Partnerium physics,
alternative (conspirative) path to naturalness searches

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Mostly from: Kats, McCullough, GP, Soreq & Thaler (2017)

Outline

- Epilogue $[(-1)^2 = 1]$: the first partnerium J/ψ .

- Higgs, top & effective naturalness, partner-accidental symmetry.

- Realisation.

- Some basic phenomenology.

- Summary.

Epilogue: the discovery of the first partnerium back to the 70's

GIM

GLASHOW, ILIOPOULOS, AND MAIANI (1970)

- Fermi theory

Why: $Br(K^+ \to \mu\nu) \gg Br(K_{S,L} \to \mu\mu)$

- While ombination of isospin sym',

  plus exchange of charge to neutral sym' currents doesn't work:

$$A(K^+ \to \mu^+\nu) \Rightarrow G_F \times f_K \times (V_{us} \sim \theta_C) \times \bar{s}u\mu^+\nu$$

$\updownarrow$ $\updownarrow$

$K_{L,S}$ $\mu^+\mu^-$ $BR(K_{L,S} \to \mu^+\mu^-) \ll BR(K^+ \to \mu^+\nu)$

Quantum/radiative problem

GLASHOW, ILIOPOULOS, AND MAIANI (1970)

- Raising two questions:

  i. why the LO NC G_F vanishes?

  ii. why diverging QM correction NC $G_F(1 + G_F^2\Lambda^2)$ so small?
  Requires $\Lambda <$ few GeV $\Leftrightarrow$ naturalness problem.

GIM & Naturalness

- First assume that LO does not exist.

- Then regarding the quantum divergent contributions, naturalness issue:

$$(\bar{s}d)_{\rm Cabibbo} \sim G_F^2 \bar{s} \left(\int d^4p \, u \times \bar{u} \right) d \sim G_F^2 \bar{s}d \times V_{ud}^* V_{us} \times \Lambda^2$$

Invent a new state, the up partner named charm, such that:

$$(\bar{s}d)_{\rm GIM} \sim G_F^2 \bar{s} \left(\int d^4p \, c \times \bar{c} \right) d \sim G_F^2 \bar{s}d \times (-V_{ud}^* V_{us} = V_{cd}^* V_{cs}) \times \Lambda^2$$

Thus: $(\bar{s}d)_{\rm Cabibbo} + (\bar{s}d)_{\rm GIM} = 0 + \mathcal{O}\left[G_f(m_c^2 - m_u^2)\right]$.

GIM's collider section, "open charm" searches

- GIM collider section:

(copious) Charm production and decay being discuss.
However, many final states and missing energy …

PDGlive:

Γ_{40}	$K^- 2\pi^+$	$(9.46 \pm 0.24)\%$
Γ_{50}	$K^0_S \pi^+ \pi^0$	$(7.24 \pm 0.17)\%$
Γ_{51}	$K^0_S \rho^+$	$(6.04^{+0.60}_{-0.34})\%$
Γ_{41}	$(K^- \pi^+)_{S\text{-wave}} \pi^+$	$(7.58 \pm 0.22)\%$
Γ_{15}	$\overline{K}^0 e^+ \nu_e$	$(8.90 \pm 0.15)\%$
Γ_{16}	$\overline{K}^0 \mu^+ \nu_\mu$	$(9.3 \pm 0.7)\%$
Γ_{17}	$K^- \pi^+ e^+ \nu_e$	$(3.91 \pm 0.11)\%$
Γ_{18}	$\overline{K}^*(892)^0 e^+ \nu_e$, $\overline{K}^*(892)^0 \rightarrow K^- \pi^+$	$(3.68 \pm 0.10)\%$

Colliders initially failed to find these states …

Partner's parity: long lived the charm

♦ GIM only requires above cancellation; details of decay aren't specified.

(it also leads to the decay so the analogy to what follows is limited here)

♦ All the relevant couplings of the charm respect accidental charm number symmetry, which has a Z_2 sym.

Finding the charm

- GIM "forgot" that there is other class of states that doesn't carry a charm number, the charmonia: $Z_2^T(c\bar{c}) = (-1)^2 = 1\,.$

- It can decay in a direct manner & lead to a clean signal: the partnerium charmonium: J/ψ a , $^3S_1$ state (as opposed to the $^1S_0$ one para-positronium).

$c\bar{c}$ **MESONS**

$J/\psi(1S)$

$I^G(J^{PC}) = 0^-(1^{--})$

$J/\psi(1S)$ MASS	3096.900 ± 0
$J/\psi(1S)$ WIDTH	92.9 ± 2.8 ke

Decay Modes

	Mode	Fraction (Γ_i / Γ)
Γ_1	hadrons	$(87.7 \pm 0.5)\%$
Γ_2	virtual $\gamma \to$ hadrons	$(13.50 \pm 0.30)\%$
Γ_3	ggg	$(64.1 \pm 1.0)\%$
Γ_4	γgg	$(8.8 \pm 1.1)\%$
Γ_5	e^+e^-	$(5.971 \pm 0.032)\%$
Γ_6	$e^+e^-\gamma$	$(8.8 \pm 1.4) \times 10^{-3}$
Γ_7	$\mu^+\mu^-$	$(5.961 \pm 0.033)\%$

The theory and the discovery

◆ Two interesting things happened in 1974:

(i) Two exp' (@SLAC & @BNL) discovered a new dilepton resonance.

E598 Collaboration, "Experimental Observation of a Heavy Particle J";

SLAC-SP-017 Collaboration, "Discovery of a Narrow Resonance in e^+e^- Annihilation" (74)

(ii) [actually before (i)] Appelquist & Politzer realized that one can use a (rough) perturbation theory to describe the bound state properties:

Appelquist & Politzer, "Orthocharmonium and e^+e^- Annihilation" (75)

$$V(r) = -C\frac{\overline{\alpha}_s}{r}\,, \qquad \text{\w } C \text{ depend on the Rep. \& } \overline{\alpha}_s \equiv \alpha_s(r_{\rm rms})\,.$$

Fragility of bound state

♦ Bound state annihilation diluted by constituent decays unless $\Gamma_X << \Gamma_{\boldsymbol{B}}$.

(celebrated example where this condition is not satisfied is the SM top quark.)

♦ While $\Gamma_{\boldsymbol{B}} \sim$ (Bohr radius)$^{-3}$ such that $\Gamma_{\boldsymbol{B}} \sim M_{\boldsymbol{B}} \times \alpha^2_{\mathrm{ann}} \times \alpha_s^3$,

(where α_{ann} is the coupling responsible for the annihilation)

♦ Thus, models \w 2-body decay & reasonable couplings will remove the charmonia.

