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Translation between PO and BSMC

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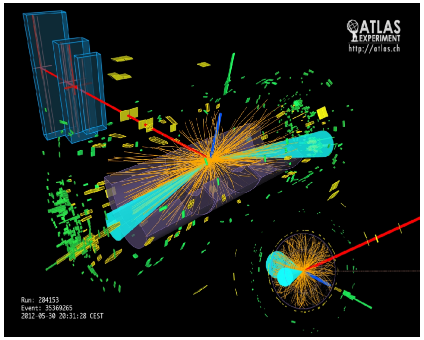
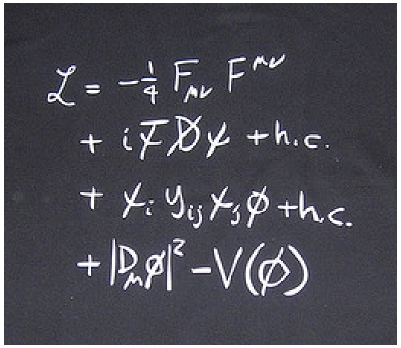
- Introduction
- PO vs. EFT
- PO for EW production and EW decays
- Matching with the BSM Characterization Lagrangian

Introduction

- PO = *encoding of the exp. results in terms of a limited number of simplified/idealized observables of easy theoretical interpretation*
  Old idea - heavily used and developed at LEP

- The experimental determination of an appropriate set of PO will "help" and not "replace" any explicit NP approach to Higgs physics (*including the EFT*)





Experimental data — Pseudo Observables — Lagrangian parameters

The PO can be computed in terms of Lagrangian parameters in any specific th. framework (SM, SM-EFT, SUSY, ...)

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$$\Downarrow$$

- The PO should be defined from kinematical properties of on-shell processes (*no problems of renormalization, scale dependence, gauge dependence,…*)

- The theory corrections applied to extract them should be universally accepted as “NP-free” (*soft QCD and QED radiation*)

Introduction

There are two main categories of PO:

A) "Ideal observables"

M_W, $\Gamma(Z \to f\!f)$, …

M_h, $\Gamma(h \to \gamma\gamma)$, $\Gamma(h \to Z\mu\mu)$, …
but also $d\sigma(pp \to hZ)/dm_{hZ}$ ...

B) "Effective on-shell couplings"

$$g_Z^{f},\ g_W^{f}, \ldots \leftrightarrow\ \Gamma(Z \to f\!f) = C\,[\,|g_Z^{f_L}|^2 + |g_Z^{f_R}|^2],\ \ldots$$

- Both categories are useful
  (*there is redundancy having both, but that's not an issue...*).
- For B) one can write an effective Feynman rule, not to be used beyond tree-level (its just a practical way to re-write, *and code in existing tools,* an on-shell amplitude).

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PO vs. EFT

PO and couplings in EFT Lagrangians are *intimately related but are not the same thing* (on-shell amplitudes vs. Lagrangians parameters) → full complementarity

- The PO are calculable in any EFT approach (*linear, non-linear, LO, NLO...*)
  - In the limit where we work at the tree-level in the EFT there is a simple linear relation between PO and EFT couplings: each PO represent a unique linear combination of couplings of the most general Higgs EFT.
  - This does not hold beyond the tree-level (the PO do not change, but their relation to EFT couplings is more involved....)

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In general (*linear, non-linear, LO, NLO...*):

