

# Toward a Rigorous Comparison Between Models & Data

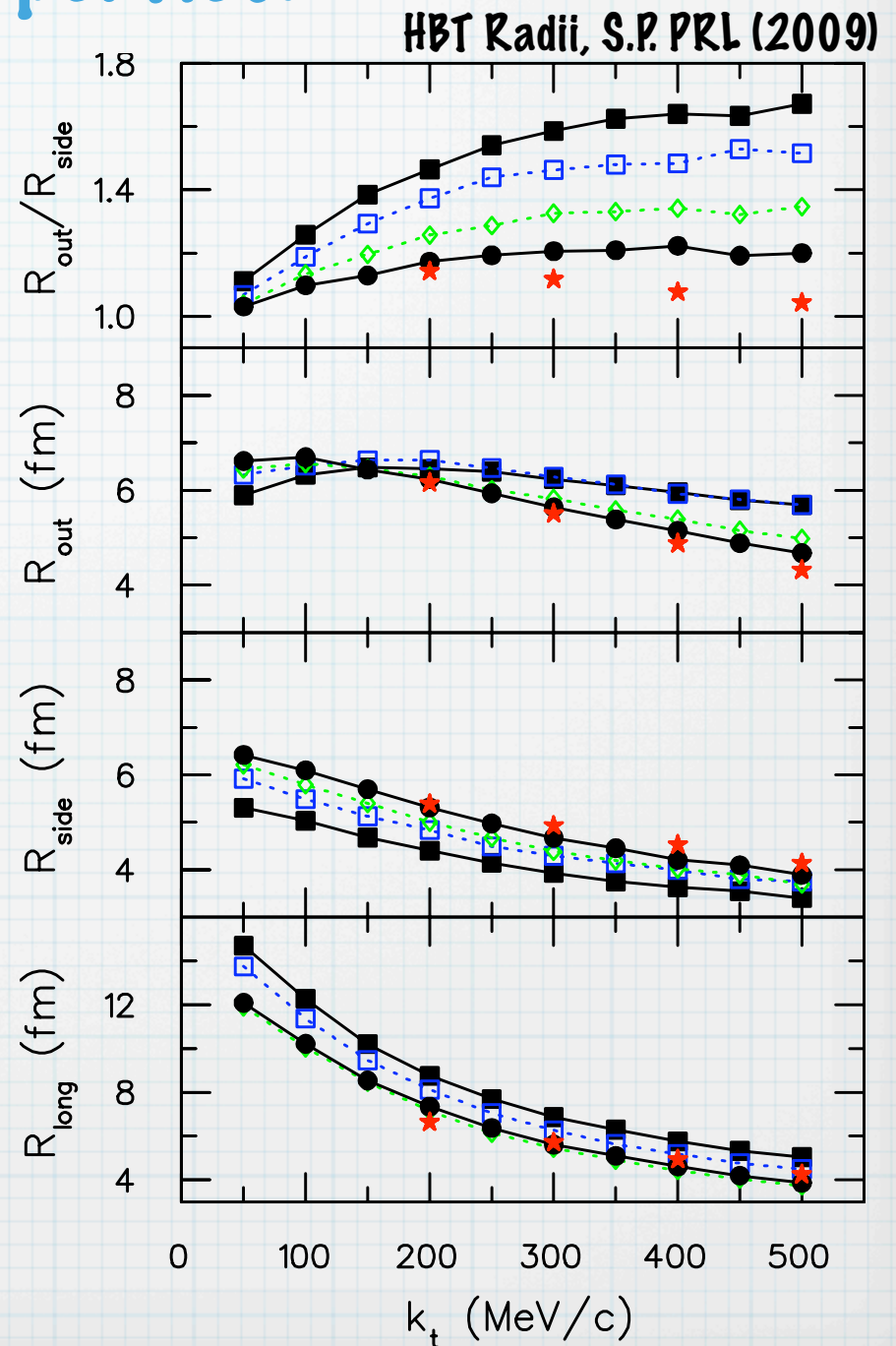
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# Lessons from RHIC (bulk properties)

