

Numerical Solution of mDGLAP Evolution and the Modification factor in DIS

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Resum Multiple scattering and induced gluon radiations.



medium-modified DGLAP (mDGLAP)

$$\frac{\partial \tilde{D}_q^h(z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\tilde{\gamma}_{q \rightarrow qg}(z, \mu^2) \tilde{D}_q^h\left(\frac{z_h}{z}, \mu^2\right) + \tilde{\gamma}_{q \rightarrow gq}(z, \mu^2) \tilde{D}_g^h\left(\frac{z_h}{z}, \mu^2\right) \right]$$

$$\frac{\partial \tilde{D}_g^h(z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\sum_{q=1}^{2n_f} \tilde{\gamma}_{g \rightarrow q\bar{q}}(z, \mu^2) \tilde{D}_q^h\left(\frac{z_h}{z}, \mu^2\right) + \tilde{\gamma}_{g \rightarrow gg}(z, \mu^2) \tilde{D}_g^h\left(\frac{z_h}{z}, \mu^2\right) \right]$$

Medium-modified splitting function:

$$\tilde{\gamma}_{a \rightarrow bc}(z, l_T^2) = \gamma_{a \rightarrow bc}(z) + \Delta \gamma_{a \rightarrow bc}(z, l_T^2)$$

X. F. Guo and X. N. Wang
 Phys. Rev. Lett. 85, 3591(2000)

B. W. Zhang and X. N. Wang
 Nucl. Phys. A 720, 429 (2003)

Modification to splitting function for quark:

$$\Delta\gamma_{q \rightarrow qg}(z, l_T^2) = \frac{1}{l_T^2} \left[C_A \frac{z(1+z^2)}{(1-z)_+} + C_F (1-z)(z+z^2) \right] \times \int_0^L 4dy^- \hat{q}(y) \sin^2 \left(x_L P^+ y^- / 2 \right) + \delta(1-z) \frac{\Delta_q(l_T^2)}{l_T^2}$$

$$\Delta\gamma_{q \rightarrow qq}(z, l_T^2) = \Delta\gamma_{q \rightarrow qg}(1-z, l_T^2)$$

Modification to splitting function for gluon:

$$\Delta\gamma_{g \rightarrow q\bar{q}}(z, l_T^2) = \frac{1}{2l_T^2} \left[z^2 + (1-z)^2 \right] \left[1 - \frac{N_c}{C_F} z(1-z) \right] \times \int_0^L 4dy^- \hat{q}(y) \sin^2 \left(x_L P^+ y^- / 2 \right)$$

$$\Delta\gamma_{g \rightarrow gg}(z, l_T^2) = \frac{2C_A N_c}{l_T^2 C_F} \frac{(1-z+z^2)^3}{z(1-z)_+} \times \int_0^L 4dy^- \hat{q}(y) \sin^2 \left(x_L P^+ y^- / 2 \right) - \delta(1-z) \frac{\Delta_g(l_T^2)}{l_T^2}$$

numerically

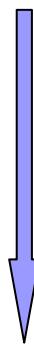
$$\tilde{D}_q^h(z, Q_0^2) \xrightarrow{\hspace{1cm}} \tilde{D}_q^h(z, Q^2)$$

Initial Conditions

$$\mu^2 = 0$$

$$D(Q_0^2)$$

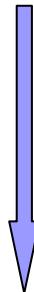
HKN
parameterization



$$\tilde{\gamma}(z, l_T^2) = \Delta\gamma(z, l_T^2), \quad \alpha_s = \alpha_s(Q_0^2)$$