Charmonia-partnerium way to go

♦ However due to residual/accidental charm-partner, Z_2^T, charms can only decay via 3-body & suppressed by the heavy W:

$$\Gamma_c \sim m_c^5 G_F^2 \sim \alpha_2^2 (m_c/m_W)^4 \times m_c$$

♦ It seems that Appelquist & Politzer understood this but decided not write it up hence the J/ψ was discovered before predicted as a dilepton resonance.

E598 Collaboration, "Experimental Observation of a Heavy Particle J";

SLAC-SP-017 Collaboration, "Discovery of a Narrow Resonance in e^+e^- Annihilation" (74)

↓

In modern language: J/ψ made of up-partners $= c\bar{c}$ predicted by naturalness.

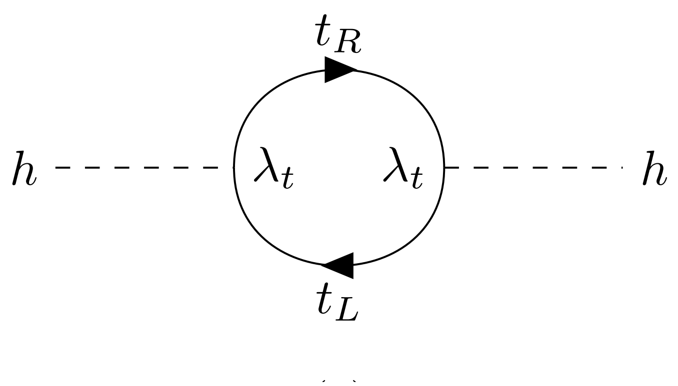
↓

The partners were first discovered as bound state $=$ Partnerium .

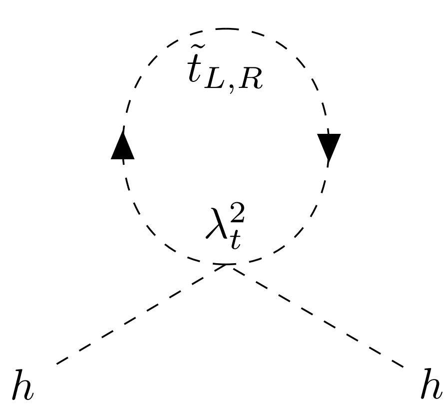
Back to the 21st century: Higgs naturalness

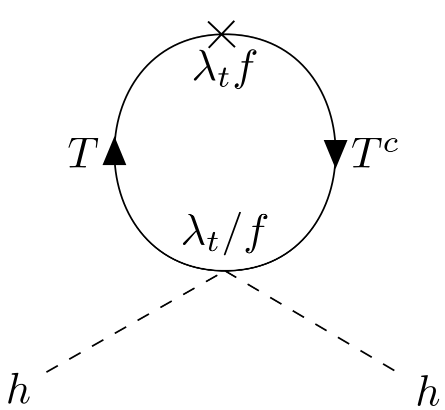
Back to naturalness

- Let's just focus on the cancellation of top divergencies need to cancel this vertex, $\mathcal{L}_{\rm SM} \supset \lambda_t q H t^c$,



- Can add partners with these contributions:





scalars vs fermions

Kats, McCullough, GP, Soreq & Thaler (2017)

- Scalar case:

$$\mathcal{L}_{\text{spin-0}} \supset -m^2_{\tilde{Q}_3}|\tilde{Q}_3|^2 - m^2_{\tilde{U}^c_3}|\tilde{U}^c_3|^2 + \lambda_t^2|H\cdot\tilde{Q}_3|^2 + \lambda_t^2|H|^2|\tilde{U}^c_3|^2$$

renormalizable, explain special-valued couplings for ex. via SUSY.

- Fermion case:

$$\mathcal{L}_{\text{spin-1/2}} \supset -\lambda_t\left(f - \frac{|H|^2}{2f}\right)TT^c$$

non-renormalizable, explain couplings value via for ex. composite Higgs

Partnerium physics

- In both cases couplings respect Z_2^T accidental symmetry.

- Here the partner-decay is model dependent as charge of partners is not set.

- Can imagine cases with no linear mixing with SM leading to possibly stable particle with charge tracks or displaced or rich final state.

- History-lesson: in case of suppressed/elusive top-partner-decay => partnerium physics might be the way to go. $\left[(-1)^2 = 1\right]$

Can one realize such construction?

Kats, McCullough, GP, Soreq & Thaler (17)

Can we find natural models that have an approximate partner-parityand lead to partnerium signals?

- In some limit even SUSY leads to stoponioum signals. (highly constrained)

  Drees & Nojiri (94); for current status see e.g: Batell & Jung (15)

- ”Sterile” partners: possibly in mirror models. (visible?)

  Chacko, Goh, & Harnik (06); Iwamoto, Lee, Shadmi & Ziegler (16)

- What about colored partners? interesting if charge is twisted.

  Burdman, Chacko, Goh, & Harnik (06); Cohen, Craig, Lou & Pinner (15); Kats, McCullough, GP, Soreq & Thaler (15)

Folded/twisted SUSY

- Begin \w folded-SUSY, double SUSY:

for "SM" N=1 fields remove scalars & for mirror ones project fermions.

Burdman, Chacko, Goh, & Harnik (06); Cohen, Craig, Lou & Pinner (15).

