

June 2nd? 2017

Based on: Giudice, MM, 2016



On Masses and Scales

There is a fundamental difference between masses and scales, sometimes overlooked.

Example: Global Symmetry Breaking

$$V = \lambda \left(\frac{f^2}{2} - |\phi|^2\right)^2 \longleftrightarrow \phi = \frac{f + \rho}{\sqrt{2}} e^{i\pi/f}$$

Massless Goldstone boson, massive radial mode:

$$m_{\pi} = 0$$
 , $m_{\rho} = \sqrt{2\lambda}f$

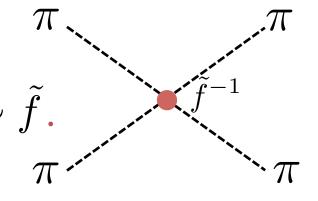
At low energies, Goldstone self-interactions $\mathcal{L} \supset rac{1}{ ilde{f}^4} (\partial \pi)^4 \quad , \ \ ilde{f}^4 = 4\lambda f^4$

On Masses and Scales

Example: Global Symmetry Breaking

If we could do Goldstone scattering at low energies, could measure this interaction scale:

May be tempted to think this points to UV-completion at $E\sim \tilde{f}$



However, UV-completion enters at $m_{
ho}$ which could be completely different:

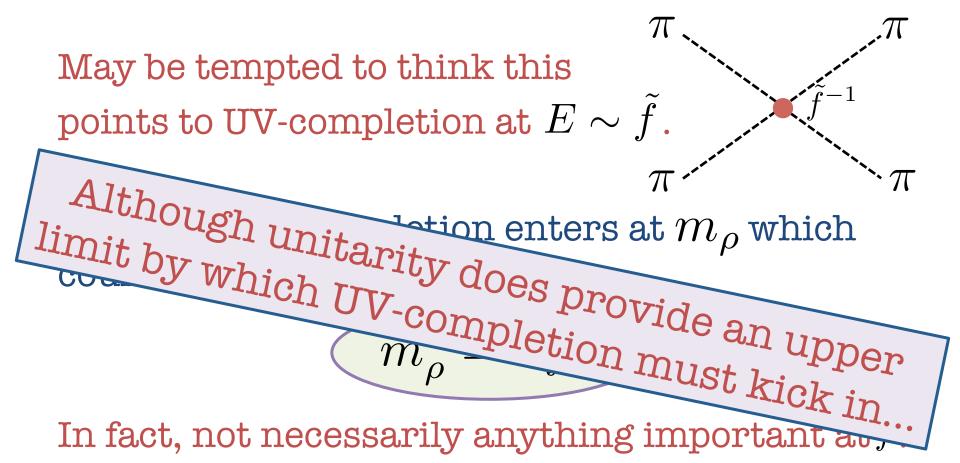
$$m_{\rho}^4 = \lambda \tilde{f}^4$$

In fact, not necessarily anything important at f.

On Masses and Scales

Example: Global Symmetry Breaking

If we could do Goldstone scattering at low energies, could measure this interaction scale:



Choi & Im, Kaplan & Rattazzi

Take N+1 copies of original story, assume $\lambda \approx 1$, such that at low energies only have Goldstones:

$$\phi_j \sim \frac{f}{\sqrt{2}} e^{i\pi_j/f} , \quad j = 0, ..., N$$

Now explicitly break N of the U(1) symmetries explicitly with spurions,

$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

This action is justified by symmetry assignments for spurions.

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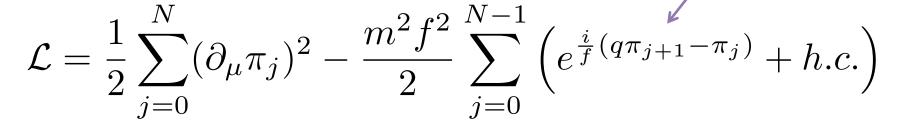
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Action given by

"Interaction basis π "



Spontaneous symmetry breaking pattern:

$$\mathrm{U}(1)^{N+1} \to \emptyset$$

So expect N + 1 Goldstones.

Explicit symmetry breaking: $U(1)^{N+1} \rightarrow U(1)$ So expect N pseudo-Goldstones and one true Goldstone.

