Physical Cosmology II

Structure Formation and Evolution

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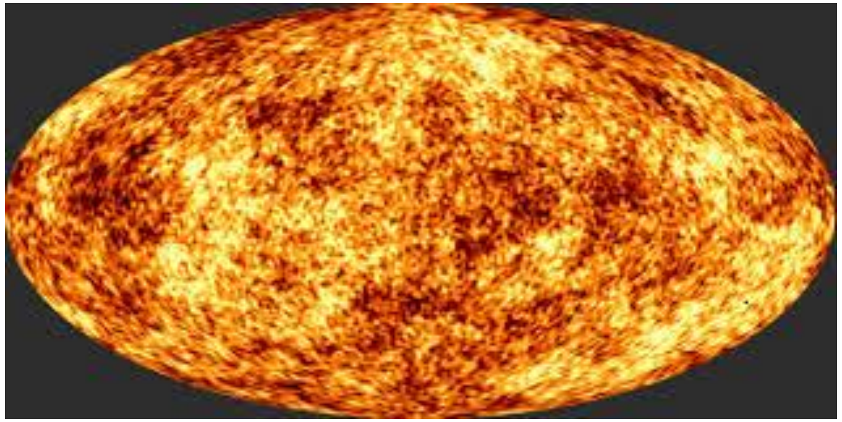
Useful refs.

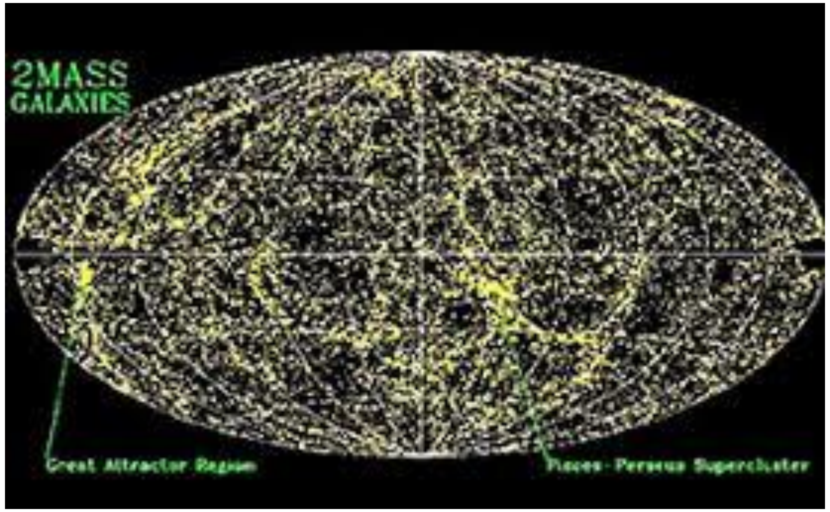
Peacock: Large Scale Surveys and Cosmic Structure (overview of structure form. from Newtonian perspective)
Wayne Hu's CMB pages: http://background.uchicago.edu/~whu/: Multilevel overview of CMB (nice graphics… used here)
Peacock: Cosmological Physics (comprehensive account; intermediate level)
Doddleson: Modern Cosmology (Relatively advanced treatment of structure formation and associated kinetics in GR)
Daniel Baumann Tripos Lectures http://www.damtp.cam.ac.uk/user/db275/Cosmology.pdf Chap 4 & 5