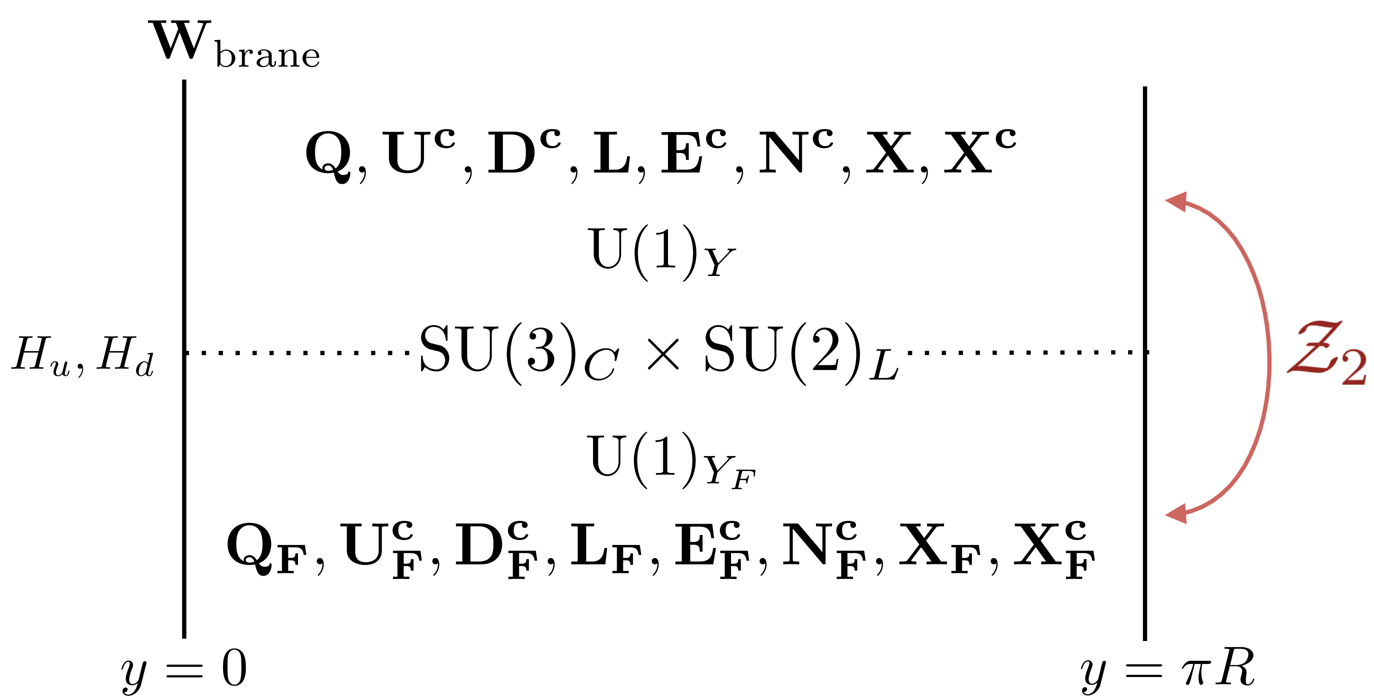


Illustration of the hypertwisted SUSY model.

$$
\begin{aligned}
\boldsymbol{W}_{\text{Yuk}} \; = \; & \lambda_u \boldsymbol{H_u} \left(\boldsymbol{Q U^c} + \boldsymbol{Q_F U^c_F} \right) - \lambda_d \boldsymbol{H_d} \left(\boldsymbol{Q D^c} + \boldsymbol{Q_F D^c_F} \right) \\
& - \lambda_l \boldsymbol{H_d} \left(\boldsymbol{L E^c} + \boldsymbol{L_F E^c_F} \right) - \lambda_N \boldsymbol{H_u} \left(\boldsymbol{L N^c} + \boldsymbol{L_F N^c_F} \right) \; .
\end{aligned}
$$

Folded/twisted SUSY, twist hyper charge for the folded sector

- For simplicity chose anomaly free spectrum: $Y_F = Y + (3q-2)(B-L)$,

now the charge of folded RH stops is a free parameter …

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Y_F}$
$\boldsymbol{H_u}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$1/2$
$\boldsymbol{H_d}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$-1/2$
$\boldsymbol{Q}, \boldsymbol{Q_F}$	$\mathbf{3}$	$\mathbf{2}$	$(\frac{1}{6}, q-\frac{1}{2})$	$(q-\frac{1}{2}, \frac{1}{6})$
$\boldsymbol{U^c}, \boldsymbol{U^c_F}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$(-\frac{2}{3}, -q)$	$(-q, -\frac{2}{3})$
$\boldsymbol{D^c}, \boldsymbol{D^c_F}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$(\frac{1}{3}, 1-q)$	$(1-q, \frac{1}{3})$
$\boldsymbol{L}, \boldsymbol{L_F}$	$\mathbf{1}$	$\mathbf{2}$	$(-\frac{1}{2}, \frac{3}{2}-3q)$	$(\frac{3}{2}-3q, -\frac{1}{2})$
$\boldsymbol{E^c}, \boldsymbol{E^c_F}$	$\mathbf{1}$	$\mathbf{1}$	$(1, 3q-1)$	$(3q-1, 1)$
$\boldsymbol{N^c}, \boldsymbol{N^c_F}$	$\mathbf{1}$	$\mathbf{1}$	$(0, 3q-2)$	$(3q-2, 0)$
$\boldsymbol{X}, \boldsymbol{X_F}$	$\mathbf{1}$	$\mathbf{1}$	$(q_X, 0)$	$(0, q_X)$
$\boldsymbol{X^c}, \boldsymbol{X^c_F}$	$\mathbf{1}$	$\mathbf{1}$	$(-q_X, 0)$	$(0, -q_X)$

Hyper twisted fermion top partners

Kats, McCullough, GP, Soreq & Thaler, to appear.

- Similar in spirit, simplest construction via $\mathrm{SU}(3)_G \times \mathrm{SU}(2)_F \times U(1)_Z$:

SU(3)/SU(2) composite "folded" Higgs model:

$$\Phi = \exp\left[-i\frac{\pi^a T^a_G}{f}\right]\begin{pmatrix}0\\0\\f\end{pmatrix} \supset \begin{pmatrix} H \\ f - \frac{H^\dagger H}{2f}\end{pmatrix}, \qquad H = -\frac{1}{2}\begin{pmatrix}\pi^5 + i\pi^4\\ \pi^7 + i\pi^6\end{pmatrix} \Rightarrow \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix},$$

- Doubling the doublet: $(\mathbf{3},\mathbf{2})_{\frac{y_T}{2}}: \quad Q = \begin{pmatrix} b & q'_d \\ -t & -q'_u \\ t' & T\end{pmatrix}, \qquad (\mathbf{1},\bar{\mathbf{2}})_{-\frac{y_T}{2}-\frac{1}{3}}: \quad Q^c = \begin{pmatrix} t^c & -T^c\end{pmatrix},$

- Twisting the charges: $T^{1,2,3}_L = T^{1,2,3}_G, \qquad Y = Z - \frac{T^8_G}{\sqrt{3}} + \left(\frac{2}{3} - y_T\right) T^3_F,$

- Projecting out the untwisted top-partners:

$$\mathcal{L}_Y = \lambda_t Q\Phi Q^c + \text{h.c.}\,, \qquad \mathcal{L}_{\text{soft}} = -M_{t'}t't'^c - M_{q'}q'q'^c\,.$$

Some phenomenology

Blum, Efrati, Nir & Frugiuele (16); Kats, McCullough, GP, Soreq & Thaler (17)