$$\mu^2 = Q_0^2$$

$$\tilde{D}(Q_0^2) = D(Q_0^2) + \Delta D(Q_0^2)$$



$$\tilde{\gamma}(z, l_T^2) = \gamma(z) + \Delta\gamma(z, l_T^2)$$

$$\alpha_s = \alpha_s(\mu^2)$$

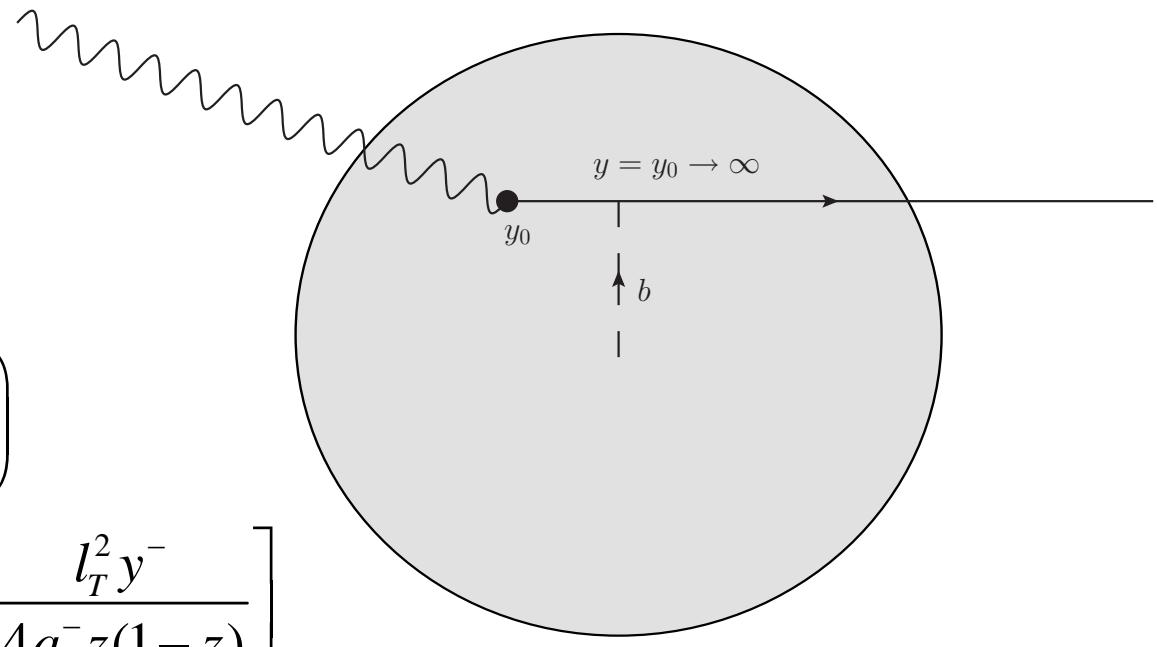
$$\mu^2 = Q^2$$

$$\tilde{D}(Q^2)$$

Averaged mFF in DIS

$$\hat{q}(y) = \hat{q}_0 \frac{\rho(y, b)}{\rho(0, 0)}$$

$$\begin{aligned} & \int 4dy^- \hat{q}(y) \sin^2 \left(\frac{l_T^2 y^-}{4q^- z(1-z)} \right) \\ &= -\frac{4\hat{q}_0}{\rho(0, 0)} \int_{y_0}^{\infty} dy^- \rho(y, b) \sin^2 \left[\frac{l_T^2 y^-}{4q^- z(1-z)} \right] \end{aligned}$$

