Gravitation and Cosmology II

Introduction to Inflation

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I- Problems with the Standard Model

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- *The Big Bang model is a very successful model in explaining many important features of our universe.*
- *This model suffers from some drawbacks few of them were persisting problems which invoked an important addition to the model that we call today "inflation"*
- *Here we are going to discuss two main problems; flatness and horizon problems.*

Flatness problem:

- *Consider 1st Freidmann equation:* $$\Omega - 1 = \frac{k}{(a\,H)2}$$
- *If k=0,* $\Omega = 1$ *at all times, but if k≠0, and* $(\Omega - 1)$ *was small at some time* t_1*, at a latter time* t_2 *it will grow large in a radiation or matter dominated universe since* $(a\,HR)2 \sim 1/a^2$ *or* $(a\,Hm)2 \sim 1/a$

I- Problems with the Standard Model

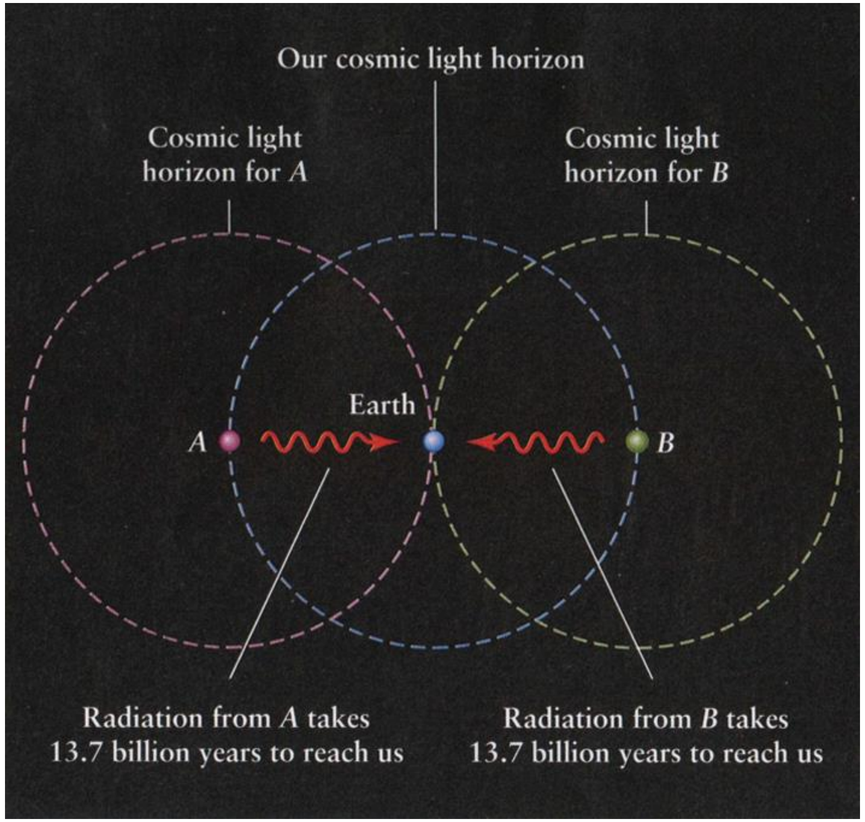
- *For example let us calculate how much this quantity will change from GUT scale till today in a matter dominated universe*
- $|\Omega - 1|_{GUT} \,/\, |\Omega - 1|_0 = a_{GUT} \,/\, a_0 = T_0 \,/\, T_{GUT} = 10^{-29}$