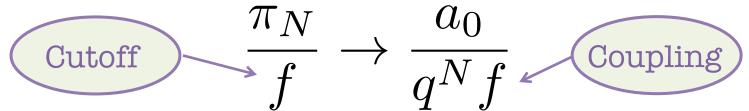
Can identify true Goldstone direction from remaining shift symmetry

$$(\pi_j \to \pi_j + \kappa/q^j)$$

Identify Goldstone <u>couplings</u> by promoting shift parameter to a field:

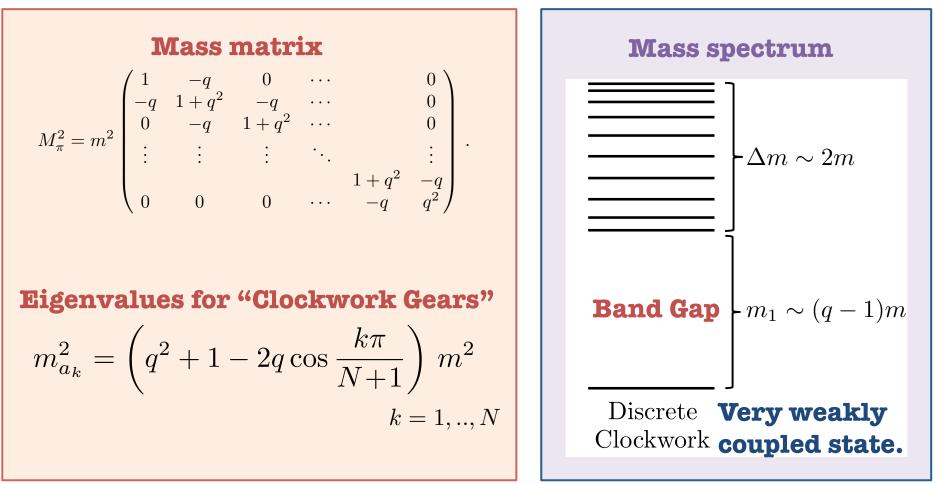
$$\pi_j \to \pi_j + a(x)/q^j$$

Now, imagine we had some fields coupled to π_N . Coupling to massless Goldstone becomes:



Exponential separation between zero mode coupling and cutoff! This is generated <u>entirely</u> from the shift symmetry, not from the form of the interaction or potential.

Peculiar spectrum, reminiscent of Condensed Matter...

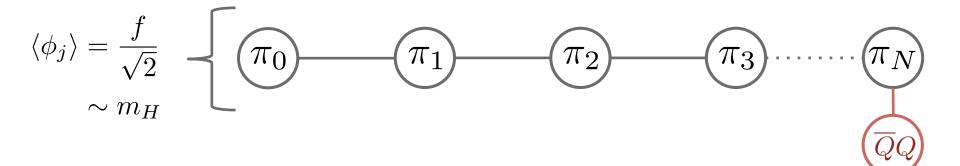


How might this be useful in practice?

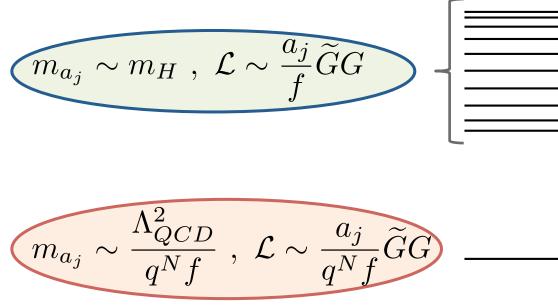
A Clockwork Axion

See also Farina et al 2016.

Imagine clockworking Peccei-Quinn at weak scale:



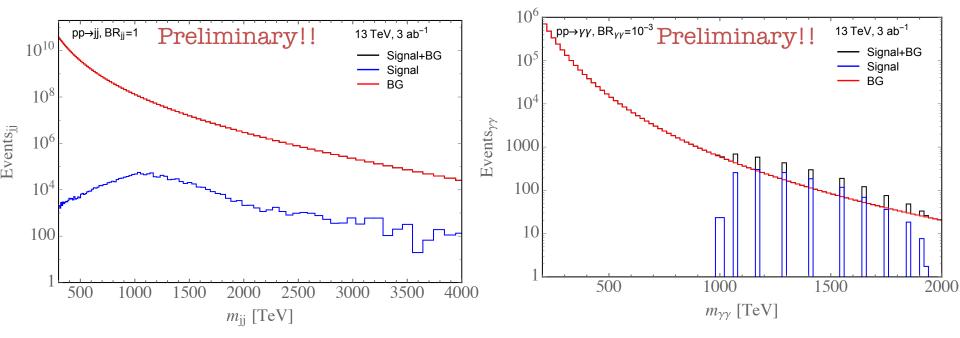
An invisible axion and band of weak-scale "gears":