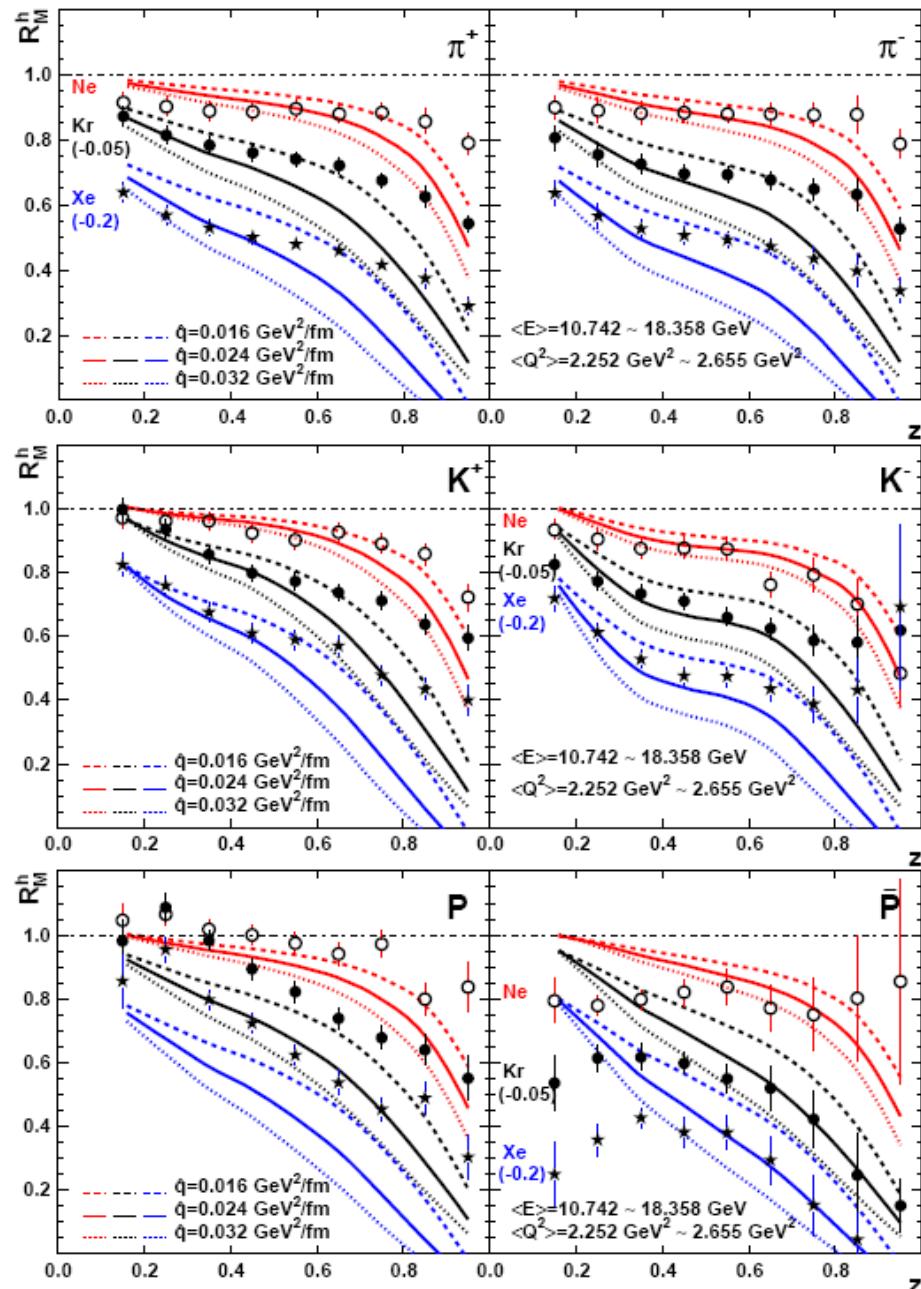


$$\tilde{D}(z) = \langle \tilde{D}(z, y_0) \rangle = \frac{\int_{-\infty}^{\infty} d^2 b \int_{-\infty}^{\infty} dy_0 \tilde{D}(z, y_0) \rho(y_0, b)}{\int_{-\infty}^{\infty} d^2 b \int_{-\infty}^{\infty} dy_0 \rho(y_0, b)}$$

$$R_M^h = \left(\frac{N^h(z, \nu)}{N^e(\nu)} |A \right) \Bigg/ \left(\frac{N^h(z, \nu)}{N^e(\nu)} |D \right)$$

