen by the symmetry. How to recognize a BGL model when written in a different WB?

The following relations

$$\Delta_1^\dagger \Delta_2 = 0 ; \quad \Delta_1 \Delta_2^\dagger = 0 ; \quad \Gamma_1^\dagger \Delta_2 = 0 ; \quad \Gamma_2^\dagger \Delta_1 = 0$$

are necessary and sufficient conditions for a set of Yukawa matrices  $\Gamma_i, \Delta_i$  to be of the BGL type, with FCNC in the down sector.

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- In a certain sense, **BGL models** are rather unique. They have **FCNC** either in the up or the down sectors but not in both.
  - If one restricts oneself to **abelian symmetries**, **BGL models** are the only 2HDM with **FCNC** at tree level, but no new flavour parameters, apart from **VCKM**.
  - **Question** - Can one generalize **BGL models** and construct a 2HDM with non-vanishing but controlled **FCNC** in both the up and down sectors? These **gBGL models** would contain **BGL models** as special cases, corresponding to specific values of the parameters of **gBGL**.

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Answer : Yes!!

The **gBGL** models are implemented through a  $\mathbb{Z}_2$  symmetry, where  $u_R$ ,  $d_R$  are even and one of the scalar doublets and one of the left-handed quark doublets are odd :

$$Q_{L3} \rightarrow -Q_{L3}$$

$$d_R \rightarrow d_R ; u_R \rightarrow u_R$$

$$\Phi_1 \rightarrow \Phi_2 ; \Phi_2 \rightarrow -\Phi_2$$

In words : Each line of  $\Gamma_j$ ,  $\Delta_j$  should couple only to one Higgs doublet.

# Structure of Yukawa Couplings

$$\Gamma_1 = \begin{pmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{pmatrix}$$

For  $\delta_{ij} = 0$  one obtains uBGL models, with

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

Similarly for  $\delta_{ij} = 0$ , one obtains dBGL

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It can be shown that  $N_d, N_u$  can be parametrized as:

$$N_d = \left[ t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1}) V^\dagger U P_3 U^\dagger V \right] M_d$$

$$N_u = \left[ t_\beta \mathbb{1} - (t_\beta + t_\beta^{-1}) U P_3 U^\dagger \right] M_u$$

$V \equiv V^{\text{CKM}}$  It is clear that in gBGL one has more

freedom, due to the presence of the **arbitrary matrix**

**$U$** . Nevertheless, there is much less freedom than one might expect, since the only quantities involving  $U$  are:

$$\left[ U P_3 U^\dagger \right]_{ij} = U_{i3} U_{j3}^*$$

# Conclusions

- The 2 HDMs are one of the most plausible extensions of the SM.
- There are 2 HDMs with naturally suppressed FCNC at tree level.
  - BGL models - only  $V^{CKM}$  appears
  - g BGL models - reduced number of parameters

• If extra scalars, either neutral or charged are discovered, an important task will be to measure the matrices  $N_u, N_d$

These measurements may may

~~help~~ help to uncover a

possible Family Symmetry