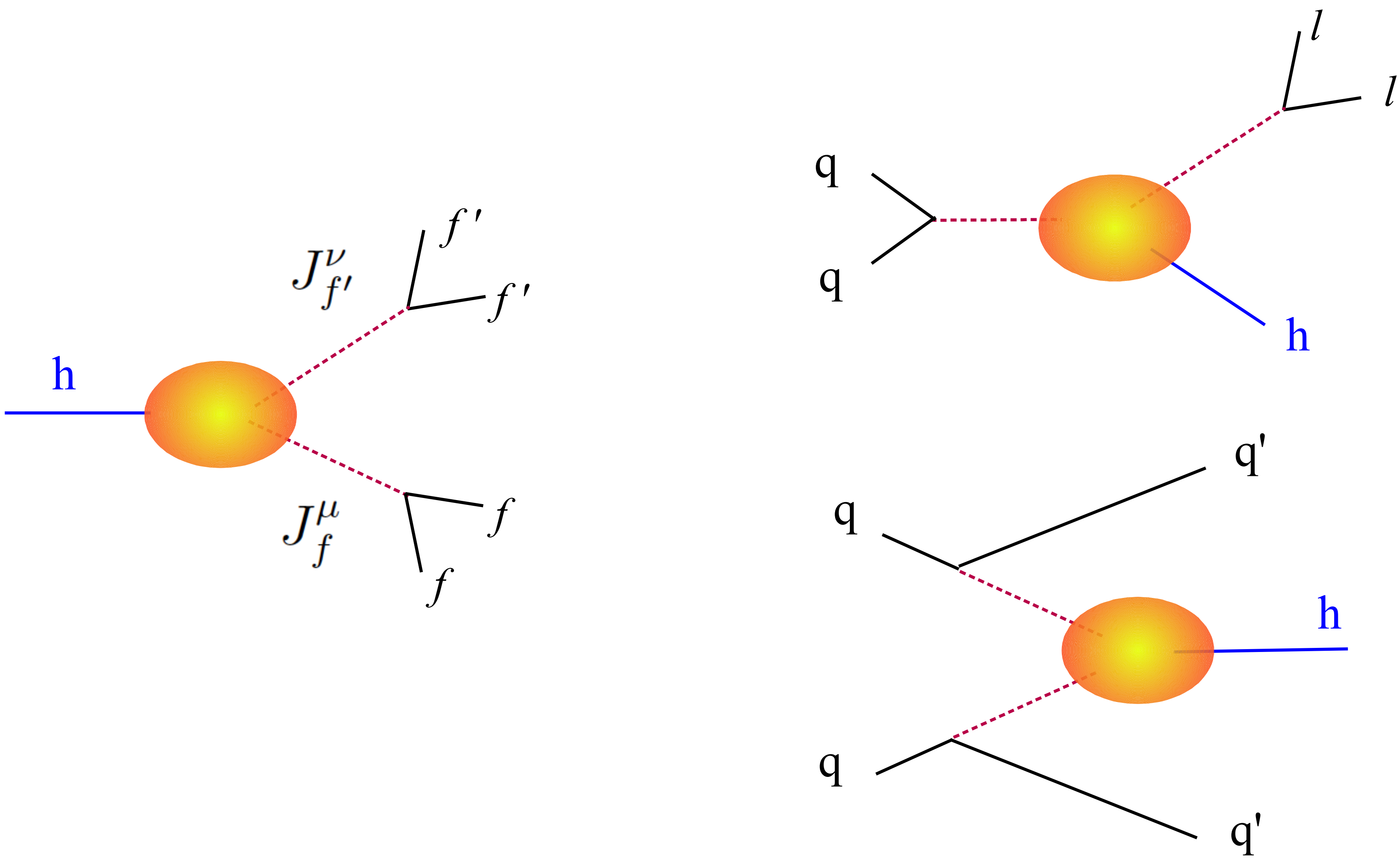
- PO → inputs for EFT coupling fits
- EFT → predictions of relations between different PO sets (that can be tested)

In each process the PO are the maximum number of independent observables that can be extracted by that process only → naturally optimized for data analyses

PO for EW production and EW decay modes

In two-body (on-shell) Higgs decays the PO are equivalent to the old kappa's.

Non-trivial aspects arises in process with non-trivial kinematics



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Form factors $\rightarrow f_i(s)$ [E.g.: $s = m^2_{\ell\ell}$]

↓

Momentum expansion of the *f.f.* around leading poles

E.g.: $$f_i^{\mathrm{SM+NP}} = \frac{\kappa_i}{s - m_Z^2 + i m_Z \Gamma_Z} + \frac{\epsilon_i}{m_Z^2} + O(s/m_Z^4)$$

Gonzales-Alonso *et al.* 1412.6038

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General decomposition of the on-shell amplitudes based on Lorentz symmetry, Crossing symmetry, and Unitarity

The $\{\kappa_i, \epsilon_i\}$ thus defined are well-defined PO [*pole decomposition → gauge-invariant terms*] → systematic inclusion of higher-order QED and QCD (soft) corrections possible (and necessary...)

PO for EW production and EW decay modes

The $\{\kappa_i, \epsilon_i\}$ can be put in one-to-one correspondence with couplings of an effective Lagrangian, written in terms of the mass-eigenstate basis for the fields, to be used only at the tree-level:

$$
\begin{aligned}
\mathcal{L}^{\rm eff}_{\rm HPO} = & \; \kappa_{ZZ}\frac{m_Z^2}{v} Z_\mu Z^\mu h + \kappa_{WW}\frac{2m_W^2}{v} W^+_\mu W^{-\,\mu} h + \\
& - \epsilon_{\gamma\gamma}\frac{h}{2v} A_{\mu\nu}A^{\mu\nu} - \epsilon_{Z\gamma}\frac{h}{v} Z_{\mu\nu}A^{\mu\nu} - \epsilon_{ZZ}\frac{h}{2v} Z_{\mu\nu}Z^{\mu\nu} - \epsilon_{WW}\frac{h}{v} W^+_{\mu\nu}W^{-\,\mu\nu} + \\
& - \epsilon^{\rm CP}_{\gamma\gamma}\frac{h}{2v} A_{\mu\nu}\widetilde{A}^{\mu\nu} - \epsilon^{\rm CP}_{Z\gamma}\frac{h}{v} Z_{\mu\nu}\widetilde{A}^{\mu\nu} - \epsilon^{\rm CP}_{ZZ}\frac{h}{2v} Z_{\mu\nu}\widetilde{Z}^{\mu\nu} - \epsilon^{\rm CP}_{WW}\frac{h}{v} W^+_{\mu\nu}\widetilde{W}^{-\,\mu\nu} + \\
& + \sum_f \sum_i \epsilon_{Zf_i}\,\frac{2h}{v} Z_\mu \bar{f}^i \gamma^\mu f^i + \\
& + \sum_{i,j} \left[\epsilon_{We^{ij}}\,\frac{2h}{v} W^+_\mu \bar{\nu}_{e^i L}\gamma^\mu e^j_L + \epsilon_{Wu^i_L d^j_L}\,\frac{2h}{v} W^+_\mu \bar{u}^i_L \gamma^\mu d^j_L + \epsilon_{Wu^i_R d^j_R}\,\frac{2h}{v} W^+_\mu \bar{u}^i_R \gamma^\mu d^j_R + \text{h.c.} \right]
\end{aligned}
$$

→ easy to perform a tree-level matching with any other effective Lagrangian and compute the PO (*at the tree level...*) in terms of couplings of specific effective operators

Translation between PO and BSMC

Doing this matching with the BSMC Lagrangian [→ see added note] we get:

$$\kappa_{ZZ} = 1 + \delta c_z + g^2 c_{z\Box} \ ,$$

$$\kappa_{WW} = 1 + \delta c_w + g^2 c_{w\Box} \ ,$$

$$\epsilon_{ZZ} = -\frac{g^2 + g'^2}{2} c_{zz} \ , \qquad \epsilon_{ZZ}^{CP} = -\frac{g^2 + g'^2}{2} \tilde{c}_{zz} \ ,$$

$$\epsilon_{Z\gamma} = -\frac{e\sqrt{g^2 + g'^2}}{2} c_{z\gamma} \ , \qquad \epsilon_{Z\gamma}^{CP} = -\frac{e\sqrt{g^2 + g'^2}}{2} \tilde{c}_{z\gamma} \ ,$$

$$\epsilon_{\gamma\gamma} = -\frac{e^2}{2} c_{\gamma\gamma} \ , \qquad \epsilon_{\gamma\gamma}^{CP} = -\frac{e^2}{2} \tilde{c}_{\gamma\gamma} \ ,$$

$$\epsilon_{WW} = -\frac{g^2}{2} c_{ww} \ , \qquad \epsilon_{WW}^{CP} = -\frac{g^2}{2} \tilde{c}_{ww} \ ,$$

$$\epsilon_{Zf} = \frac{g^3}{2c_W} \left(T_f^3 - Q_f s_W^2\right) c_{z\Box} + \frac{g g' e}{2} Q_f c_{\gamma\Box} + \sqrt{g^2 + g'^2} \delta g_{L,R}^{hZf} \ ,$$

$$\epsilon_{We^{ij}} = \frac{g^3}{2\sqrt{2}} (\mathbf{1} + \delta g_L^{W\ell})_{ij} c_{w\Box} + \frac{g}{\sqrt{2}} \delta (g_L^{hW\ell})_{ij} \ ,$$

$$\epsilon_{Wu_L^i d_L^j} = \frac{g^3}{2\sqrt{2}} (\mathbf{V}_{\rm CKM} + \delta g_L^{Wq})_{ij} c_{w\Box} + \frac{g}{\sqrt{2}} \delta (g_L^{hWq})_{ij} \ ,$$

$$\epsilon_{Wu_R^i d_R^j} = \frac{g^3}{2\sqrt{2}} (\delta g_R^{Wq})_{ij} c_{w\Box} + \frac{g}{\sqrt{2}} \delta (g_R^{hWq})_{ij} \ .$$

Translation between PO and BSMC

Alternatively, one can try to invert these relations and derive the BSMC couplings in terms of PO (*i.e. with the purpose of simulating the effect of a PO using a MC written in terms of BSMC couplings*). However, this process is not unique given the redundancy in the BSMC couplings.

