Raffaele Tito D’Agnolo

What’s going on at the weak scale? - Jeju - June 2017

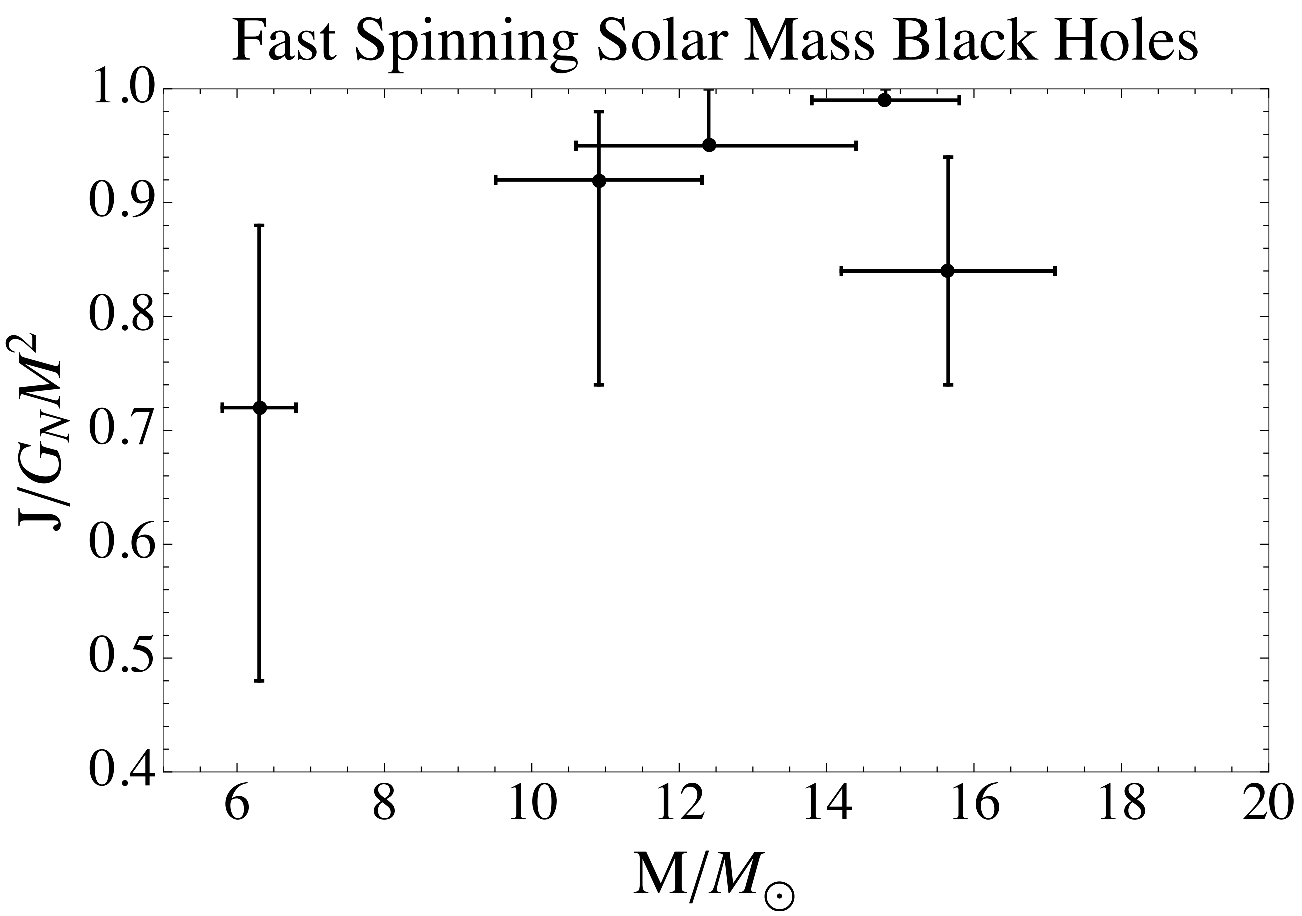