Twisted-partneriums at the LHC

- Naturalness => partnerium => di-electroweak/Higgs resonance signals.

$$\mathcal{L} \supset -\kappa\, v\, h\, \tilde{t}_F^*\, \tilde{t}_F\,, \quad \mathcal{L} \supset \frac{\kappa v}{2m}\, h\, \bar{T}\, T \quad .$$

- Scalar partners:

Large partial annihilation widths to *WW*/*ZZ*/*hh*. (see e.g.Martin (08)) & reduction in diphoton branching fraction. (binding due to Higgs exchange is negligible, *gg* production)

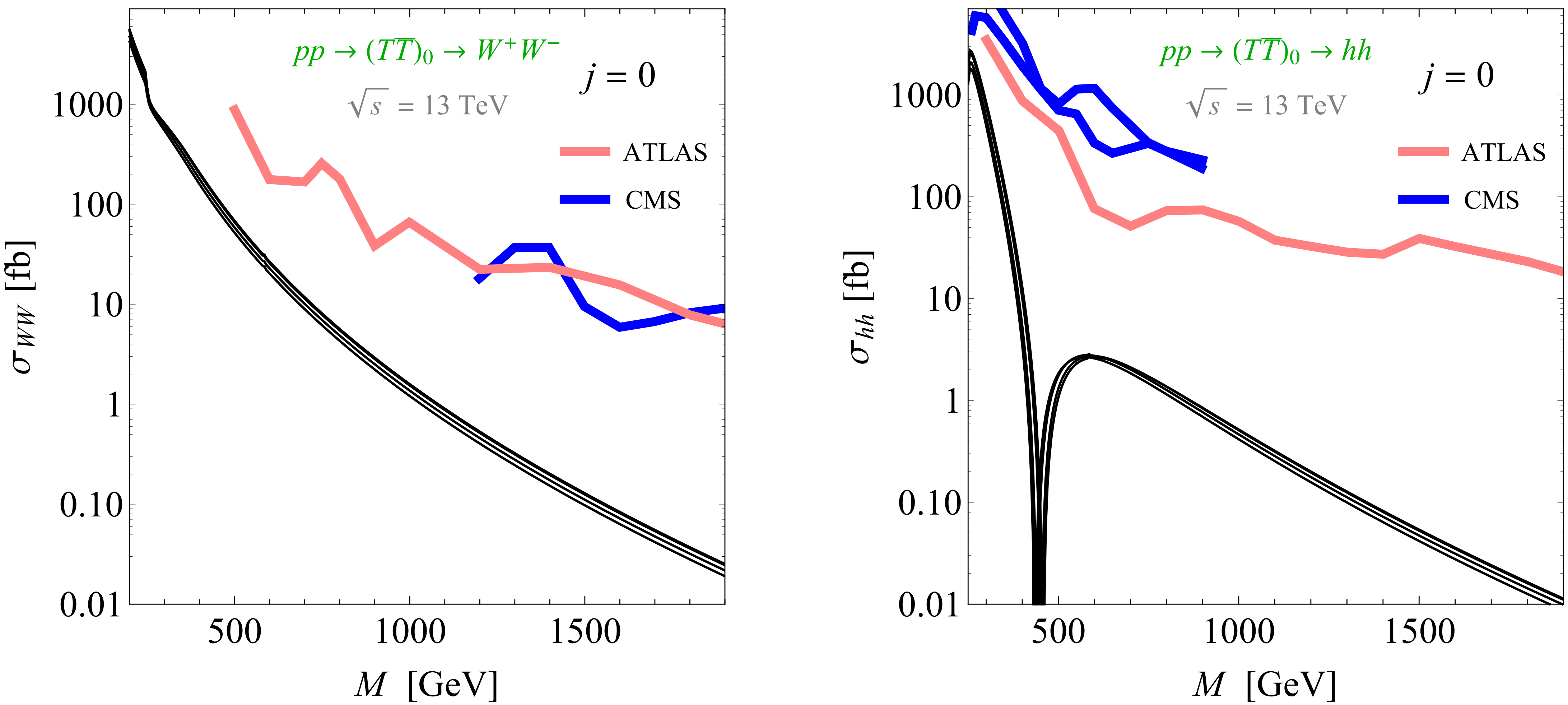
- However, fermionic partners - selection-rules exclude natural couplings:

Spin-0 (s-wave) bound-state is a pseudoscalar => cannot annihilate to *WW*, *ZZ* or hh.

The other state is simply a vector … (*gg* production & associated/EW production respectively)

Spin-0 partnerium signals

Kats, McCullough, GP, Soreq & Thaler (17)



Partnerium signals in the WW and hh channels for $SU(2)_L$-singlet scalars (solid black),as a function of the partnerium mass, M. (diiferent black curves correspond to different constituent charges)

The dip in *hh* dist' is due to a cancellation between 4 diagrams (contact interaction, s-channel higgs, and t- and u-channel stop).