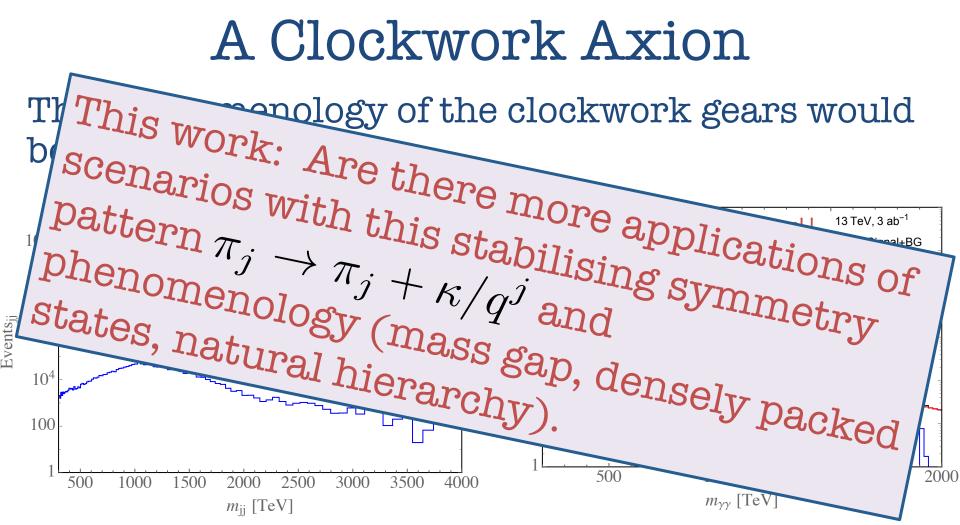
- Clockwork gears could show up as a band of states at colliders.
- Cosmology / thermal history of invisible axion radically altered: stays in thermal equilibrium to late times.

A Clockwork Axion

The phenomenology of the clockwork gears would be very exotic:



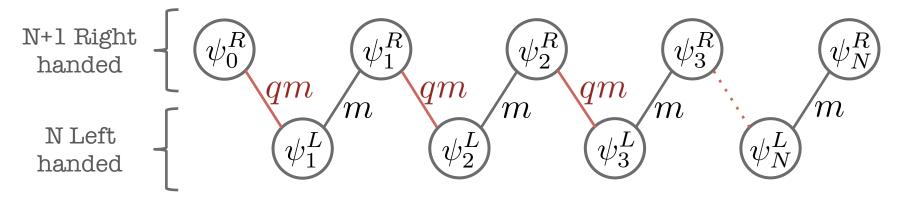
Dijet spectrum likely too smeared, and background too large, to reveal anything here. Perhaps diphotons could reveal gears.



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Clockwork Fermion

Can also construct analogous fermion models:



One Weyl fermion left over to be massless. If last site is the RHD Neutrino, then clockworked interaction is:

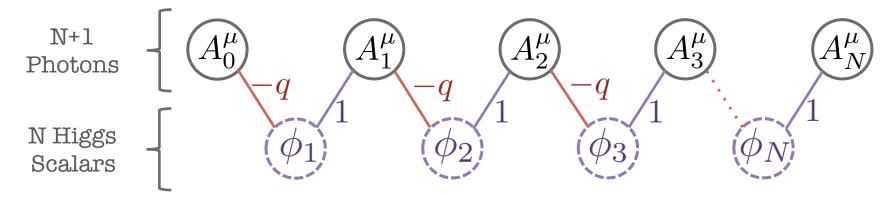
$$\mathcal{L} = -\lambda H \bar{L}_L \psi_N^R \longrightarrow \mathcal{L} = -\frac{1}{q^N} \lambda H \bar{L}_L \tilde{\psi}$$

Tiny Dirac neutrino masses! Again, much interesting phenomenology to look into. See Hambye, Teresi, Tytgat

Clockwork Photon

First proposed by Saraswat.