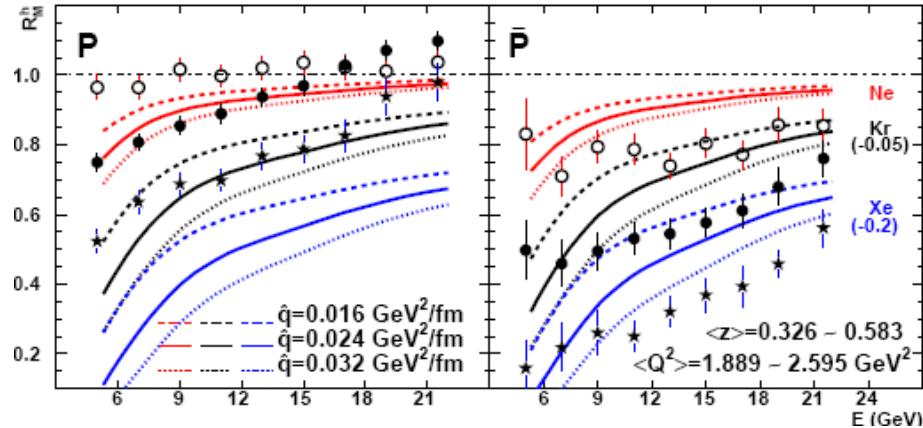
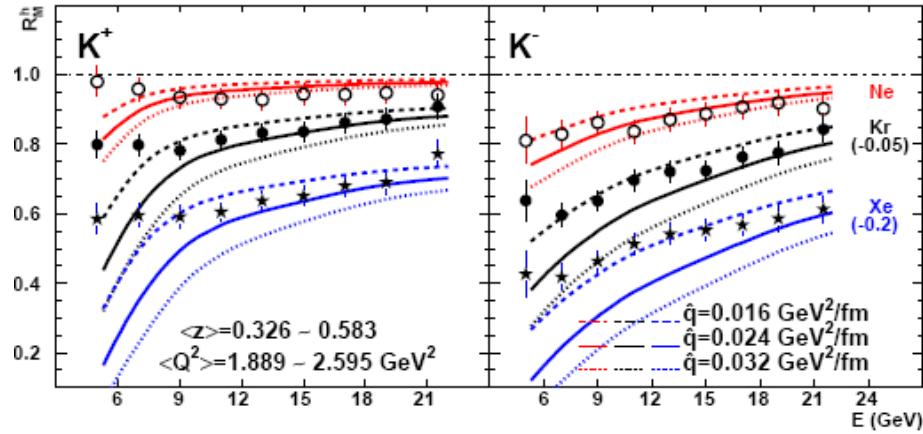
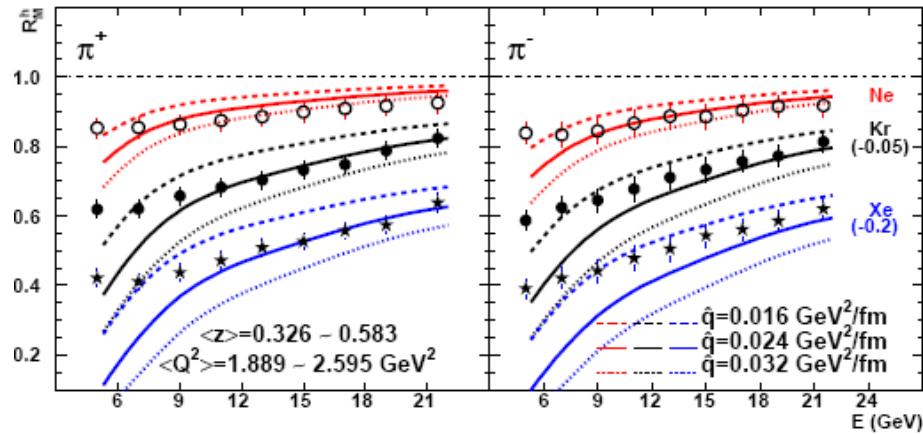
$$= \frac{\sum e_f^2 q_f(x_B, Q^2) D_f^h(z, Q^2)}{\sum e_f^2 q_f^A(x_B, Q^2)} |A$$