A possible simple mapping is given by

$$c_{zz} = \frac{-2}{g^2+g'^2}\epsilon_{ZZ} , \quad \tilde{c}_{zz} = \frac{-2}{g^2+g'^2}\epsilon^{CP}_{ZZ} ,$$

$$c_{z\gamma} = \frac{-2}{e\sqrt{g^2+g'^2}}\epsilon_{Z\gamma} , \quad \tilde{c}_{z\gamma} = \frac{-2}{e\sqrt{g^2+g'^2}}\epsilon^{CP}_{Z\gamma}$$

$$c_{z\Box} = c_{w\Box} = c_{\gamma\Box} = 0 ,$$

$$\delta c_z = \kappa_{ZZ} - 1 ,$$

$$\delta c_w = \kappa_{WW} - 1 ,$$

$$c_{\gamma\gamma} = \frac{-2}{e^2}\epsilon_{\gamma\gamma} , \quad \tilde{c}_{\gamma\gamma} = \frac{-2}{e^2}\epsilon^{CP}_{\gamma\gamma} ,$$

$$c_{ww} = \frac{-2}{g^2}\epsilon_{WW} , \quad \tilde{c}_{ww} = \frac{-2}{g^2}\epsilon^{CP}_{WW} ,$$

$$\delta g^{hZf}_{L,R} = \frac{1}{\sqrt{g^2+g'^2}}\epsilon_{Zf} , \quad (\delta g^{hW\ell}_{L})_{ij} = \frac{\sqrt{2}}{g}\epsilon_{We^{ij}} ,$$

$$(\delta g^{hWq}_{L})_{ij} = \frac{\sqrt{2}}{g}\epsilon_{Wu^i_L d^j_L} , \quad (\delta g^{hWq}_{R})_{ij} = \frac{\sqrt{2}}{g}\epsilon_{Wu^i_R d^j_R} ,$$

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- With BSMC this inversion is possible but is not unique given the redundancy in the BSMC couplings.
- With the original Higgs Characterization Lagrangian this inversion is not possible given the basis of operators is not complete (not only because of flavor-universality)

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Recall...

- In each process the PO are the maximum number of independent observables that can be extracted by that process only → naturally optimized for data analyses
- By construction PO are gauge- and basis-indpendent

This is why we believe a MC directly written in terms of PO is preferable (and we are working for it...)


Backup

Parameter counting & symmetry limits

Number of independent PO for EW Higgs decays

PO set with maximal symmetry [CP + Lepton Univ + cust.] → no symmetry

Minimal set:

Without custodial symm.:

h → 4μ, 4e, 2e2μ, 2μ2ν, 2e2ν, eμ2ν, γγ, eeγ, μμγ

$\kappa_{ZZ}, \kappa_{Z\gamma}, \epsilon_{ZZ}$

$\kappa_{\gamma\gamma}, \epsilon_{Ze_L}, \epsilon_{Ze_R}, \epsilon_{We_L}$

$\kappa_{WW}, \epsilon_{WW}$

$\epsilon_{Z\nu_\mu}$

7 → 10 (no CS) → 20 (no symm.)

Parameter counting & symmetry limits

Number of independent PO for EW production

PO set with maximal symmetry [CP + Lepton Univ + cust.] ⟶ no symmetry

VBF, Zh, Wh

	Minimal set:	*Without custodial symm.:*
	$\kappa_{ZZ}, \kappa_{Z\gamma}, \epsilon_{ZZ}$	$\kappa_{WW}, \epsilon_{WW}$
	$\epsilon_{Zu_L}, \epsilon_{Zu_R}, \epsilon_{Zd_L}, \epsilon_{Zd_R}$	ϵ_{Wu_L}

7 ⟶ 10 (no CS) ⟶ 20 (no symm.)

Parameter counting & symmetry limits

Number of independent PO for EW Higgs decays + EW production + Yukawa modes (h → ff):

PO set with maximal symmetry [CP + Lepton Univ + cust.] → no symmetry

	Minimal set:	*Without custodial symm.:*
Prod. & decays	$\kappa_{ZZ}, \kappa_{Z\gamma}, \epsilon_{ZZ}$	$\kappa_{WW}, \epsilon_{WW}$
EW decays only	$\kappa_{\gamma\gamma}, \epsilon_{Ze_L}, \epsilon_{Ze_R}, \epsilon_{We_L}$	$\epsilon_{Z\nu_\mu}$
EW prod. only	$\epsilon_{Zu_L}, \epsilon_{Zu_R}, \epsilon_{Zd_L}, \epsilon_{Zd_R}$	ϵ_{Wu_L}
		11 → 15 (no CS) → 32 (no symm.)
Yukawa modes	$\kappa_b, \kappa_\tau, \kappa_c, \kappa_\mu$	4 → 8 (no symm.)
		(as in the original κ-formalism)
gg→h & ttH	κ_g, κ_t	2 → 4 (no symm.)