Order from Disorder

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Work in progress with D. Sutherland

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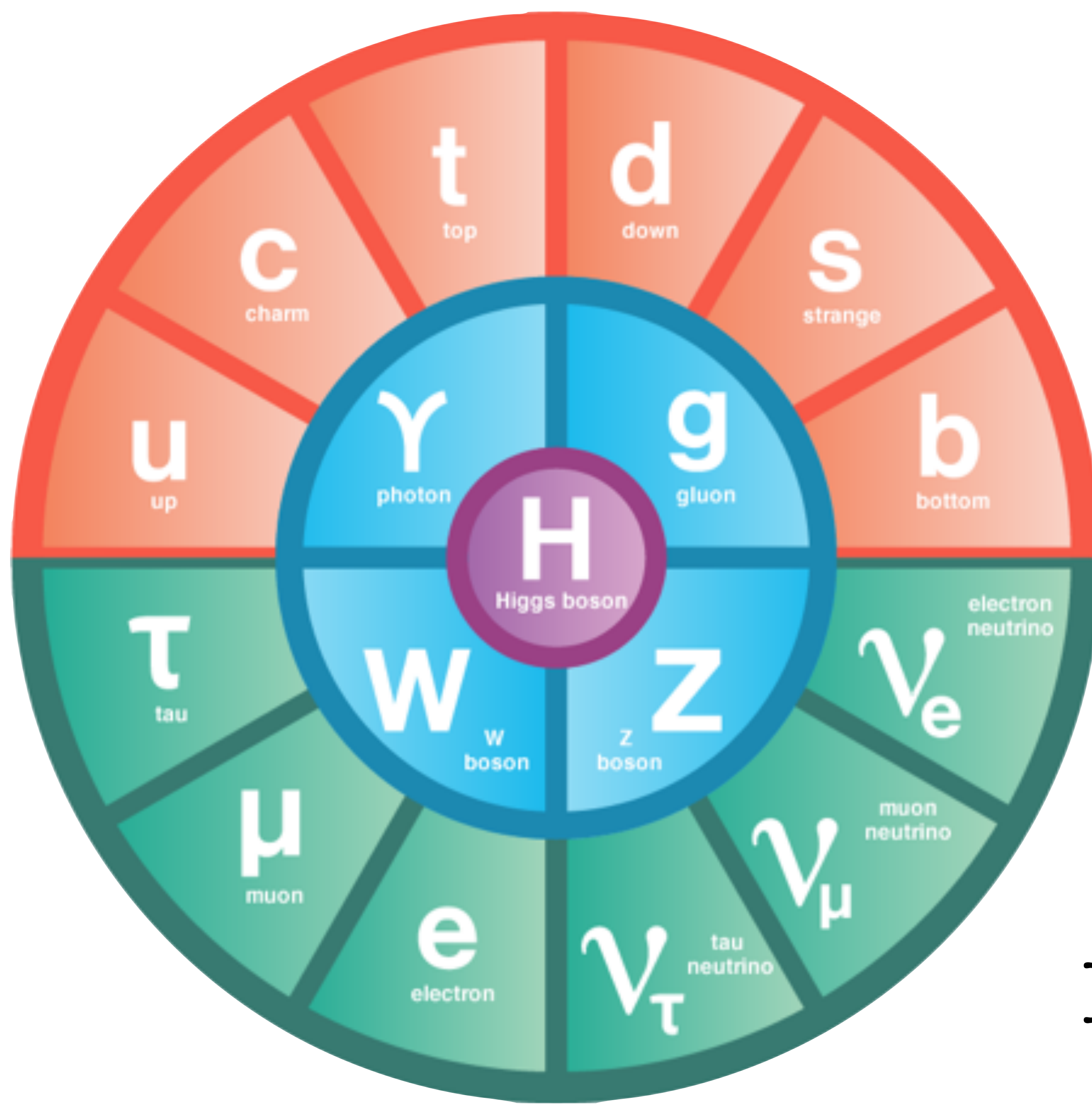
The illusion of order

We see signs of order in the structure of the Standard Model & beyond

Patterns, small couplings, apparent violations of naturalness, etc.