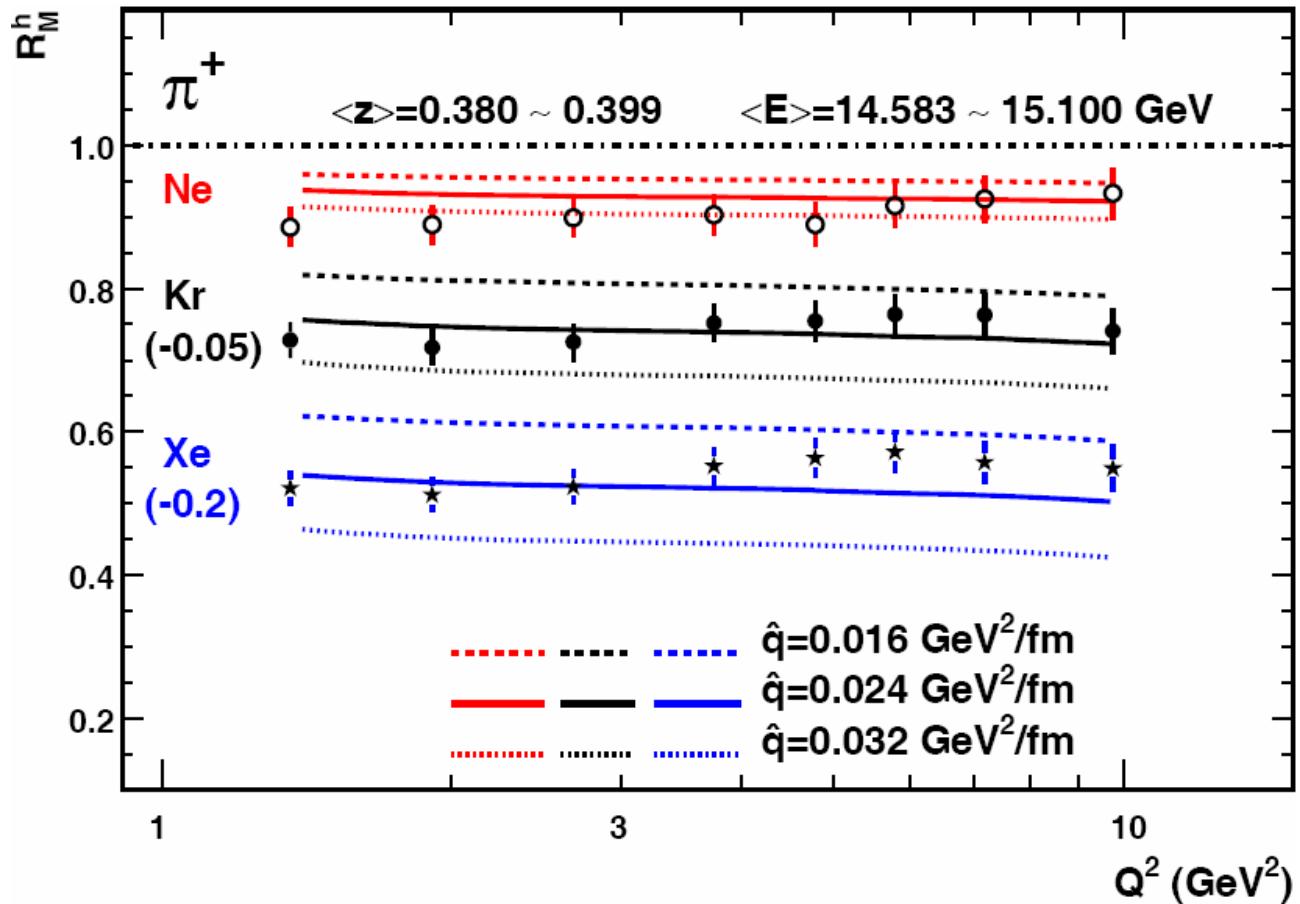
$$= \frac{\sum e_f^2 q_f(x_B, Q^2) D_f^h(z, Q^2)}{\sum e_f^2 q_f^A(x_B, Q^2)} |D$$