Can even have clockwork photons:



If all scalars get vevs $\langle \phi_j \rangle = \frac{f}{\sqrt{2}}$, vector action becomes

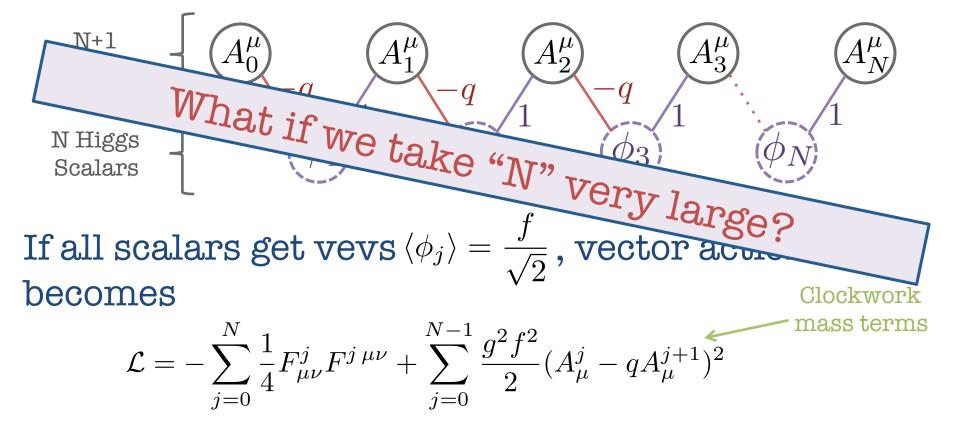
$$\mathcal{L} = -\sum_{j=0}^{N} \frac{1}{4} F_{\mu\nu}^{j} F^{j\,\mu\nu} + \sum_{j=0}^{N-1} \frac{g^2 f^2}{2} (A_{\mu}^{j} - q A_{\mu}^{j+1})^2 \qquad \text{mass terms}$$

Interesting applications: millicharges, dark forces, etc...

Clockwork Photon

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Looking back to scalar

$$\mathcal{L} = \frac{1}{2} \sum_{j=0}^{N} (\partial_{\mu} \pi_{j})^{2} - \frac{m^{2} f^{2}}{2} \sum_{j=0}^{N-1} \left(e^{\frac{i}{f} (q \pi_{j+1} - \pi_{j})} + h.c. \right)$$

This action exhibits a single continuous $U(1)_{CW}$ symmetry, under which the complex scalars have charge $Q_{CW} = 1, 1/q, ..., 1/q^N$

The "j'th" field of carries charge $Q_j^{CW} = q^{-j}$ under U(1)_{CW}. Axion of spontaneously broken symmetry couples proportional to charge, thus

$$\Delta \mathcal{L} = \frac{\partial_{\mu} a_0}{f} Q^{CW} J_{CW} \to \frac{\partial_{\mu} a_0}{q^N f} J_{CW}$$

This sets discrete gauge symmetry of axion.

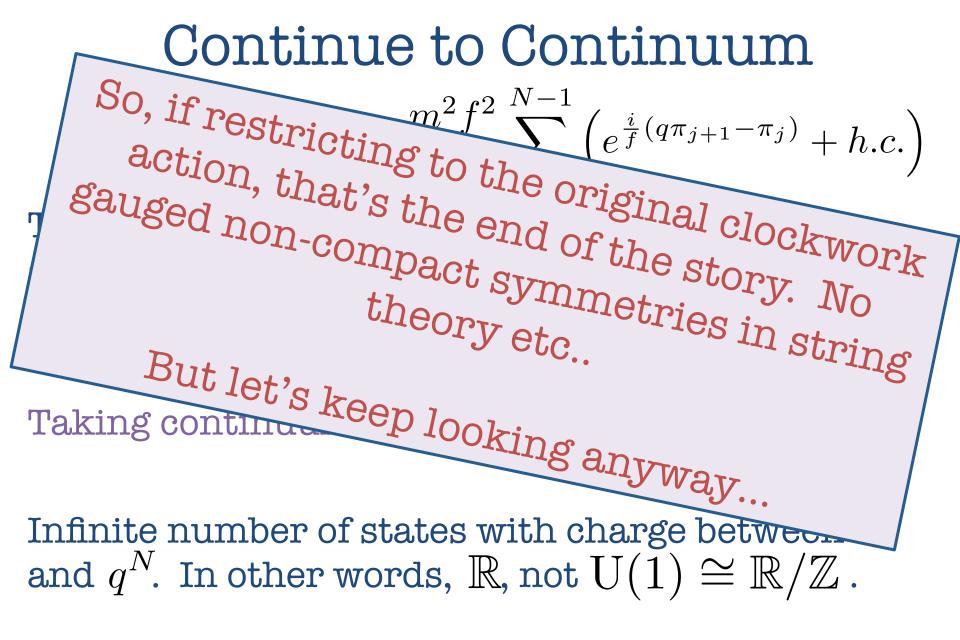
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This action exhibits a single continuous $U(1)_{CW}$ symmetry, under which the complex scalars have charge $Q_{CW} = 1, 1/q, ..., 1/q^N$

Taking continuum limit, with q^N fixed.