Flavor

Neutrinos



DARK MATTER

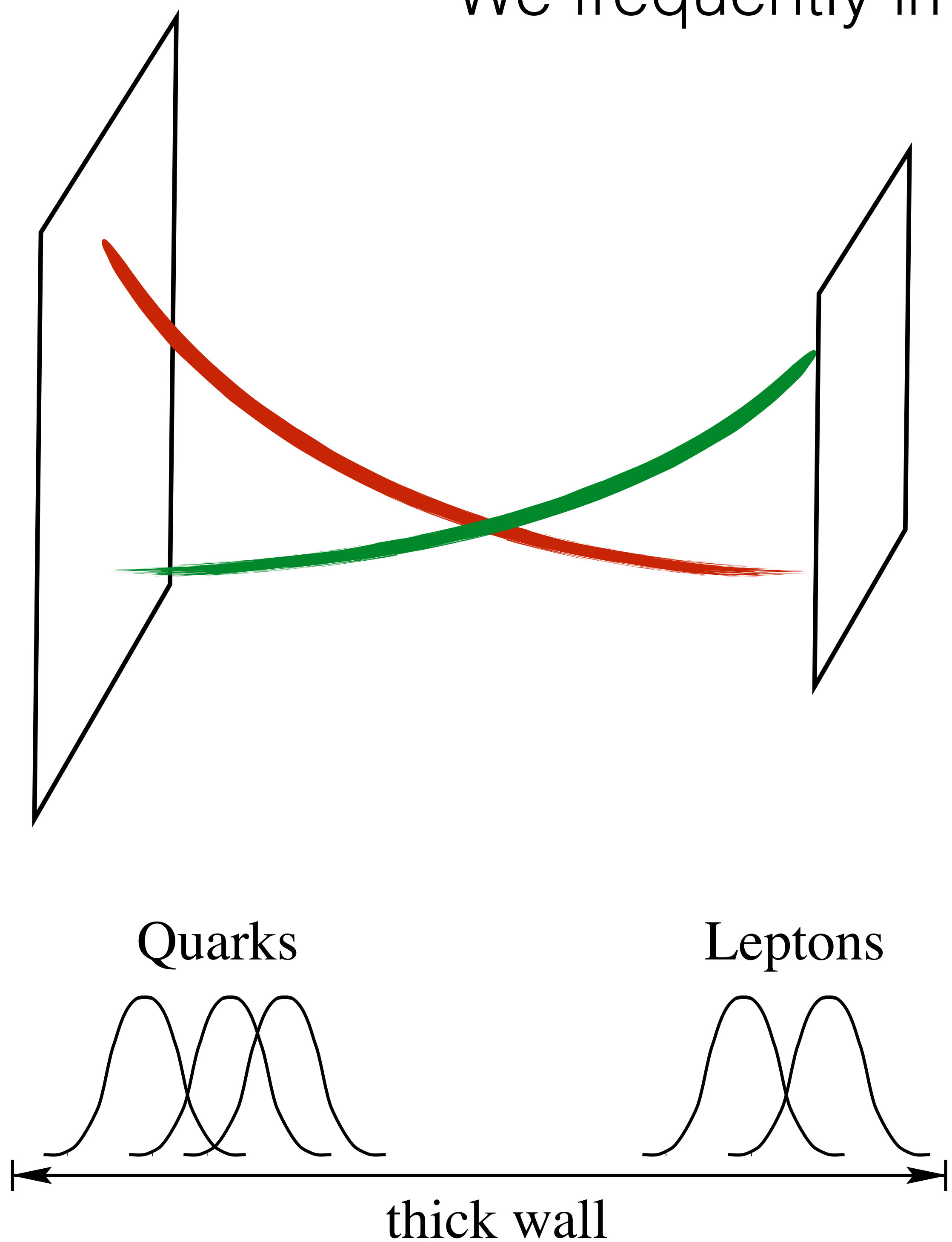
HIERARCHY PROBLEM

Baryogenesis

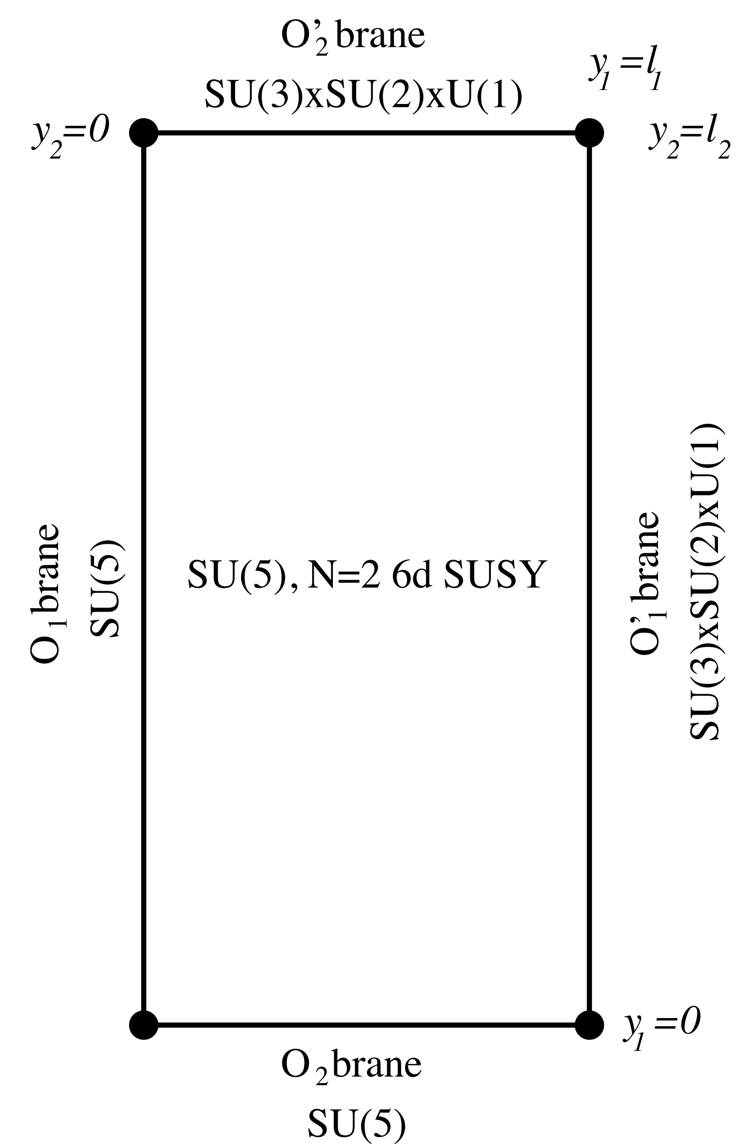
Much of BSM physics amounts to explaining the origin of this order.