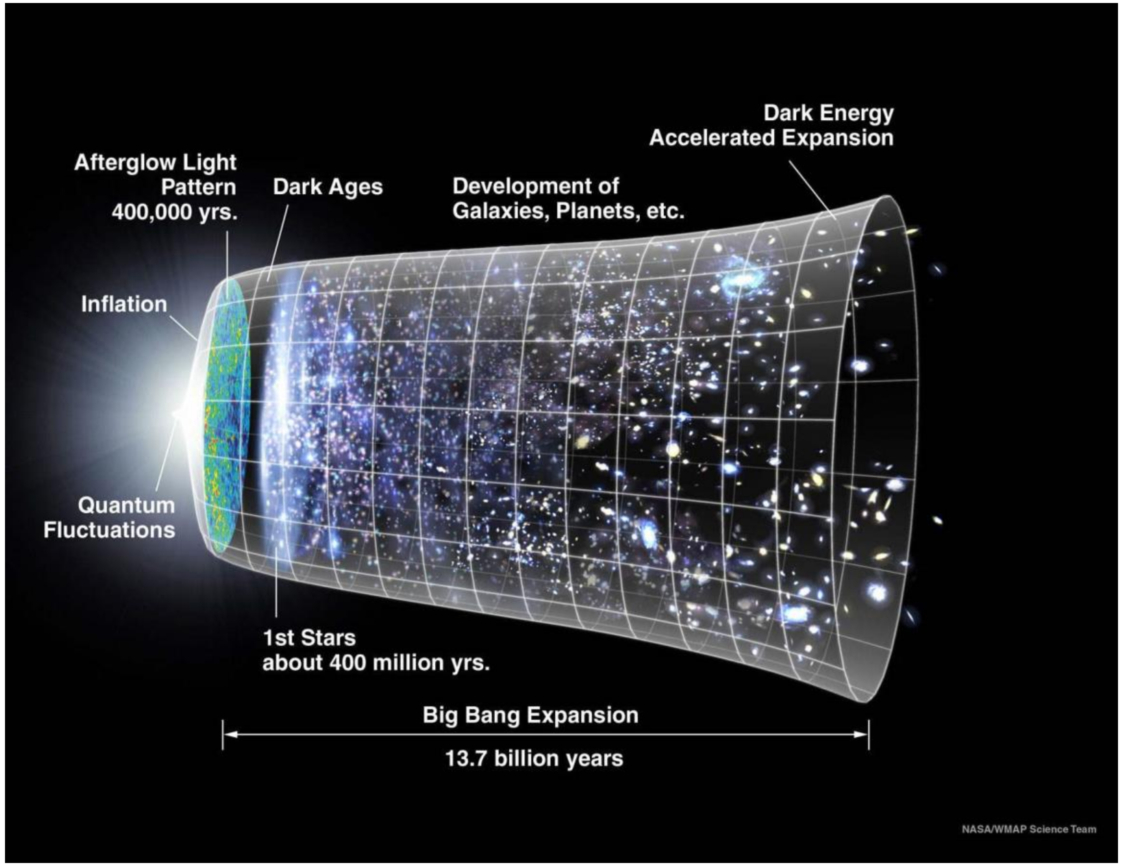
Forming Structure from Fluctuations

** THE CMB 300 000 YRS



**GALAXY DISTRIBUTION (14 GYR)



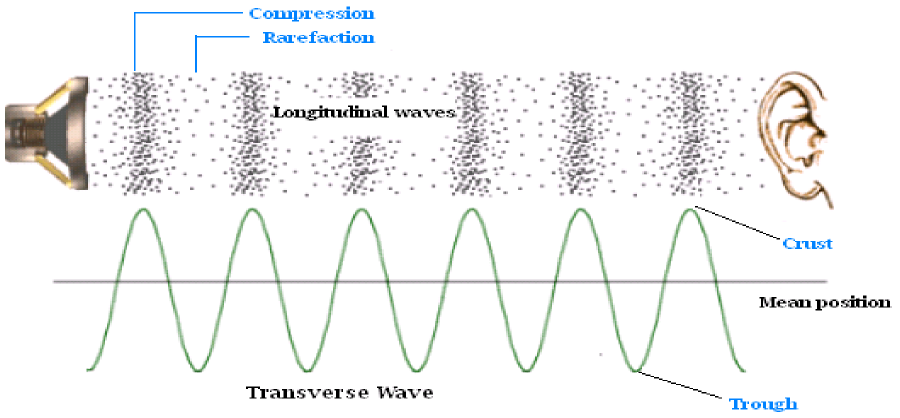


Fundamental Aspect of Gravity : Clustering Instability

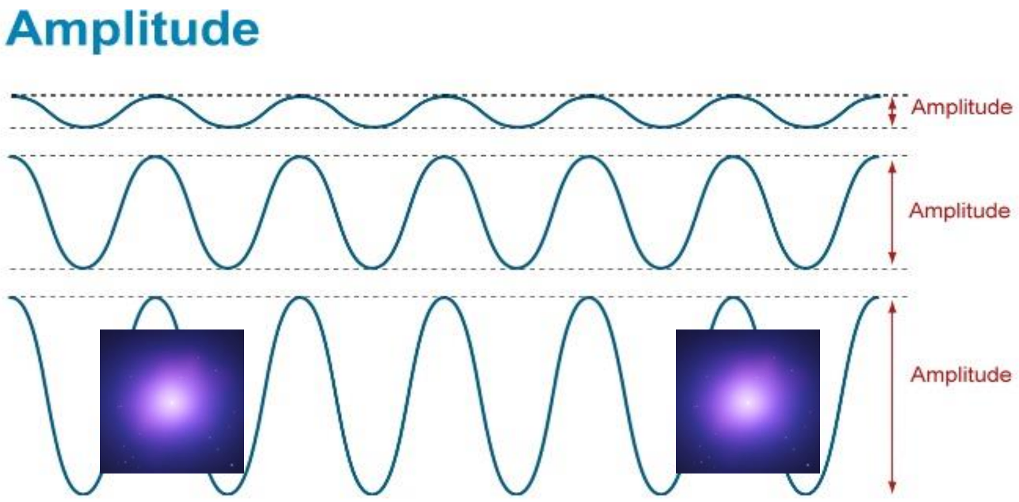
Gravity:

i) always attractive   ii) long range

I) 'Normal' sound wave



Gravity against Pressure



ii) GRAVITY BEATS PRESSURE

→ COLLAPSE!

Fluids with Gravity

- Mass Conservation (continuity eq.):

$$\partial_t \rho = -\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot (\rho \boldsymbol{u})$$

- Momentum Conservation (Euler eq.):

$$(\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}})\, \boldsymbol{u} = -\frac{\boldsymbol{\nabla}_{\boldsymbol{r}} P}{\rho} - \boldsymbol{\nabla}_{\boldsymbol{r}} \Phi$$

$$D_t \boldsymbol{u} \equiv (\partial_t + \boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u}$$

- Poisson Eq. (Gauss law in differential form):

$$\nabla_{\boldsymbol{r}}^2 \Phi = 4\pi G \rho$$

Small Perturbation Propagation → Linearise

- Meaning: Expand variables in power series and only take first order terms (as in Taylor expansion)… $\rho(t,\boldsymbol{r}) = \bar{\rho}(t) + \delta\rho(t,\boldsymbol{r})$

  (much of the physics you learned is a consequence of this 'trick')

First Ignore gravity

$$\partial_t \delta\rho = -\boldsymbol{\nabla}_{\boldsymbol{r}} \cdot (\bar{\rho}\boldsymbol{u})$$

$$\bar{\rho}\, \partial_t \boldsymbol{u} = -\boldsymbol{\nabla}_{\boldsymbol{r}} \delta P \ .$$

++ Introduce adiabatic sound speed $\delta P = c_s^2 \delta\rho$

→ **Wave equation**: $\left(\partial_t^2 - c_s^2 \nabla^2\right) \delta\rho = 0$

→ Solutions in terms of Fourier modes $\delta\rho = A \exp[i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})]$ and $\boxed{\omega^2 = c_s^2 k^2}$

Add Gravity and Start to Cluster

$$\left(\partial_t^2 - c_s^2 \nabla_r^2\right) \delta\rho = 4\pi G \bar{\rho} \delta\rho$$

Gravitational source term
"Loading"

$$\Rightarrow \quad \omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}$$

$$k_{\rm J} \equiv \frac{\sqrt{4\pi G \bar{\rho}}}{c_s} \, .$$

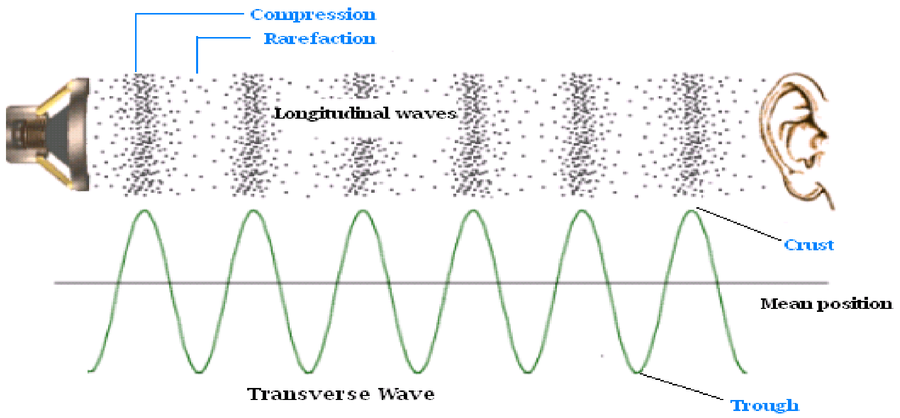
→ Jeans (clustering or collapse) scale ~ Dynamical time / sound horizon