Infinite number of states with charge between 1 and q^N . In other words, \mathbb{R} , not $\mathrm{U}(1) \cong \mathbb{R}/\mathbb{Z}$.

Continuum clockwork is non-compact, non-fun?



Continuum clockwork is non-compact, non-fun?

Take original clockwork model

$$\mathcal{L} = -\frac{f^2}{2} \sum_{j=0}^{N} (\partial_{\mu} \pi_j)^2 - m^2 f^2 \sum_{j=0}^{N-1} \cos\left[\frac{1}{f}(\pi_j - q\pi_{j+1})\right] + \frac{\pi_j}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$
Perform a field redefinition
$$+ \frac{1}{a^2} G^{\mu\nu} G_{\mu\nu}$$

$$\pi_j \to \pi_j/q^j$$

 $q^2 U$

in a 5D interval of length πR . Scalar action is

$$\mathcal{L} = -\frac{1}{2} \sum_{j=0}^{N} q^{-2j} (\partial_{\mu} \pi_{j})^{2} - m^{2} f^{2} \sum_{j=0}^{N-1} \cos\left(\frac{q^{-j}}{f} (\pi_{j+1} - \pi_{j})\right)$$
$$\frac{1}{g^{2}} G^{\mu\nu} G_{\mu\nu} + q^{-j} \frac{\pi_{j}}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$

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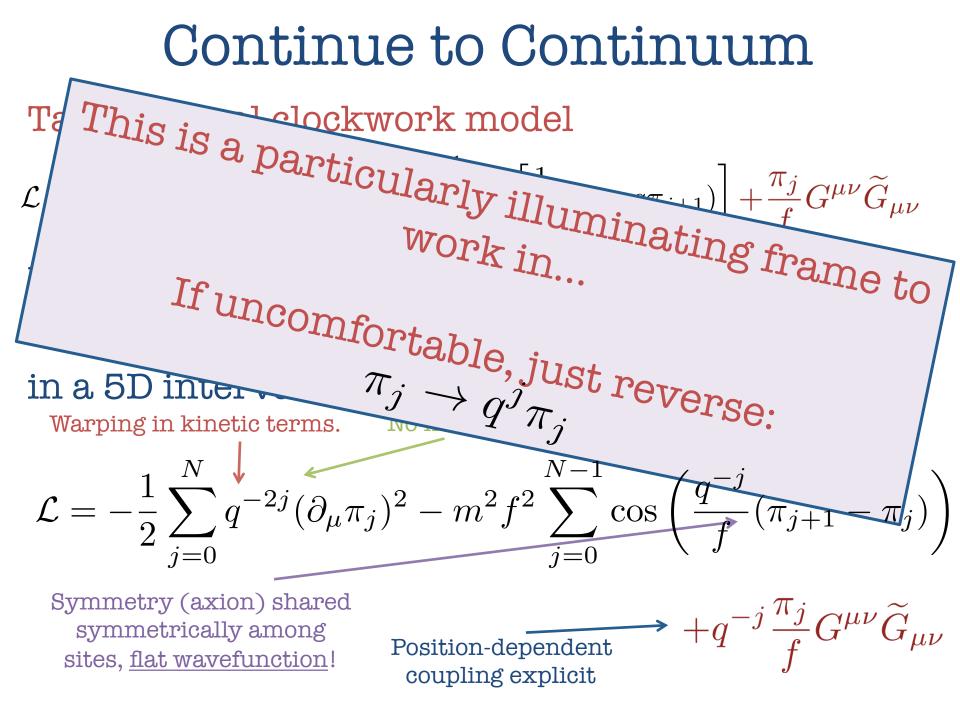
Warping in kinetic terms. No more "by hand" than this.

$$\mathcal{L} = -\frac{1}{2} \sum_{j=0}^{N} q^{-2j} (\partial_{\mu} \pi_{j})^{2} - m^{2} f^{2} \sum_{j=0}^{N-1} \cos\left(\frac{q^{-j}}{f}(\pi_{j+1} - \pi_{j})\right)$$

Symmetry (axion) shared symmetrically among sites, <u>flat wavefunction</u>!

