General Continuum Clockwork

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Based on work in progress with
Kiwoon Choi, and Sang Hui Im [arXiv:1706.0xxxx]

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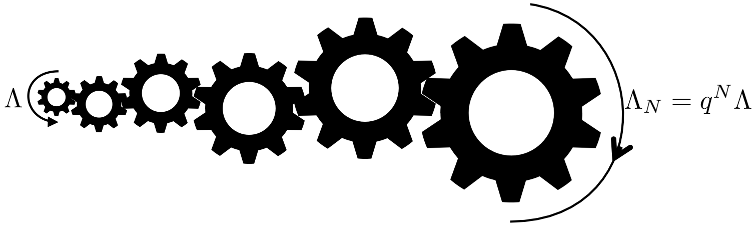
Clockwork

- *The simplified clockwork model and its zero mode distribution*

[Choi, Kim, Yun 14]
[Choi, Im 15]
[Kaplan, Rattazzi 15]

$$\mathcal{L} = -\frac{1}{2}\left[\sum_{i=0}^{N}(\partial_\mu\phi_i)^2 + \sum_{i=0}^{N-1} m^2(\phi_i - q\phi_{i+1})^2\right]$$

$$\boxed{\phi_l(x) = q^{-l} Z_0 \phi^{(0)}(x) + \cdots}$$

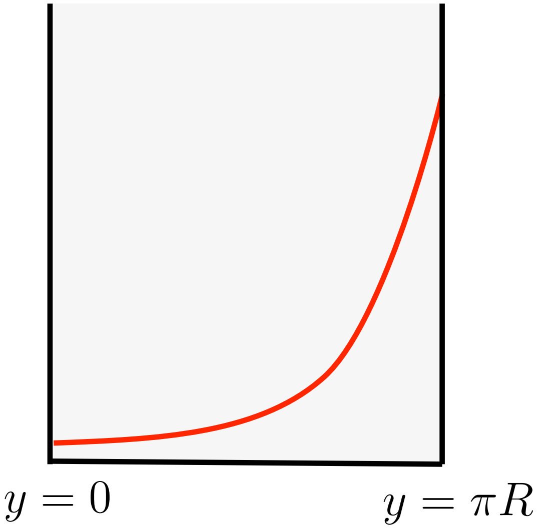


- *Naturally realized in the continuum limit*

[Giudice, McCullough 16]

$$\mathcal{L} = -\frac{1}{2}\int_0^{\pi R} dy\left[(\partial_\mu\Phi)^2 + \underbrace{(\partial_y\Phi - m\Phi)}_{=0}{}^2\right]$$

$$\boxed{\Phi(x,y) = e^{my} Z_0 \phi^{(0)}(x) + \cdots}$$



Deconstructed Ex-dim on the orbifold S/Z$_2$

- *The Lagrangian could be written as several forms*

$$\mathcal{L} = -\frac{1}{2}\int_0^{\pi R} dy \left[(\partial_\mu \Phi)^2 + (\partial_y \Phi - m\Phi)^2\right]$$

Integration by parts with boundary localized terms (M: μ,y)

$$-\frac{1}{2}\int_0^{\pi R} dy \left[(\partial_M \Phi)^2 + m^2\Phi^2 + 2m\Phi^2(\delta(y) - \delta(y-\pi R))\right]$$

Field redef. ($\Phi \rightarrow e^{my}\,\Phi$) with a nontrivial metric, $ds^2 = e^{4my/3}(dx^2+dy^2)$

$$-\frac{1}{2}\int_0^{\pi R} dy\, e^{2my}(\partial_M \tilde{\Phi})^2 = -\frac{1}{2}\int_0^{\pi R} dy\, \sqrt{-G} G^{MN}(\partial_M \tilde{\Phi})(\partial_N \tilde{\Phi})$$

Coordination transformation, $dz = e^{2my}\,dy$ ($e^{2\pi Rm} >> 1$)

$$-\frac{1}{2}\int_{\frac{1}{2m}}^{\frac{e^{2\pi Rm}}{2m}} dz \left[(\partial_\mu \tilde{\Phi})^2 + (2mz)^2(\partial_z \tilde{\Phi})^2\right]$$

$$ds^2 = (2mz)^{2/3} dx^2_{(4)} + (2mz)^{-4/3} dz^2$$

- *Profiles in field/coordinate space depend on the basis*
  Nontrivial geometry and/or bulk-boundary mass parameters in 5D models is needed