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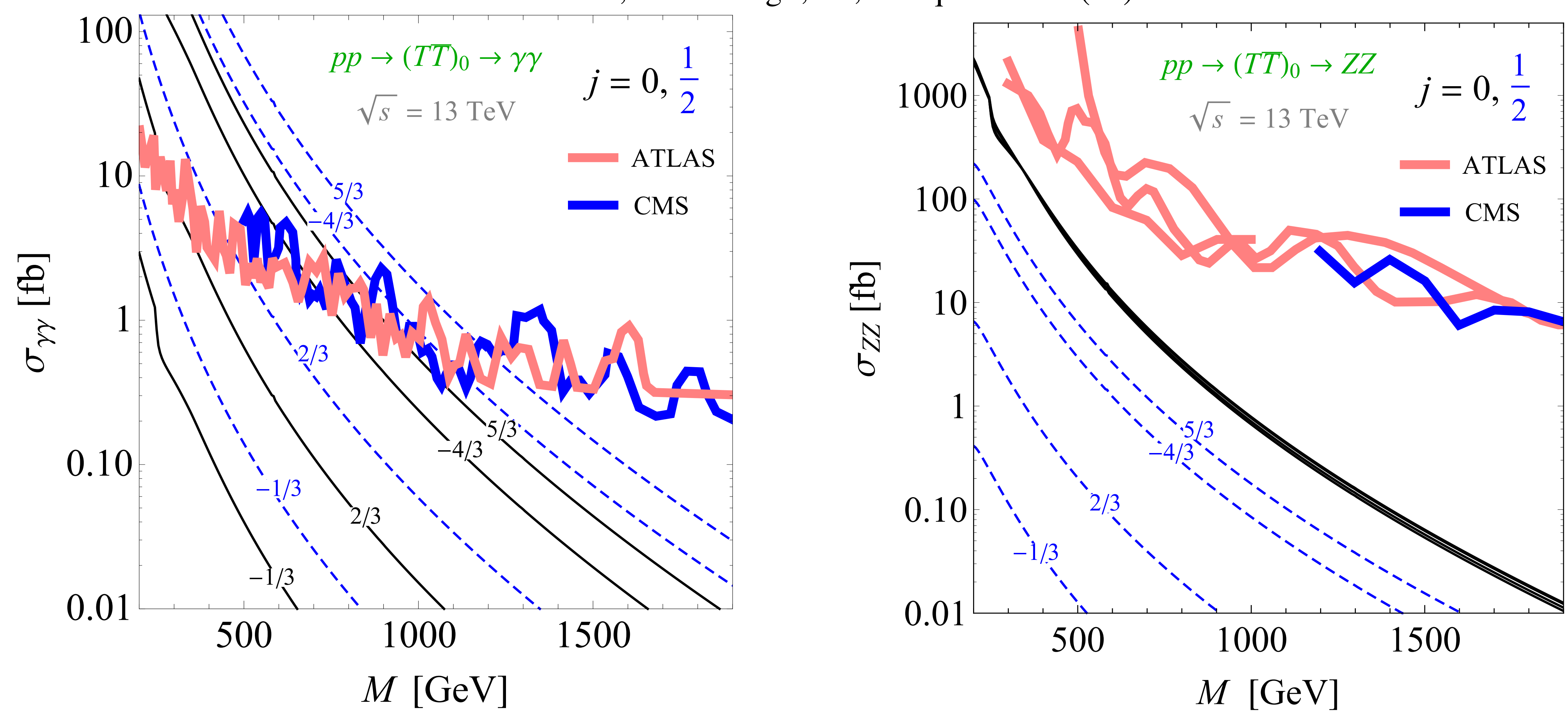
Spin-0 partnerium signals

Kats, McCullough, GP, Soreq & Thaler (17)



Partnerium signals in the γγ/ZZ channels for $SU(2)_L$-singlet scalars (solid black) and fermions (dashed blue) for electric charge values indicated on each curve.

ATLAS Collaboration, "Search for scalar diphoton resonances with 15.4 fb^{-1} of data collected at $\sqrt{s} = 13$ TeV in 2015 and 2016 with the ATLAS detector," Tech. Rep. ATLAS-CONF-2016-059, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206154.

CMS Collaboration, "Search for resonant production of high mass photon pairs using 12.9 fb^{-1} of proton-proton collisions at $\sqrt{s} = 13$ TeV and combined interpretation of searches at 8 and 13 TeV," Tech. Rep. CMS-PAS-EXO-16-027, CERN, Geneva, 2016. http://cds.cern.ch/record/2205245.

ATLAS Collaboration, "Search for new phenomena in the $Z(\to \ell\ell) + E_T^{miss}$ final state at $\sqrt{s} = 13$ TeV with the ATLAS detector," Tech. Rep. ATLAS-CONF-2016-056, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206138.

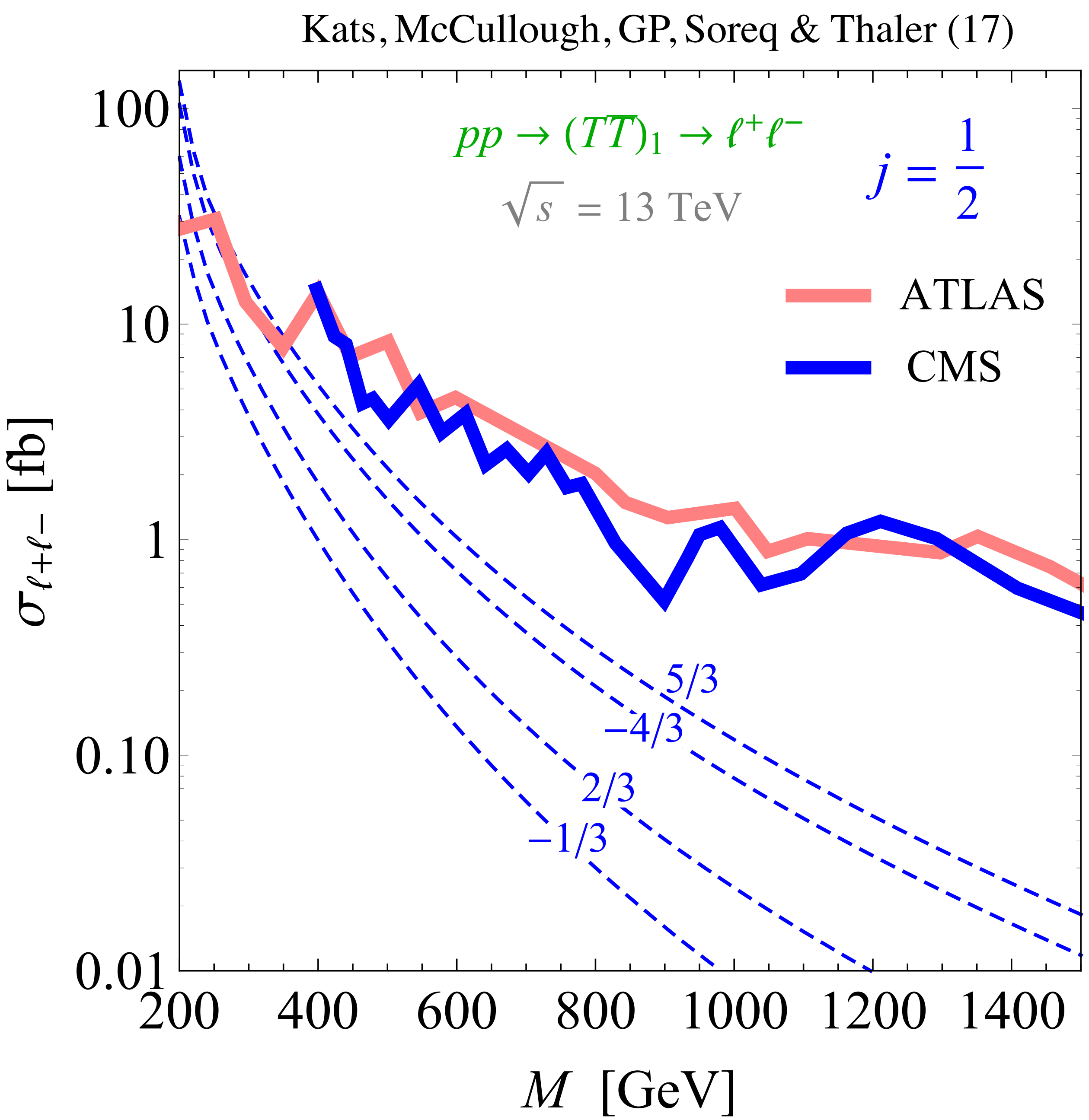
ATLAS Collaboration, "Searches for heavy ZZ and ZW resonances in the $\ell\ell qq$ and $\nu\nu qq$ final states in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector," Tech. Rep. ATLAS-CONF-2016-082, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206275.