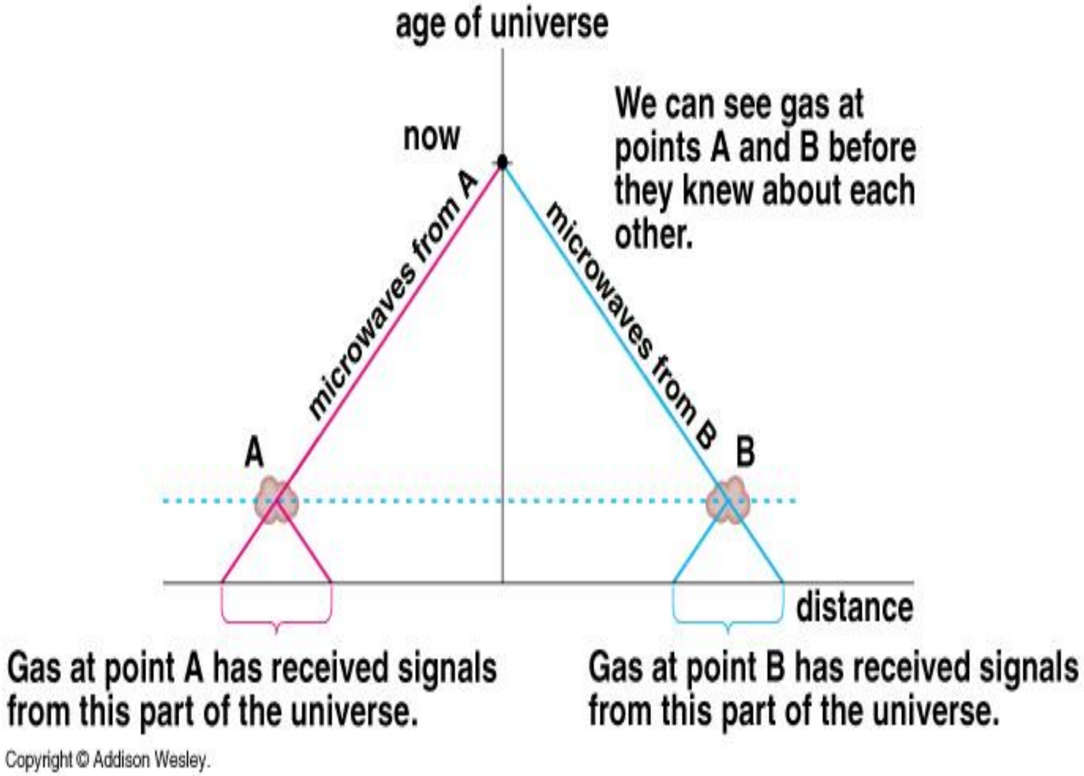
  Where we used $a \sim 1/T$, $T_{GUT} = 10^{16}$ *GeV and* $T_0 = 10^{-13}$ *GeV.*
- *One can think of this problem as a fine-tuning problem or initial condition problem.*

Horizon problem:

- *Think of the very isotropic CMB received from different parts of the sky, it is the same up to 1 part in 100,000!!*
- *Why do we get the same Planckian BB distribution for patches of the universe that were not in causal contact at any time in the past !! How could they have the same temperature without being in thermal equilibrium?*

I- Problems with the Standard Model





II Inflation

II- Inflation

Inflation: *is a fast growing period during which the universe size grows (exponentially in most models) about 30 orders of magnitude.*

- *Although inflation has been proposed to solve the above mentioned problems it has provided us with the source or the seeds of initial inhomogeneities which formed galaxies and stars latter on.*

Flatness problem: er

$$a(t) = e^{Ht} \quad , H = const. \quad \rightarrow \quad \dot{a}/a \sim H \quad \rightarrow \Omega - 1 \sim /(aH)^2 \rightarrow 0.$$

- *This might explain why $\Omega - 1$ is tiny.*

Horizon problem:

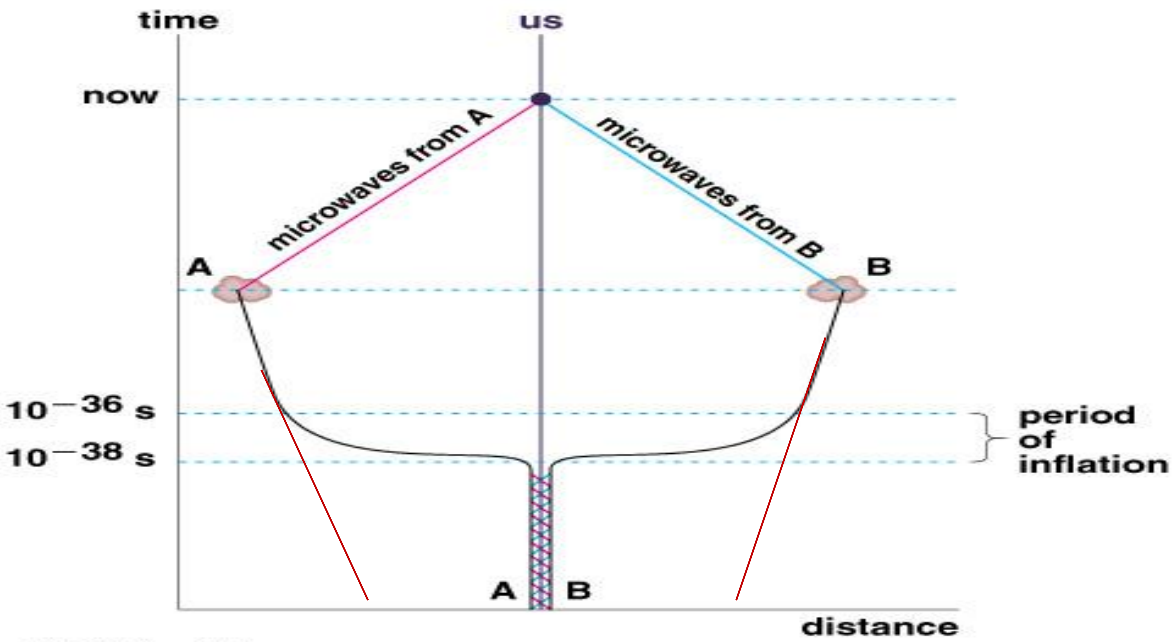
- *We can explain why two regions*

appear to be casually disconnected

if we extrapolate the evolution

back in time using the usual

FRW cosmology



II- Inflation

Conditions for inflation:

- *Accelerated expansion,*

$$\ddot{a} > 0 \rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G(\rho+3P)}{3} > 0 \rightarrow p < -\rho/3 \quad \text{(–ve pressure)}$$

- *Slowly varying Hubble rate (slow-roll parameters),*

$$\varepsilon = -\dot{H}/H^2 = \frac{d\ \ln H}{d\ \ln a} = \frac{d\ \ln H}{d\ N} << 1\ ,\ N = \ln a \quad \text{(number of e-folds)}$$

- *One might think it would be ideal to have de Sitter (Cosm. Const.) universe, but inflation has to end, therefore, $\varepsilon <<1$ but $\varepsilon \neq 0$.*

- *For inflation to last enough time (to get a large enough number of e-folds)*

$$\eta = \frac{d\ln\varepsilon}{dN} = \frac{\dot{\varepsilon}}{\varepsilon H}$$

II- Inflation

Scalar Field Inflation

- We consider a scalar field ϕ coupled to a metric $g_{\mu\nu}$ with the following action

$$S = \int d^4x \sqrt{-g}\,\mathcal{L} = \int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right],$$

- Euler-Lagrange equation is

$$\partial^\mu \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\,\partial^\mu\phi} - \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\phi} = 0,$$

- Which reads

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V' = 0}\,.$$

$$V'(\phi) = (dV(\phi)/d\phi).$$

- Also we have Einstein Field eqns

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} \equiv G_{\mu\nu} = 8\pi G\,T_{\mu\nu}$$