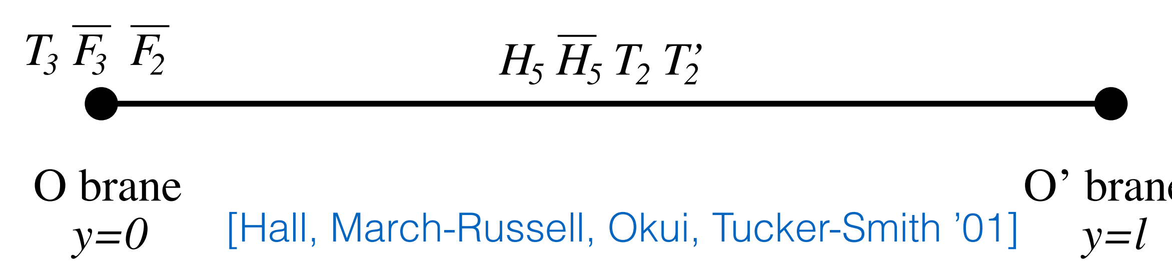
Order from geometry

We frequently impose order with geometry



[Arkani-Hamed, Schmaltz '99]

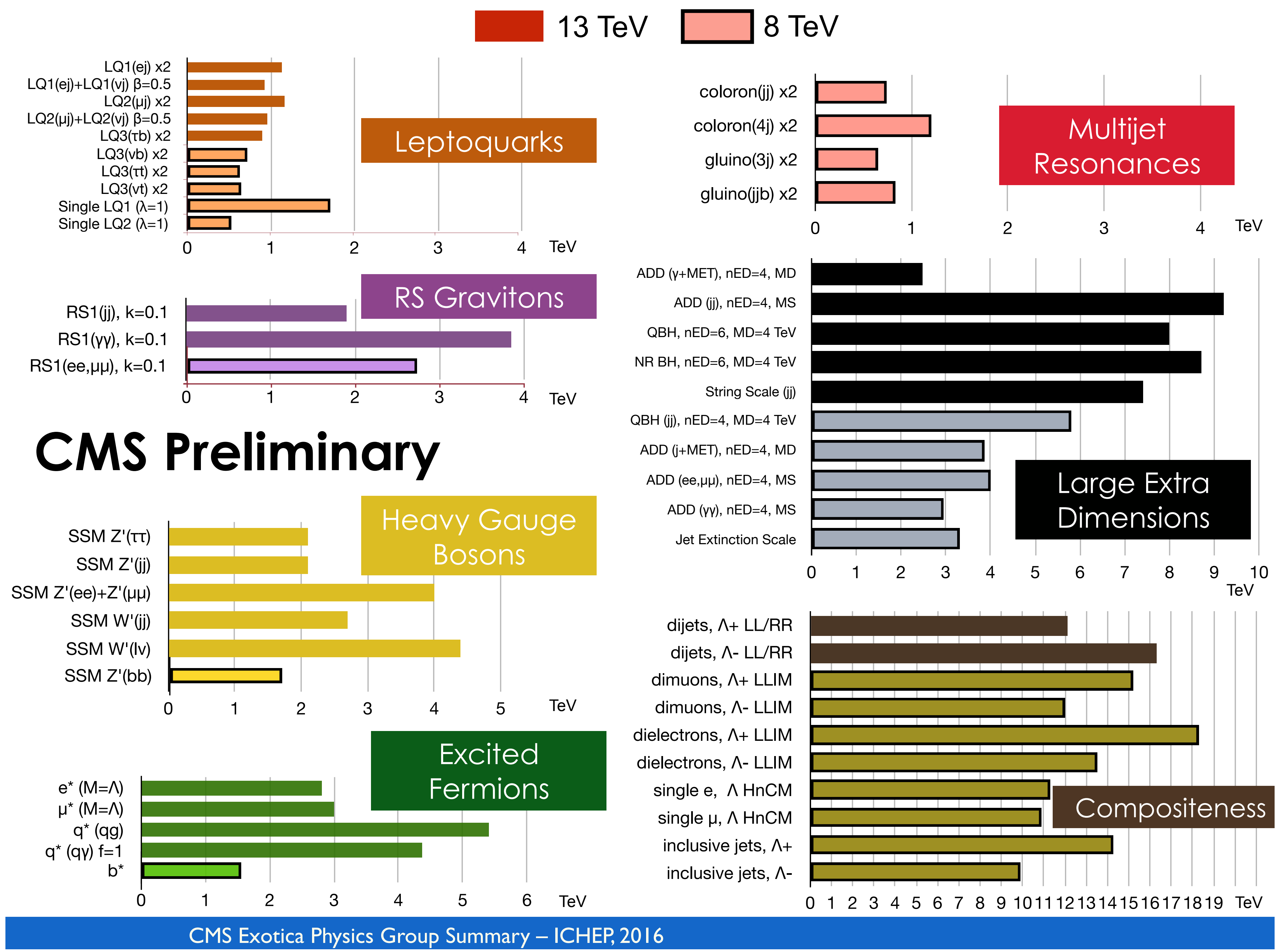




[Hall, March-Russell, Okui, Tucker-Smith '01]

Geometry and its discontents

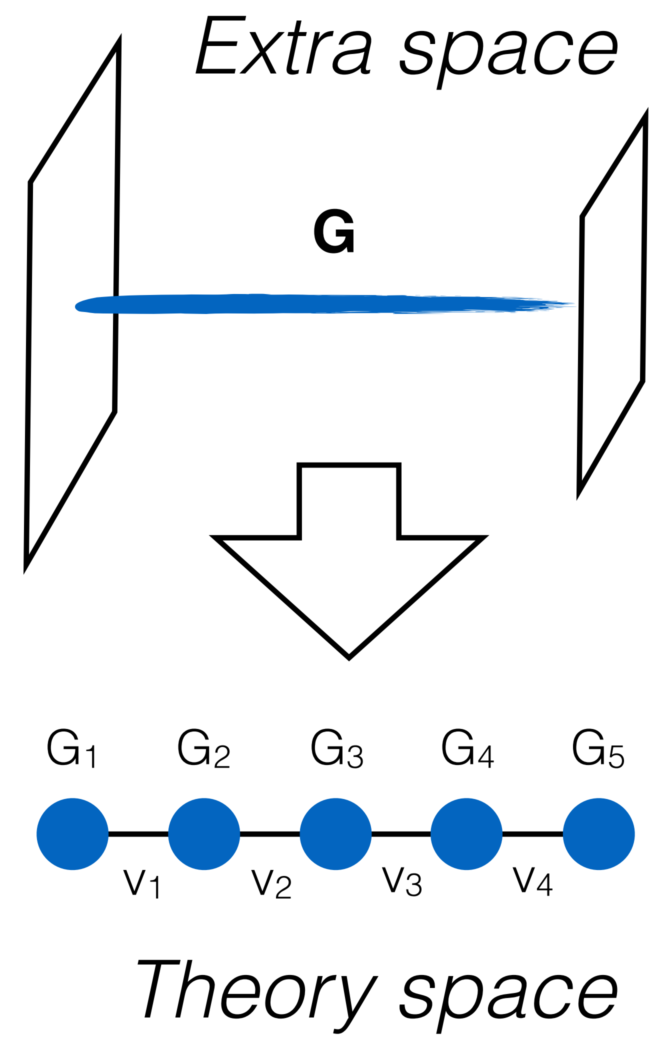
…but we have yet to find evidence for any of it.