ATLAS Collaboration, "Search for diboson resonance production in the $\ell\nu qq$ final state using pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector at the LHC," Tech. Rep. ATLAS-CONF-2016-062, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206199.

ATLAS Collaboration, "Search for pair production of Higgs bosons in the $b\bar{b}b\bar{b}$ final state using proton−proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector," Tech. Rep. ATLAS-CONF-2016-049, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206131.

CMS Collaboration, "Search for resonant Higgs boson pair production in the $b\bar{b}\tau^+\tau^-$ final state using 2016 data," Tech. Rep. CMS-PAS-HIG-16-029, CERN, Geneva, 2016. https://cds.cern.ch/record/2204936.

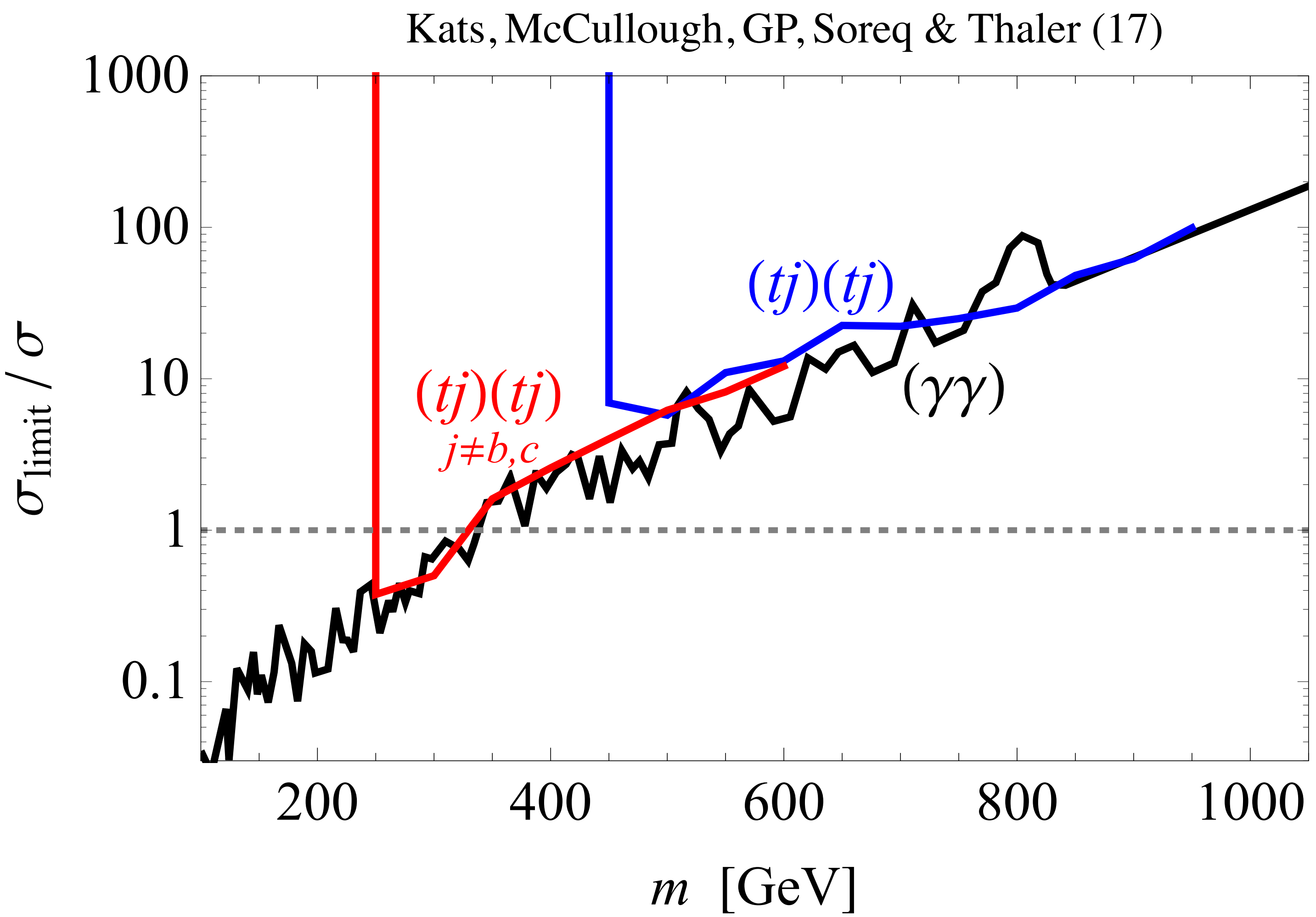
Spin-1 twisted partnerium fermion-only signals for the LHC



ATLAS Collaboration, "Search for new high-mass resonances in the dilepton final state using proton-proton collisions at $\sqrt{s}$ = 13 TeV with the ATLAS detector," Tech. Rep. ATLAS-CONF-2016-045, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206127.

CMS Collaboration, "Search for a high-mass resonance decaying into a dilepton final state in 13 fb^{-1} of pp collisions at $\sqrt{s}$ = 13 TeV," Tech. Rep. CMS-PAS-EXO-16-031, CERN, Geneva, 2016. http://cds.cern.ch/record/2205764.

Partnerium vs open partners



CMS Collaboration, "Search for pair production of resonances decaying to a top quark plus a jet in final states with two leptons," Tech. Rep. CMS-PAS-B2G-12-008, CERN, Geneva, 2013. http://cds.cern.ch/record/1630845.

CMS Collaboration, V. Khachatryan *et al.*, "Searches for R-parity-violating supersymmetry in pp collisions at $\sqrt{s} = 8$ TeV in final states with 0–4 leptons," *Phys. Rev.* **D94** no. 11, (2016) 112009, arXiv:1606.08076 [hep-ex].