If Gravity can start collapse before pressure can act → collapse

Fundamental Aspect of Gravity : Clustering Instability

I) Small scales

Wavelength << $c_s \sqrt{\frac{\pi}{G\bar{\rho}}}$



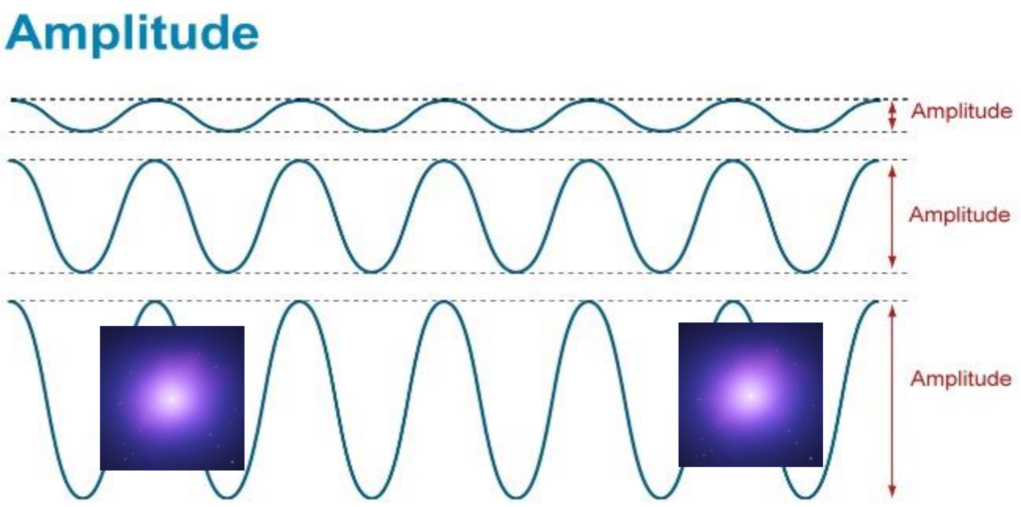
II) Larger Scales

** Slower Propagation

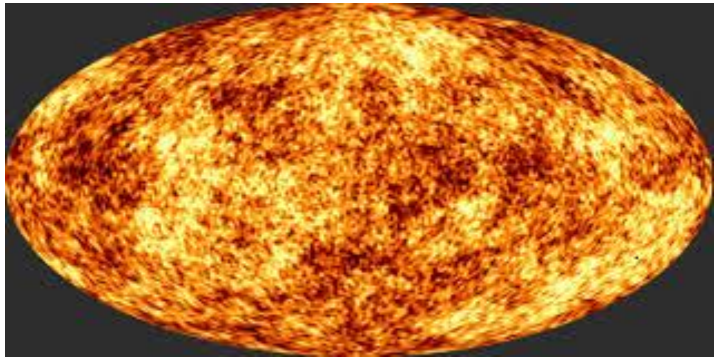
** Larger Amplitude

→ Then collapse

Wavelength > $c_s \sqrt{\frac{\pi}{G\bar{\rho}}}$



Sound Waves in a Photon Fluid



- **Remember recombination?**
- Before that Baryons tightly coupled with photon gas.
- The latter behaves as ideal fluid with $P = \frac{1}{3}\,\rho$.
- It obeys a continuity equation for photon number ~ T^3

++ a momentum conservation equation.

$$\Theta_{\ell=0,m=0}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}}\Theta(\mathbf{k}),$$

→ Wave Eq. for Temp. Perturbations:

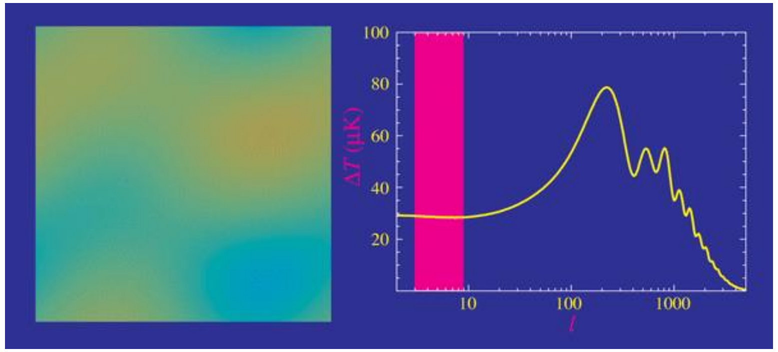
$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0,$$

** Transformed wave equation obtained earlier → **acoustic waves travelling at $\sqrt{\frac{1}{3}}$ c**

****If system expanding**, equation remains with t → $\eta \equiv \int dt/a(t)$

→ Valid in terms of this 'conformal time' (comoving dist light travels since *t=0*)

Acoustic Peaks



- **Photon fluid is permeated by sound waves**
- **Frozen at recombination**

→ Temperature fluctuations: $\Theta(\eta_*) = \Theta(0)\cos(ks_*)$,

where $s = \int c_s d\eta \approx \eta/\sqrt{3}$ and s_* and indicates recomb. era.

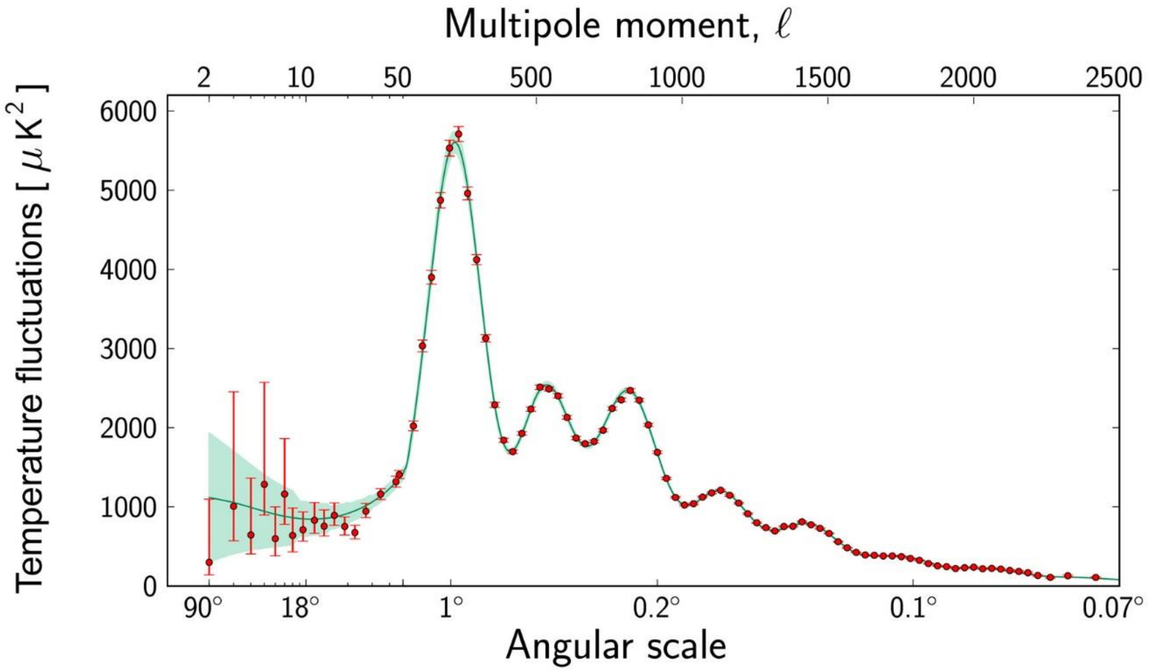
** Peaks indicate $k = n\pi/s_*$

First peak → 1st Compression

Second → 1st Rarefication

Third → 2nd Compression

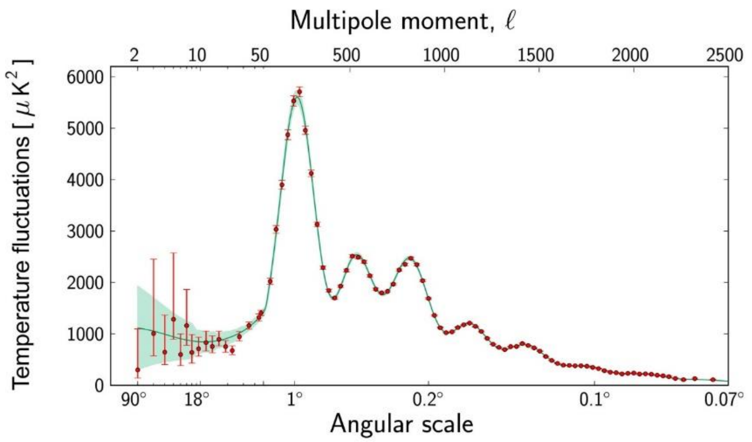
more oscillations→ damping



Location of the Peaks

- An object of length λ appears of angular size θ appears at D:

If $\quad \theta = \frac{\lambda}{D}$, and $\quad k = n\frac{\pi}{s_*} \rightarrow \theta = \frac{2}{n}\frac{s_*}{D}$