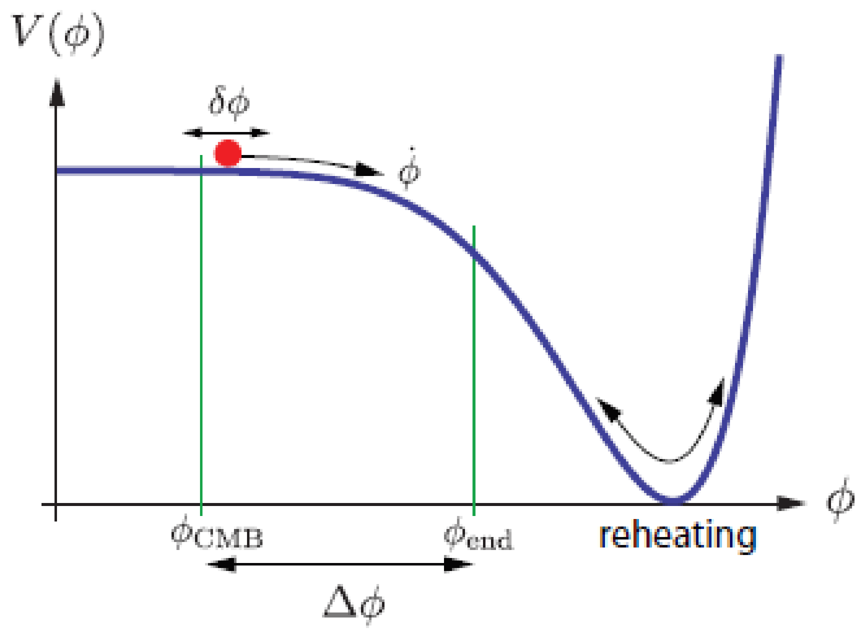
$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\,\mathcal{L}.$$

II- Inflation

- Density and pressure caused by field ϕ are

$$T_{00} = \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$T_{ii} = p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$



- To have $p_\phi \simeq -\rho_\phi$ → $V(\phi) \gg \dot{\phi}^2$
- This means that

$$H^2 \simeq \frac{8\pi G}{3} V(\phi),$$

- But in order to K.E. small compared with the total E. The acceleration $\ddot{\phi}$ has to be small too, this leads to

$$3H\dot{\phi} = -V'(\phi)$$

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + V' = 0}\ .$$

II- Inflation

- Defining a dimensionless acceleration

$$\frac{\ddot{\phi}}{3H\dot{\phi}} << 1$$

- Now let us collect the

$$\begin{aligned}\epsilon &= -\frac{\dot{H}}{H^2} = 4\pi G\frac{\dot{\phi}^2}{H^2} = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 ,\\ \eta &= \frac{1}{8\pi G}\left(\frac{V''}{V}\right) = \frac{1}{3}\frac{V''}{H^2},\\ \delta &= \eta - \epsilon = -\frac{\ddot{\phi}}{H\dot{\phi}} .\end{aligned}$$

$$\boxed{\epsilon_{\rm v} \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad |\eta_{\rm v}| \equiv M_{\rm pl}^2\frac{|V''|}{V}} .$$

- Number of e-folding

$$\boxed{N(\phi) = \int_{\phi_{\rm end}}^{\phi}\frac{{\rm d}\phi}{\sqrt{2\varepsilon}} \approx \int_{\phi_{\rm end}}^{\phi}\frac{{\rm d}\phi}{\sqrt{2\epsilon_{\rm v}}}} .$$

II- Inflation

Naïve example:

Consider the simple potential $V(\phi) = \frac{1}{2}m^2\phi^2$.

- The ϵ_V parameter is

$$\boxed{\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2},$$

- $\epsilon_V = 2\left(\frac{M_{pl}}{\phi}\right)^2$
- Notice that inflation ends at $\epsilon_V = 1$, or $\phi_{end} = \sqrt{2}M_{pl}$
- Number of e-folding, $N_{tot} \sim 60$
- $N(\phi) = \left(\frac{\phi}{2\,M_{pl}}\right)^2 - \frac{1}{2}$ → $\phi_{start} = 15.5\ M_{pl}$
- This an example of large field inflation which is not favored by Planck experiments.