Motivation 1

- *The interaction terms (beyond the quadratic) represent model dependence*

See [Craig, Garcia, Sutherland 17]
[Giudice, McCullough 17]

$$\mathcal{L} = -\frac{1}{2}\int_0^{\pi R} dy \left[(\partial_\mu \Phi)^2 + (\partial_y \Phi - m\Phi)^2\right] \quad + g(y_*)\Phi(x, y_*)\mathcal{O}_p(x)$$

- *Predict the interactions based on the symmetry in Ex-dim models.*

- *A Linear Dilaton Model is a good starting point since it could give natural variation of the warped geometry,* $ds^2 = e^{2k_1 y}dx_{(4)}^2 + e^{2k_2 y}dy^2$

$$k_1 = k_2 \ (\mathrm{CW}), \ \ k_1 \gg k_2 \ (\mathrm{RS}), \ \ k_1 \ll k_2 \ (\mathrm{LED})$$

from the shift symmetry of the dilaton field, S, and the soft breaking massive parameters.

$$\frac{M_5^3}{2}\int d^5x\sqrt{-G}\left[\mathcal{R} - G^{MN}\partial_M S\partial_N S + 4(e^{-cS}k)^2\right]$$

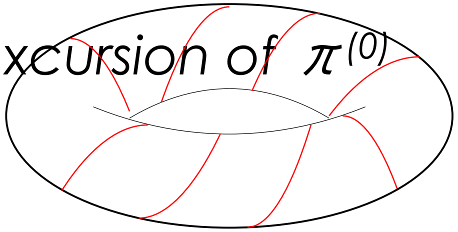
$$\boxed{S(x,y) \to S(x,y) + \alpha, \ \ m_{\mathrm{soft}} \to \exp(c\alpha)m_{\mathrm{soft}}}$$

Periodicity in a clockwork axion

- *For the fields, $\{\pi_i\}$ with a period $2\pi f$, (q_0, q_1) should be integers.*

$$\mathcal{L} = -\frac{1}{2}\left[\sum_{i=0}^{N}(\partial_\mu \pi_i)^2 - 2\sum_{i=0}^{N-1} m_\pi^2 f^2 \cos\left(\frac{q_0\pi_i - q_1\pi_{i+1}}{f}\right)\right]$$

- *One cycle along π_N corresponds to the large field excursion of $\pi^{(0)}$*



$$\pi_N \to \pi_N + 2\pi f \;\; \Rightarrow \pi^{(0)}(x) \to \pi^{(0)}(x) + 2\pi(q_1/q_0)^N f$$

- *The additional potential of π_N gives the potential of $\pi^{(0)}$ with an exponentially large period, $f_{eff} = (q_1/q_0)^N f$.*

$$\Delta\mathcal{L} = V(\pi_N) = V(\pi_N + 2\pi f) \Rightarrow V_{\text{eff}}(\pi^{(0)}) = V_{\text{eff}}(\pi^{(0)} + 2\pi f_{\text{eff}})$$

- *Useful for inflationary, relaxion related model buildings, etc.*
- *Deconstruction of the 5D model ?*

$$y \equiv \epsilon i,\ \pi R \equiv \epsilon N,\ m \equiv -(q_1/q_0 - 1)/\epsilon$$

$$\pi(x,y) = \pi_i(x)$$

$$\epsilon \to 0,\ \ N \to \infty,\ (q_0, q_1 \in \mathbb{Z}) \to \infty$$

$$\pi(x,y) \to \pi(x,y) + 2\pi f$$

Periodicity in a clockwork axion

- *The effective continuum Lagrangian*

$$\mathcal{L} = -\frac{1}{2}\int_0^{\pi R} dy \left[(\partial_\mu \pi)^2 - 2m_\pi^2 f^2 \cos\left(\frac{\epsilon q_0}{f}(\partial_y \pi - m\pi)\right)\right]$$

- *Taking* εq_0 *finite :*
  the finite period, breakdown of 5D Lorentz symmetry

- *Taking* q_0/f *finite :*
  could recover the 5D Lorentz symmetry, the infinite period (f→∞)