$$x_L = \frac{l_T^2}{2 p^+ E z(1-z)} < 1$$

$$\Rightarrow z(1-z) > \frac{l_T^2}{2 p^+ E}$$

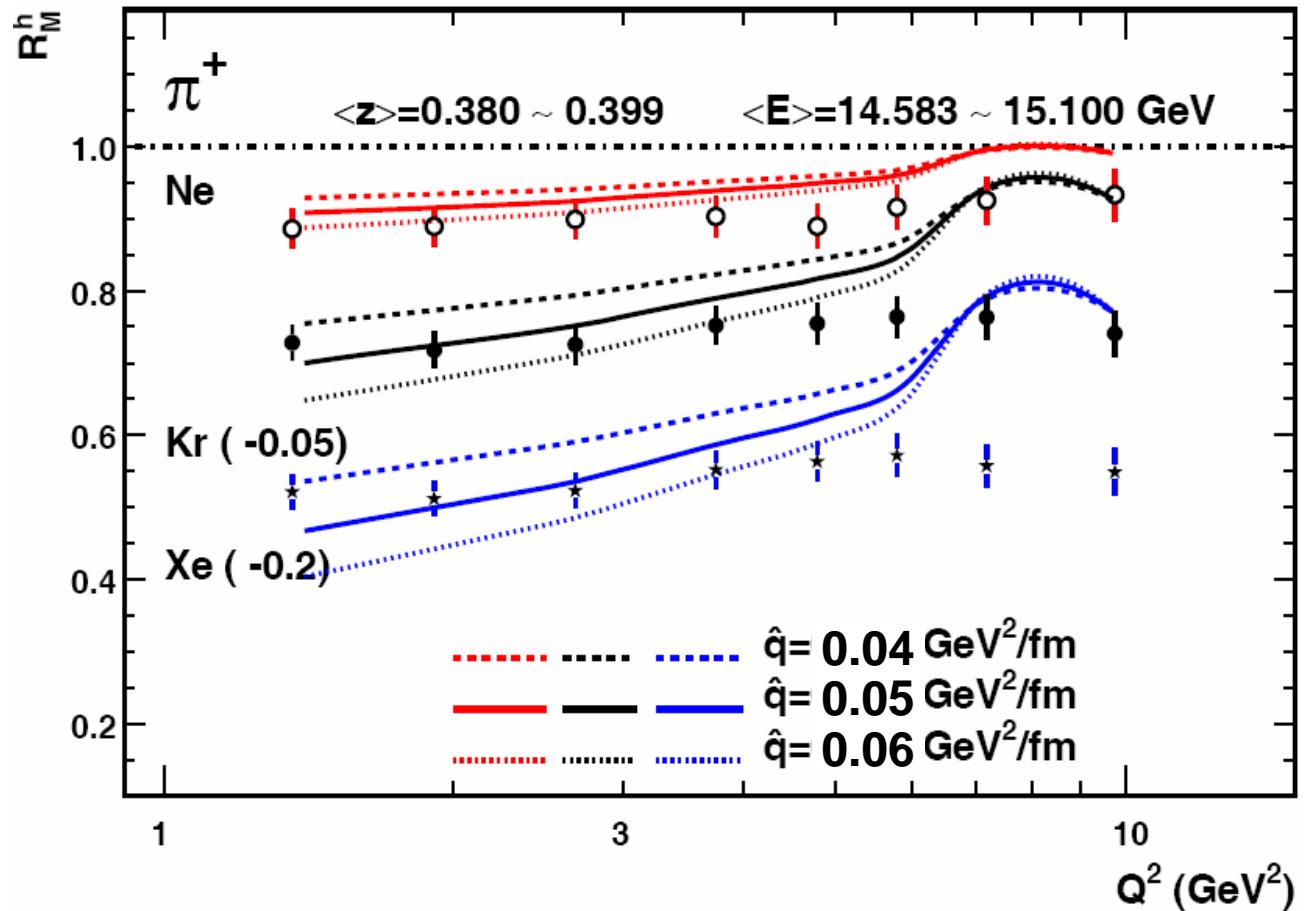




Our combined result
 $\hat{q} = 0.024 \text{ GeV}^2 / \text{fm}$



$\langle \Delta q_T^2 \rangle = 0.016 A^{1/3} \text{ GeV}^2$ in Drell-Yan
 $\sim \hat{q} = 0.018 \text{ GeV}^2 / \text{fm}$

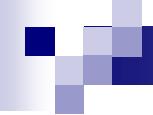


If $\tilde{D}(Q_0^2) = D(Q_0^2)$

Summary and Outlook

- Multiple parton scattering \rightarrow mDGLAP
- Modification factor in DIS.
- R_{AA} in Heavy Ion Collisions is available soon.