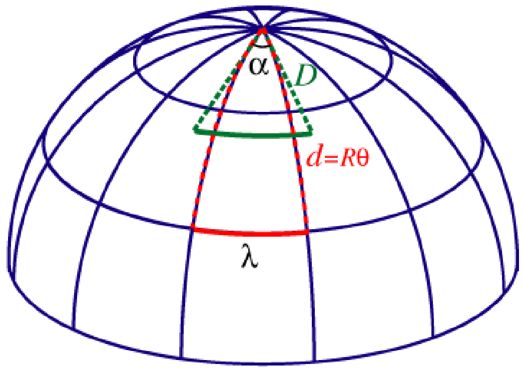
In an expanding flat universe D (last scattering) = $\eta_0 - \eta_{rec} \sim \eta_0$

and $\theta = \frac{2}{\mathrm{n}}\frac{\eta_{rec}}{\sqrt{3}\,\eta_0}$ $\rightarrow$ in critical matter dom. "Einstein- de Sitter" Uni. $a \sim t^{\frac{2}{3}}$

so that $\eta \sim a^{\frac{1}{2}}$ and with $\frac{a_{rec}}{a_0} \sim \left(\frac{1}{1000}\right)$ one gets and angle ~ 2 degrees for n=1

Curvature and Cosmological Constant

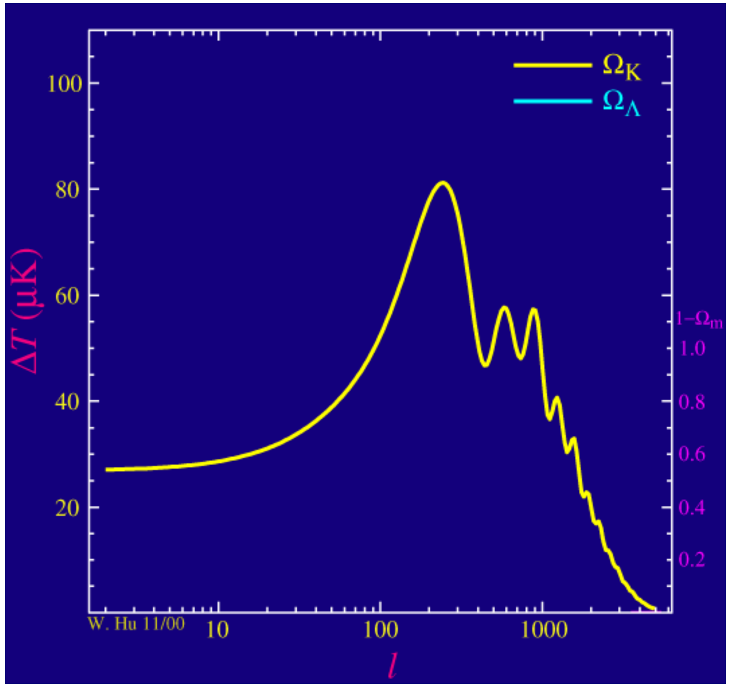
• In a closed universe objects appear closer → peaks shifted to **larger angles** as $D = R\sin(d/R)$.



where ***R*** is the radius of curvature

$$R = H_0^{-1}|\Omega_{tot} - 1|^{-1/2}$$

In an open universe things will appear further away (the sin → sinh)



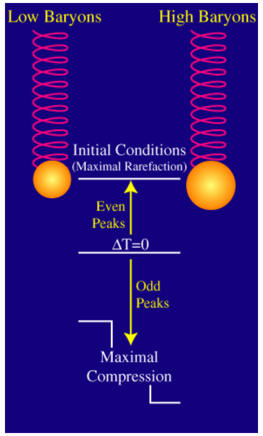
Also in a universe with Λ since $\eta_0 \to \eta_0(1 + \ln \Omega_m^{0.085})$

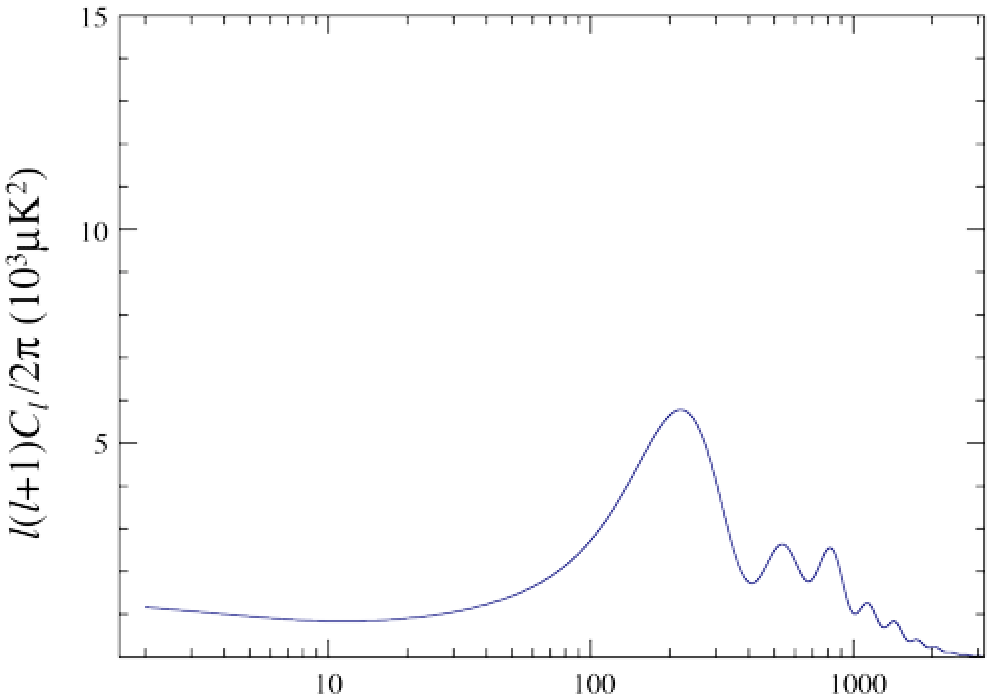
Baryon Loading

- Perturbations exist even if there's no initial temperature fluctuation (due to potential fluctuations from Inflation)

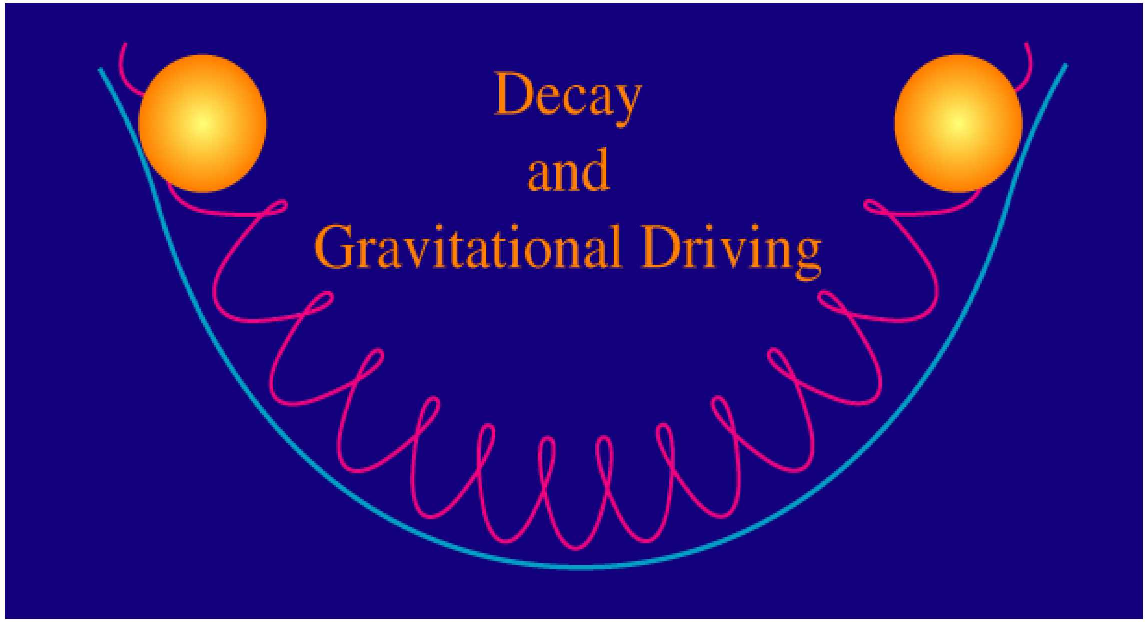
- Adding baryons → additional source (gravitational potential) term in wave Eq.