Position-dependent coupling now explicit

 $\rightarrow + q^{-j} \frac{\pi_j}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$



Take continuum limit

$$m^2(a) = \frac{1}{a^2}$$
, $q(a) = e^{ka}$

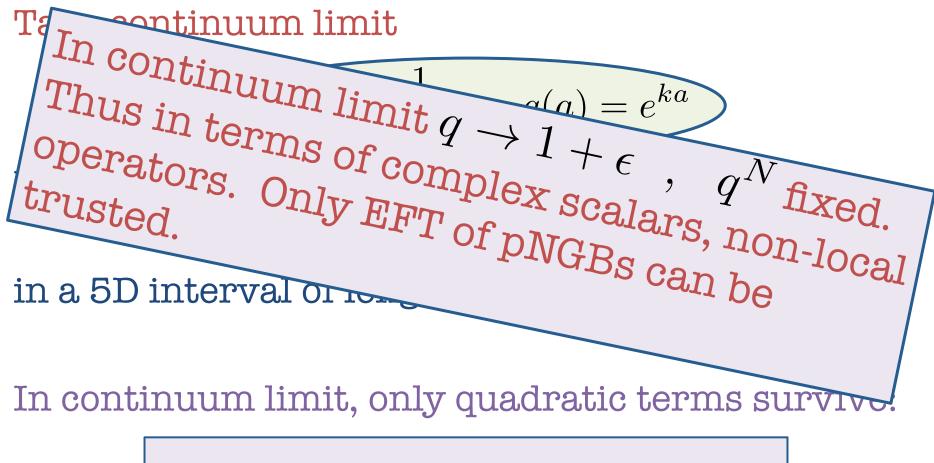
Including derivatives

$$(\pi_{j+1} - \pi_j \to a\partial_y \pi)$$

in a 5D interval of length πR

In continuum limit, only quadratic terms survive:

$$\left[\frac{1}{a^2}\cos\left(\frac{a}{\kappa}\partial_y\pi\right)\right]_{a\to 0}\to \left(\frac{1}{\kappa}\partial_y\pi\right)^2$$



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$$+ q^{-j} \frac{\pi_{j}}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$
In continuum limit, only quadratic terms survive:
$$\mathcal{L} = -\frac{1}{2} \int dy \, e^{-2ky} \left[\partial_{\mu} \pi \, \partial^{\mu} \pi + (\partial_{y} \pi)^{2}\right] + e^{-ky} \frac{\pi}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$

Interaction explicitly breaks discrete gauge symm.

$$\begin{aligned} & \text{Linue to Continuum} \\ & \text{Deconstruct this action, with appropriate} \\ & \text{irrelevant operators, and you recover the} \\ & \text{original clockwork model.} \end{aligned} \\ & \text{If uncomfortable with basis, reverse field} \\ & \text{redefinition:} \\ & \pi \to e^{ky} \tilde{\pi} , \quad \tilde{\pi} \to \tilde{\pi} + \kappa e^{-ky} \\ & \text{In continuum lime,} \quad \tilde{\pi} \to \tilde{\pi} + \kappa e^{-ky} \\ & \mathcal{L} = -\frac{1}{2} \int dy \, e^{-2ky} \left[\partial_{\mu} \pi \, \partial^{\mu} \pi + (\partial_{y} \pi)^{2} \right] + e^{-\omega} f \end{aligned}$$

Interaction explicitly breaks discrete gauge symm.

Connection with the linear dilaton model:

$$\mathcal{L} = -\frac{1}{2} \int dy \, e^{-2ky} \left[\partial_{\mu} \pi \, \partial^{\mu} \pi + (\partial_{y} \pi)^{2} \right] + e^{-ky} \frac{\pi}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}$$

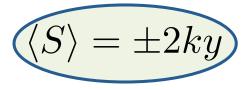
Where $\langle S \rangle = \pm 2ky$. Interaction term arises from "k"-like parameter. This is the direct continuum limit of the original clockwork model.

What's the linear dilaton model?

See e.g. Antoniadis, Dimopoulos, Giveon, 2001.

$$S = \int d^4x \, dy \, \sqrt{-g} \, \frac{M_5^3}{2} e^S \left(\mathcal{R} + g^{MN} \partial_M S \, \partial_N S + 4k^2 \right)$$

Solution of Einstein's equations:



Connection with the linear dilaton model:

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If coupled at different sites, for example, $y_a = 0$, $y_b = \log(2)/2k$

Then compact discrete shift symmetry explicitly broken by brane couplings (not bulk action). Symmetry is <u>non-compact</u>.