Geometry and its discontents

4D equivalents appealing, but often either ad hoc or incalculable

Dimensional deconstruction
[Arkani-Hamed, Cohen, Georgi '01; Hill, Pokorski, Wang '01]



E.g. bulk gauge field in 5D

$$\mathcal{L} = -\frac{1}{4}\sum_{i=0}^{N} F^{a}_{i\mu\nu}F^{\mu\nu a}_{i} + \sum_{i=1}^{N} D_\mu \Phi_i^\dagger D^\mu \Phi_i$$

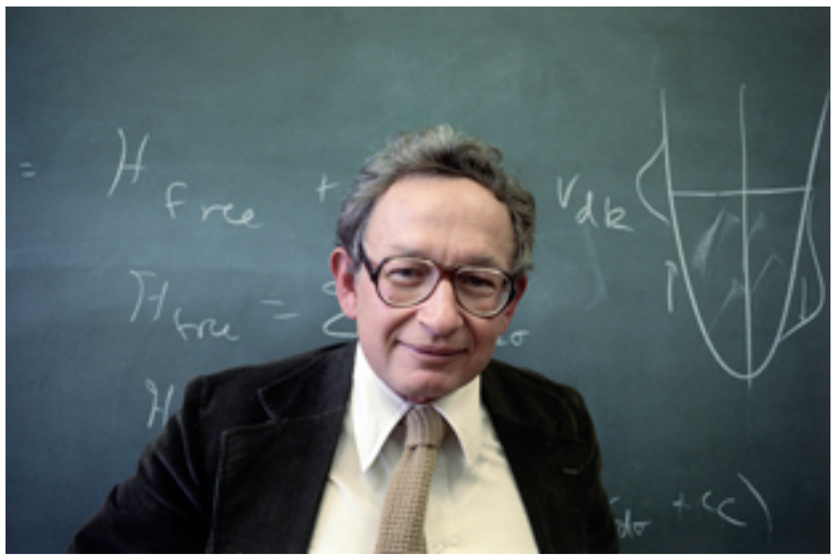
Flat	AdS	Linear dilaton
$v_{j+1} = v_j$	$v_{j+1} = e^{-ka} v_j$	$v_{j+1} = e^{-ka} v_j$
$g_{j+1} = g_j$	$g_{j+1} = g_j$	$g_{j+1} = e^{ka} g_j$

Nice exception: **clockwork** [Choi & Im, Kaplan & Rattazzi '15]
See talks by McCullough, Shin, & Kats

Perhaps it’s entirely random

But maybe disorder ⇒ order

Anderson Localization



$$H = \sum_i \epsilon_i |i\rangle\langle i| - \sum_{ij} t_{ij} |i\rangle\langle j| + \text{h.c.}$$

Impurities

Bound state energies

Tunneling

Simplify:

$$t_{ij} = t\left(\delta^j_{i+1} + \delta^j_{i-1}\right)$$

Nearest-neighbor hopping

$$\epsilon_i \in [-W/2, W/2]$$

Random impurities

Tight-binding model

$$H = \begin{pmatrix} \epsilon_1 & -t & 0 & \dots & 0 & 0 \\ -t & \epsilon_2 & -t & \dots & 0 & 0 \\ 0 & -t & \epsilon_3 & & & \\ \vdots & \vdots & & \ddots & & \\ 0 & 0 & & & \epsilon_{N-1} & -t \\ 0 & 0 & & & -t & \epsilon_N \end{pmatrix}$$

Anderson Localization

S.E. for energy eigenstates $\psi_E = \sum_i \psi_i |i\rangle$ gives (t=-1) $\psi_{i+1} + \psi_{i-1} = (E - \epsilon_i)\psi_i$

All eigenstates are **localized** in presence of disorder, $\psi(r) \propto \exp[-r/L_{loc}]$

but localization lengths not identical

Analytic results for weak localization, $\sigma \ll 1$ (for $\varepsilon_i \in [-W/2, W/2]$, $\sigma^2 = W^2/12$)

States fill a band of $E \in [-2,2]$

(allowed energies of Bloch waves for $\varepsilon_i = 0$)

In bulk of band, localization length given by Thouless result

$$L_{loc}^{-1} = \frac{W^2}{96(1 - E^2/4)}$$

Anomalous scaling near band edges E =±2

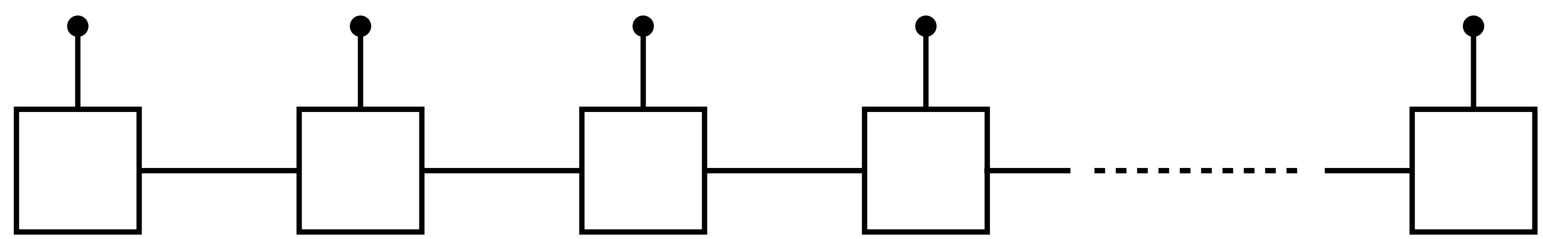
$$L_{loc}^{-1} = \frac{6^{1/3}\sqrt{\pi}}{2\Gamma(1/6)}\sigma^{2/3} \approx 0.13 W^{2/3}$$

States at band edges more sharply localized than generic eigenstates at weak disorder

A Scalar Toy Model

Can we Anderson localize in theory space?

A toy model: *N* U(1) global symmetries, softly broken



$$\mathcal{L} = \sum_i \left(\partial_\mu \Phi_i^\dagger \partial^\mu \Phi_i + t \Phi_i^\dagger \Phi_{i+1} + \frac{1}{4} \epsilon_i \Phi_i^2 + \text{h.c.} + V(\Phi_i) \right)$$

In general all terms could vary randomly; for illustration we randomly vary the ε.

Potential leads to vevs $\quad \Phi_i \to U_i \equiv f e^{i\pi_i/(\sqrt{2}f)}$

$$\to \sum_i \left((\partial_\mu \pi_i)^2 - \frac{1}{2} t (\pi_{i+1} - \pi_i)^2 - \frac{1}{2} \epsilon_i \pi_i^2 + \ldots \right)$$

Looks familiar…

Tight Binding in Theory Space

Mass matrix is of the tight binding form

(Offset from hopping terms doesn't alter story, guarantees positivity)

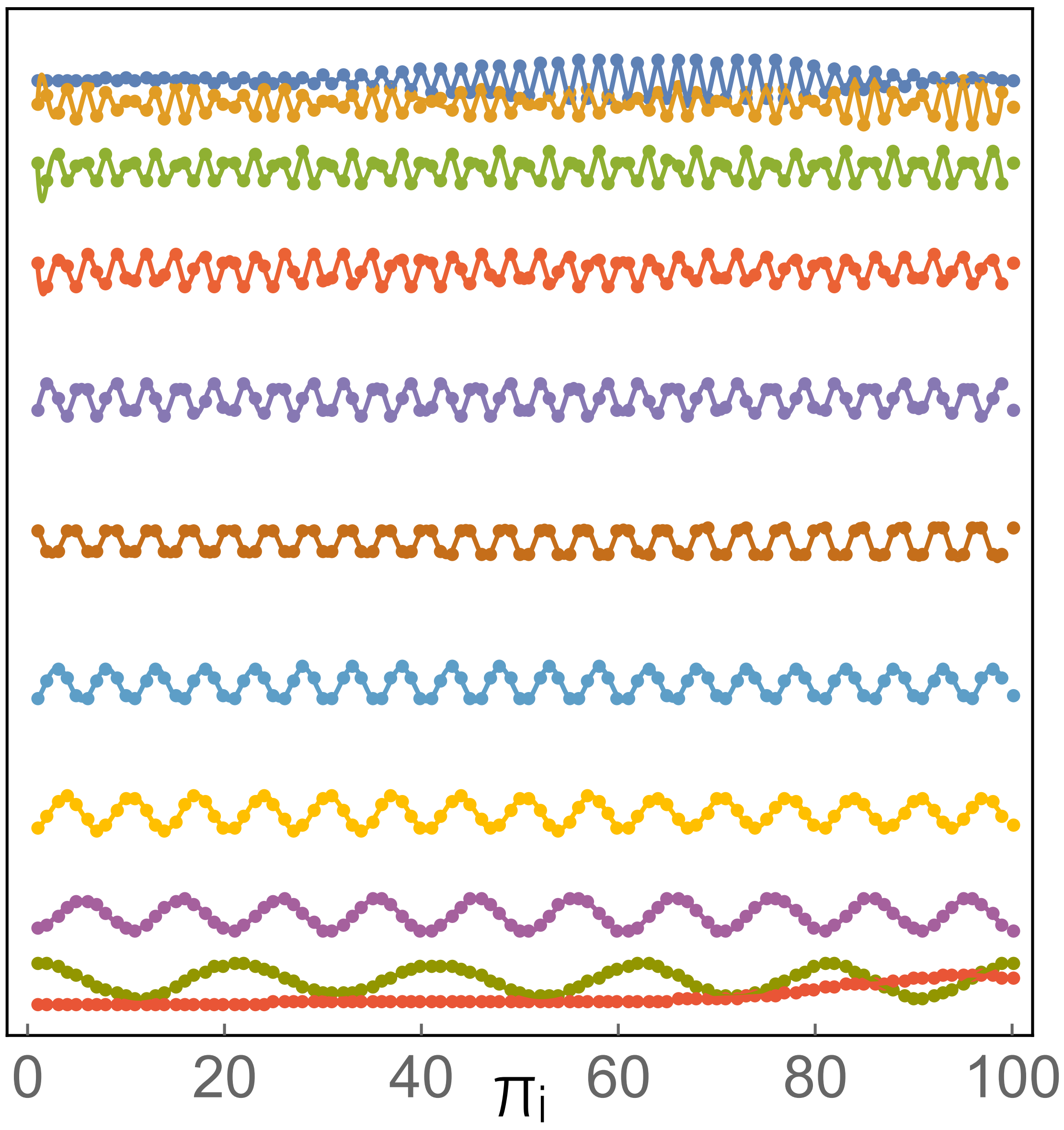
$$M^2 = \begin{pmatrix} t+\epsilon_1 & -t & 0 & \dots & 0 & 0 \\ -t & 2t+\epsilon_2 & -t & \dots & 0 & 0 \\ 0 & -t & 2t+\epsilon_3 & & & \\ \vdots & \vdots & & \ddots & & \\ 0 & 0 & & & 2t+\epsilon_{N-1} & -t \\ 0 & 0 & & & -t & t+\epsilon_N \end{pmatrix}$$

Wavefunctions of the mass eigenstates will be exponentially localized in theory space.

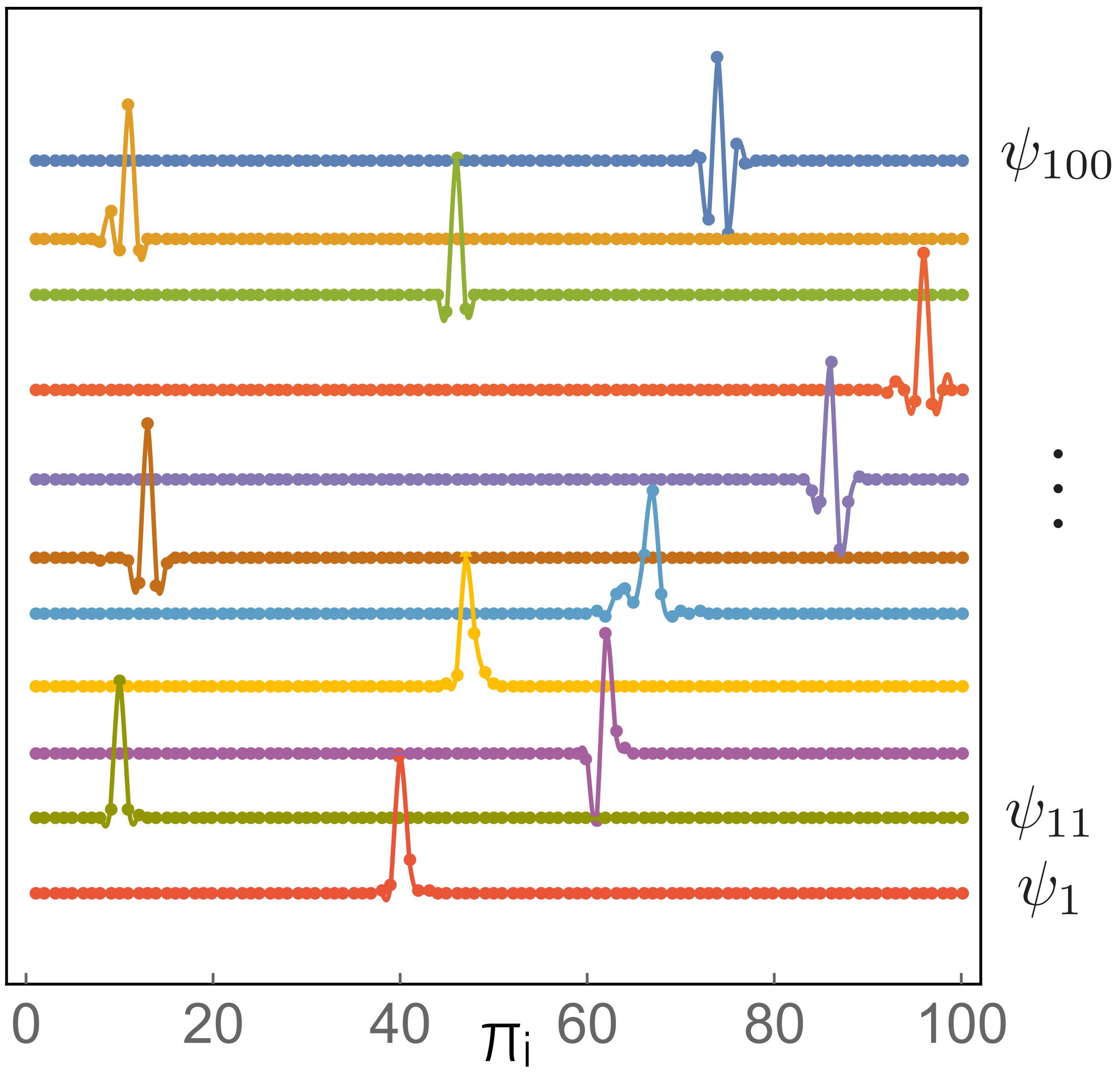
Note: this breaks all symmetries, zero mode light but not protected.

Generalizes naturally to spin-1.

(De)Localization



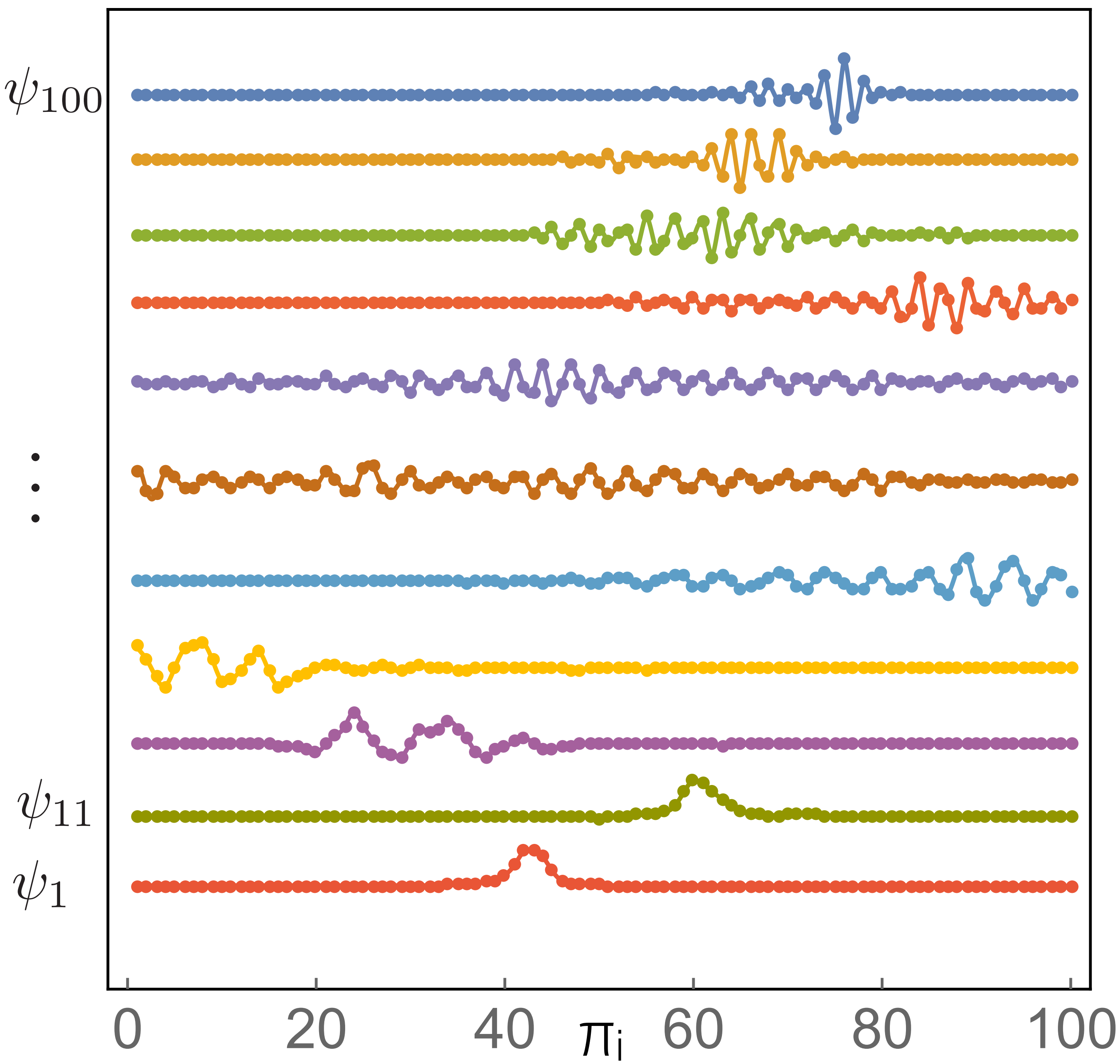
N=100, t=10, $\varepsilon_i \in [0,1]$



N=100, t=0.05, $\varepsilon_i \in [0,1]$

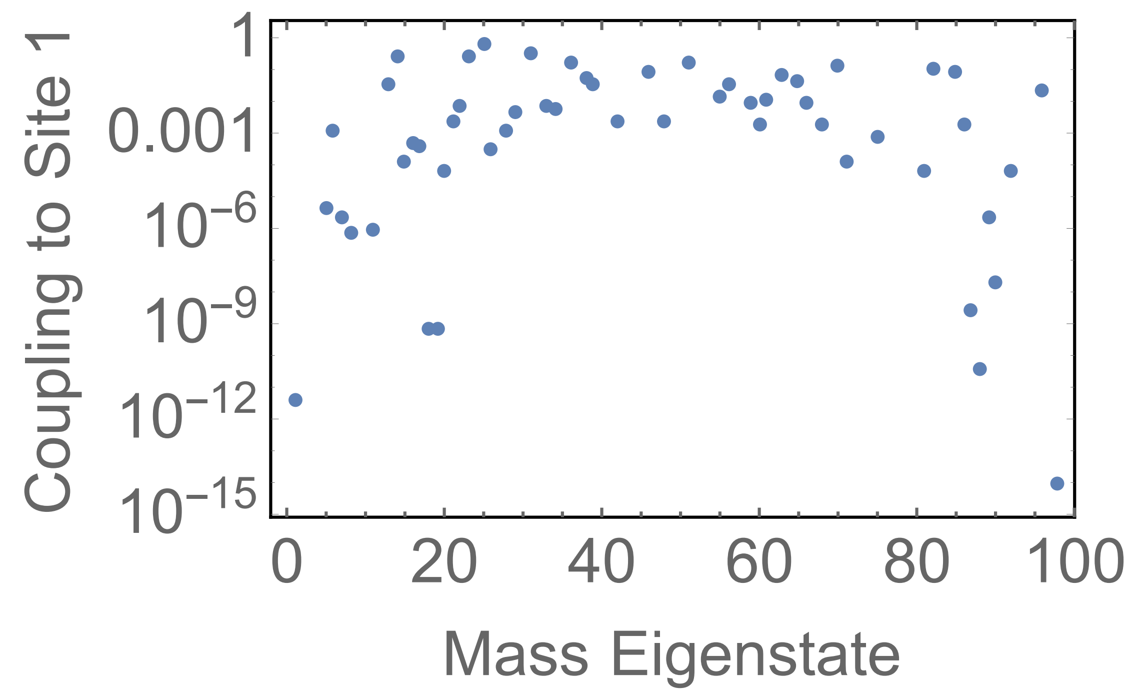
Note anomalous localization at “band edges”, i.e., top & bottom of spectrum @ weak disorder

Order from disorder



$N=100$, $t=0.5$, $\varepsilon_i \in [0,1]$

Exponential profile for lightest eigenstate with short localization length



Mid-band eigenstates have random couplings corresponding to less localization

Many applications…

Fermion toy model

Take *N* LH Weyl fermions & *N* RH Weyl fermions

$$L = (L_1, L_2, \ldots, L_N)^T \qquad R = (R_1, \ldots, R_N)^T$$

$$\mathcal{L} = \text{kinetic terms} - \bar{L}^T M R + \text{h.c.}$$

$$M = \begin{pmatrix} \epsilon_1 & -t & 0 & \ldots & 0 & 0 \\ -t & \epsilon_2 & -t & \ldots & 0 & 0 \\ 0 & -t & \epsilon_3 & & & \\ \vdots & \vdots & & \ddots & & \\ 0 & 0 & & & \epsilon_{N-1} & -t \\ 0 & 0 & & & -t & \epsilon_N \end{pmatrix}$$

Tight-binding model for fermions; zero mode localizes

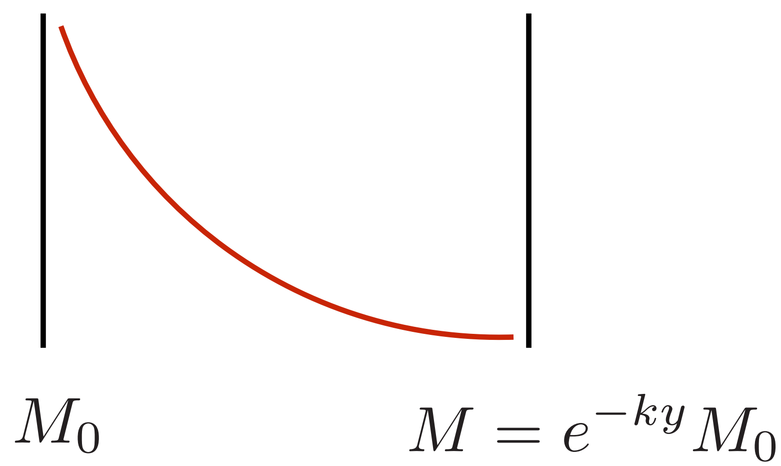
Many possible applications: dark matter, neutrino masses,…

The hierarchy problem?

What you've been asking yourself during this whole talk.

How does RS solve hierarchy problem?
Curvature localizes the graviton zero mode.

→ Fields localized at different points in 5th dimension see different fundamental scales

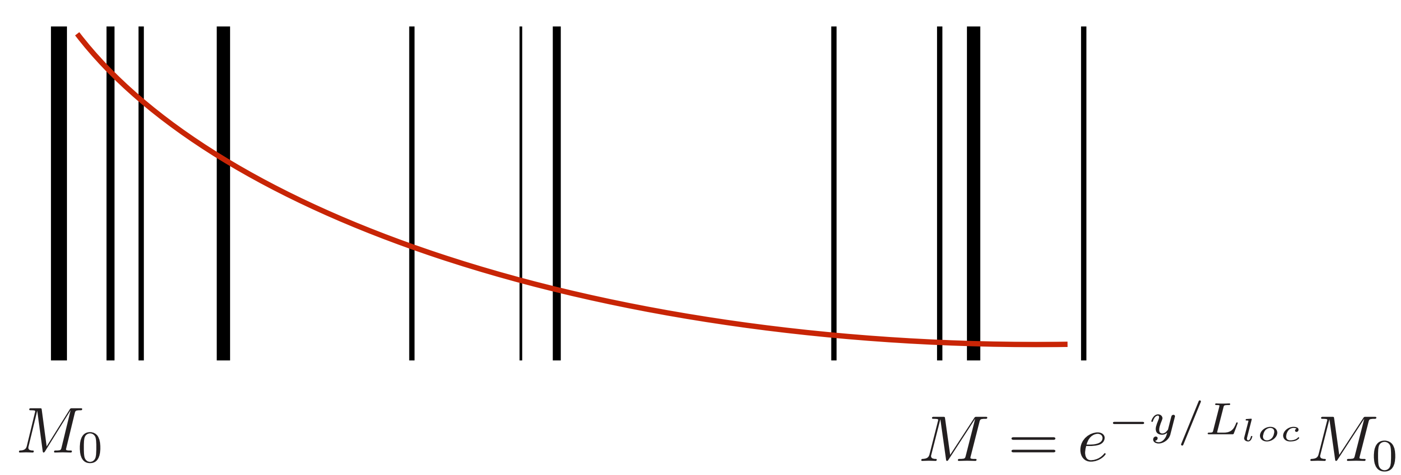


[Rothstein '12]: *Can achieve the same outcome in a flat fifth dimension by localizing graviton w/ disorder*

$$S = -\int d^5x \sqrt{G}(M_\star^3 \mathcal{R}) + \sum_{\langle ij \rangle} M_\star^4 V(|X_i - X_j|) - \sum_i \int d^4x \sqrt{g} f_i$$

In this case disorder = randomly spaced & tensioned branes

The challenge: *naive tight-binding model in theory space does not preserve diffeomorphism invariance*



Conclusions

- Nature hasn't obviously warmed up to our attempts to impose external order on the structure of the Standard Model & BSM.
- Many of the features we seek from geometry or dynamics can be reproduced by disorder in theory space (or disorder in a flat extra dimension).
- A novel source of exponential hierarchies in four dimensions without small or tuned parameters in the fundamental theory.
- The statistical nature of spectrum & couplings leads to diverse experimental signatures; lots of "who ordered that?"
- Much more to think about…

Thank you!