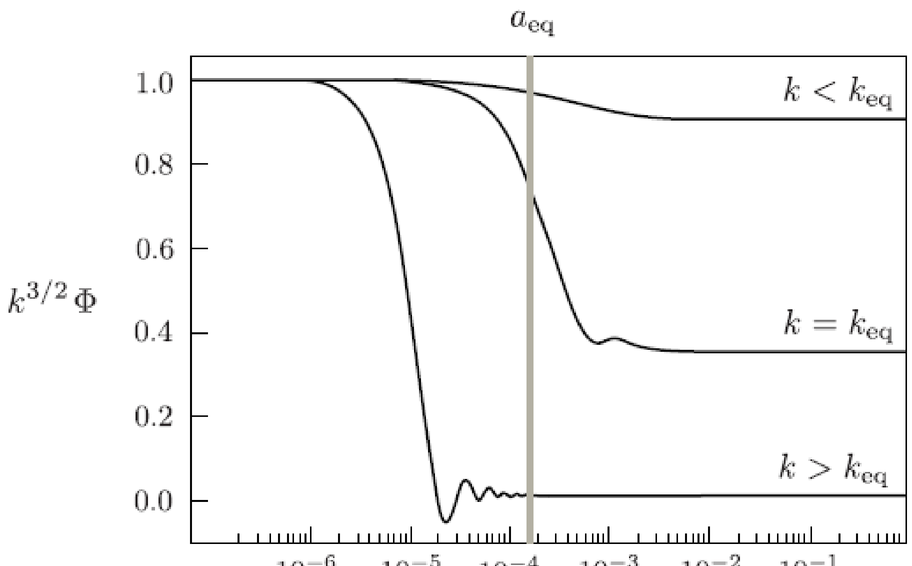
→ compression larger than rarefaction (but no collapse as Jeans scale is of order of horizon for coupled baryons)