- \* Matter is rather stiff:  
(no large latent heat,  
but softer than  $\pi$  gas)
- \* Early flow seems important:  
(otherwise difficult to fit HBT)
- \* Viscosity is low:  
(mostly from elliptic flow)

$$\frac{\eta}{s} \approx \left( \frac{1}{2} \text{ to } 3 \right) \times \frac{\hbar}{4\pi}$$

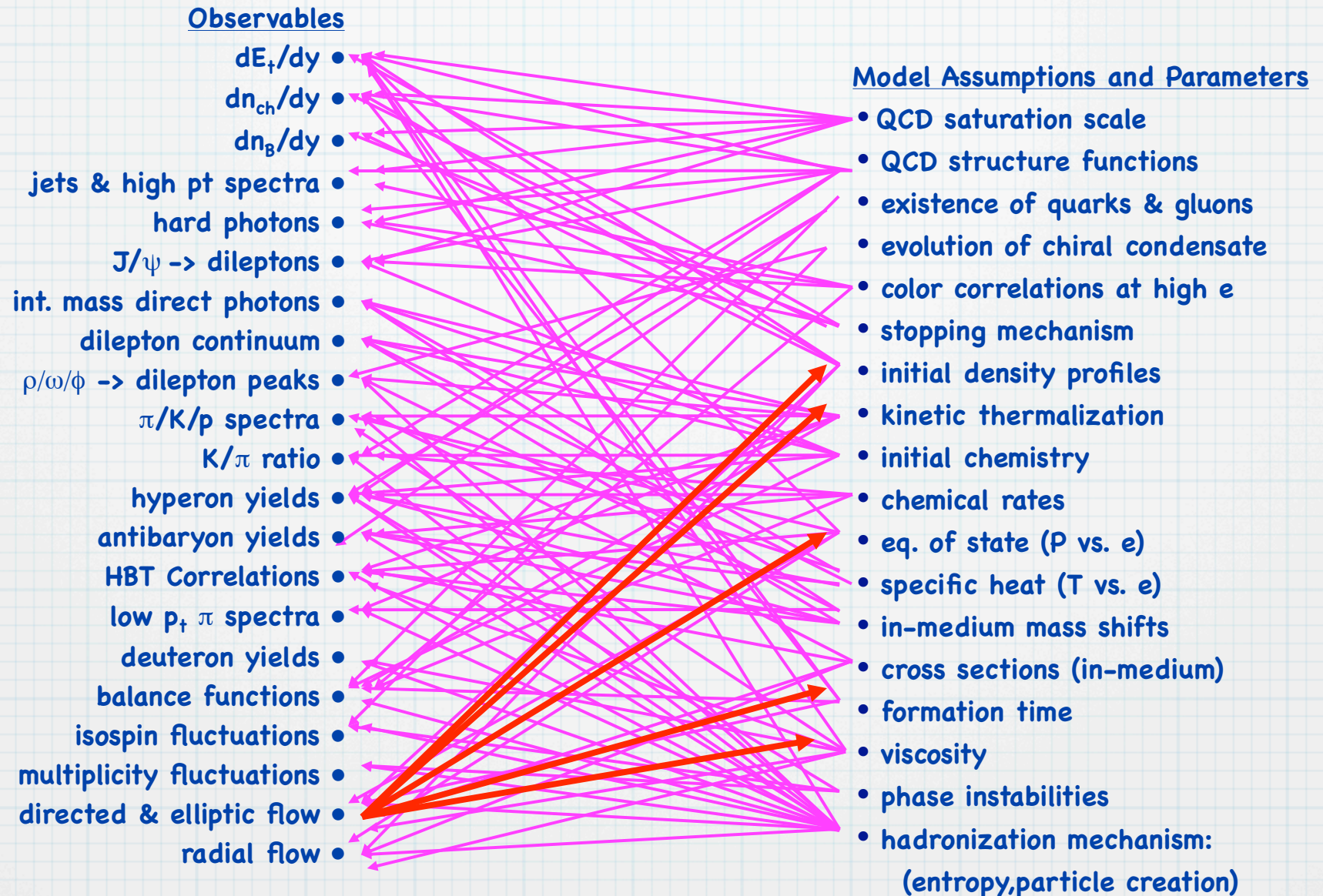


... but nothing is quantitative or rigorous

- \* EOS (min  $c_s^2$ , width of soft region, max  $c_s^2$ )
- \*  $??? < \eta / s < ???$  (energy dependence?)
- \*  $\varepsilon$  for  $\tau < 0.5$  fm/c uncertain by factor of 2