CMS Collaboration, S. Chatrchyan *et al.*, "Search for pair production of excited top quarks in the lepton + jets final state," *JHEP* **06** (2014) 125, arXiv:1311.5357 [hep-ex].

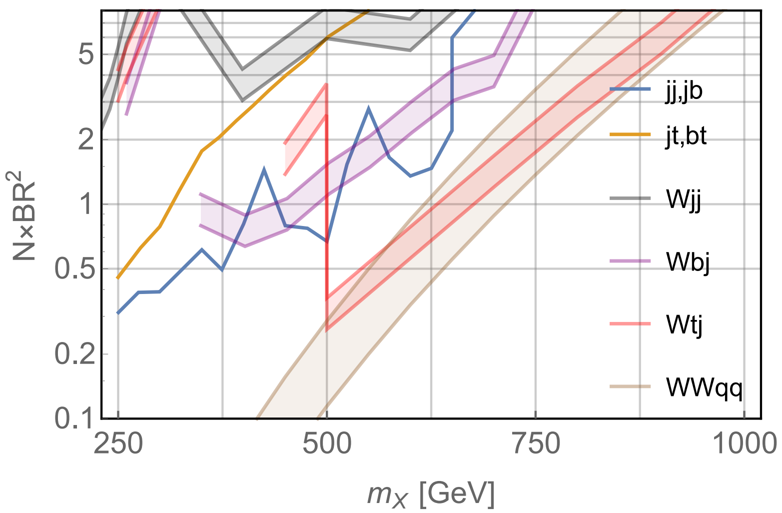
ATLAS Collaboration, "A search for pair produced resonances in four jets final states in proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS experiment," Tech. Rep. ATLAS-CONF-2016-084, CERN, Geneva, Aug, 2016. http://cds.cern.ch/record/2206277.

CMS Collaboration, "Search for low-mass pair-produced dijet resonances using jet substructure techniques in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 13$ TeV," Tech. Rep. CMS-PAS-EXO-16-029, CERN, Geneva, 2016. http://cds.cern.ch/record/2231062.

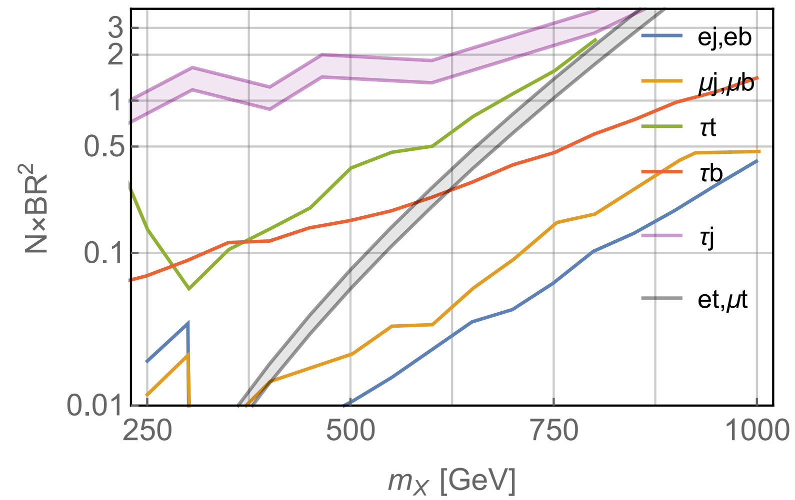
Cross section limits on a color-triplet scalar with electric charge −4/3, as a function of its mass m. Shown are CMS limits on top+jet decays (red & blue), using the 8 TeV dataset. The red limits do not apply when the jet is a charm since the analysis employs loose b-tag vetoes. Also shown is the above limit on the bound state diphoton signal.

Partnerium vs open scalar partners

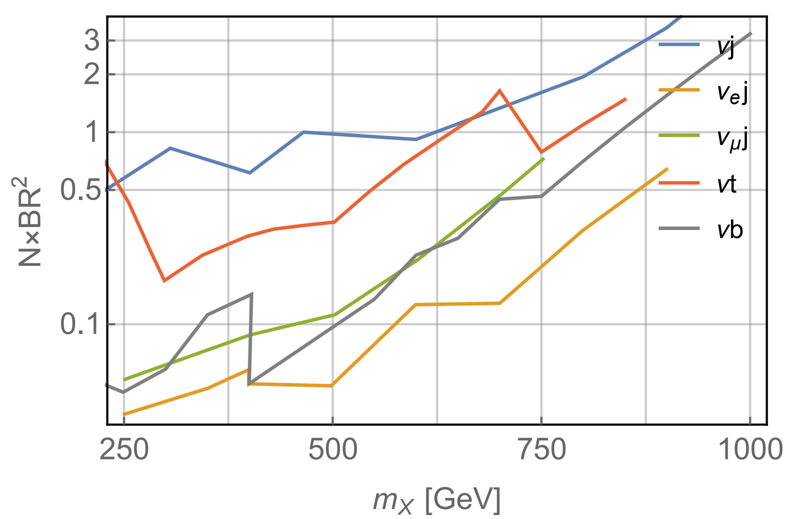
Blum, Efrati, Nir & Frugiuele (16)



(a)qq final states



(b)$q\ell$ final states



(c)$q\nu$ final states

1(a) dijet decays, Fig. 1(b) jet and charged lepton signals, and Fig. 1(c) neutrino-jet topology. In each figure we show the current limit on the pair-production cross section times BR^2, normalized to the NLO+NLL cross section. Presented this way, when a single mode dominates the decay (namely BR = 1), the y axis corresponds to the number of copies of the X representation that are experimentally allowed.

Conclusions

- Epilogue: the first partnerium J/ψ .

- Higgs, top & effective naturalness, partner-accidental symmetry.

- Proof of concept: twisted fermion & twisted scalar partner.

- Some basic phenomenology, seems as lightish partners \w partnerium signal are viable.

- More systematic study is required: shouldn’t we look for the above?