- *m=0 (finite* $q_0=q_1$*)*
  5D Lorentz invariant with a finite period, no exponentially large f_{eff}

$$y \equiv \epsilon i,\ \pi R \equiv \epsilon N,\ m \equiv -(q_1/q_0 - 1)/\epsilon$$
$$\pi(x,y) = \pi_i(x)$$
$$\epsilon \to 0,\ \ N \to \infty,\ (q_0, q_1 \in \mathbb{Z}) \to \infty$$

$$\pi(x,y) \to \pi(x,y) + 2\pi f$$

Motivation 2

- *Is it possible to have the property*

$$\boxed{\pi(x,y) \to \pi(x,y) + 2\pi f, \quad \pi(x,y) = e^{my} Z_0 \pi^{(0)}(x) + \cdots}$$

- *the 5th component of the 5D U(1) gauge field C_M , $C_5(x, y)$ yields the Wilson line as the scalar zero mode,*

$$\exp\left(i\int_0^{\pi R} dy C_5\right) \qquad a(x) \to a(x) + 2\pi$$

which has the non-trivial wave-function profile, $C_5(x,y) = e^{my} Z_0 \phi^{(0)}(x) +$

- *The charged complex scalar, Φ(x,y) with a VEV v(y) breaks 5D U(1).*

The zero mode of the 5D Goldstone of

$$\Phi(x,y) = v(y) \exp\left(i\frac{\pi(x,y)}{v}\right)$$

could be mixed with the zero mode of C_5

with suitable boundary conditions, $\boxed{C_\mu(x,0) = C_\mu(x,\pi R) = 0}$

$$G^{MN} D_M \pi D_N \pi \Rightarrow \quad (\partial_\mu \pi - qvC_\mu)^2 + (\partial_y \pi - qvC_5)^2$$

Motivation summary

- *In a clockwork*

  a) hierarchically large period of the axion zero mode (f_{eff} >> f)

  $$V(\pi_N) = V(\pi_N + 2\pi f) \Rightarrow V_{\text{eff}}(\pi^{(0)}) = V_{\text{eff}}(\pi^{(0)} + 2\pi f_{\text{eff}})$$

  Trans-Planckian?

  (e.g. $f_{eff} \sim M_{Pl}$ >> $f \sim M_5$ in [Giudice, McCullough 16010.07952])

  b) hierarchically different interaction strengths (α_{eff} << α)

  via quasi localization of zero modes : generalization to other spins

  $$\Delta L = (\partial_\mu \pi_N)\mathcal{O}_N \Rightarrow q^{-N} Z_0 (\partial_\mu \pi^{(0)})\mathcal{O}_N$$

  c) observable degenerated KK spectrums, $m_{KK}^{(2)}/m_{KK}^{(1)} \ll m_{KK}^{(1)} \sim \mathcal{O}(\text{TeV})$

  related with the solution to the hierarchy problem

- *Each parts could be independent. Still it would be interesting to study the possibilities through the extra-dimensional realization*

Linear dilaton model

- *CW geometry in a linear dilaton model with brane localized terms*

[Antoniadis, Arvanitaki, Dimopoulos, Giveon 11]

$$S^{[J]}_{\rm gravity} = \frac{M_5^3}{2}\int d^5x\sqrt{-\tilde{G}}e^{\tilde{S}}\Big[\mathcal{R} + \tilde{G}^{MN}\partial_M\tilde{S}\partial_N\tilde{S} + 4k^2 + \frac{8\tilde{k}}{\sqrt{G_{55}}}\Big(\delta(y) - \delta(y-\pi R)\Big)\Big]$$

- *In the Einstein frame with a canonical normalization of S,*

$$S_{\rm gravity} = \frac{M_5^3}{2}\int d^5x\sqrt{-G}\Big[\mathcal{R} - G^{MN}\partial_M S\partial_N S + 4(e^{-\frac{1}{\sqrt{3}}S}k)^2 + \frac{8(e^{-\frac{1}{\sqrt{3}}S}\tilde{k})}{\sqrt{G_{55}}}\Big(\delta(y) - \delta(y-\pi R)\Big)\Big]$$

$$\boxed{ds^2 = e^{2k_1 y}(dx^2_{(4)} + dy^2)}$$

- *Softly broken shift symmetry*

$$\boxed{S \to S + \alpha,\ \ (k, \tilde{k}) \to e^{\frac{1}{\sqrt{3}}\alpha}(k, \tilde{k})}$$