ROTATING BLACK HOLES AND CONFORMAL QUANTUM MECHANICS

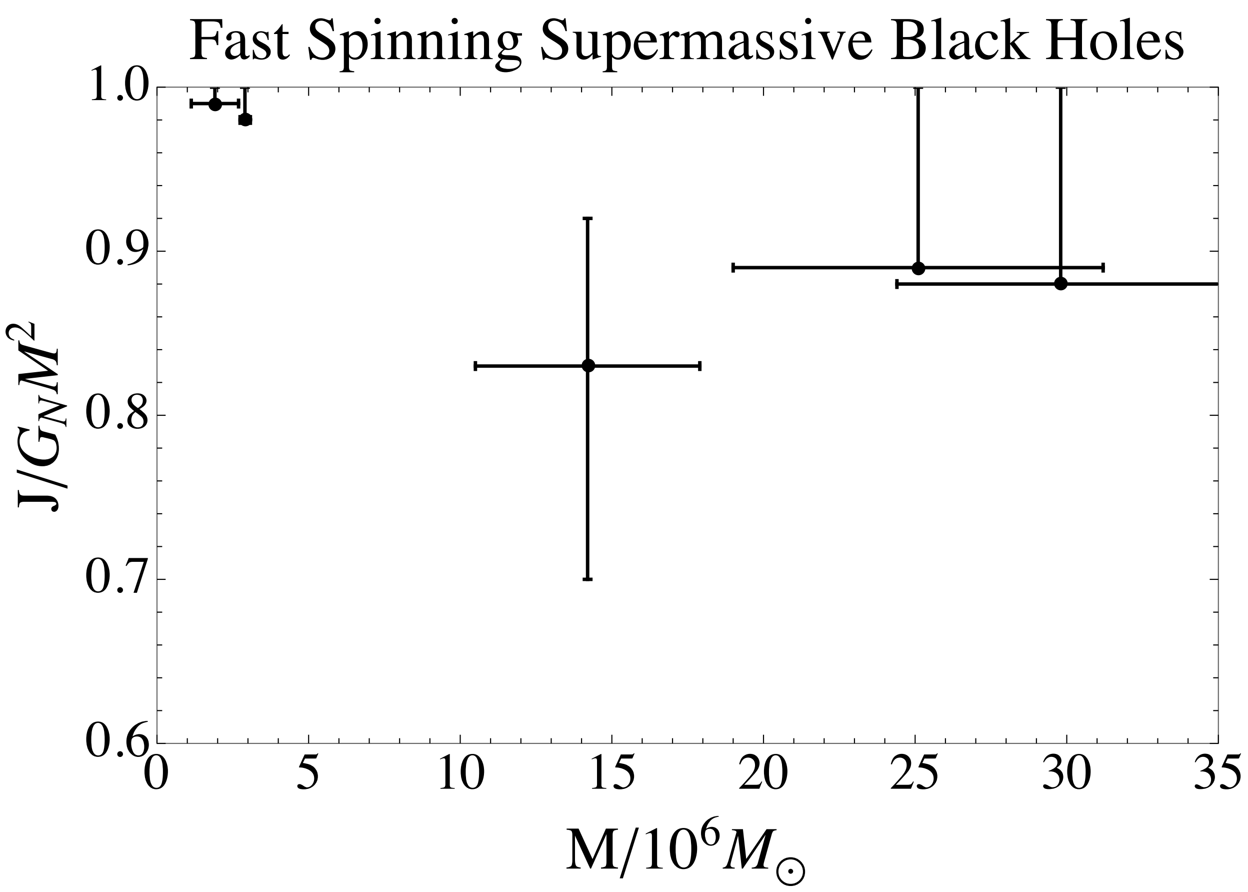
work in progress with
D. Anninos (IAS) and T. Anous (UBC)

OUTLINE

- MOTIVATION
- NEAR THE HORIZON OF EXTREMAL BLACK HOLES
- DISORDERED LARGE-N QUANTUM MECHANICS

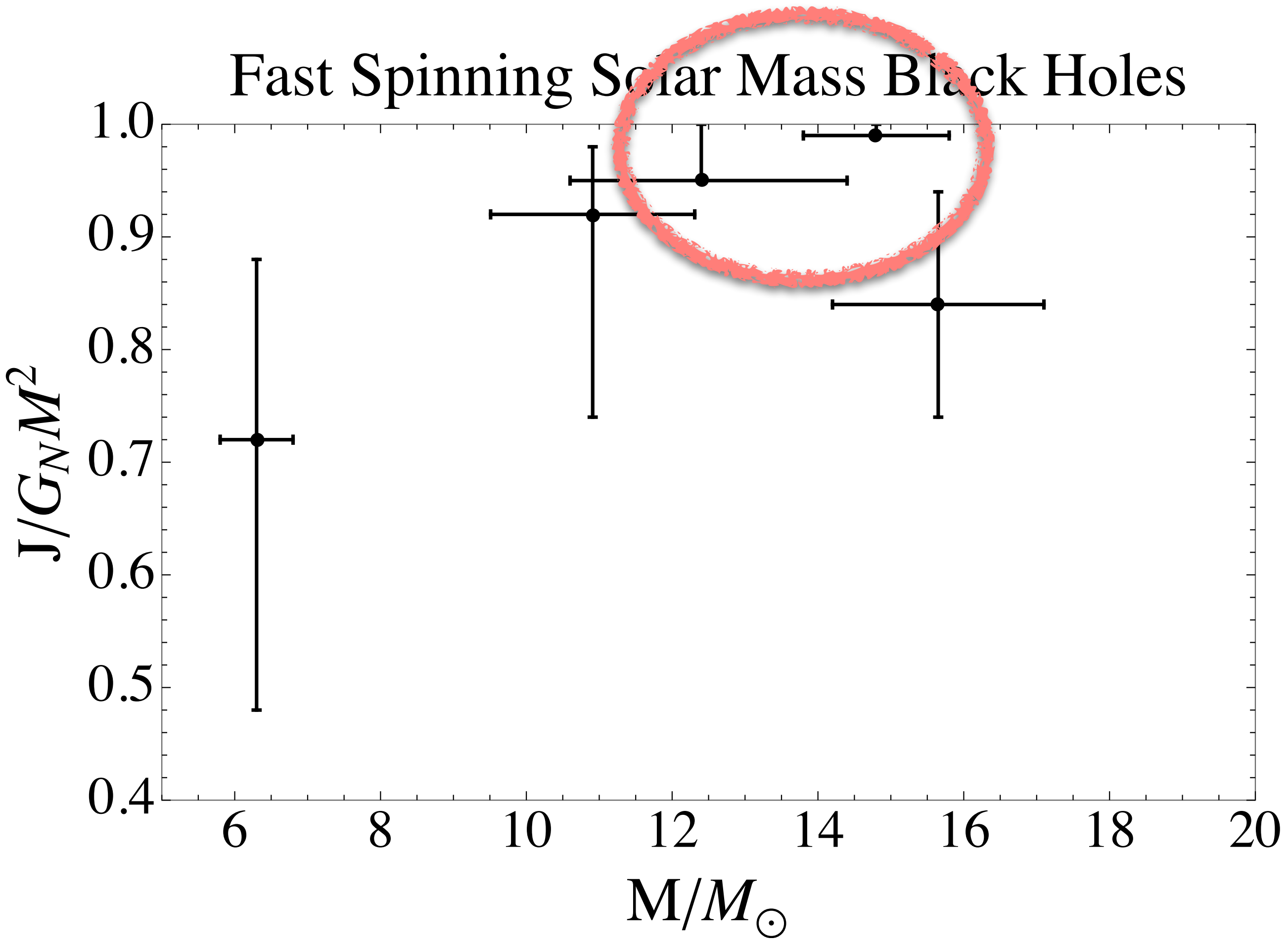
EXTREMAL BLACK HOLES

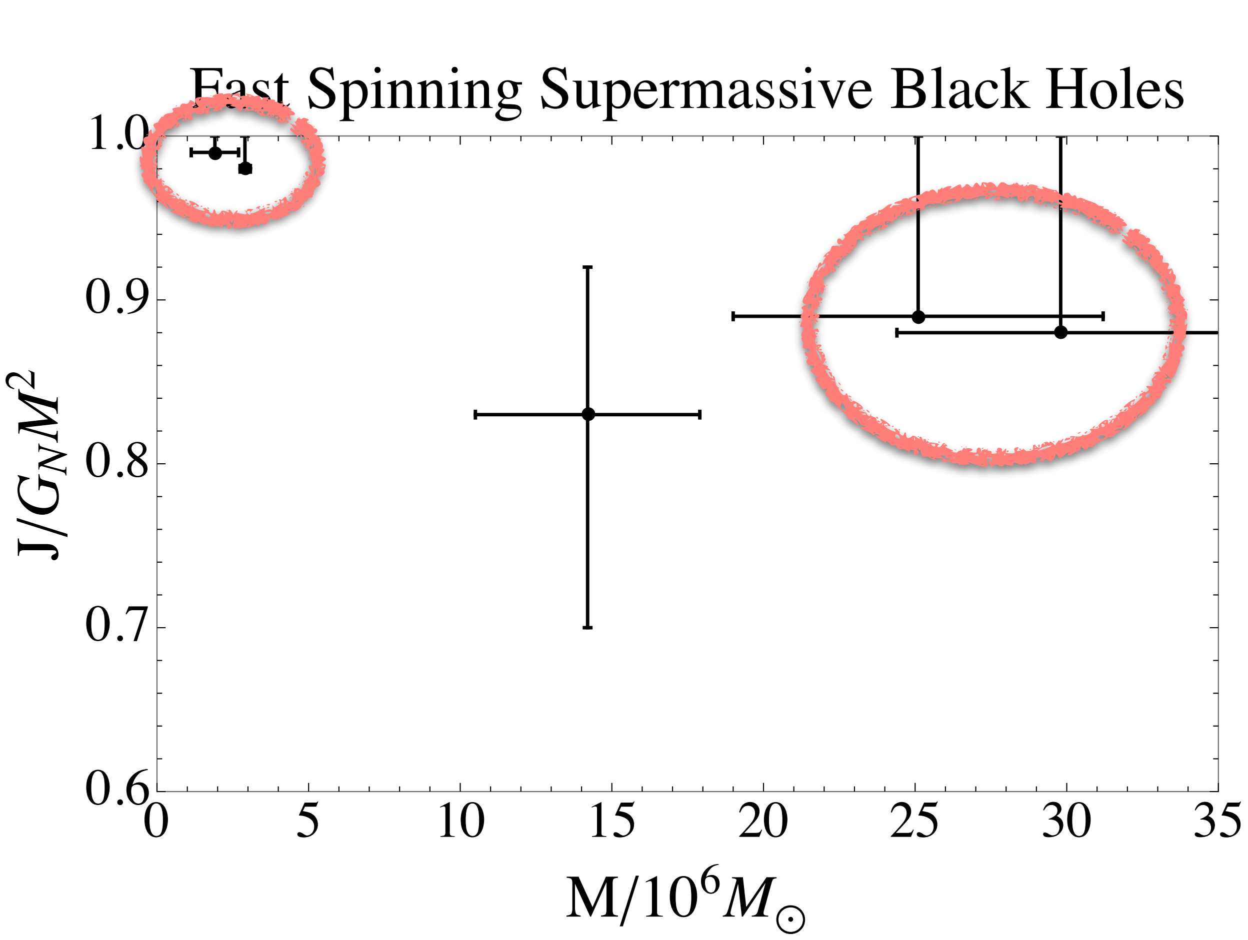




https://arxiv.org/pdf/1704.05081.pdf

EXTREMAL BLACK HOLES





THE MAIN CHARACTERS

4D ASYMPTOTICALLY FLAT

$$M, Q, J$$

$$\hbar = c = G_N = 1$$

’30-’60
Birkhoff
Israel
Carter-Robin
Hawking, Wald

THE MAIN CHARACTERS

$$M, Q, J$$

	$Q = 0$	$Q \neq 0$
STATIC $J = 0$	Schwarzschild $r_+ = 2M$	
STATIONARY $J \neq 0$		

THE MAIN CHARACTERS

$$M, Q, J$$

	$Q = 0$	$Q \neq 0$
STATIC $J = 0$	Schwarzschild $r_+ = 2M$	Reissner-Nordström $r_+ = M + \sqrt{M^2 - Q^2}$
STATIONARY $J \neq 0$	Kerr $r_+ = M + \sqrt{M^2 - J^2/M^2}$	Kerr-Newman $r_+ = M + \sqrt{M^2 - Q^2 - J^2/M^2}$

NEAR HORIZON LIMIT

EXTREMAL RN BLACK HOLE $M = Q$

$$ds^2 = \left(1 - \frac{M}{r}\right)^2 dt^2 - \frac{dr^2}{\left(1 - \frac{M}{r}\right)^2} - r^2 d\Omega^2_{(2)}$$

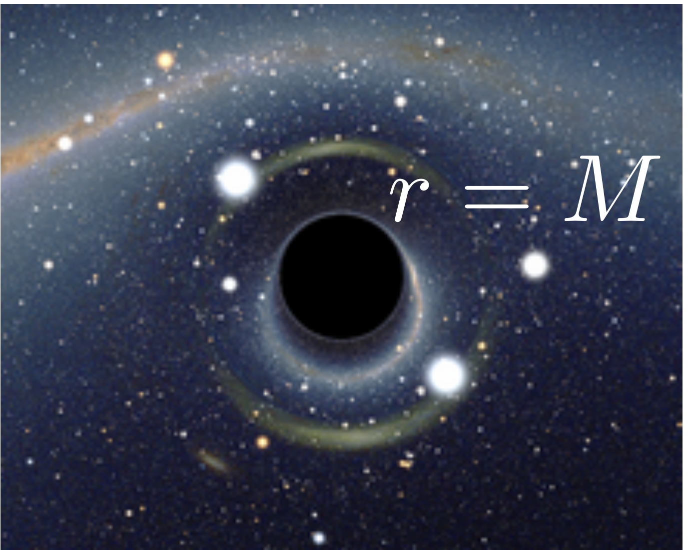
S_2

$SU(2)$

NEAR HORIZON LIMIT

EXTREMAL RN BLACK HOLE $M = Q$

$$ds^2 = \left(1 - \frac{M}{r}\right)^2 dt^2 - \frac{dr^2}{\left(1 - \frac{M}{r}\right)^2} - r^2 d\Omega^2_{(2)}$$

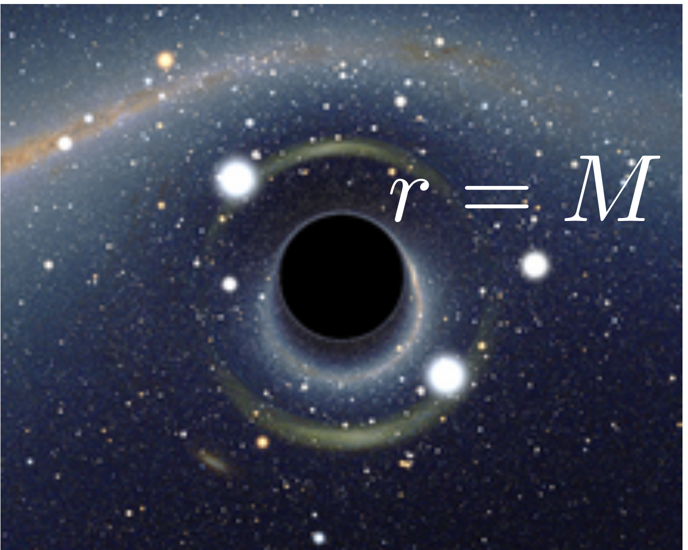


$$r = M\left(1 + \epsilon/z\right), \quad t = MT/\epsilon$$

NEAR HORIZON LIMIT

EXTREMAL RN BLACK HOLE $M = Q$

$$ds^2 = \frac{M^2}{z^2}\left(dT^2 - dz^2\right) - M^2 d\Omega^2_{(2)} + \mathcal{O}(\epsilon)$$



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NEAR HORIZON LIMIT

EXTREMAL RN BLACK HOLE $M = Q$

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$$\mathrm{AdS}_2 \quad \times \quad S_2$$

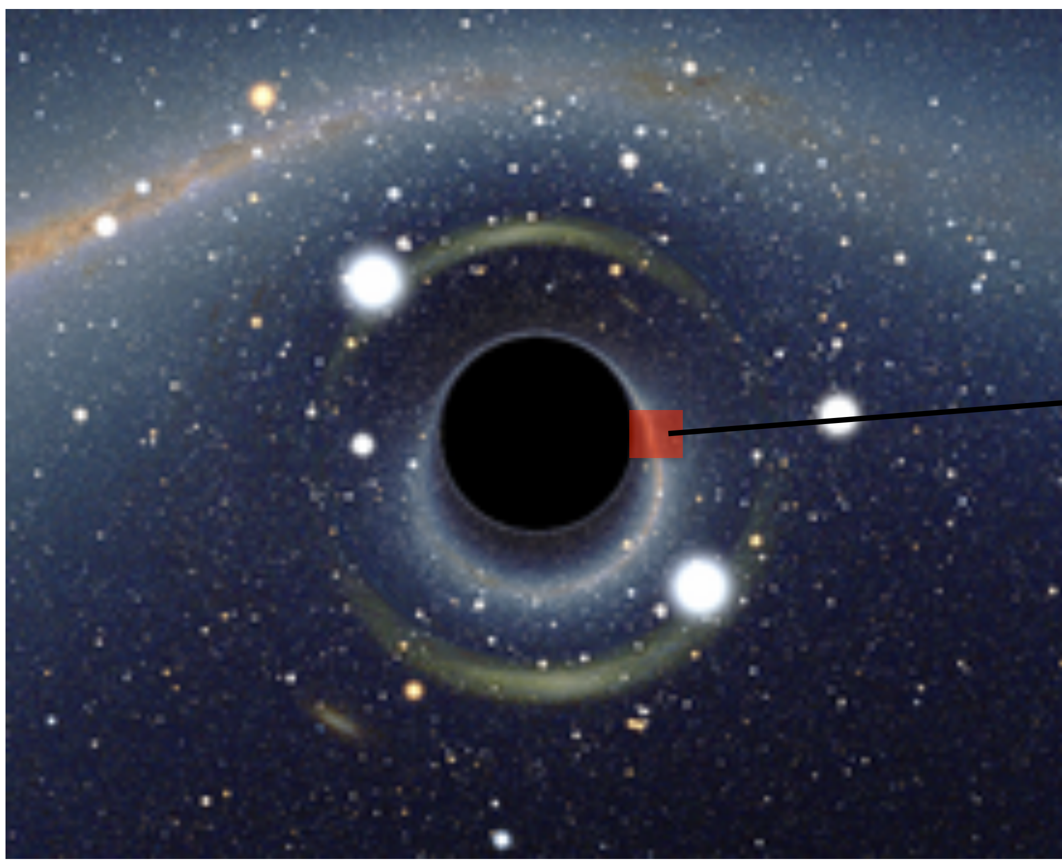
NEAR HORIZON LIMIT

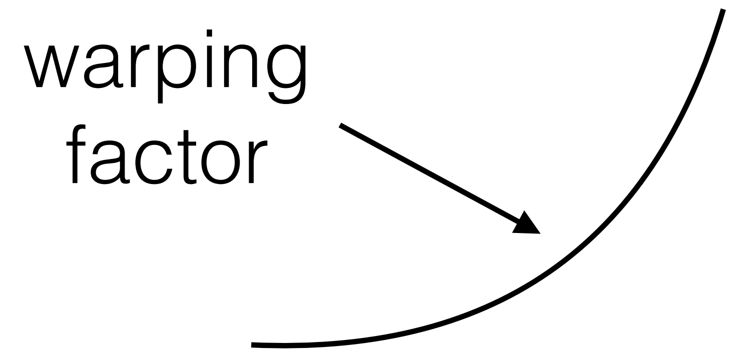
FOR ANY EXTREMAL M, Q, J

SAME SYMMETRIES AS

$$\mathrm{AdS}_2$$

NEAR HORIZON LIMIT





$$\mathrm{AdS}_2$$

AdS boundary $z = 0$

BH Horizon $z = \infty$

N.B.

$$g_{tt} = \left(1 - \frac{M}{r}\right)^2 \to 0$$

$$r \to r_+$$

SYMMETRIES

$$\mathrm{AdS}_2$$

TIME REPARAMETRIZATIONS $\quad T \to f(T) \quad$ BOUNDARY

$SL(2,R)$

$$\partial_T\,,\, z\partial_z + T\partial_T$$

$$\frac{1}{2}\left(T^2 + z^2\right) + Tz\partial_z$$

BULK

SYMMETRIES

$$\mathrm{AdS}_2$$

VIRASORO

TIME REPARAMETRIZATIONS

$$T \to f(T) \qquad f(T)\partial_T = \sum_n f_n T^n \partial_T$$

1D CONFORMAL

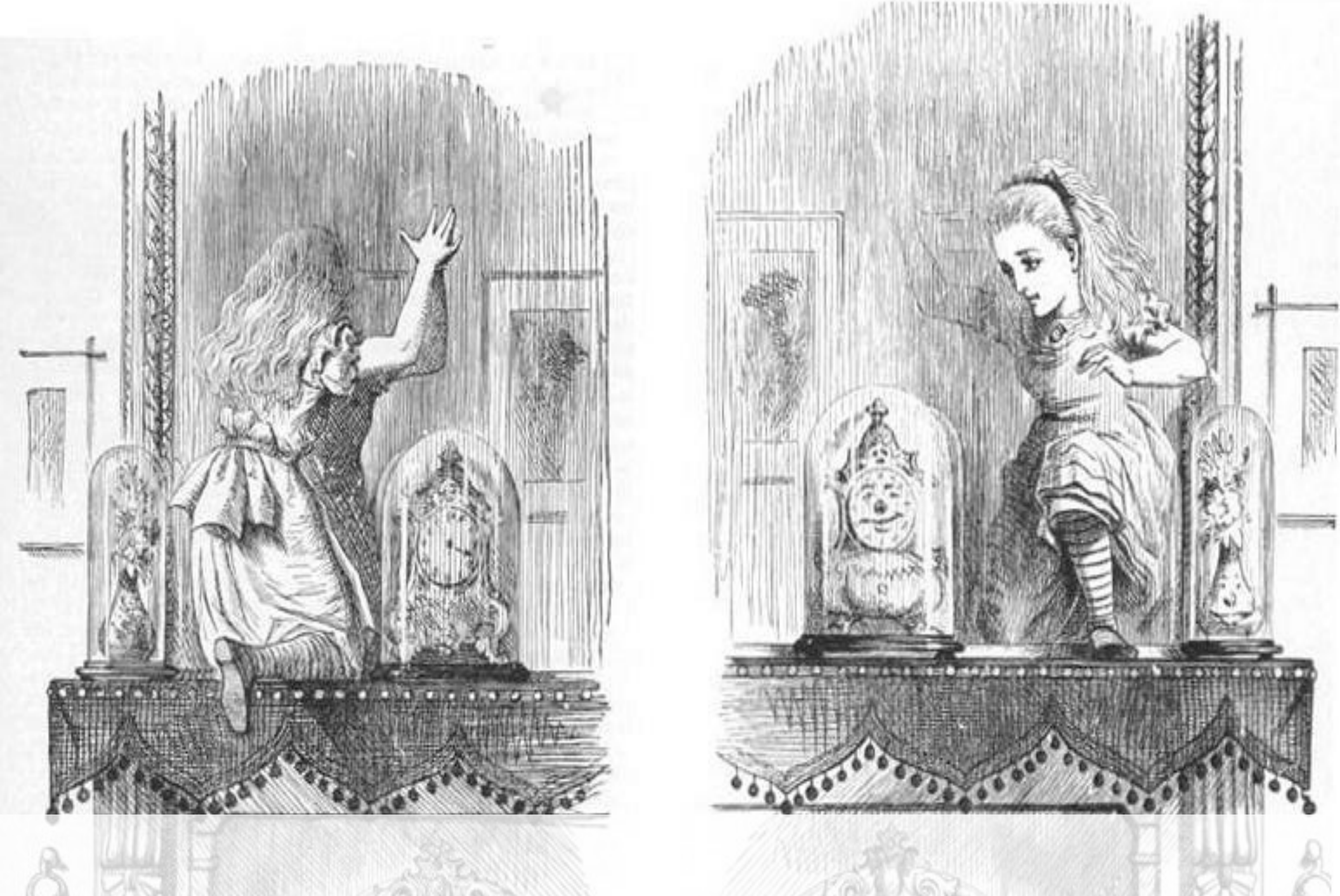
$SL(2,R)$

$$\partial_T,\ z\partial_z + T\partial_T$$

$$\frac{1}{2}\left(T^2 + z^2\right) + Tz\partial_z$$

$$\partial_T,\ T\partial_T,\ T^2\partial_T$$

$$z \to 0$$



THROUGH THE LOOKING GLASS

A SIMPLE QUANTUM MECHANICAL MODEL

$$S_E = \int d\tau \left[\psi^{\dagger,\dot{a}}_\alpha \dot{\psi}^a_\alpha + \left(\frac{\omega_{\alpha\beta}}{2} \psi^a_\alpha \epsilon_{ab} \psi^b_\beta + \text{h.c.} \right) \right]$$

$$\alpha, \beta = 1, ..., N$$

$\dot{a}, a = 1, 2$ global $SU(2)$

$\omega_{\alpha\beta}$ Gaussian variables with

$$\langle \omega \rangle = 0$$

$$\langle \omega^2 \rangle = \frac{\Omega^2}{N}$$

REPLICA TRICK

$$S_E = \int d\tau \left[\psi_\alpha^\dagger \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2} \psi_\alpha \psi_\beta + h.c. \right) \right]$$

At large N we can average over the noise

$$\overline{F} = -T\overline{\log Z} = -T \lim_{n \to 0} \frac{\overline{Z^n} - 1}{n}$$

$$\overline{Z^n} = \int d\omega d\bar{\omega} P(\omega, \bar{\omega}) Z^n$$

NEW VARIABLES

$$S_E = \int d\tau \left[\psi^\dagger_\alpha \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2}\psi_\alpha\psi_\beta + h.c.\right)\right]$$

For convenience

$$S^a_{AB}(\tau,\tau') = \delta_{AB} S^a(\tau,\tau') = \frac{1}{N}\langle \bar{\psi}^{\dot{a}}_\alpha(\tau)\psi^a_\alpha(\tau')\rangle$$

$$1 = \prod_{A,B}\int \mathcal{D}S_{AB}\mathcal{D}\Lambda_{AB} e^{i\int d\tau d\tau' \Lambda_{AB}(\tau,\tau')\left[S_{AB}(\tau,\tau') - \psi^\dagger_{\alpha A}(\tau)\psi_{\alpha B}(\tau')\right]}$$

THE EFFECTIVE LAGRANGIAN

$$S_E = \int d\tau \left[\psi^\dagger_\alpha \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2}\psi_\alpha\psi_\beta + h.c.\right)\right]$$

Large N saddle point

$$\frac{S_E}{Nn} = \sum_a \mathrm{Tr}\log S^a + \int d\tau d\tau' \left[-\frac{\Omega^2}{2} S^1 S^2 + \delta(\tau-\tau')\sum_a \partial_\tau S^a\right]$$

SYMMETRIES

$$\frac{S_E}{Nn} = \sum_a \mathrm{Tr}\log S^a + \int d\tau d\tau' \left[-\frac{\Omega^2}{2} S^1 S^2 + \delta(\tau - \tau') \sum_a \partial_\tau S^a \right]$$

AT LOW ENERGIES

TIME REPARAMETRIZATIONS $\times SU(2)$

$$\tau \to f(\tau) \ , \quad S^a(\tau, \tau') \to f'(\tau)^{1/2} S^a(f(\tau), f(\tau')) f'(\tau')^{1/2}$$

SPONTANEOUSLY BROKEN TO $SL(2,R) \times SU(2)$

$$S^a(\tau, \tau') \approx \frac{1}{\pi\Omega(\tau - \tau')}$$

SYMMETRIES

$$S_E = \int d\tau \left[\psi_\alpha^\dagger \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2}\psi_\alpha\psi_\beta + h.c.\right)\right]$$

NEAR HORIZON BH

LOW ENERGY

$$\tau \to f(\tau)$$

BOUNDARY

$$T \to f(T)$$

SPONTANEOUSLY BROKEN TO

$$SL(2,R)$$

BULK

$$SL(2,R)$$

N.B. True also in SYK

$$H = (i)^{q/2} \sum J_{i_1 i_2 \dots . i_q} \psi_{i_1} \psi_{i_2} \dots \psi_{i_q}$$

SYMMETRIES

$$S_E = \int d\tau \left[\psi^\dagger_\alpha \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2}\psi_\alpha\psi_\beta + h.c.\right)\right]$$

NEAR HORIZON BH

EXPLICIT BREAKING

UV Effects

$\partial_\tau S^a$

Deviations from extremality and leading near horizon corrections

THERMODYNAMICS

$$S_E = \int d\tau \left[\psi^\dagger_\alpha \dot{\psi}_\alpha + \left(\frac{\omega_{\alpha\beta}}{2}\psi_\alpha\psi_\beta + h.c.\right)\right]$$

NEAR HORIZON BH

$$C \sim T$$

$$C \sim T$$

Massless (large N) modes
from the spontaneous breaking of

$$SL(2,R)$$

$$S \propto \int d\tau \, \{f(\tau),\tau\} \qquad \{f(\tau),\tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

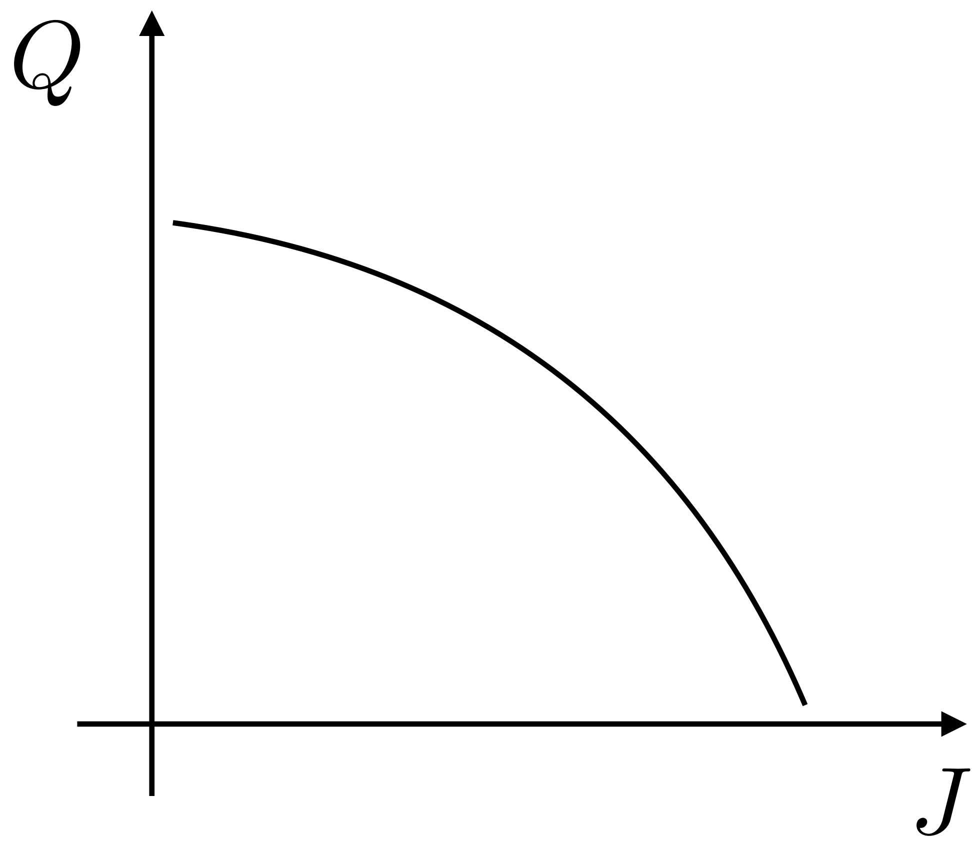
J. Maldacena, D. Stanford arXiv:1606.01857, arXiv:1604.07818

EXTREMALITY CONTINUED

$$M, Q, J$$

	$Q = 0$	$Q \neq 0$
STATIC $J = 0$		
STATIONARY $J \neq 0$		Kerr-Newman $M^2 = Q^2 + \frac{J^2}{M^2}$

EXTREMALITY AND MARGINAL DEFORMATIONS



$$SU(2) \to U(1)$$

$$z \int d\tau\, \bar{\psi}^{\dot{a}}_{\alpha}(\tau) \boldsymbol{\sigma}^{\dot{a}b}_{z} \psi^{b}_{\alpha}(\tau)$$

MARGINAL DEFORMATIONS

$$z \int d\tau \, \bar{\psi}^{\dot{a}}_{\alpha}(\tau) \boldsymbol{\sigma}_z^{\dot{a}b} \psi^b_\alpha(\tau)$$

$$z < 2\Omega$$

$$S^1(\tau,\tau') = \frac{1}{\Omega} \frac{\sqrt{1-\left(\frac{z}{2\Omega}\right)^2}}{\pi(\tau-\tau')} + \text{contact}$$

$$z > 2\Omega$$

$$S^1(\tau,\tau') \propto \delta(\tau-\tau')$$

EXTREMALITY AND MARGINAL DEFORMATIONS

$$z \int d\tau \, \bar{\psi}^{\dot{a}}_{\alpha}(\tau) \boldsymbol{\sigma}^{\dot{a}b}_{z} \psi^{b}_{\alpha}(\tau)$$

$$z < 2\Omega$$

$$J = M\sqrt{M^2 - Q^2}$$

Continuum of CTFs (N.B. large N) ~
1 parameter family of extremal BHs

$$z > 2\Omega$$

$$J > M\sqrt{M^2 - Q^2}$$

BH solution ceases to exist ~
Gapped fermions that saturate J
(all “spins” are aligned)

CONCLUSION

- SEVERAL ROTATING BLACK HOLES CONSISTENT WITH EXTREMALITY HAVE BEEN OBSERVED IN THE UNIVERSE
- THERE ARE NON TRIVIAL, FASCINATING ANALOGIES BETWEEN EXTREMAL BLACK HOLES IN 4D AND SIMPLE LARGE-N QUANTUM MECHANICAL MODELS
- THE CORRESPONDENCE IS STILL FAR FROM BEING FORMULATED PRECISELY, BUT WE MIGHT BE ON THE RIGHT PATH

BACKUP

NON-ANALYTIC BEHAVIOR AT SMALL TEMPERATURE

$$z < 2\Omega$$

$$\log Z = \frac{N\pi}{6\Omega^2\beta}\sqrt{4\Omega^2 - z^2} + \ldots$$

$$J < M\sqrt{M^2 - Q^2}$$

$$F = \frac{Q}{s+1}\left[\sqrt{\frac{(s+1)(s+3)^3}{32}} + \mathcal{O}(T)\right]$$

$$s \equiv \sqrt{1 - 8\Omega_H^2 Q^2}$$

NON-ANALYTIC BEHAVIOR AT ZERO TEMPERATURE

$$T = 0 \qquad \frac{1}{N}\frac{d^2}{dz^2}\langle \hat{J}_z \rangle_{\beta=\infty} = -\frac{z}{\Omega^2\sqrt{4\Omega^2 - z^2}}$$

$$J = M\sqrt{M^2 - Q^2} \qquad \log Z_0 = \frac{Q^2}{2}\sqrt{4\pi^2 - \beta_L^2}$$

Chemical potential for J at extremality

EXTREMAL BLACK HOLES

$$M, Q, J$$

	$Q = 0$	$Q \neq 0$
STATIC $J = 0$	$M \geq 0$	$M \geq Q$
STATIONARY $J \neq 0$	$M^2 \geq J$	$M^2 \geq Q^2 + \frac{J^2}{M^2}$