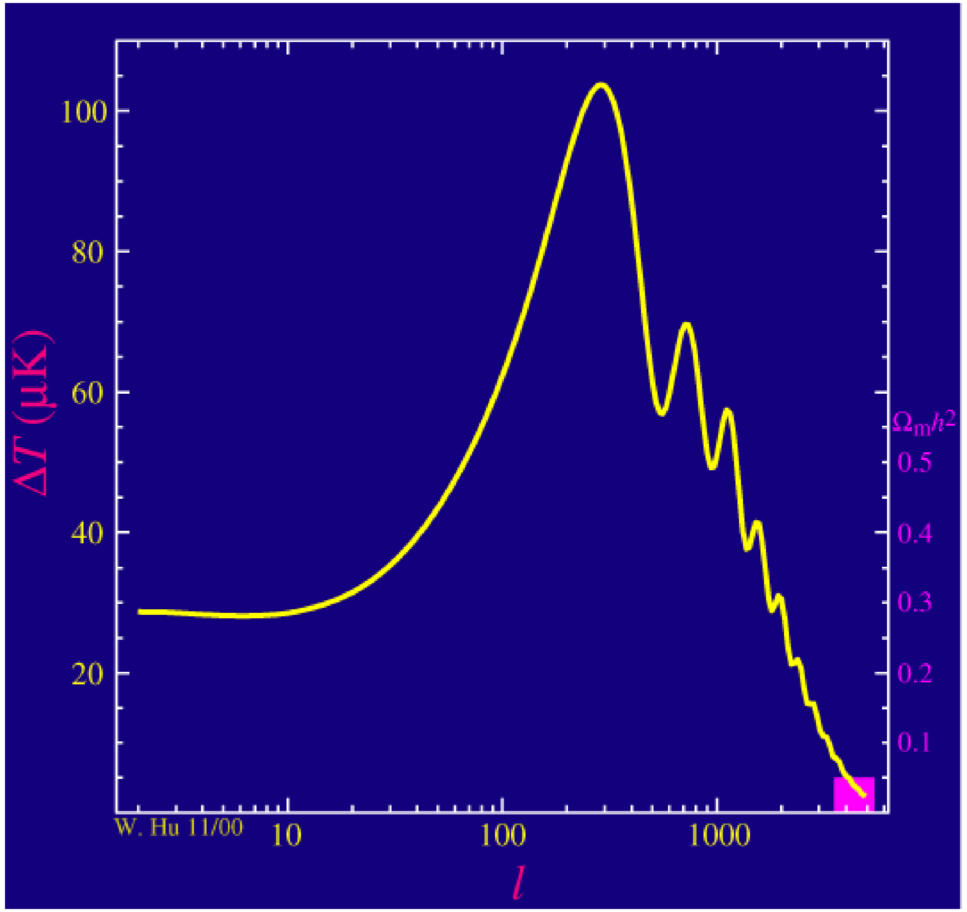




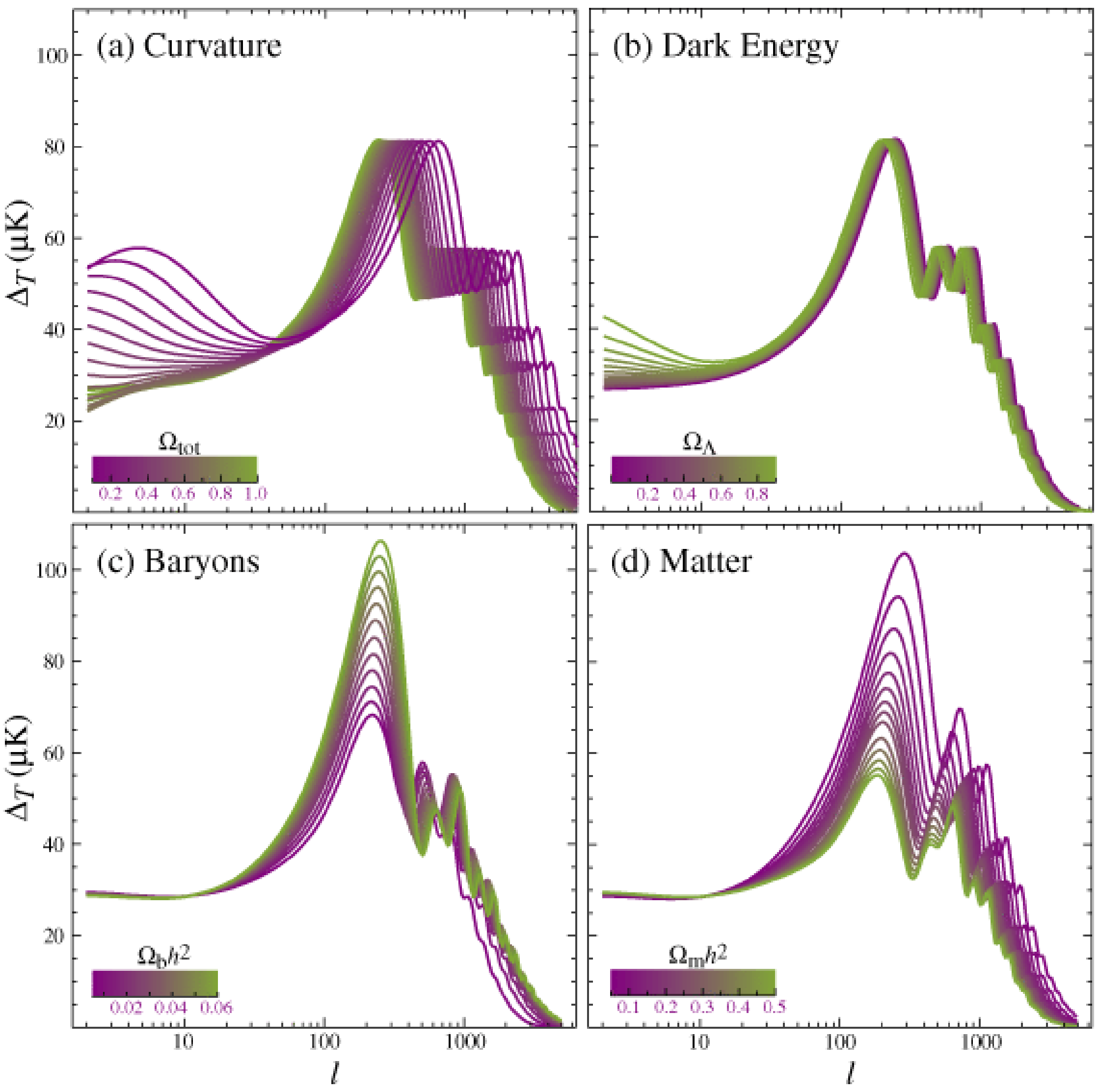
Radiation Driving and DM fraction







Summary of CMB Parameter Sensitivity



Now remember the WIMPS? Not coupled to photon bath and are heavy and slow moving!

Pressureless matter $(P \ll \rho)$ → Jeans scale 0 in expanding background

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_0\,\delta$$

In matter dominated E-dS

$$4\pi G\rho_0 = 3H^2/2 = 2/3t^2 \quad \text{and} \quad \delta(t) \propto t^{2/3}$$

In radiation dom. growth very slow, since radiation does not cluster and the matter density small (putting $\rho_0 = 0$ above gives δ = const)

Shape of Matter Spectrum of Fluctuations

- Use Poisson equation (formally in GR will be valid for fluctuations in the commoving gauge)

$$\nabla^2\Phi = 4\pi G\rho_0\delta \quad \Rightarrow \quad \Phi_k = -4\pi G\rho_0\delta_k/k^2$$

$$\Phi(\mathbf{r}) = \frac{V}{(2\pi)^3}\int \phi_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}} d\mathbf{k},$$

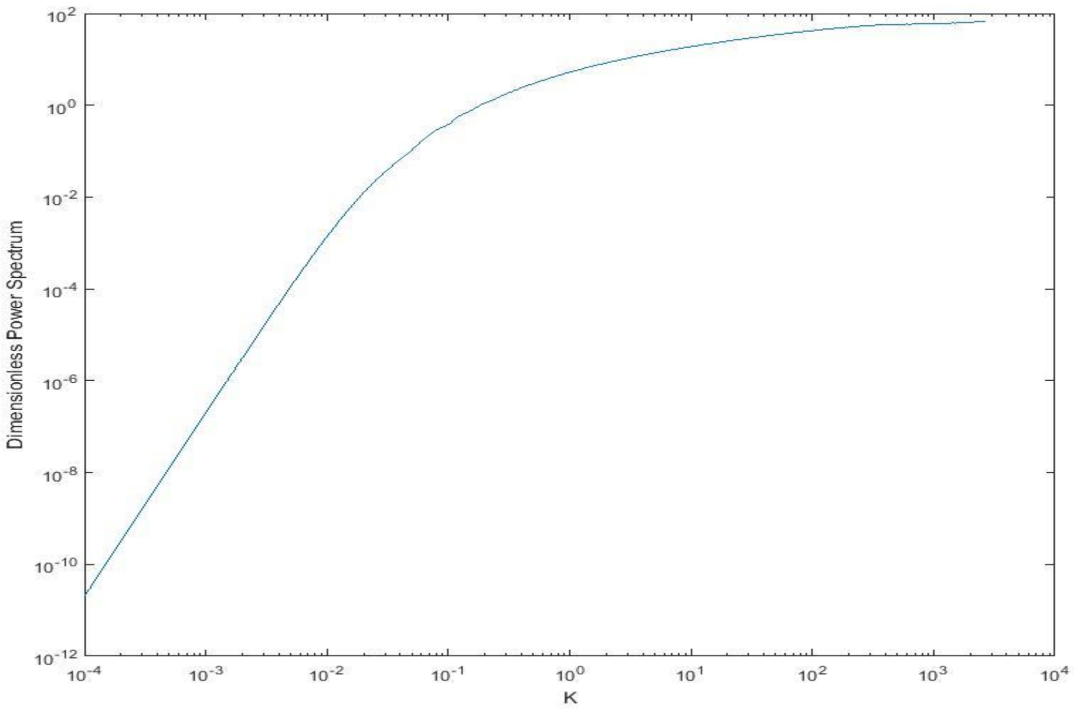
and

$$\delta(\mathbf{r}) = \frac{V}{(2\pi)^3}\int \delta_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}} d\mathbf{k}.$$

$$\Delta^2_\Phi \sim k^3\ \phi_k^2 \sim \text{constant}$$

(as predicted by inflation)

$$\rightarrow \Delta^2 \sim k^3\delta_k \sim k^4$$

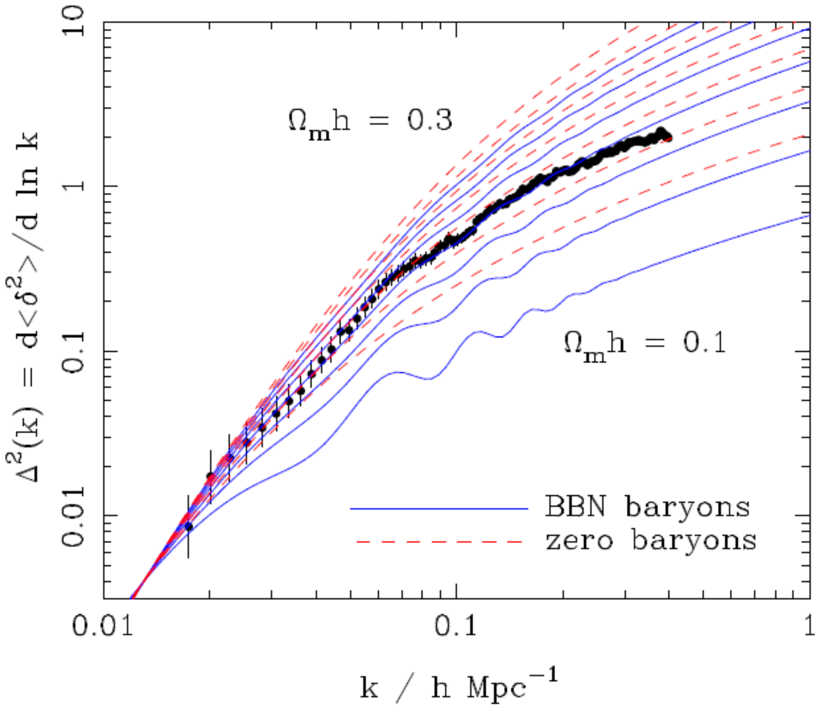


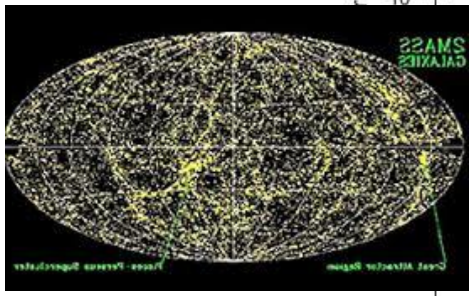
- But (in frame comoving with perturbations)