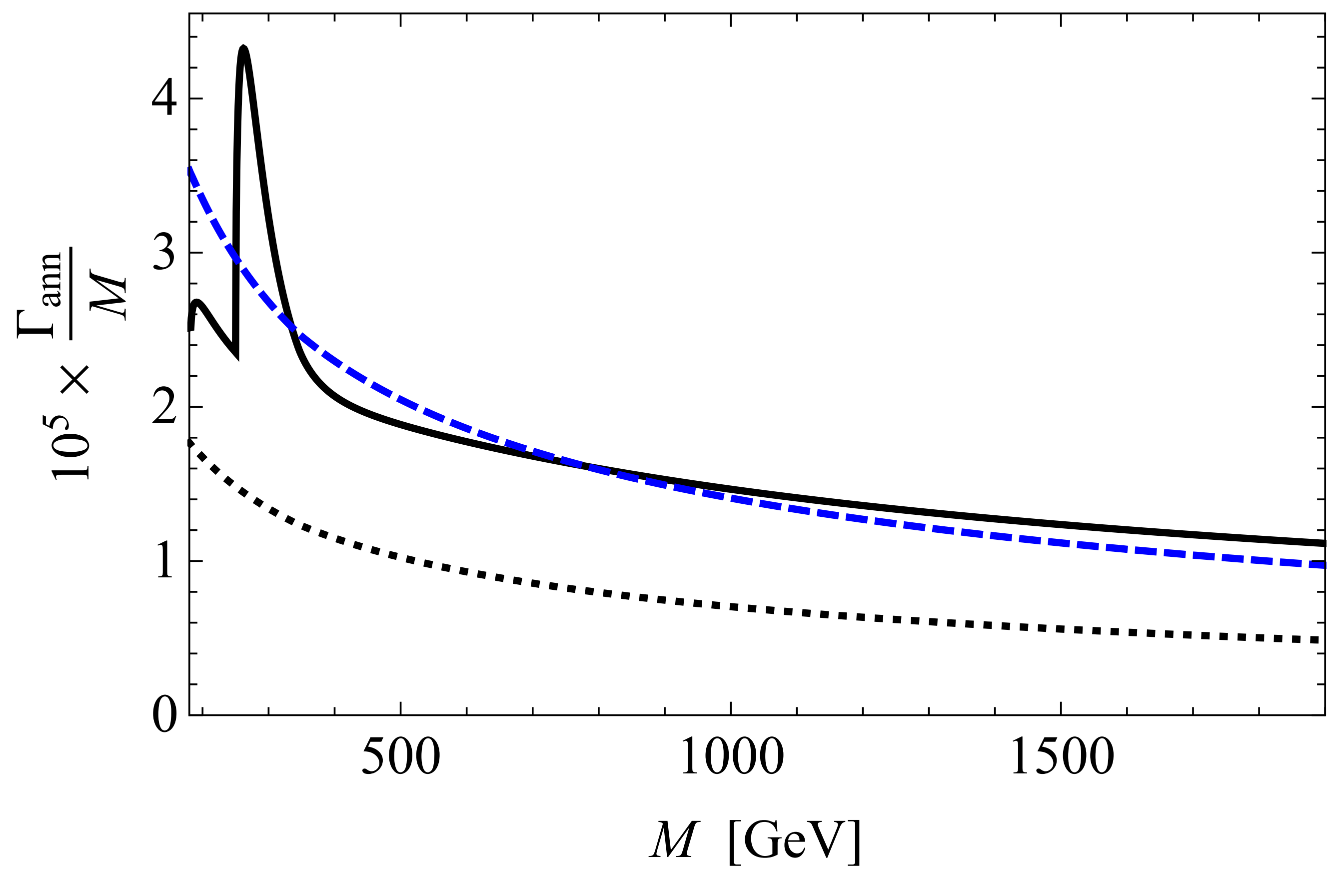


FIG. 6: Bound-state annihilation width as a function of the bound-state mass for $\mathrm{SU}(2)_L$-singlet constituents with electric charge $Q = -4/3$, assuming they are scalars which are either top partners (solid black) or have no coupling to the Higgs (dotted black), or fermions (dashed blue).

Backups

Hyper twisted fermion top partners

The interaction term of Eq. (18) leads to the following interaction of the Higgs with the SM and partner fermions:

$$\mathcal{L}_Y \supset \lambda_t q H t^c - \lambda_t \left(f - \frac{H^\dagger H}{2f} \right) T T^c - \lambda_t q' H T^c + \lambda_t \left(f - \frac{H^\dagger H}{2f} \right) t' t^c + \mathcal{O}(1/f^2)\,, \tag{20}$$

which matches to Eqs. (1) and (3). Combining Eqs. (18) and (19) one can write the fermion mass matrices

$$M_{2/3} = \begin{pmatrix} t & t' \end{pmatrix} \begin{pmatrix} -\lambda_t f s_\epsilon & 0 \\ \lambda_t f c_\epsilon & -M_{t'} \end{pmatrix} \begin{pmatrix} t^c \\ t'^c \end{pmatrix}\,, \qquad M_{y_T} = \begin{pmatrix} T & q'_u \end{pmatrix} \begin{pmatrix} -\lambda_t f c_\epsilon & 0 \\ \lambda_t f s_\epsilon & -M_{q'} \end{pmatrix} \begin{pmatrix} T^c \\ q'^c_u \end{pmatrix}\,, \tag{21}$$

The GIM mechanism

- Straight forward to realise:

$$\left\{V^\dagger \;\left[G_F\,\delta_{ij} + G_F^2\,\mathrm{diag}(m_u^2, m_c^2)\right] V\right\}_{d_i\,,d_j}$$

- For charm mass < few x GeV protection is obtained.

Theory of charmonia

Appelquist & Politzer, "Orthocharmonium and e^+e^- Annihilation" (75)

- Once the potential understood the rest follows:

Consider X-$\overline{X}$ the bound state, the binding energies & wavefunctions at origin for ground state (n = 1) & its radial excitations (n = 2, 3, . . .) are given by -

$$E_b = -\frac{1}{4n^2}\, C^2 \overline{\alpha}_s^2 m\,, \qquad |\psi(\mathbf{0})|^2 \equiv \frac{1}{4\pi}\, |R(0)|^2 = \frac{C^3 \overline{\alpha}_s^3 m^3}{8\pi\, n^3}$$

The cross-section for the bound state ***B*** to be produced by initial-state partons *a* and *b* is:

Kats & Strassler (12)

$$\hat{\sigma}_{ab\to\mathcal{B}}(\hat{s}) = \frac{8\pi}{m}\, \frac{\hat{\sigma}^{\text{free}}_{ab\to X\overline{X}}(\hat{s})}{\beta(\hat{s})}\, |\psi(\mathbf{0})|^2\, 2\pi\, \delta(\hat{s} - M^2)$$

$\hat{\sigma}^{\text{free}}_{ab\to X\overline{X}}(\hat{s})$ is the production cross section for a free pair at threshold (i.e., for $\beta(\hat{s}) \to 0$, where $\beta(\hat{s})$ is the velocity of X or $\overline{X}$ in their COM.

(Annihilation fall like $1/n^3$ => excited states vulnerable to decays of constituent X's => them annihilation, thus ignored.)