$$R^{-1} \ll (k,\ \tilde{k}) \ll M_5 \lll M_{\rm Pl} \sim M_5 e^{3k_1\pi R/2}$$

Linear dilaton model

- *Taking a parameter, c*

$$\boxed{S \to S + \alpha, \;\; (k, \tilde{k}) \to e^{c\alpha}(k, \tilde{k})}$$

$$S_{\text{gravity}} = \frac{M_5^3}{2} \int d^5x \sqrt{-G} \left[\mathcal{R} - G^{MN} \partial_M S \partial_N S + 4(e^{-cS}k)^2 \right.$$
$$\left. + \frac{8(e^{-cS}\tilde{k})}{\sqrt{G_{55}}} \Big(\delta(y) - \delta(y - \pi R) \Big) \right]$$

the metric solution has the form as

$$ds^2 = e^{2k_1 y} dx_{(4)}^2 + e^{2k_2 y} dy^2$$

where $k_1 = \frac{2k}{\sqrt{12 - 9c^2}}$, $k_2 = 3c^2 k_1$, *and* $e^{-c\langle S \rangle} = e^{-k_2 y} = \sqrt{G^{55}}$

- *c² → 1/3 (CW limit),*

  c² → 0 (RS limit : dilaton is decoupled from the gravity sector)

General quadratic action for the scalar

- *Kinetic term*

$$-\frac{1}{2}\int_0^{\pi R} dy\sqrt{-G}e^{nS}G^{MN}\partial_M\Phi\partial_N\Phi \qquad (n=0)$$

- *Interactions between the scalar and the dilaton, <S> ~ k y*

$$\int_0^{\pi R} dy\sqrt{-G}\left(\kappa_1\underbrace{G^{MN}(\partial_M S)\Phi\partial_N\Phi}_{\text{effective boundary mass}}+\overset{\text{vanishing}}{\overline{\kappa_2(\Box S)\Phi^2}}+\kappa_3\underbrace{G^{MN}(\partial_M S)(\partial_N S)\Phi^2}_{\text{effective bulk mass}}\right)$$

- *Bulk and brane soft mass terms*

$$-\int_0^{\pi R} dy\sqrt{-G}\left[\frac{1}{2}e^{-2cS}m^2\Phi^2+\frac{e^{-cS}}{\sqrt{G_{55}}}(m_0\delta(y)-m_\pi\delta(y-\pi R))\Phi^2\right]$$

- *With a tuning of boundary mass terms, we get*

$$\boxed{\mathcal{L}_\Phi=-\int_0^{\pi R} dy\, e^{(2k_1+k_2)y}\left[\frac{1}{2}(\partial_\mu\Phi)^2+\frac{1}{2}e^{2(k_1-k_2)y}(\partial_y\Phi-M\Phi)^2\right]}$$

Interactions

- *The zero mode profile*

$$\Phi(x,y)=\frac{e^{My}}{e^{(k_1+k_2/2+M)\pi R}}\phi^{(0)}(x)+\cdots$$

- *Brane localized interactions with dilatonic shift symmetry,* $n=m=0$

$$\int_0^{\pi R}dy\sqrt{-G}\frac{\delta(y-y_*)}{\sqrt{G_{55}}}\left[-\frac{e^{nS}}{4g^2}G^{\mu\nu}G^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}+\frac{e^{mS}\Phi(x,y)}{\sqrt{-G_{(4)}}M_5}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

- *Then,*

$$\Delta\mathcal{L}=-\frac{1}{4}F^c_{\mu\nu}F^{c\mu\nu}+\frac{g^2\Phi(x,y_*)}{M_5}F^c_{\mu\mu}\tilde{F}^{c\mu\nu}$$

- *Overall suppression by* $e^{-(2k_1+k_2)\pi R}$*, the hierarchical couplings by* e^{My}
- *m*

m_{soft} in denominator ? ($M_5\to e^{-cS}m_{\rm soft}$?)