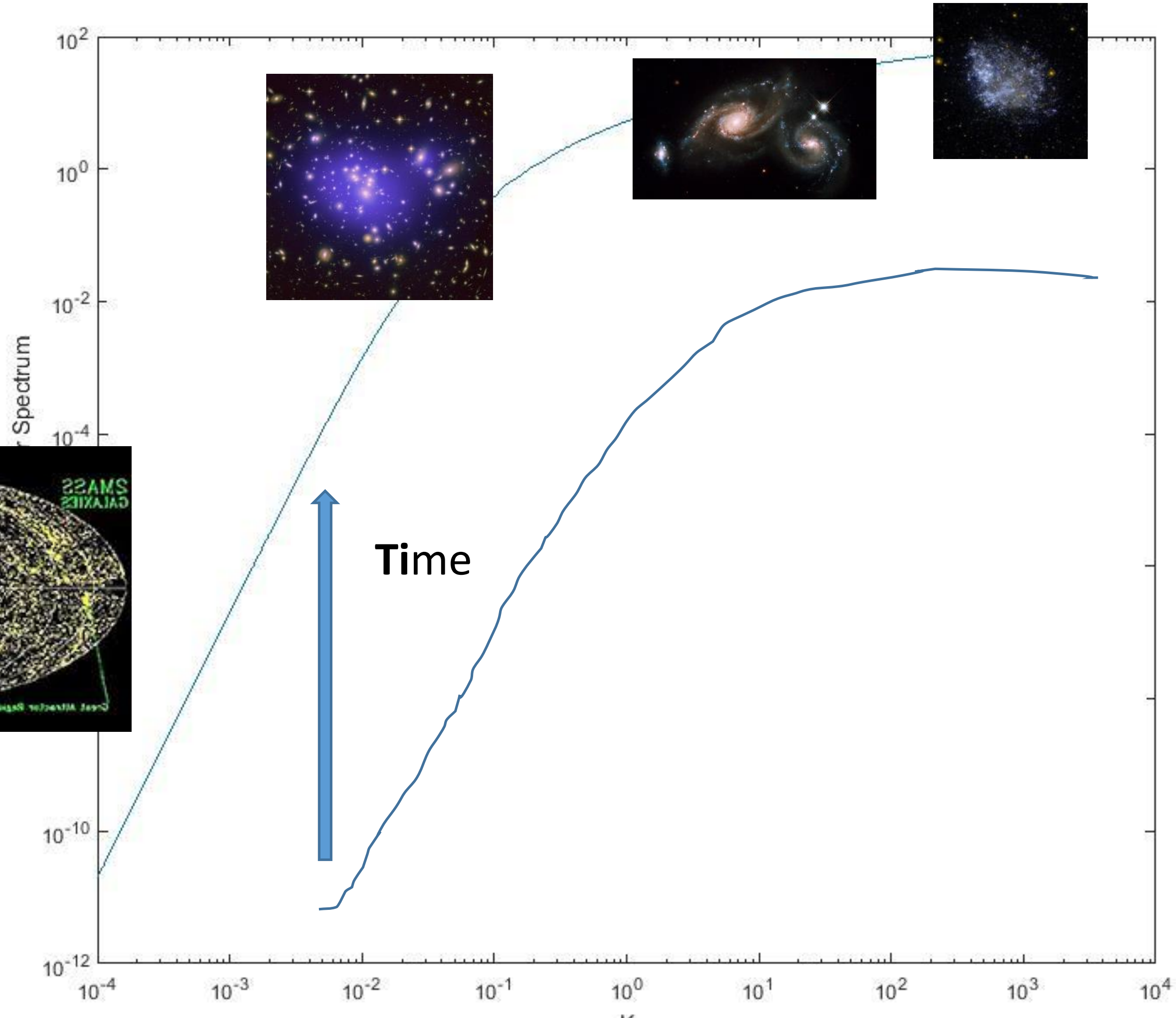
Perturbations grow outside horizon during radiation era… but not inside!

CDM Seeding of Hierarchical Formation

Linear regime scales grow independently
→ Small scales are favoured →
→ When overdensity reaches ~1
→ Collapse into structures
→ Largest scales still linear
→ They probe primordial spectrum

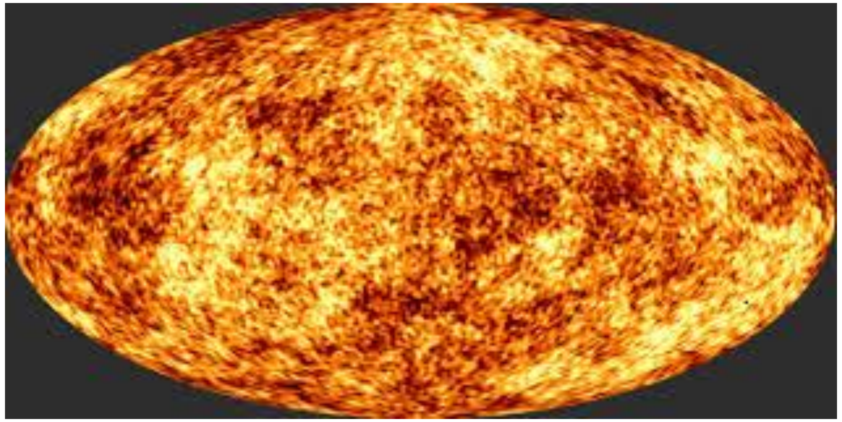






Modelling Cosmic Structure Formation

** COSMIC MICROWAVE BACKGROUND 400 000 YRS



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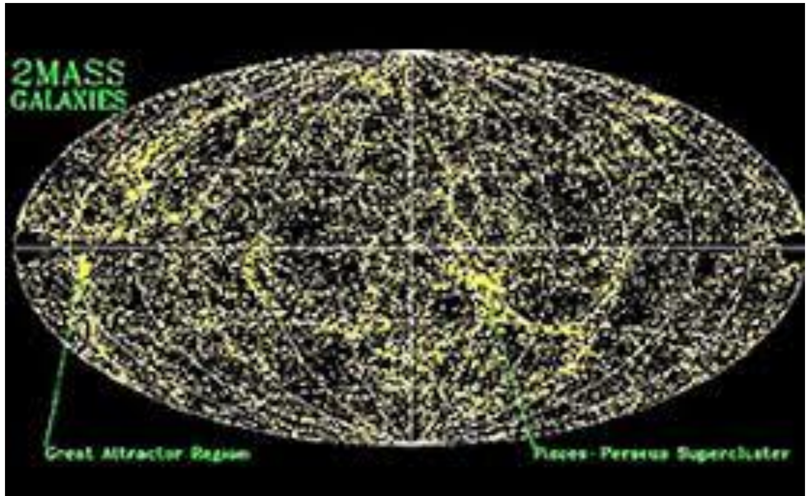
**PRESENT GALAXY DISTRIBUTION (~14 GYR)