Connection with the linear dilaton model:

$$\mathcal{L} \underbrace{\int dy \, e^{-2ky} \left[\partial_{\mu} \pi \, \partial^{\mu} \pi + (\partial_{y} \pi)^{2} \right] + e^{-ky} \frac{\pi}{f} G^{\mu\nu} \widetilde{G}_{\mu\nu}}_{Non-compact, as expected}$$

Where $\langle S \rangle$ where $\langle S \rangle$ where $\langle S \rangle$ solution term arises from the parameter. This is expected, the second second

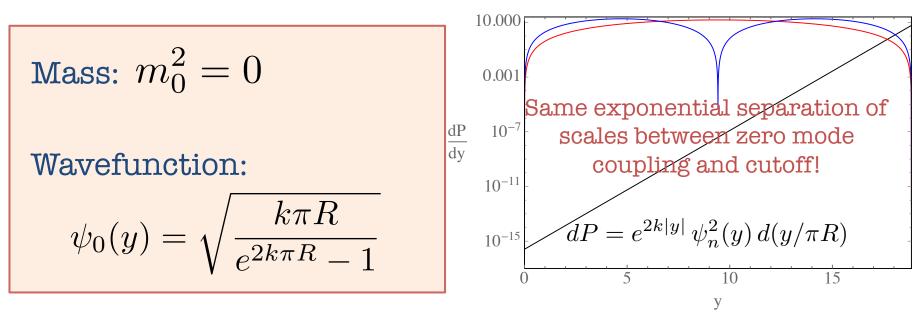
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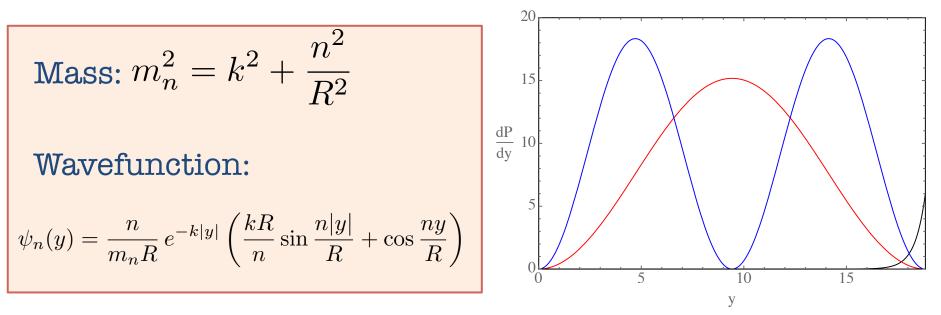
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Consistent the linear dilaton model. Mass spectrum, wavefunction, coupling Connection with the linear dilaton model: pattern all continuum limit of discrete model, see QCD application earlier. limit OI LING coupled at y = 0, to particular Mass: $m_n^2 = k^2 + \frac{n^2}{D^2}$ 15 $\frac{dP}{dv}$ 10 Wavefunction: 5 $\psi_n(y) = \frac{n}{m_m R} e^{-k|y|} \left(\frac{kR}{n} \sin \frac{n|y|}{R} + \cos \frac{ny}{R}\right)$ 10 15 У

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Where $\langle S \rangle = \pm 2ky$. If coupled only at y = 0 all features physically identical to being at the end of the clockwork chain, since this is the continuum of the clockwork.

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Could also remove dilaton factor from topological term, but then zero mode couplings become position-independent: <u>No longer a continuum limit</u> <u>of the clockwork.</u>

Connection with the linear dilaton model:

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Localisation in terms of zero-mode coupling to gluons no longer varies exponentially with position. This is the central objection of Craig et al, with regard to the original clockwork model, and we are in complete agreement on this. But can we still use it for other purposes?

Connection with the linear dilaton model:

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Bulk unchanged, properties of continuum limit of:

$$\mathcal{L} = \frac{1}{2} \sum_{j=0}^{N} (\partial_{\mu} \pi_{j})^{2} - \frac{m^{2} f^{2}}{2} \sum_{j=0}^{N-1} \left(e^{\frac{i}{f} (q \pi_{j+1} - \pi_{j})} + h.c. \right)$$

preserved, including clockworked shift symmetry.

Connection with the linear dilaton model:

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Localisation and hierarchy of zero-mode coupling

$$\mathcal{L} = \frac{1}{2} \sum_{j=0}^{N} (\partial_{\mu} \pi_{j})^{2} - \frac{m^{2} f^{2}}{2} \sum_{j=0}^{N-1} \left(e^{\frac{i}{f} (q \pi_{j+1} - \pi_{j})} + h.c. \right)$$

to cutoff the same. Can now have compact symm.