Properties are neither DETERMINED  
nor VALIDATED rigorously

# RHIC Analysis Challenge



**Individual elements cannot be isolated!!**  
**complex, non-linear network**

# Uncertainties and Parameters

Initial State	6	Energy density, profile shape, rapidity width, pressure, anisotropy of $T_{ij}$ , quark/gluon content
Hadronic Boltzmann	2-4	Mass changes
Eq. of State / Viscosity	3-8	Might be constrained by lattice, hadron gas
Chemical	3-6	Quark density, relaxation rates, hadronic scattering reduction
Jet Quenching	2-4	Dissipation rates
Systematic Experimental	?	Efficiencies, calibrations...

**$\approx 30$  parameters**

**Some are unimportant**

**Some combinations are unimportant**

*Turn ALL the knobs!!!!*



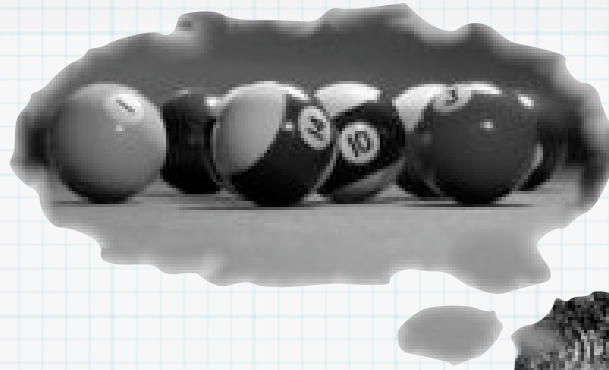
# Bayesian Analysis

WIKIPEDIA: Bayesian inference is statistical inference in which evidence or observations are used to update or to newly infer the probability that a hypothesis may be true. The name "Bayesian" comes from the frequent use of Bayes' theorem in the inference process.

$$P(H | E) = \frac{P(E | H)P(H)}{P(E)}$$

- $P(H)$  is probability (in absence of  $E$ ) for parameter set  $H$   
a.k.a. the "prior distribution"
- $P(E|H)$  is probability of  $E$  given  $H$ , i.e.,
$$P \sim \exp\left(-\sum \delta_i^2 / 2\sigma_i^2\right)$$
- $P(E)$  is net probability of  $E$ , i.e., a normalization factor
- $P(H|E)$  is probability of parameter set  $H$  given  $E$

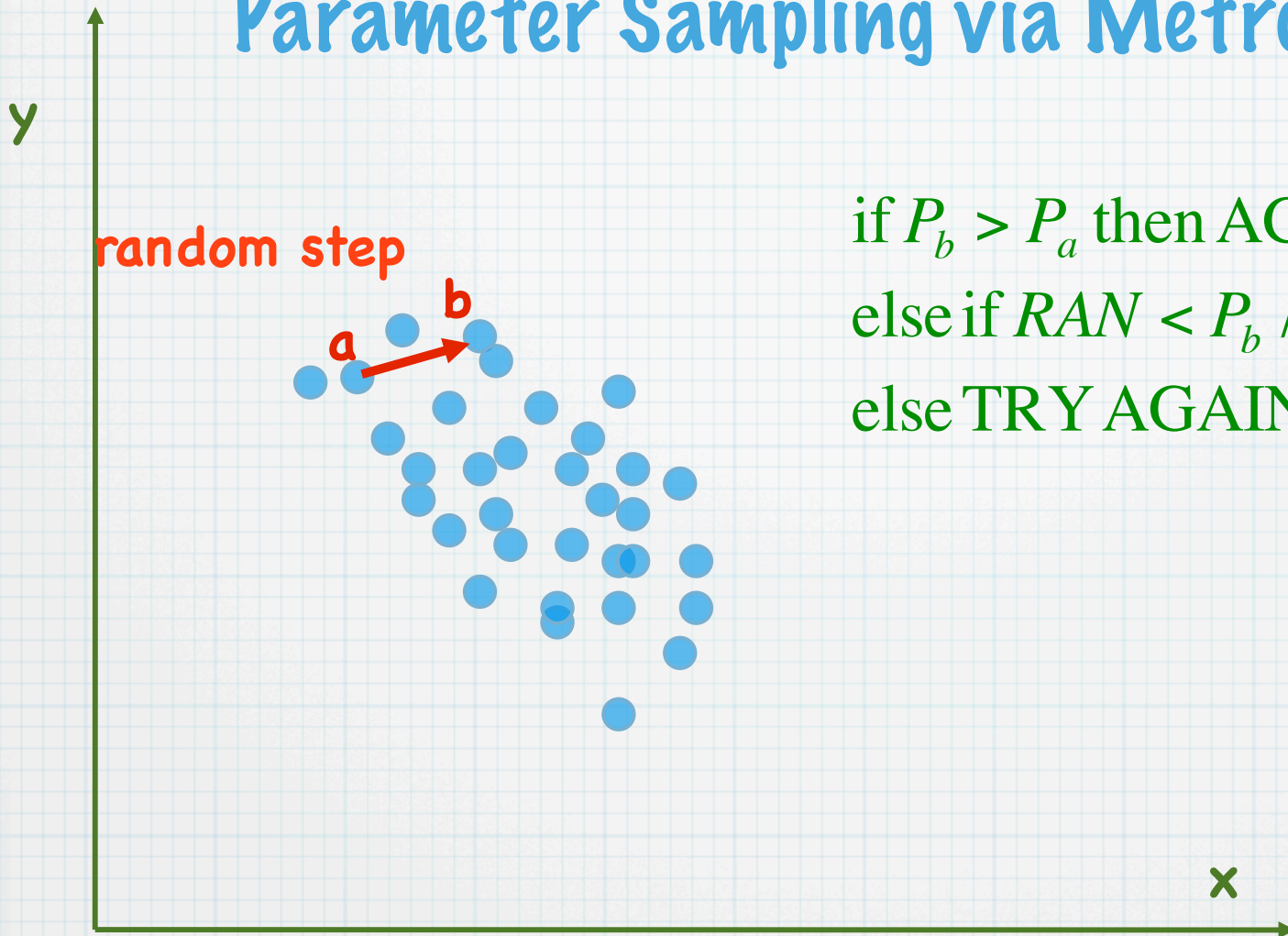
# Bayes' Theorem



$$P(E \& H) = P(E | H) \cdot P(H) = P(H | E) \cdot P(E)$$

$$P(H | E) = \frac{P(E | H) \cdot P(H)}{P(E)}$$

# Parameter Sampling via Metropolis



if  $P_b > P_a$  then ACCEPT

else if  $RAN < P_b / P_a$  ACCEPT

else TRY AGAIN

Can find disjoint regions  
No problem with undeterminable parameters

# Surrogate Models (a.k.a. Emulators, Meta-Models)

## Brute Force:

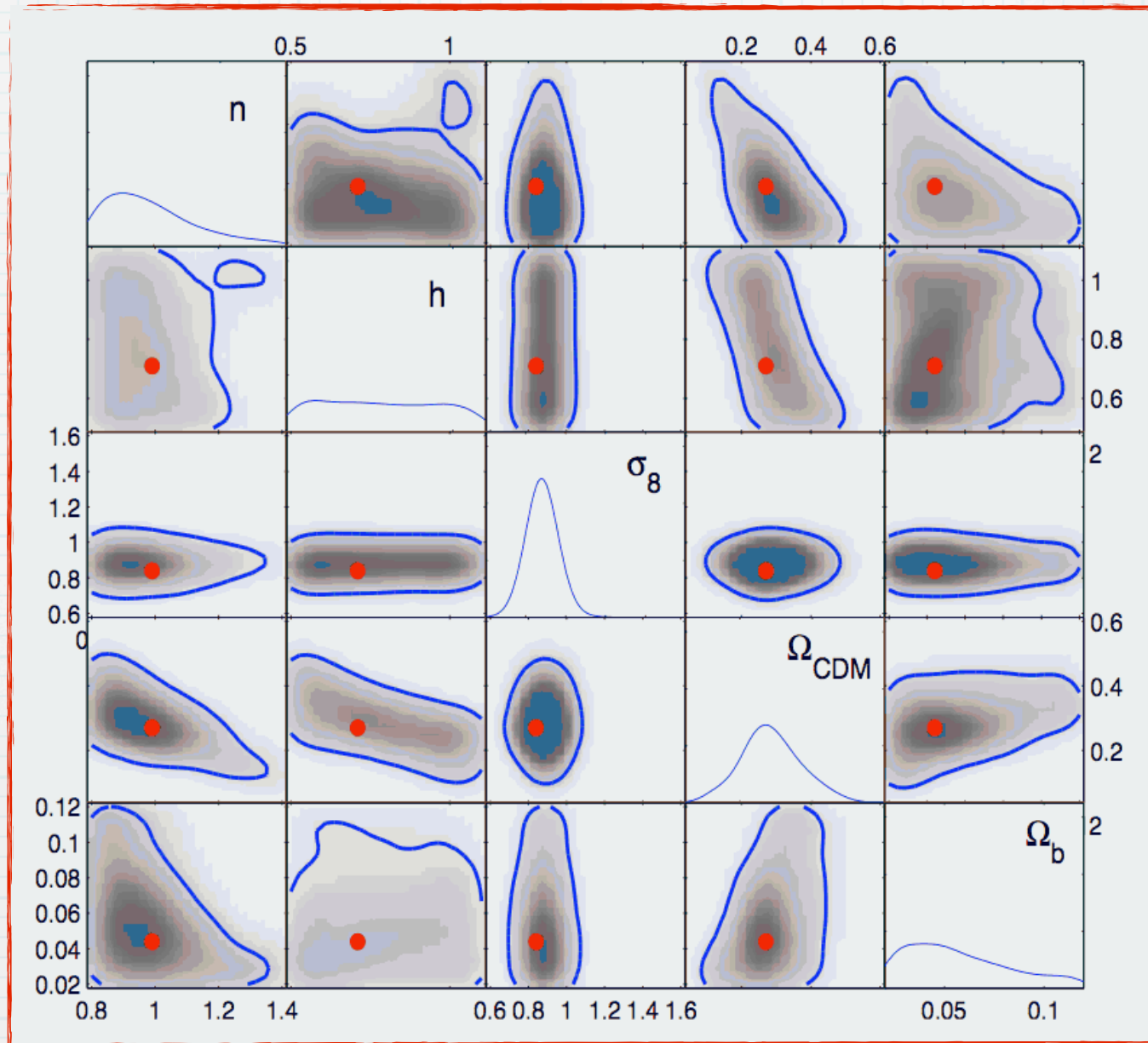
- Sampling requires millions of runs
- Each run requires 1 work-station day

## Alternative:

- Run  $10^2 - 10^3$  times at various points
- "Interpolate" to find values at all other points
- Competing "interpolation" schemes:
  - Gaussian fields
  - Multi-dimensional splines

*An Emerging Science*

Other fields can do it....



Cosmological parameters (Habib et al, astro-ph/0702348 1)

# Can this work for Relativistic Heavy Ion Collisions?

- \* **Must be amenable to parameterization**
  - \* Model must contain basic truth
  - \* Not too many competing theories
- \* **Must have well stated errors**
  - \* Statistical & systematic for both theory and experiment
  - \* Cross correlated errors

# STRATEGY

- \* **First Pass At "Bulk" Observables**
  - \* Spectra, Yields, HBT, Flow
- \* **Intimate Theory/Experiment Discussions**
  - \* Re-express experimental errors
- \* **"Professionalize" hydro/cascade code**
  - \* validated, open-source, modular, flexible...
- \* **Dedicated team to develop comparison software**
  - \* Theory/experiment/statistics/computation expertise

# OUTCOME

- \* Rigorous Quantitative Conclusions about Bulk Properties**
- \* Validated Base from which to Calculate Jet Energy Loss, Fluctuations, Rare Probes...**