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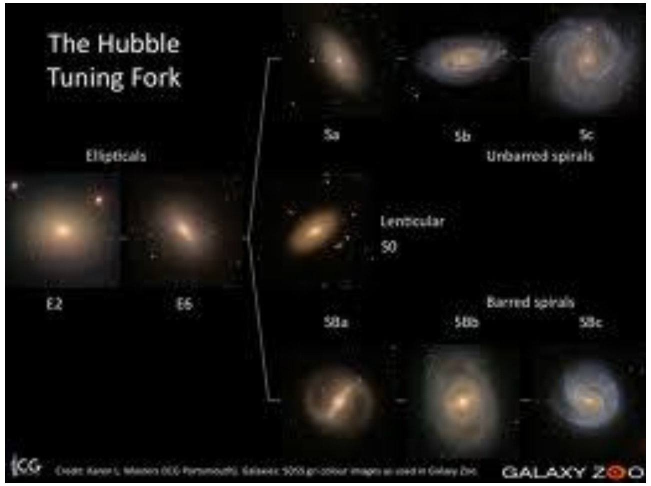
- As in hydrodynamics
  - → Baryons are tracers of DM on large scales
  - → Success in predicting the large scale galaxy distribution

- Gas cools and condenses in centers of haloes,.. Forms stars (efficiently) when self gravitating ~ dominant in central region

  most galaxies form hierarchically



The Galaxy Population

- Determined by mass, rotation, history of star formation
- → largely determined by the hierarchical DM halo buildup

In turn

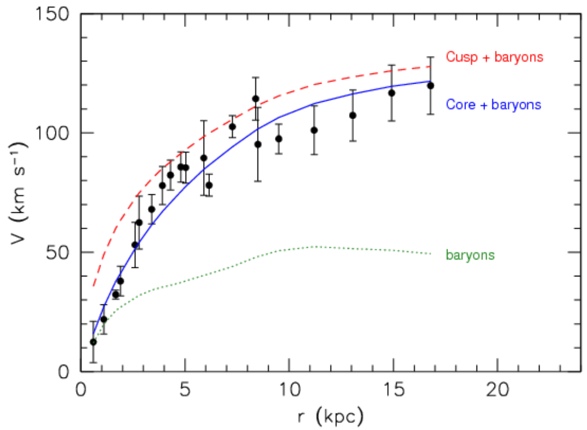
→ ~ Determined by the statistics of initial fluctuations

- Some predictions appear borne out: → e.g, fraction of large disk galaxies increases with time

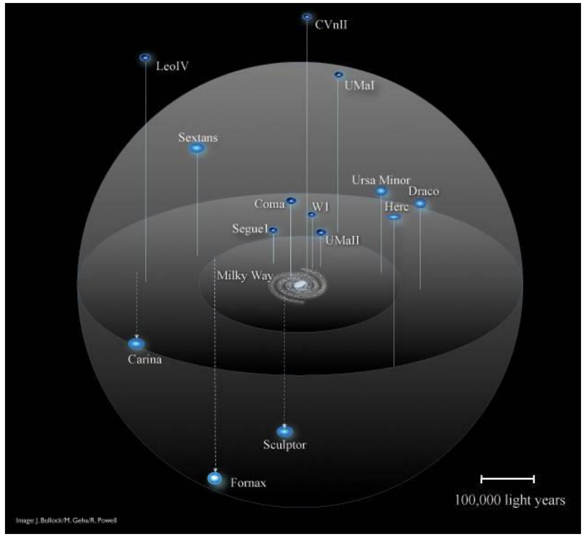
→

Small Scale Failures of CDM

- The central parts of CDM structures are too dense.



- Too many small structures **(also of wrong mass profile!)**



Proposed solutions

Pump energy → decrease DM density:

** Warm DM (smaller mass) → preheat!

(e.g., Lowell et. al. 2013; El-Zant, Sil & Khalil 2015)

** Self interacting DM → Conduction

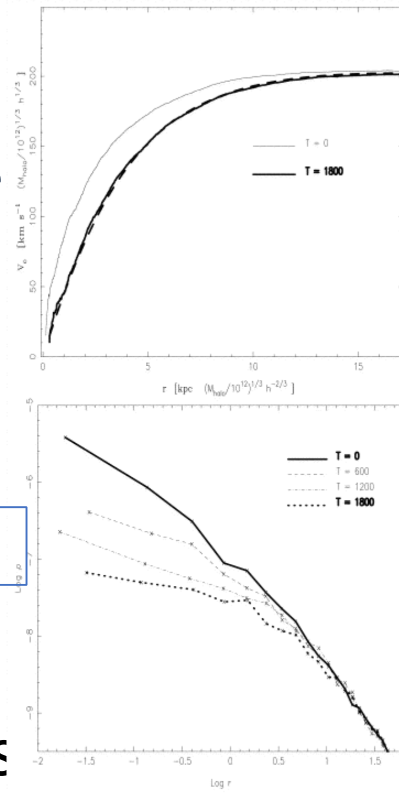
(Spergel & Steinhardt 2000; Elbert et. al. 2015)

** Quantum effects (on kpc scale!)

** Baryonic solutions: gas gives off energy to DM as it settles into halo (El-Zant et. al 2001;2004;2016)

Rotation Curve

Density prof.



Toward a Consistent Picture of Cosmic Evolution and Fundamental Physics

Our model of the universe →

- Precise: explains spatial distribution and many galaxy properties from primordial (quantum) fluctuations

- BUT most of it is missing: we know the parameters but not content

- Non-gravitational physics of galaxy formation and internal not well understood *(intense research ongoing on small scale problem and other conundrums…)*

→ stringent tests as we obtain complete time maps

- All salvageable; or completely new physics? (e.g., serious modifications to gravity theory)

The Power Spectrum

- Fourier analyse as we did with temp of CMB

$$P(k) \equiv \langle |\delta_k|^2 \rangle$$

$$\Phi(\mathbf{r}) = \frac{V}{(2\pi)^3} \int \phi_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}} d\mathbf{k},$$

and

$$\delta(\mathbf{r}) = \frac{V}{(2\pi)^3} \int \delta_{\mathbf{k}} e^{-i\mathbf{k}.\mathbf{r}} d\mathbf{k}.$$

- Dimensionless Power spectrum (measures fluctuations per log. Interval in k)

$$\Delta^2(k) \equiv \frac{V}{(2\pi)^3} 4\pi k^3 P(k)$$

Dynamical Equilibrium for Self Gravity

$$\frac{GM^2}{R} \sim M\left\langle V^2 \right\rangle \sim Nk_B T$$

$$\rightarrow R \sim \lambda_J \sim \frac{GM}{\langle V^2 \rangle} \sim \frac{G\,R^3\rho}{\langle V^2 \rangle}$$

$$\boxed{R \sim \lambda_J \sim \sqrt{\frac{\langle V^2 \rangle}{G\rho}}}$$

If speeds are too large for given density system unravels.
→ Size determined by completion density .vs. 'pressure'