This means that any massless field (scalar, fermion, vector, graviton) placed in the linear dilaton background

$$\mathcal{L} = -\frac{1}{2} \int dy \, e^{-2ky} \left[\partial_{\mu} \pi \, \partial^{\mu} \pi + (\partial_{y} \pi)^{2} \right]$$

Has the same physical localisation, mass spectrum, and hierarchy between zero-mode coupling, all canonically normalized fields obey symmetry

$$\langle \pi_j \to \pi_j + \kappa/q^j \rangle$$

For us, all of these features are clockworky enough, but, as pointed out by Craig, fq Garcia-Garcia, and Sutherland, now position-independent means charges not clockworked, but this allows to go from non-compact to compact. Has the same page spectrum, and hierarchy be coupling, all canonically normalized no symmetry

 $\pi_j
ightarrow \pi_j + \kappa/q^j$

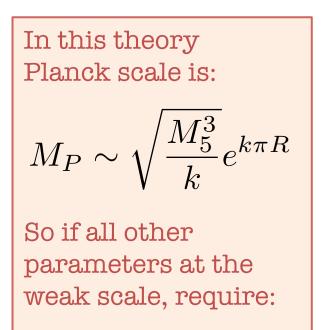
Things get really interesting when looking to the phenomenology...

See: Work in progress with Giudice, <u>Kats</u>, Torre, Urbano.

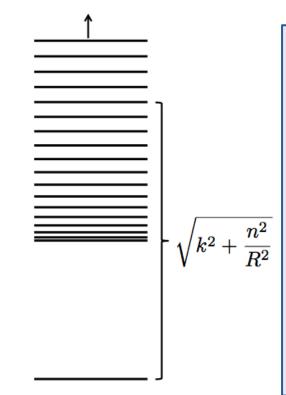
Previous related studies:

- Antoniadis, Arvanitaki, Dimopoulos, Giveon, 2011. (Large-k)
- Baryakhtar, 2012. (All-k)
- Cox, Gherghetta, 2012. (Dilatons)
- Giudice, Plehn, Strumia, 2004. Franceschini, Giardino, Giudice, Lodone, Strumia, 2011. (Large extra dimensions, pheno similar.)

Irreducible prediction:



$$kR\sim 11$$

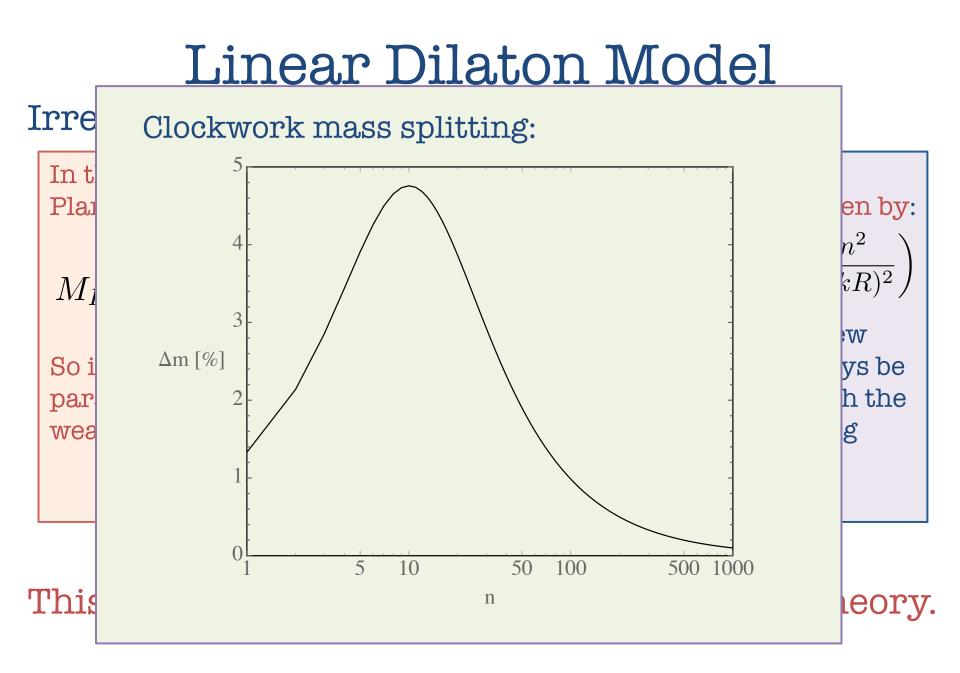


But the mass spectrum is given by: (n^2)

$$m_n \sim k \left(1 + \frac{n^2}{2(kR)^2} \right)$$

Thus the first few states will always be split by %'s, with the relative splitting decreasing for heavier modes.

This splitting is thus a key prediction of the theory.



At colliders would look something like:

