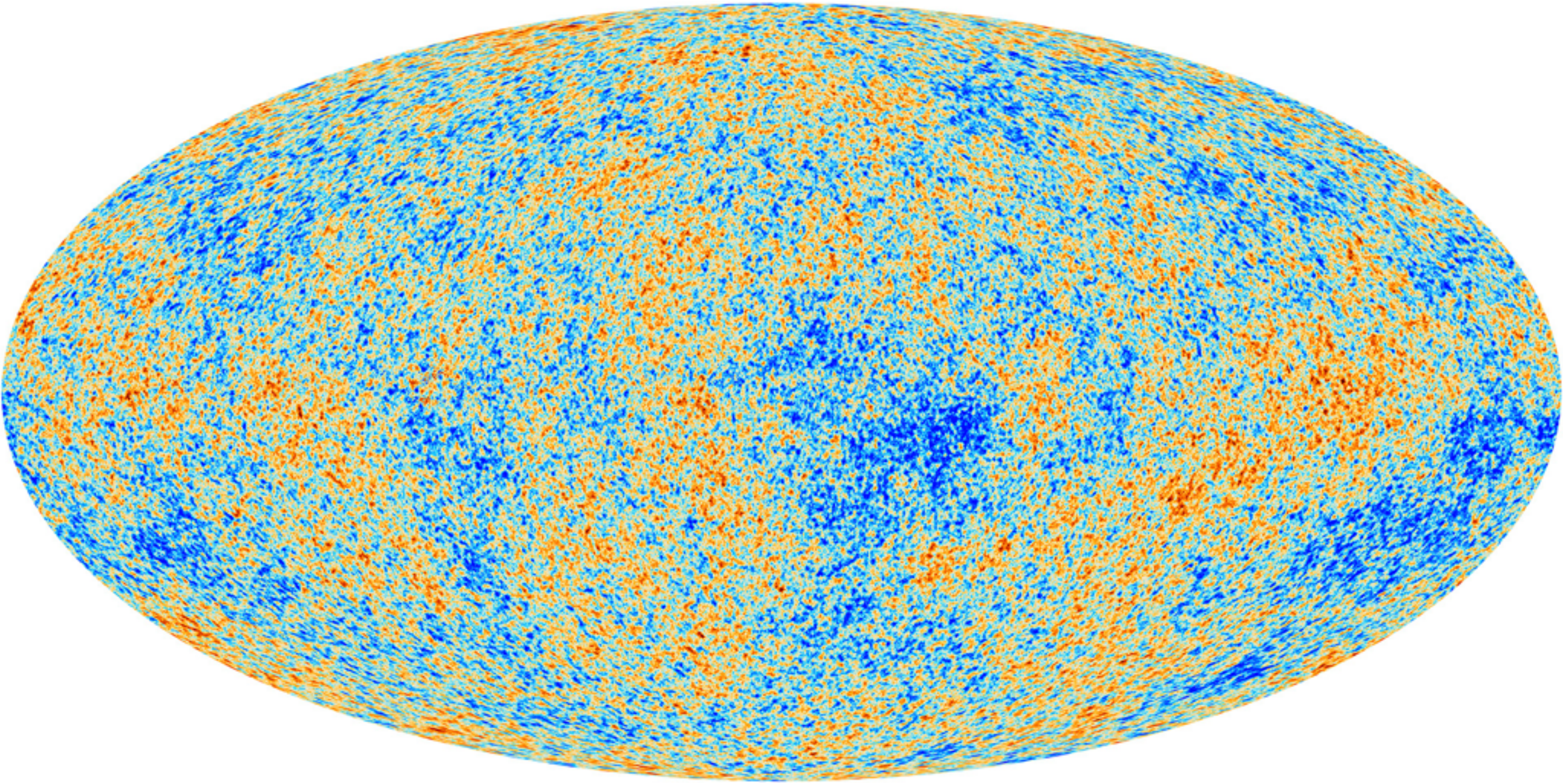
Strongly interacting massive particle with ALP

Hyungjin Kim

KAIST & IBS-CTPU

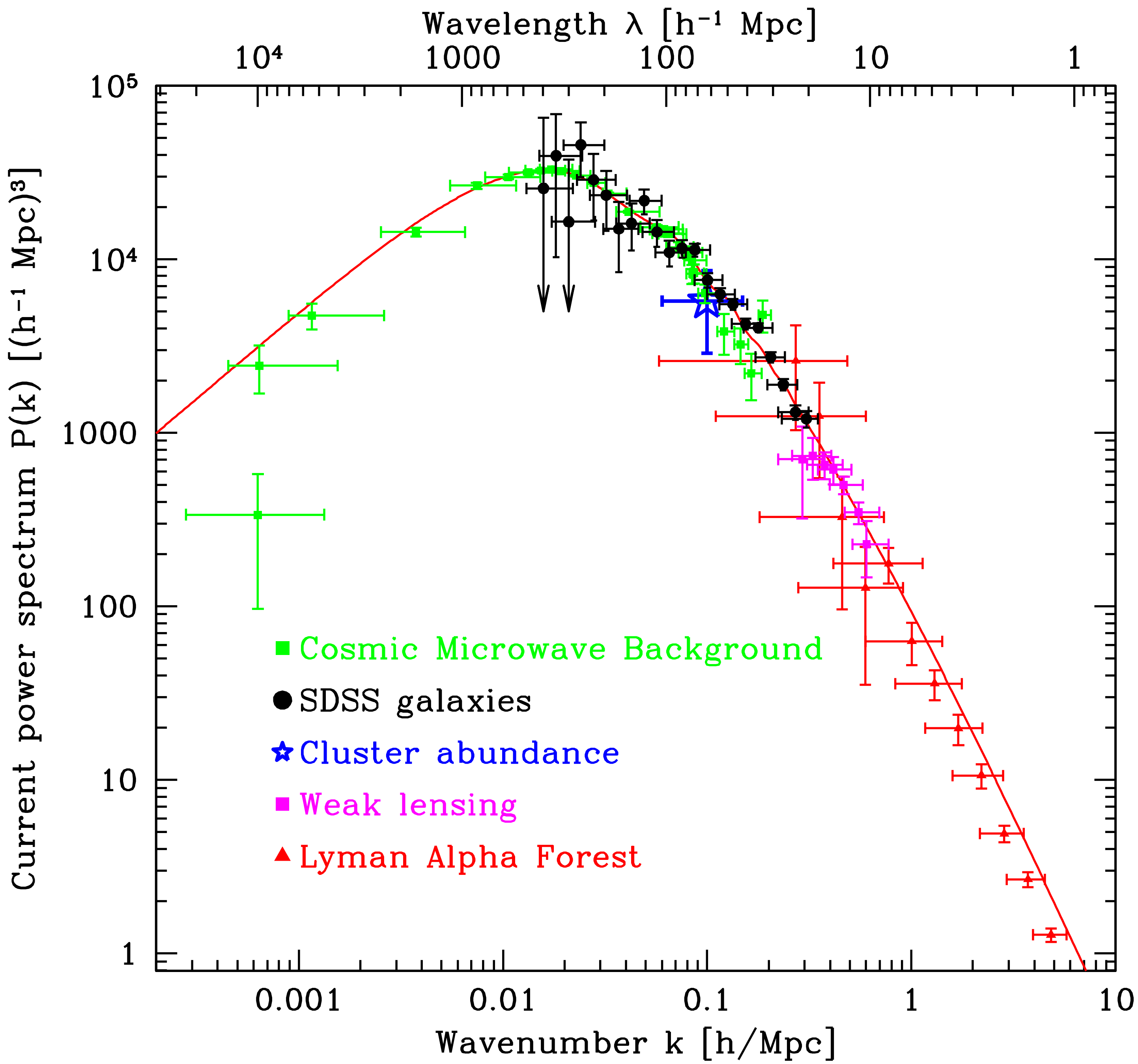
based on 1704.04505
with A. Kamada & T. Sekiguchi

June 2017 @ CERN-CKC Workshop



[Planck Collaboration, 2013]

CDM is successfully on large scale



[Tegmark *et al.*, 2003]

However, on smaller scales

- *core-cusp problem*
- *missing satellite problem*
- *Too big to fail problem*
- *unexpected diversity problem*

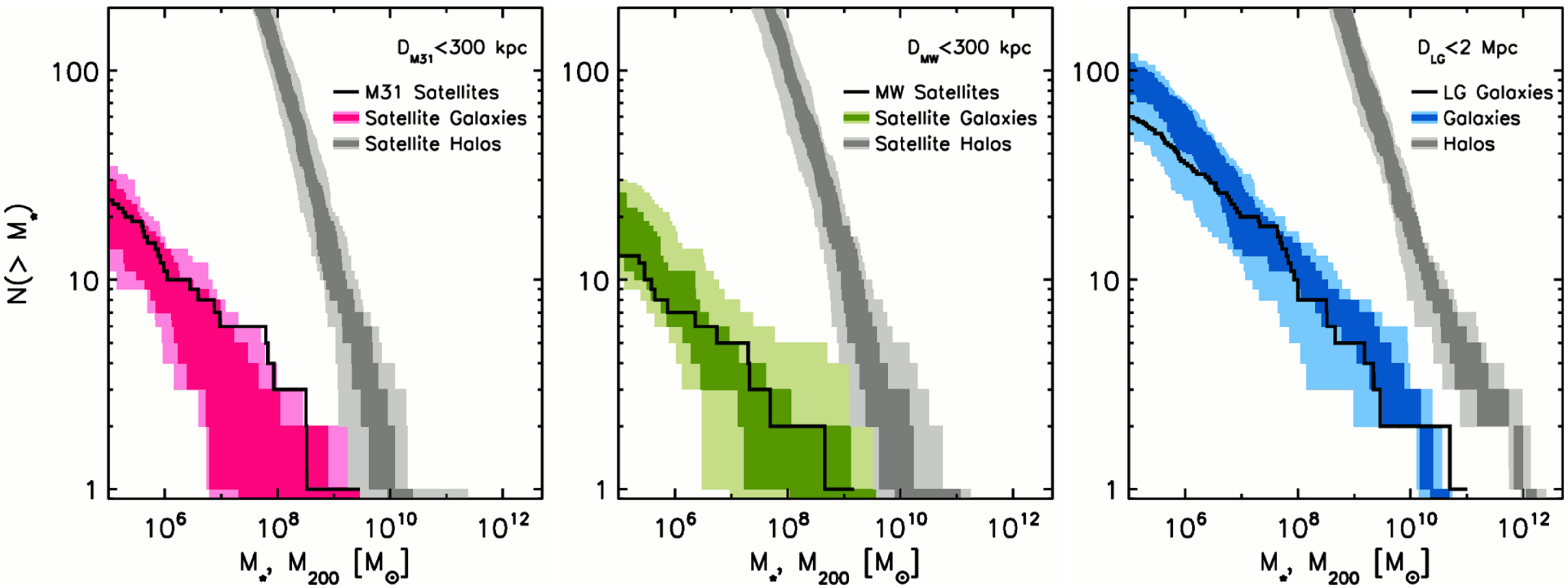
*discrepancies between **CDM**-simulation and **observations***

these small scale issues might be attributed to

baryonic effects *(star formation, SN feedback, … …)*

these small scale issues might be attributed to

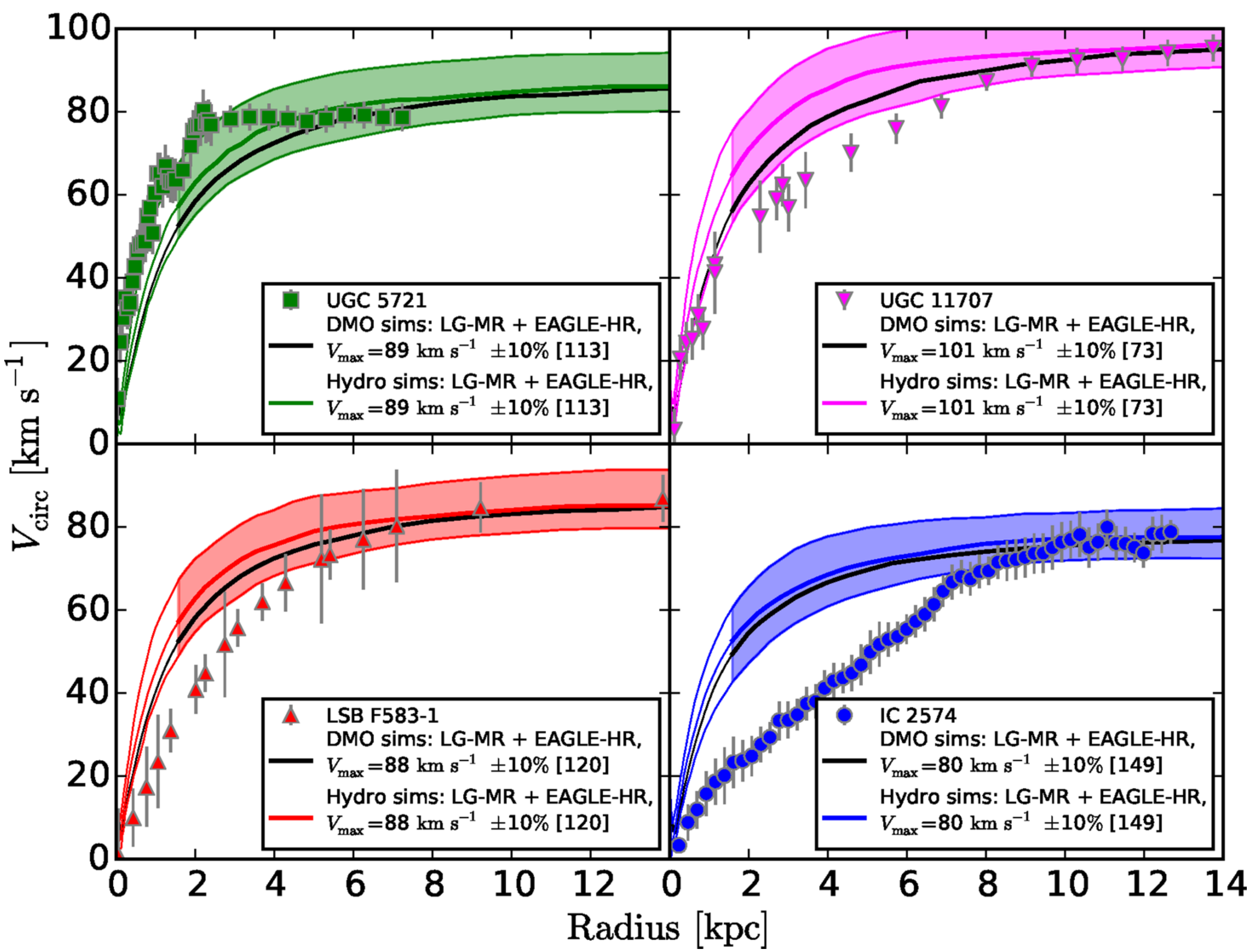
baryonic effects *(star formation, SN feedback, … …)*



[Sawala et al., 15]

these small scale issues might be attributed to

baryonic effects *(star formation, SN feedback, … …)*



[Oman et al., 15]

these

baryonic effects

or may be hinting to alternatives of CDM

these

baryonic effects

or may be hinting to alternatives of CDM

- *warm dark matter*

these

baryonic effects

or may be hinting to alternatives of CDM

- *warm dark matter*
- *fuzzy dark matter (FDM)* [Hu *et al.*, 00]

these

baryonic effects

or may be hinting to alternatives of CDM

- *warm dark matter*
- *fuzzy dark matter (FDM)* [Hu *et al.*, 00]
- *self-interacting dark matter (SIDM)* [Hall *et al.*, 92] [Spergel and Steinhardt, 01]

these

baryonic effects

or may be hinting to alternatives of CDM

- *warm dark matter*
- *fuzzy dark matter (FDM)* [Hu *et al.*, 00]
- *self-interacting dark matter (SIDM)* [Hall *et al.*, 92] [Spergel and Steinhardt, 01]

these

baryonic effects

or may be hinting to alternatives of CDM

- *warm dark matter*
- *fuzzy dark matter (FDM)* **[Hu *et al.*, 00]**
- *self-interacting dark matter (SIDM)* **[Hall *et al.*, 92]** **[Spergel and Steinhardt, 01]**

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\rm self}/m \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

[For recent review, Tulin and Yu, 17]

Large self-interaction is required:

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

What particle physics models can realize this?

Large self-interaction is required:

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

What particle physics models can realize this?

*An option : **Strongly Interacting Massive Particle (SIMP)***

[Hochberg *et al.*, 14]

Consider QCD-like hypercolor dynamics of $SU(N_c)$

with $SU(N_f) \times SU(N_f)$ *flavour symmetry*

$$\mathcal{L} = \bar{q}_i i \not{D} q_i - m_q \bar{q}_i q_i$$

fermion condensation forms at some scale

$$q_{Li} q^{\dagger}_{Rj} = \mu^3 U_{ij}$$

with a matrix of Goldstone bosons

$$U = \exp\left[2 i \pi^a T^a / f_\pi\right]$$

dark meson; dark matter

Effective description of meson states

$$\mathcal{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger) + \mathcal{L}_{\mathrm{WZW}}$$

Expanding them, we find

Effective description of meson states

$$\mathcal{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger) + \mathcal{L}_{\mathrm{WZW}}$$

Expanding them, we find

$$\mathcal{L}_{\mathrm{self}} \supset \frac{1}{f_\pi^2}(\partial\pi)^2(\pi)^2, \quad \frac{m_\pi^2}{f_\pi^2}(\pi)^4$$

*(**self-interaction**; responsible for small scale issues)*

Effective description of meson states

$$\mathcal{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger) + \mathcal{L}_{\mathrm{WZW}}$$

Expanding them, we find

$$\mathcal{L}_{\mathrm{self}} \supset \frac{1}{f_\pi^2}(\partial\pi)^2(\pi)^2, \quad \frac{m_\pi^2}{f_\pi^2}(\pi)^4$$

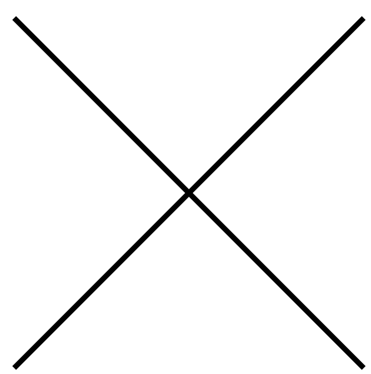
*(**self-interaction**; responsible for small scale issues)*

$$\mathcal{L}_{\mathrm{WZW}} \supset \frac{\epsilon^{\mu\nu\rho\sigma}}{f_\pi^5}(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi)$$

*(**number-changing process**; responsible for relic density)*

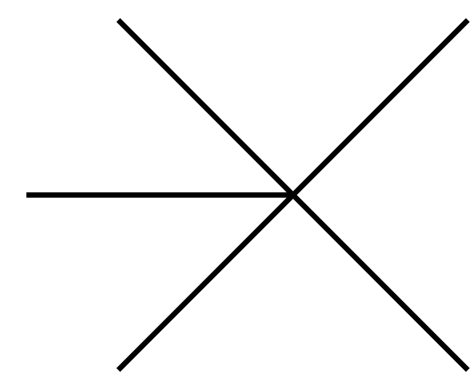
There are two relevant processes

self-interaction



$$\mathcal{L}_{\rm self} \supset \frac{1}{f_\pi^2}(\partial\pi)^2(\pi)^2, \quad \frac{m_\pi^2}{f_\pi^2}(\pi)^4$$

3-to-2 process



$$\mathcal{L}_{\rm WZW} \supset \frac{\epsilon^{\mu\nu\rho\sigma}}{f_\pi^5}(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi)$$

These interactions should be able to … …

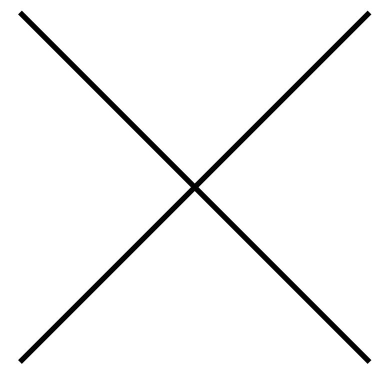
(I) predict the correct relic abundance

$$\Omega_{\rm CDM} h^2 \simeq 0.12$$

(II) provide sufficient self-scattering cross section

$$0.1\,{\rm cm}^2/{\rm g} \lesssim \sigma_{\rm self}/m_\pi \lesssim 1\,{\rm cm}^2/{\rm g}$$

self-interaction



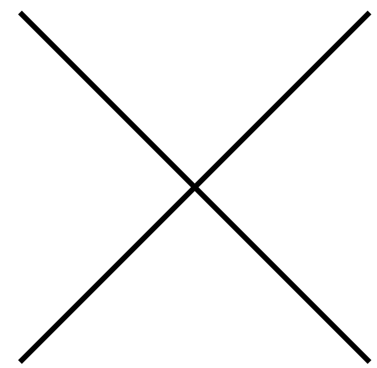
$$\mathcal{L}_{\rm self} \supset \frac{1}{f_\pi^2}(\partial\pi)^2(\pi)^2, \quad \frac{m_\pi^2}{f_\pi^2}(\pi)^4$$

cross-section

$$\sigma_{\rm self}/m_\pi \sim \alpha_\pi^4/m_\pi^3$$

with $\alpha_\pi \equiv (m_\pi/f_\pi)$

self-interaction



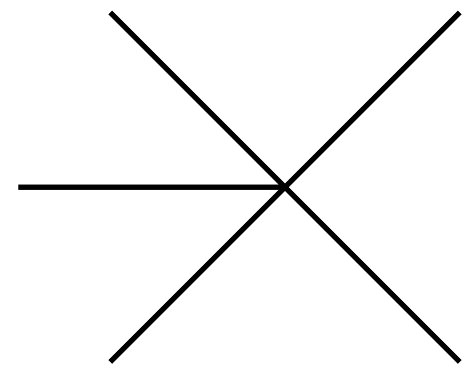
$$\mathcal{L}_{\rm self} \supset \frac{1}{f_\pi^2}(\partial\pi)^2(\pi)^2, \quad \frac{m_\pi^2}{f_\pi^2}(\pi)^4$$

cross-section

$$\sigma_{\rm self}/m_\pi \sim 1\,{\rm cm}^2/{\rm g}\left(\frac{\alpha_\pi}{5}\right)^4\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^3$$

with $\alpha_\pi \equiv (m_\pi/f_\pi)$

3-to-2 process



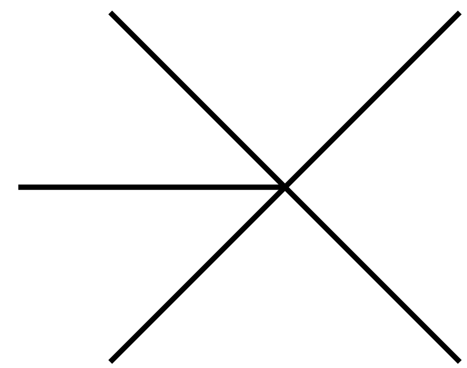
$$\mathcal{L}_{\rm WZW} \supset \frac{\epsilon^{\mu\nu\rho\sigma}}{f_\pi^5}(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi)$$

cross-section

$$(\sigma_{3\to 2}v^2) \sim \alpha_\pi^{10}/m_\pi^5$$

with $\alpha_\pi \equiv (m_\pi/f_\pi)$

3-to-2 process



$$\mathcal{L}_{\rm WZW} \supset \frac{\epsilon^{\mu\nu\rho\sigma}}{f_\pi^5}(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi)$$

cross-section

$$n_\pi\langle\sigma_{3\to 2}v^2\rangle|_{T_{\rm fo}} \sim 3\cdot 10^{-26}\,{\rm cm}^3/{\rm sec}\left(\frac{\alpha_\pi}{5}\right)^5\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^5$$

with $\alpha_\pi \equiv (m_\pi/f_\pi)$

self-scattering cross-section

$$\sigma_{\rm self}/m_\pi \sim 1\,{\rm cm}^2/{\rm g}\left(\frac{\alpha_\pi}{5}\right)^4\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^3$$

3 to 2 cross-section

$$n_\pi\langle\sigma_{3\to 2}v^2\rangle|_{T_{\rm fo}} \sim 3\cdot 10^{-26}\,{\rm cm}^3/{\rm sec}\left(\frac{\alpha_\pi}{5}\right)^5\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^5$$

self-scattering cross-section

$$\sigma_{\rm self}/m_\pi \sim 1\,{\rm cm}^2/{\rm g}\left(\frac{\alpha_\pi}{5}\right)^4\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^3$$

3 to 2 cross-section

$$n_\pi\langle\sigma_{3\to2}v^2\rangle|_{T_{\rm fo}} \sim 3\cdot10^{-26}\,{\rm cm}^3/{\rm sec}\left(\frac{\alpha_\pi}{5}\right)^5\left(\frac{400\,{\rm MeV}}{m_\pi}\right)^5$$

This model is controlled by two parameters

$$\alpha_\pi = (m_\pi/f_\pi) \quad \& \quad m_\pi$$

Assuming the standard freeze-out and taking

$$\alpha_\pi = (m_\pi / f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

(II) provide sufficient self-scattering cross section

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

Assuming the standard freeze-out and taking

$$\alpha_\pi = (m_\pi / f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

(II) provide sufficient self-scattering cross section

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

Assuming the standard freeze-out and taking

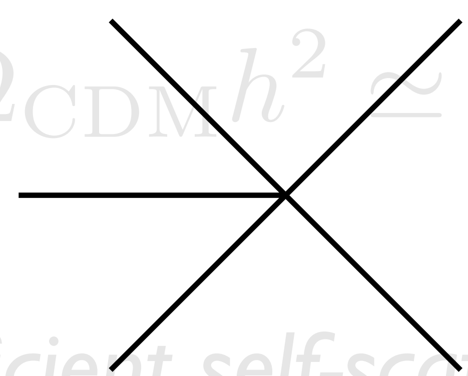
$$\alpha_\pi = (m_\pi / f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

Not a valid assumption because

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$



$$T_d \neq T$$

$$T_d \propto \ln^{-1} a$$

(II) provide sufficient self-scattering cross section

[Carlson, Machacek and Hall, 92]

heats up DM particles

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

Assuming the standard freeze-out and taking

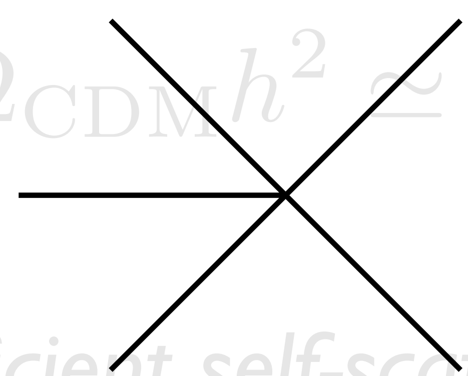
$$\alpha_\pi = (m_\pi/f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

Not a valid assumption because

$$\Omega_{\rm CDM} h^2 \simeq 0.12$$



$$T_d \neq T$$

$$T_d \propto \ln^{-1} a$$

(II) provide sufficient self-scattering cross section

[Carlson, Machacek and Hall, 92]

heats up DM particles

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\rm self}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

sensible only when SIMP is in kinetic equilibrium with SM

Assuming the standard freeze-out and taking

$$\alpha_\pi = (m_\pi / f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

(II) provide sufficient self-scattering cross section

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

Assuming the standard freeze-out and taking

$$\alpha_\pi = (m_\pi/f_\pi) \simeq 5$$

$$m_\pi \simeq 400\,\mathrm{MeV}$$

(I) predict the correct relic abundance

Too large for the effective Lagrangian …

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

$$\alpha_\pi = (m_\pi/f_\pi) \lesssim (\Lambda/f_\pi) \sim 2\pi$$

(II) provide sufficient self-scattering cross section

$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

Assuming the standard freeze-out and taking

$$\alpha_\pi = (m_\pi / f_\pi) \simeq 8$$

$$m_\pi \simeq 600\,\mathrm{MeV}$$

(i) kinetic equilibrium between SIMP and SM

(I) predict the correct relic abundance

$$\Omega_{\mathrm{CDM}} h^2 \simeq 0.12$$

(ii) perturbativity issue

(II) provide sufficient self-scattering cross section

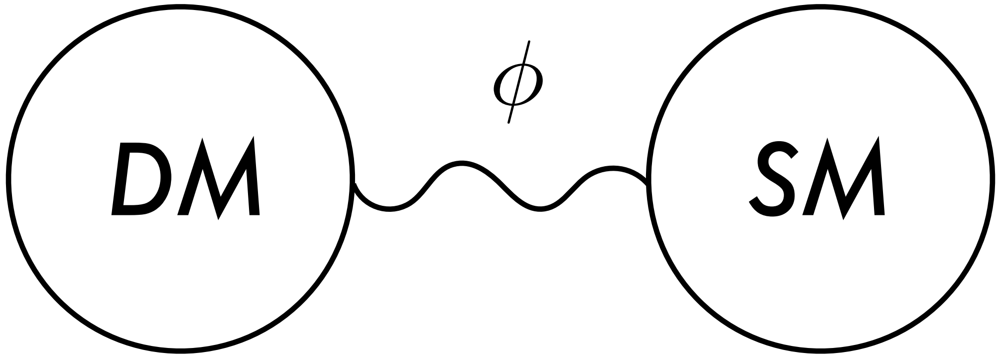
$$0.1\,\mathrm{cm}^2/\mathrm{g} \lesssim \sigma_{\mathrm{self}}/m_\pi \lesssim 1\,\mathrm{cm}^2/\mathrm{g}$$

consider an axion-like particle

[Kamada, *HK* and Sekiguchi, 17]

consider an axion-like particle

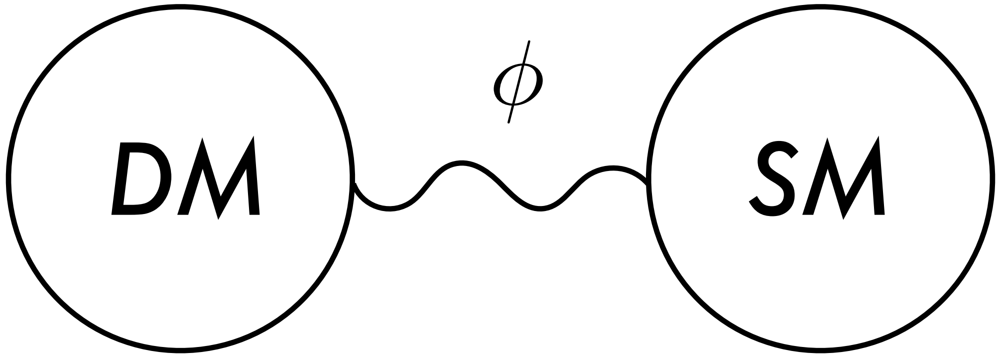
- *connecting DM sector with SM sector* ***(kinetic eq.)***



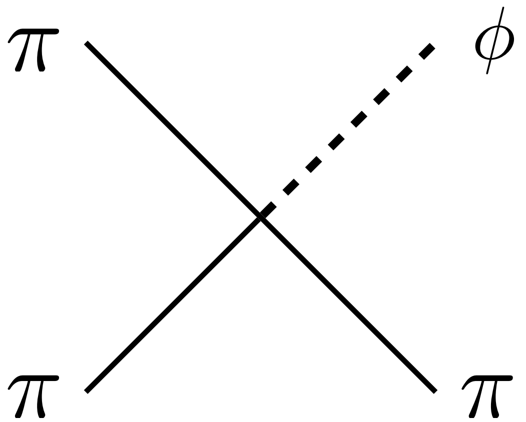
[Kamada, *HK* and Sekiguchi, 17]

consider an axion-like particle

- *connecting DM sector with SM sector* ***(kinetic eq.)***



- *openning up a new annihilation channel* ***(perturbativity)***



[Kamada, *HK* and Sekiguchi, 17]

let's see how ALP interacts with DM and SM

$$\mathcal{L} = \bar{q}_i i \not{D} q_i - m_q \bar{q}_i q_i$$

$$+ \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{g_H^2}{32\pi^2}\frac{\phi}{f} H_{\mu\nu}\widetilde{H}^{\mu\nu}$$

$$+ C_{\phi\gamma\gamma}\frac{\alpha_{\rm em}}{4\pi}\frac{\phi}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

[Kamada, *HK* and Sekiguchi, 17]

let's see how ALP interacts with DM and SM

$$\mathcal{L} = \bar{q}_i i \not{D} q_i - m_q \bar{q}_i q_i$$

$$+\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{g_H^2}{32\pi^2}\frac{\phi}{f}H_{\mu\nu}\widetilde{H}^{\mu\nu}$$

$$+C_{\phi\gamma\gamma}\frac{\alpha_{\rm em}}{4\pi}\frac{\phi}{f}F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

[Kamada, *HK* and Sekiguchi, 17]

let's see how ALP interacts with DM and SM

$$\mathcal{L} = \bar{q}_i i \not{D} q_i - m_q \bar{q}_i q_i$$

$$+\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \frac{g_H^2}{32\pi^2}\frac{\phi}{f} H_{\mu\nu}\widetilde{H}^{\mu\nu}$$

$$+C_{\phi\gamma\gamma}\frac{\alpha_{\rm em}}{4\pi}\frac{\phi}{f} F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

At low energies, effective Lagrangian is

$$\mathcal{L} = \frac{f_\pi^2}{16}{\rm Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2 f_\pi^2}{16}{\rm Tr}(e^{i\phi/(N_f f)}U + {\rm h.c.}) + \mathcal{L}_{\rm WZW}$$

[Kamada, *HK* and Sekiguchi, 17]

Expanding the chiral Lagrangian,

we see not only operators for self-scattering and 3-to-2 process

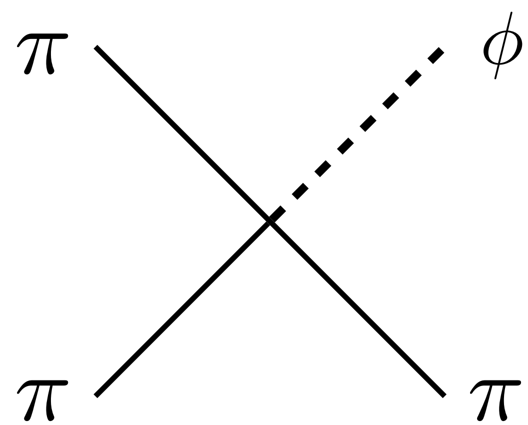
[Kamada, *HK* and Sekiguchi, 17]

Expanding the chiral Lagrangian,

we see not only operators for self-scattering and 3-to-2 process

but also an operator for ***semi-annihilation***

[D'Emaro and Thaler, 10]



$$\mathcal{L}_{\rm semi} \sim \frac{m_\pi^2}{N_f f_\pi f} d_{abc}(\pi^a \pi^b \pi^c)\phi$$

[Kamada, *HK* and Sekiguchi, 17]

*We want the **semi-annihilation** to be in equilibrium until freeze-out*

$$\langle \sigma_{\rm semi} v \rangle \sim \alpha_\pi^2 / f^2 \sim 10^{-36}\,{\rm cm}^2$$

$$\Rightarrow \quad f \lesssim \mathcal{O}({\rm TeV})$$

We want the ***semi-annihilation***
to be in equilibrium until freeze-out

$$\langle \sigma_{\rm semi} v \rangle \sim \alpha_\pi^2 / f^2 \sim 10^{-36}\,{\rm cm}^2$$

$$\Rightarrow \quad f \lesssim \mathcal{O}({\rm TeV})$$

For ALP mass, restrict our attention to

$$10\,{\rm MeV} \lesssim m_\phi \lesssim m_\pi$$

*We want the **semi-annihilation** to be in equilibrium until freeze-out*

$$\langle \sigma_{\rm semi} v \rangle \sim \alpha_\pi^2 / f^2 \sim 10^{-36}\,{\rm cm}^2$$

$$\Rightarrow \quad f \lesssim \mathcal{O}({\rm TeV})$$

For ALP mass, restrict our attention to

$$10\,{\rm MeV} \lesssim m_\phi \lesssim m_\pi$$

Constraints from cosmology (BBN, CMB)

We want the ***semi-annihilation***
to be in equilibrium until freeze-out

$$\langle \sigma_{\rm semi} v \rangle \sim \alpha_\pi^2 / f^2 \sim 10^{-36}\,{\rm cm}^2$$

$$\Rightarrow \quad f \lesssim \mathcal{O}({\rm TeV})$$

For ALP mass, restrict our attention to

$$10\,{\rm MeV} \lesssim m_\phi \lesssim m_\pi$$

kinematics of semi-annihilation

There are total four parameters

$$\alpha_\pi$$

$$m_\pi$$

$$m_\phi$$

$$f$$

$\alpha_\pi = (m_\pi / f_\pi)$

There are total four parameters

$$\alpha_\pi \sim \mathcal{O}(1)$$

$$m_\pi \sim \mathcal{O}(100)\,\mathrm{MeV}$$

$$m_\phi$$

$$f$$

$$\alpha_\pi = (m_\pi / f_\pi)$$

There are total four parameters

$$\alpha_\pi \sim \mathcal{O}(1)$$

$$m_\pi \sim \mathcal{O}(100)\,\mathrm{MeV}$$

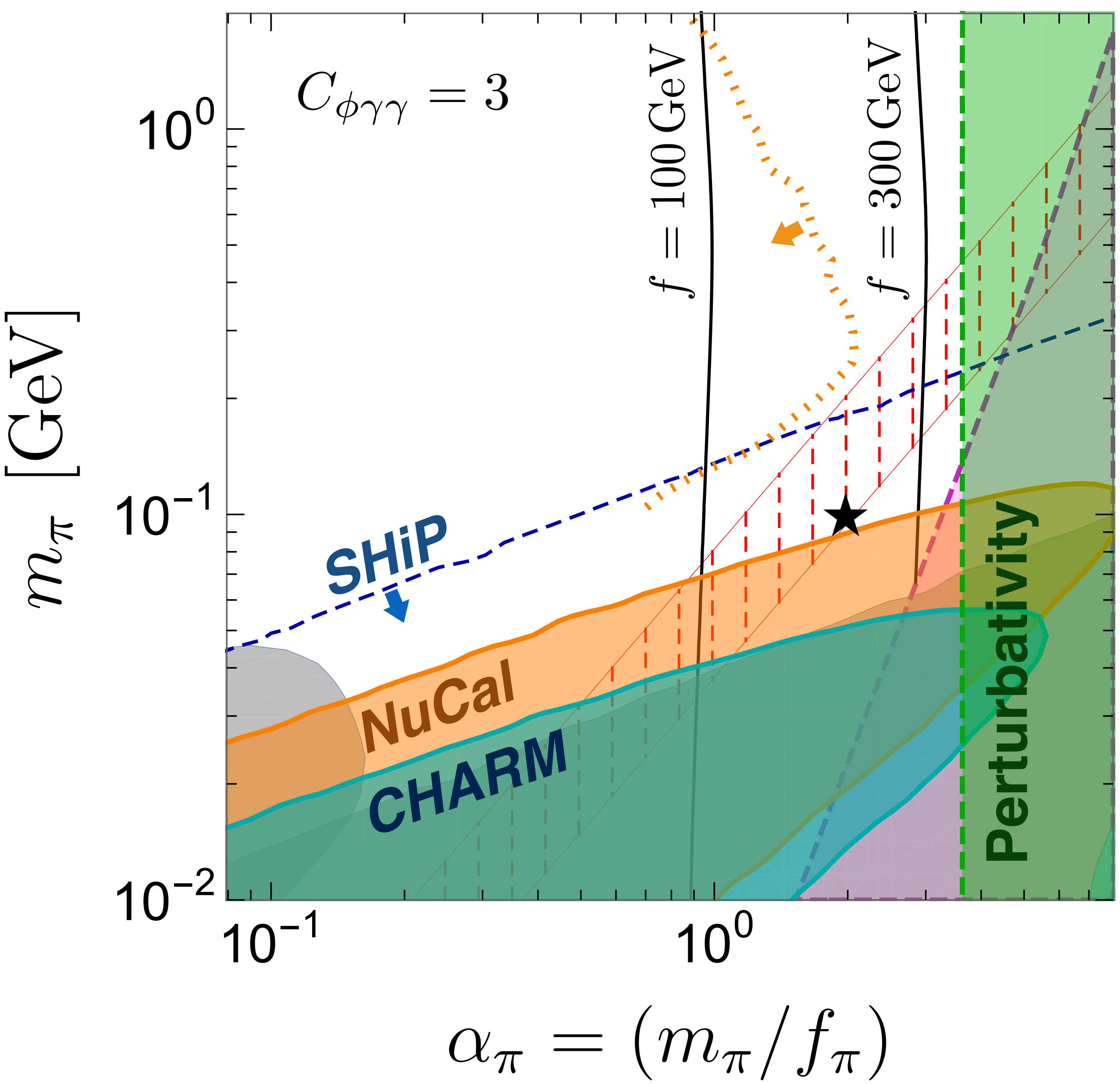
$$m_\phi \sim \mathcal{O}(100)\,\mathrm{MeV}$$

$$f \sim \mathcal{O}(100)\,\mathrm{GeV}$$

$$\alpha_\pi = (m_\pi/f_\pi)$$

taking $m_\phi = m_\pi$

$N_c = 3, \;\; N_f = 4, \;\; \theta_H = 0$

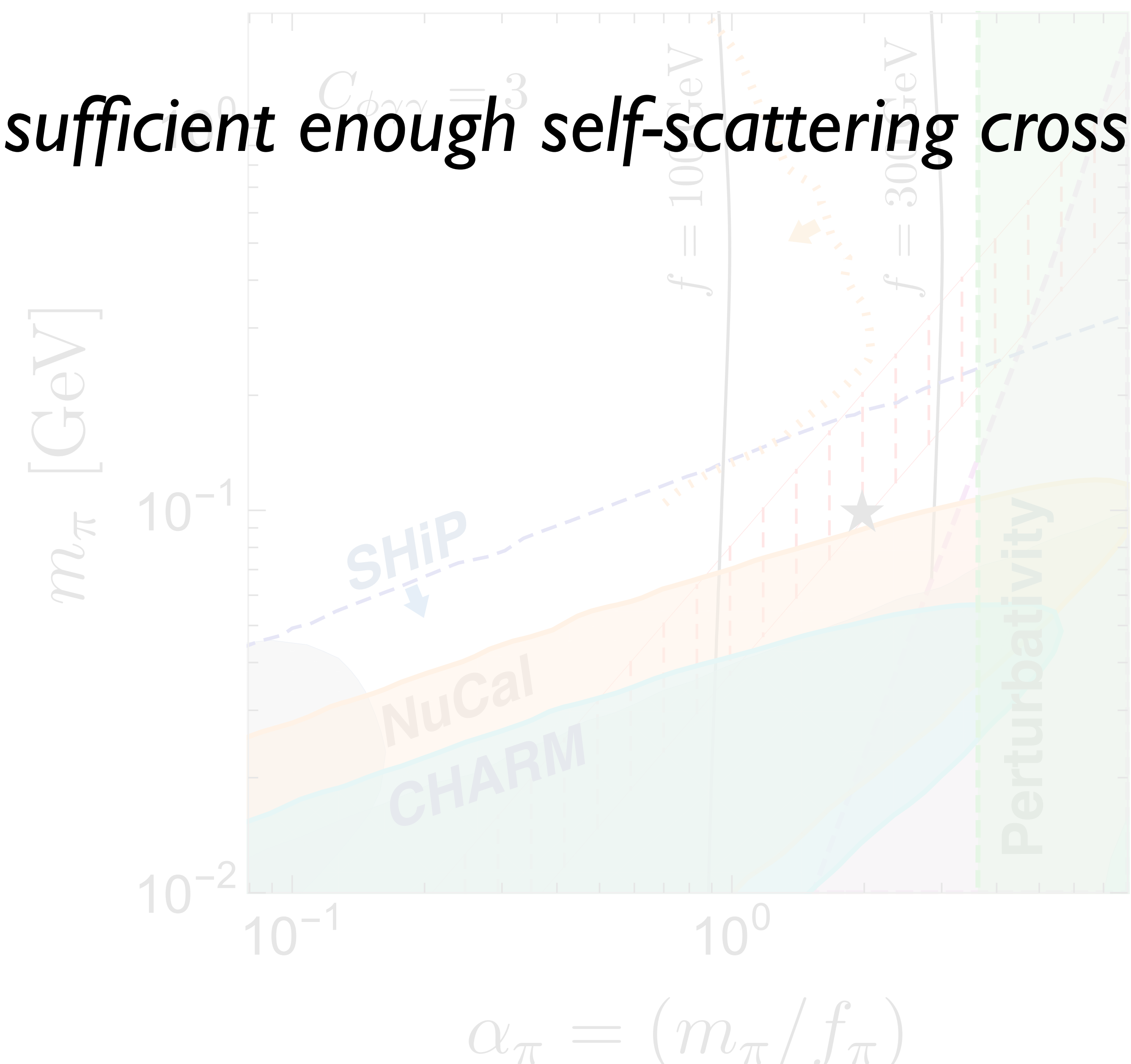


[Kamada, *HK* and Sekiguchi, 17]

taking $m_\phi = m_\pi$

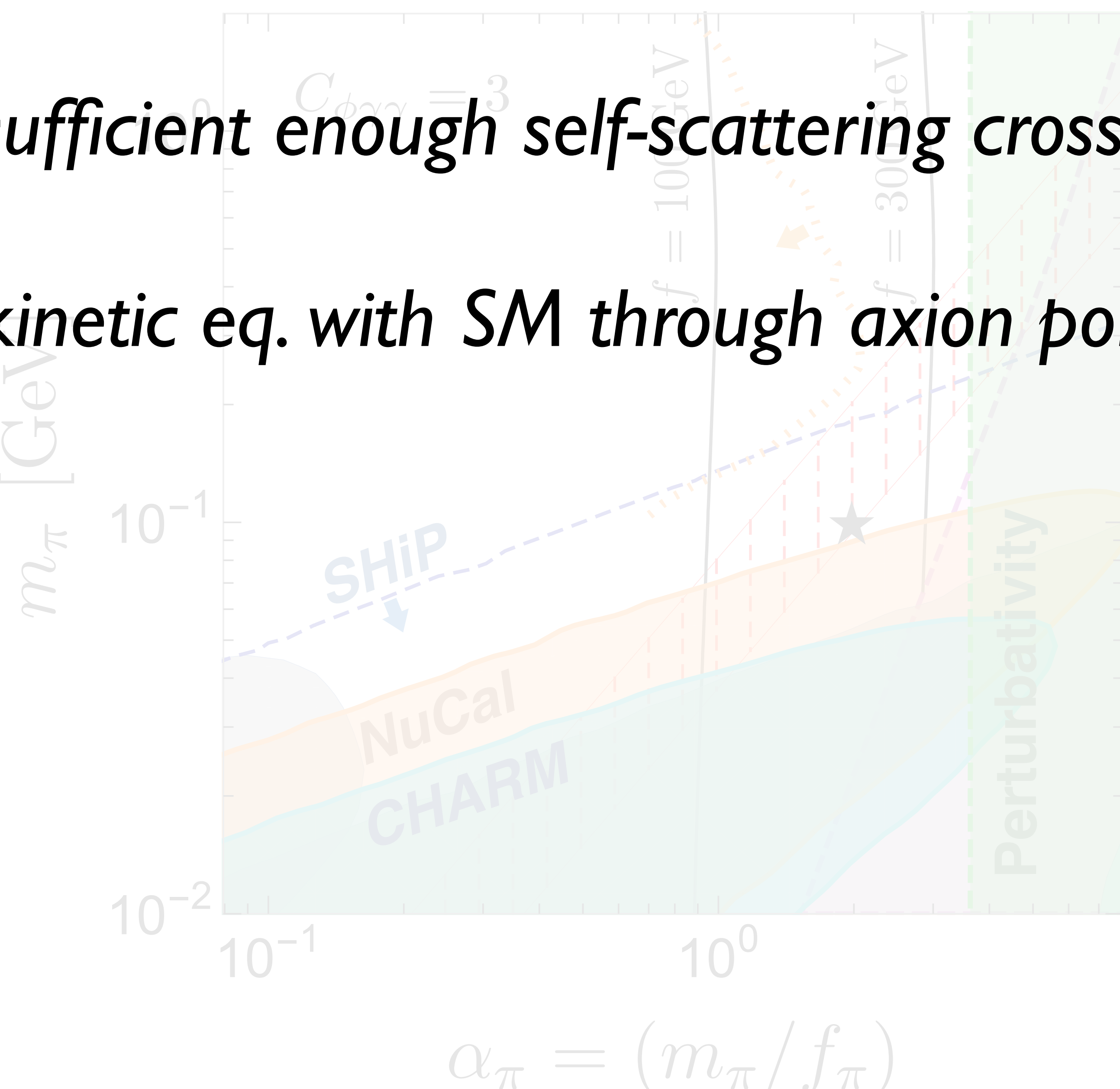
$N_c = 3, \;\; N_f = 4, \;\; \theta_H = 0$

I. sufficient enough self-scattering cross section



taking $m_\phi = m_\pi$

$N_c = 3, \;\; N_f = 4, \;\; \theta_H = 0$

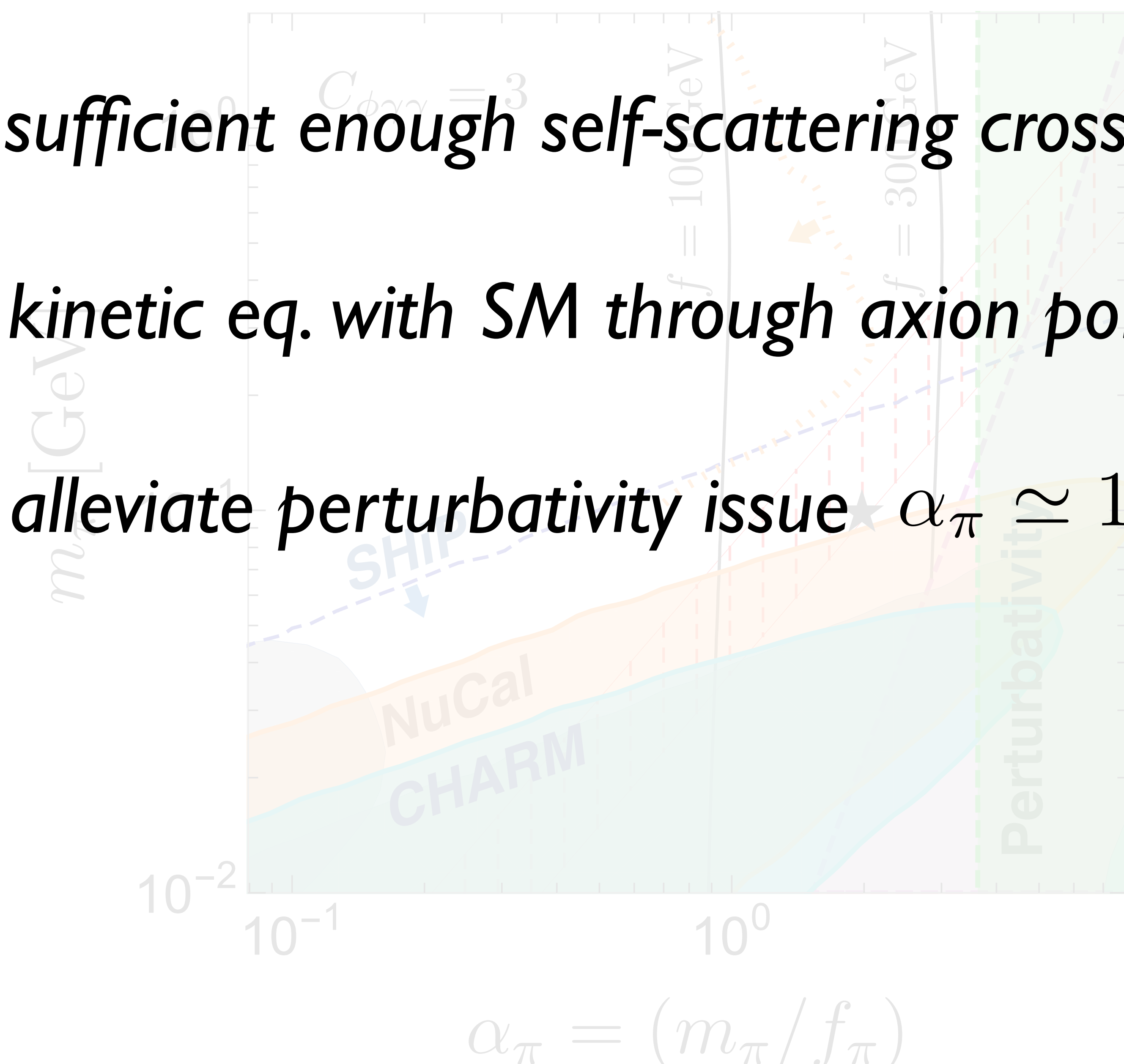


I. sufficient enough self-scattering cross section

II. kinetic eq. with SM through axion portal

taking $m_\phi = m_\pi$

$N_c = 3, \;\; N_f = 4, \;\; \theta_H = 0$



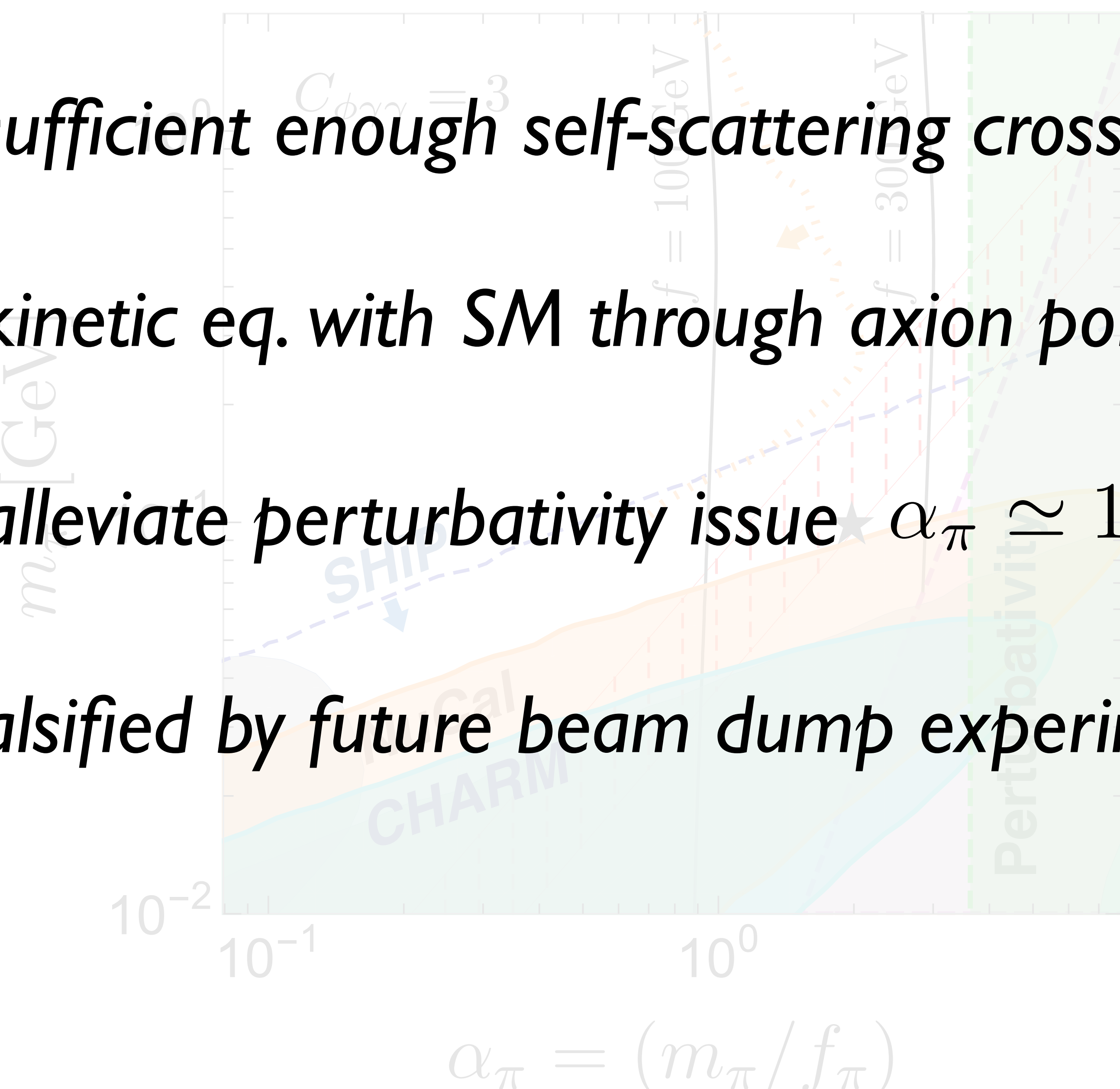
I. sufficient enough self-scattering cross section

II. kinetic eq. with SM through axion portal

III. alleviate perturbativity issue $\alpha_\pi \simeq 1 < 2\pi/\sqrt{N_c}$

taking $m_\phi = m_\pi$

$N_c = 3,\ \ N_f = 4,\ \ \theta_H = 0$



I. sufficient enough self-scattering cross section

II. kinetic eq. with SM through axion portal

III. alleviate perturbativity issue $\alpha_\pi \simeq 1 < 2\pi/\sqrt{N_c}$

IV. falsified by future beam dump experiment

taking $m_\phi = m_\pi$

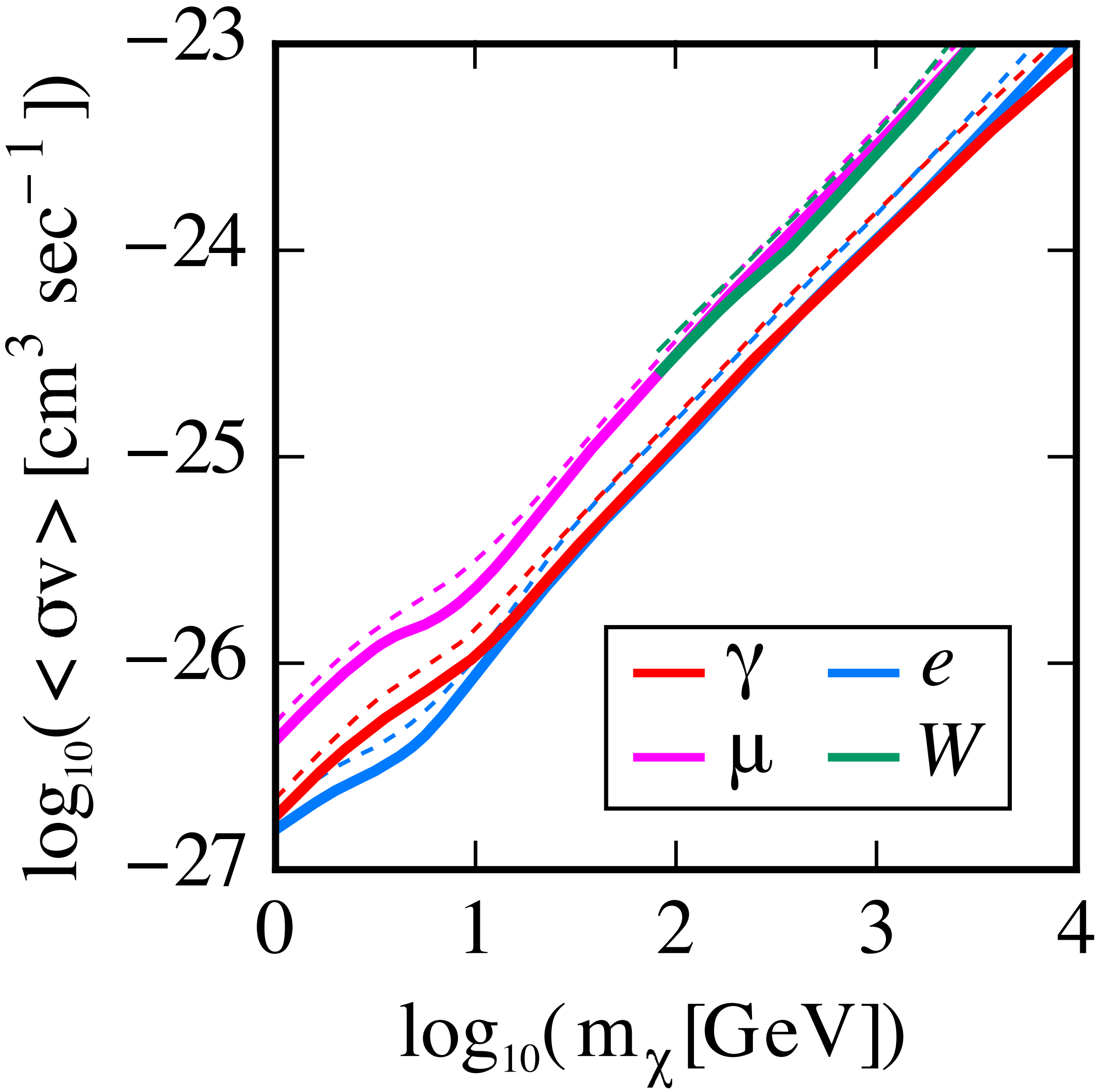
$N_c = 3, \quad N_f = 4, \quad \theta_H = 0$

not arbitrarily chosen but required …

[Kamada,

drawback : a certain mass degeneracy is required because of

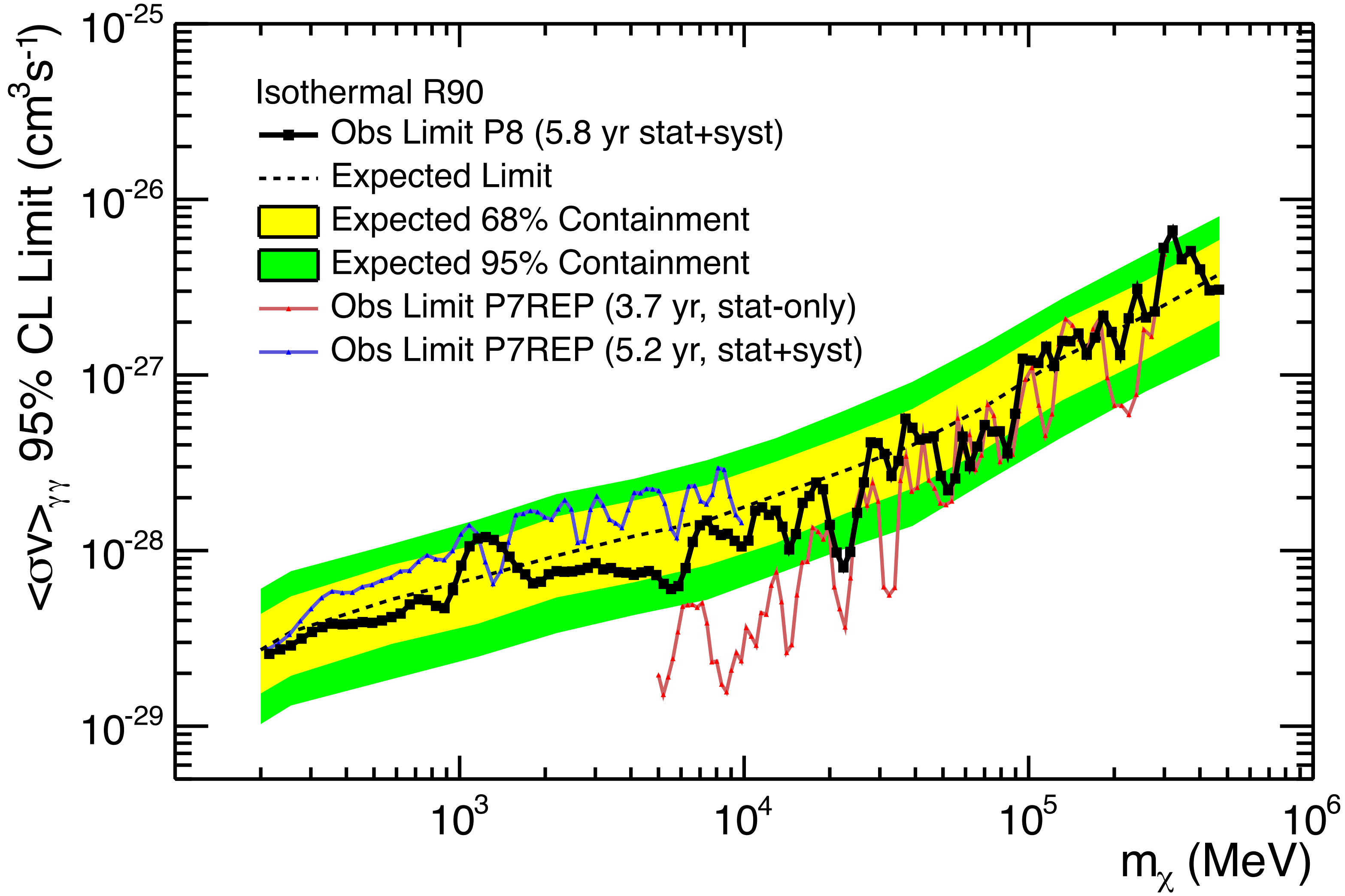
strong constraints on semi-annihilation cross section from ***CMB***



[Kawasaki et al., 15]

drawback : a certain mass degeneracy is required because of

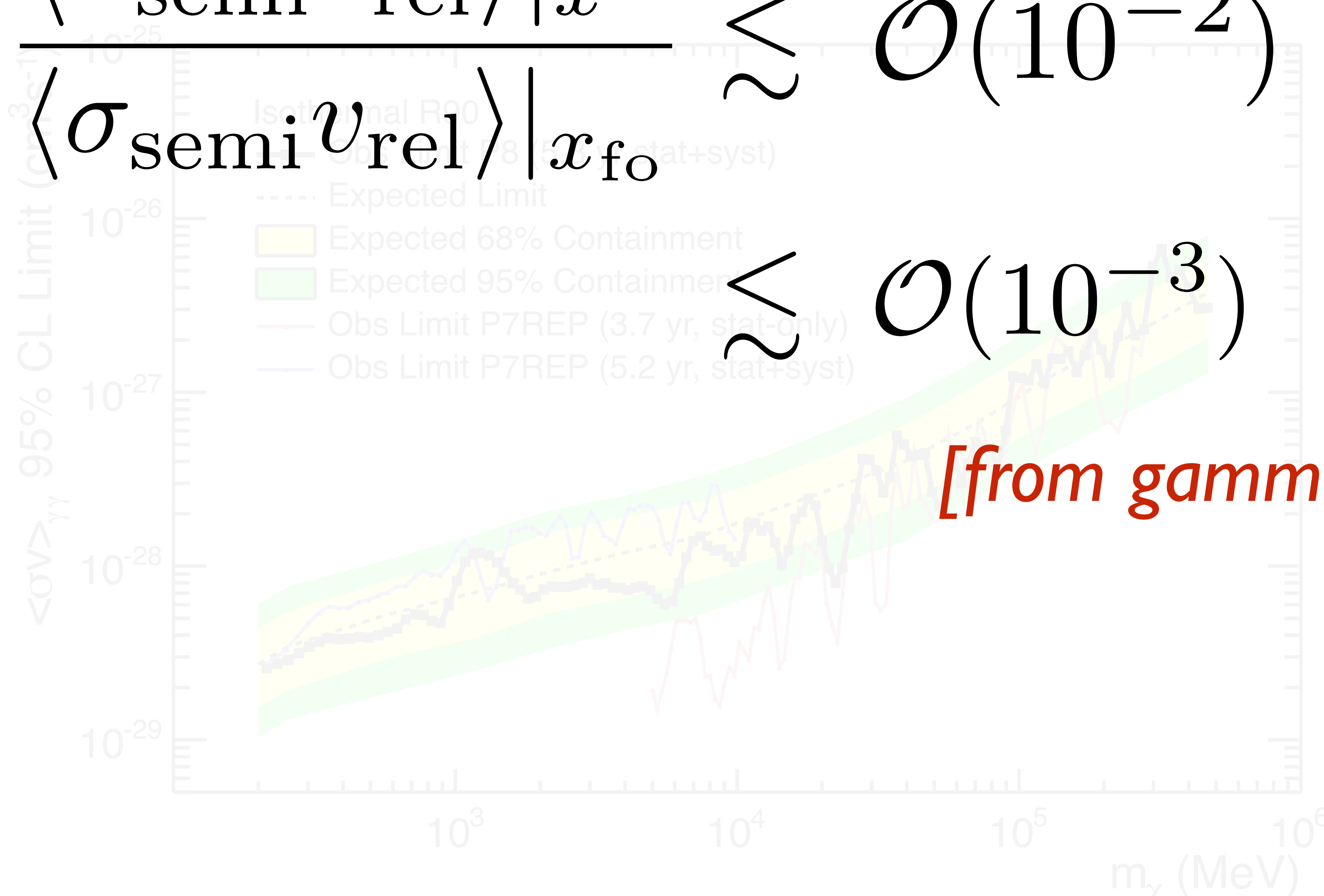
strong constraints on semi-annihilation cross section from ***gamma-ray searches***



[Ackermann et al., 15]

drawback : degenerated mass is required because of

strong constraints on semi-annihilation cross section from CMB and gamma-ray searches



$$\frac{\langle \sigma_{\rm semi} v_{\rm rel} \rangle |_x}{\langle \sigma_{\rm semi} v_{\rm rel} \rangle |_{x_{\rm fo}}} \lesssim \mathcal{O}(10^{-2})$$ *[from CMB]*

$$\lesssim \mathcal{O}(10^{-3})$$ *[from gamma-ray searches]*

[Ackermann et al., 15]

suppression can be obtained if masses are degenerate

if ALP is ***lighter*** *than dark matter*

$$\Delta m = m_\pi - m_\phi > 0$$

suppression can be obtained if masses are degenerate

*if ALP is **lighter** than dark matter*

$$\Delta m = m_\pi - m_\phi > 0$$

cross section in non-rel. limit is suppressed as

$$\langle \sigma_{\rm semi} v_{\rm rel} \rangle \propto \max(v_{\rm rel}, \sqrt{\Delta m / m_\pi})$$

suppression can be obtained if masses are degenerate

*if ALP is **lighter** than dark matter*

$$\Delta m = m_\pi - m_\phi > 0$$

cross section in non-rel. limit is suppressed as

$$\langle \sigma_{\rm semi} v_{\rm rel} \rangle \propto \max(v_{\rm rel}, \sqrt{\Delta m / m_\pi})$$

required mass degeneracy is

$$\Delta m / m_\pi \lesssim \mathcal{O}(10^{-(5\text{–}7)})$$

[from gamma-ray searches: EGRET & Fermi-LAT]

suppression can be obtained if masses are degenerate

if ALP is heavier than dark matter [i.e. forbidden DM]

[D'Agnolo and Ruderman, 15]

$$\Delta m = m_\pi - m_\phi < 0$$

suppression can be obtained if masses are degenerate

if ALP is heavier than dark matter [i.e. forbidden DM]

[D'Agnolo and Ruderman, 15]

$$\Delta m = m_\pi - m_\phi < 0$$

cross section in non-rel. limit is suppressed as

$$\langle \sigma_{\rm semi} v \rangle \propto \exp(-|\Delta m|/T)$$

suppression can be obtained if masses are degenerate

if ALP is heavier than dark matter [i.e. forbidden DM]

[D'Agnolo and Ruderman, 15]

$$\Delta m = m_\pi - m_\phi < 0$$

cross section in non-rel. limit is suppressed as

$$\langle \sigma_{\rm semi} v \rangle \propto \exp(-|\Delta m|/T)$$

no fine-tuning as in the previous case is required but

$$|\Delta m| < T_{\rm fo}$$

not to have big suppression during freeze-out process

taking $m_\phi = m_\pi$

$N_c = 3, \ \ N_f = 4, \ \ \theta_H = 0$

I. sufficient enough self-scattering cross section

II. kinetic eq. with SM through axion portal

III. alleviate perturbativity issue $\alpha_\pi \simeq 1 < 2\pi/\sqrt{N_c}$

IV. falsified by future beam dump experiment

V. a certain mass degeneracy is required