Needs UV construction, For M=0, Φ(x,y) as the phase field. If we impose the periodicity as Φ(x,y) → Φ(x,y) + 2 πv, one needs position dependent charge of the quarks or guage coupling

$$\bar{q}\left[i\gamma^\mu(\partial_\mu-ie^{-(n+m)S/2}A_\mu)+e^{-cS}ve^{i\gamma_5\Phi/v}\right]_{y=y_*}q$$

Scalar from 5th component of 5D vector

- *U(1) gauge theory,* $C_M \to C_M + \partial_M \Lambda(x,y)$. *C_5 is the 4D scalar*

$$-\int_0^{\pi R} dy \sqrt{-G} \left(\frac{1}{4g^2} G^{MN} G^{PQ} C_{MP} C_{NQ} \right)$$

- *Boundary conditions leaving only the scalar zero mode without vector zero mode (including the gauge parameter)*

$$C_\mu(x,0) = C_\mu(x,\pi R) = 0, \quad \Lambda(x,0) = \Lambda(x,\pi R) = 0$$

- *Integrating out all massive vector KK modes, the effective Lagrangian for the scalar zero mode is*

$$\mathcal{L}_{\rm eff} = -\frac{1}{2g^2} \int_0^{\pi R} dy\, e^{(2k_1-k_2)y} C_{\mu 5} C^{\mu 5}$$

[Choi 03]

[Flacke, Gripaios, March-Russell, Maybury 06]

where

$$C_{\mu 5} = (\partial_\mu C_5 - \partial_y C_\mu) = e^{-(2k_1-k_2)y} Z_0 \partial_\mu a(x)$$

$$\boxed{a(x) = \int_0^{\pi R} dy C_5}$$

- *Without introducing other soft terms, the quasi-localization in the 5D canonical field basis arises*

Scalar from 5th component of 5D vector

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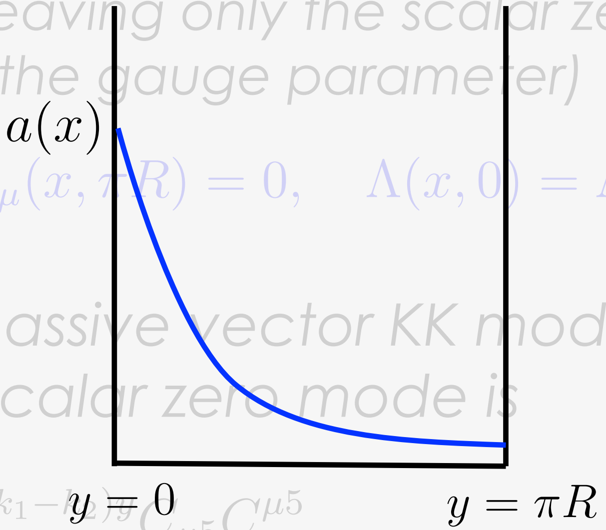

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[Flacke, Gripaios, March-Russell, Maybury 06]

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$$\boxed{a(x) = \int_0^{\pi R} dy C_5}$$

- *Without introducing other soft terms, the quasi-localization in the 5D canonical field basis arises*

Gauge fixing

- *Before integrating out KK modes, we can remove the mixing between C_5 and C_μ by introducing the gauge fixing term*

$$\mathcal{L}_{g.f.} = -\int_0^{\pi R} dy\sqrt{-G}\,\frac{1}{2g^2\xi}\left[G^{\mu\nu}\partial_\mu C_\nu + \xi G^{55}\chi^{-1}\partial_y(\chi C_5)\right]^2$$

$$\boxed{\chi = e^{(2k_1-k_2)y}}$$

- *Taking the Feynman gauge (ξ=1), the Lagrangian for C_5*

$$-\int_0^{\pi R} dy\, e^{(2k_1-k_2)y}\left[\frac{1}{2g^2}(\partial_\mu C_5)^2 + \frac{1}{2g^2}e^{2(k_1-k_2)y}(\partial_y C_5 + (2k_1-k_2)C_5)^2\right]$$

$$C_5(x,y) = \frac{e^{-(2k_1-k_2)y}}{e^{-(k_1-k_2/2)\pi R}}\phi^{(0)}(x) + \cdots$$

- *Can we use this as the clockwork axion? Well… forbidden by gauge symmetry,* $C_5(x,y) \to C_5(x,y) + \partial_y\Lambda(x,y)|_{y=0,\pi R}$

$$\int_0^{\pi R} dy\sqrt{-G}\,\frac{\delta(y-y_*)}{\sqrt{G_{55}}}\left(\cancel{\frac{C_5}{\sqrt{-G}M_5}F_{\mu\nu}\tilde{F}^{\mu\nu}} + \underline{\frac{1}{M_5}C_{\mu5}J^{\mu5}}\right)$$

hierarchies among the couplings

Spontaneous breaking of 5D U(1)

- *5D Higgs mechanism → 5D Stuckelberg field at low energies*

$$\pi(x,y) \to \pi(x,y) + qv\Lambda(x,y)$$

$$-\frac{1}{2}\int_0^{\pi R} dy\sqrt{-G}e^{-2cS}G^{MN}(\partial_M\pi - qvC_M)(\partial_N\pi - qvC_N)$$

$$\boxed{m_V = gqv}$$

- *A linear combination of the KK modes of π(x,y) and C_5(x,y) is absorbed by the KK modes of C_μ(x,y) by Higgs mechanism.*

$$\mathcal{L}_{\text{KK}} = -\sum_{n=1}\left\{\frac{1}{4}(C_{\mu\nu}^{(n)})^2 + \frac{1}{2}\left[\partial_\mu(\underline{c_n\pi^{(n)} + s_nC_5^{(n)}}) + m^{(n)}C_\mu^{(n)}\right]^2\right\}$$

- *There are two zero modes that are not absorbed by the vector KK modes. One way to see: integrating out KK modes, see the effective action for the zero modes.*

- *Other way: see the 5D action adding the gauge fixing term (ξ=1)*

Lagrangian for π and C$_5$

- *The gauge fixing term that removes the mixing with C$_\mu$(x,y) does not remove the mixing between π and C$_5$. Taking the field basis as*

$$\pi(x,y) = \Phi_+ \cos\theta + \Phi_- \sin\theta$$
$$g^{-1}C_5(x,y) = \Phi_- \cos\theta - \Phi_+ \sin\theta$$

$$\boxed{\tan 2\theta = -\frac{m_V}{k_1 - k_2/2}}$$

we get

$$-\frac{1}{2}\int_0^{\pi R} dy e^{(2k_1-k_2)y}\left[(\partial_\mu\Phi_+)^2 + e^{2(k_1-k_2)y}(\partial_y\Phi_+ - M_+\Phi_+)^2 \right.$$
$$\left. +(\partial_\mu\Phi_-)^2 + e^{2(k_1-k_2)y}(\partial_y\Phi_- - M_-\Phi_-)^2\right]$$

where

$$M_\pm = -(k_1 - k_2/2) \pm \sqrt{(k_1-k_2/2)^2 + m_V^2}$$

- *Since M$_+$ > 0, M$_-$ < 0, two zero modes are quasi-localized in different branes*

Lagrangian for π and C_5

- *The gauge fixing term that removes the mixing with $C_\mu(x,y)$ does not remove the mixing between π and C_5. Taking the field basis as*

$$\pi(x,y) = \Phi_+\cos\theta + \Phi_-\sin\theta$$
$$g^{-1}C_5(x,y) = \Phi_-\cos\theta - \Phi_+\sin\theta$$

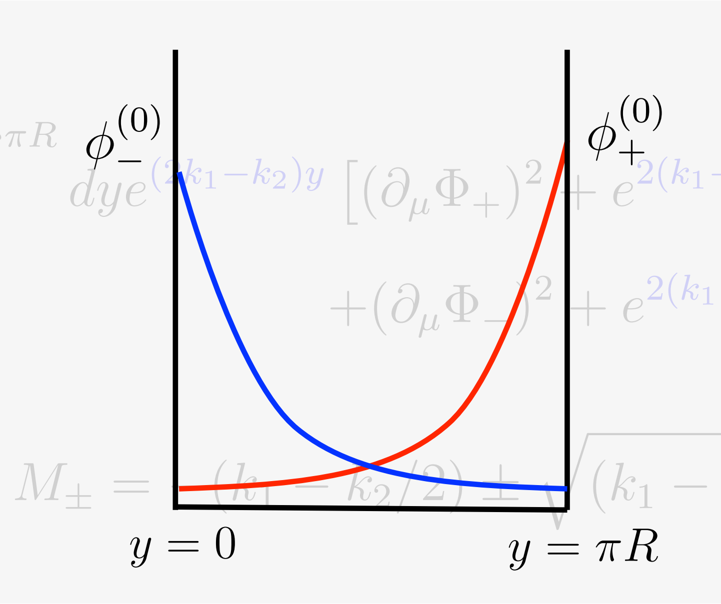
$$\boxed{\tan 2\theta = -\frac{m_V}{k_1 - k_2/2}}$$

we get

$$-\frac{1}{2}\int_0^{\pi R} dy e^{(2k_1-k_2)y}\left[(\partial_\mu\Phi_+)^2 + e^{2(k_1-k_2)y}(\partial_y\Phi_+ - M_+\Phi_+)^2 + (\partial_\mu\Phi_-)^2 + e^{2(k_1-k_2)y}(\partial_y\Phi_- - M_-\Phi_-)^2\right]$$

where

$$M_\pm = (k_1 - k_2/2) \pm \sqrt{(k_1 - k_2/2)^2 + m_V^2}$$



- *Since $M_+ > 0$, $M_- < 0$, two zero modes are quasi-localized in different branes*

- *Brane localized interaction is allowed by gauge symmetry since $\Lambda(x,0)=\Lambda(x,\pi R)=0$*

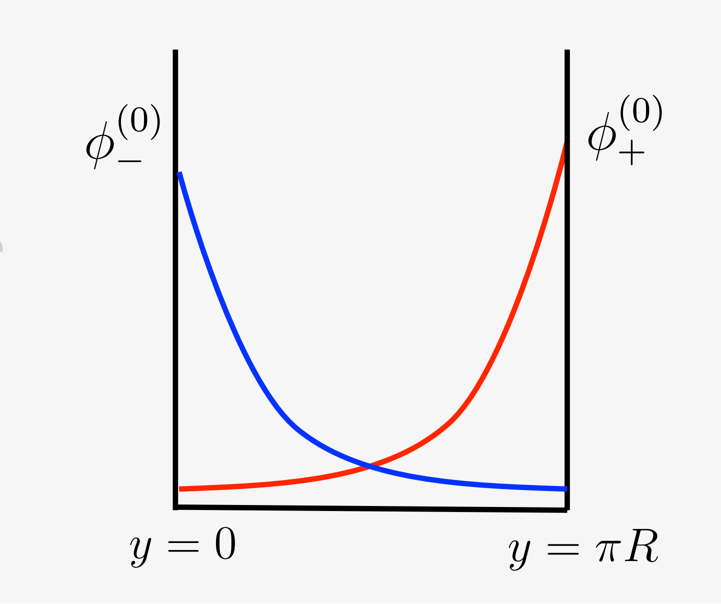
$$\frac{g^2}{16\pi^2}\frac{\pi(x,y)}{v}F_{\mu\nu}\tilde{F}^{\mu\nu}|_{y=0,\pi R}$$

Two axion model

- *Zero mode profiles:*

$$\pi(x,y) \sim \frac{e^{M_+ y}}{e^{(k_1-k_2/2+M_+)\pi R}}\phi_+^{(0)} + e^{-|M_-|y}\phi_-^{(0)}$$

- *Possible forms of the potentials from boundaries and the bulk for the Wilson line*



$$\Lambda_0^4 \cos\left(\frac{n_0\pi(x,0)}{qv}\right) + \Lambda_\pi^4 \cos\left(\frac{n_\pi\pi(x,\pi R)}{qv}\right)$$

$$+\Lambda_{\rm bulk}^4 \cos\left(\frac{n_a(\pi(x,0)-\pi(x,\pi R))}{qv}\right) + \cdots$$

$$\boxed{\int_0^{\pi R} dy C_5}$$

$$(n_0,\, n_\pi,\, n_a) \in \mathbb{Z}$$

Each Λs are exponentially sensitive to the size of gauge couplings of the boundary gauge group, the masses of the charged fields in the bulk.

Difficult to get a trans-Planckian period. Can have the hierarchically different couplings for the axions (in a mass eigenbasis)

Outlook

- *The continuum clockwork is the natural way to explain the highly ordered (engineered) interactions in the 4D discrete clockwork system. However considering it in the extra-dimensional set-up will give the extra-constraints on the couplings.*

- *I have generalized the clockwork in the generic warped background with brane/bulk masses by imposing a certain dilatonic shift symmetry in a linear dilaton model. Imposing the symmetry will specify the form of the interactions which could be more consistent with the UV theory. This approach is easily generalized to the higher spins.*

- *The system of 5D goldstone and the 5th component of the gauge field shows a different behavior compared to the massless 5D real scalar model regarding the quasi-localization. The hierarchically different couplings between the zero modes and the operators localized in different branes are naturally possible. But the physical meaning of such quasi-localization is not clearly understood yet.*

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