Back up slides

Numerical Methods

$$\begin{aligned}\frac{\partial q(x, Q^2)}{\partial \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} P(z, \alpha_s(Q^2)) q\left(\frac{x}{z}, Q^2\right) \\ &= \frac{\alpha_s(Q^2)}{2\pi} P(x, \alpha_s(Q^2)) \otimes q(x, Q^2)\end{aligned}$$

→ The HOPPET (High Order Perturbative Evolution Toolkit)

- higher-order Runge-Kutta method
- x-spaces grid.

Euler method:

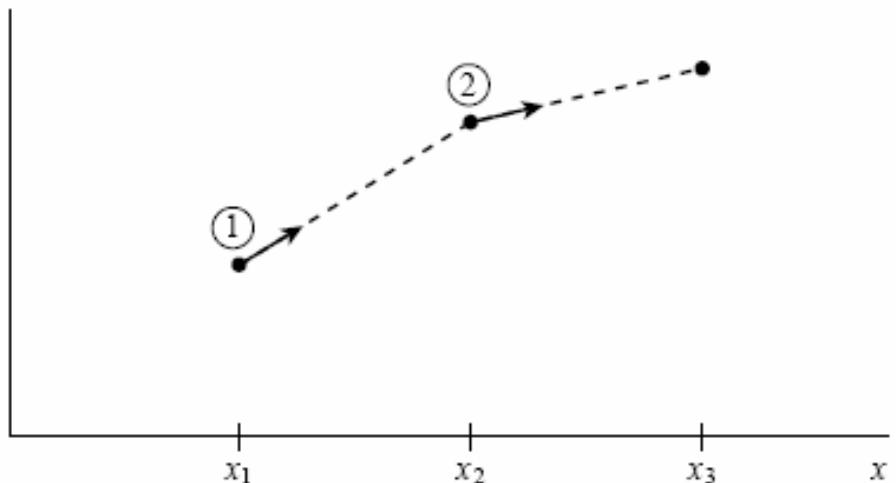
$$\begin{cases} u' = f(t, u) \\ u|_{t=0} = u(0) \end{cases}$$

Euler method:

$$u_{n+1} = u_n + f(t_n, u_n)h + O(h^2)$$

Taylor Series:

$$u(t) = u(t_0) + u'(t_0)(t - t_0) +$$

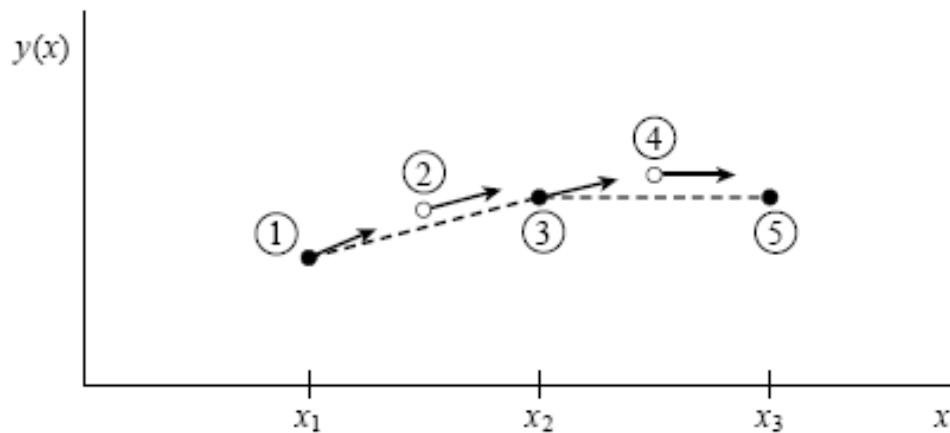


Second-Order Runge-Kutta method:

$$k_1 = hf(t_n, u_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, u_n + \frac{k_1}{2}\right)$$

$$u_{n+1} = u_n + k_2 + O(h^3)$$



Fourth-Order Runge-Kutta method:

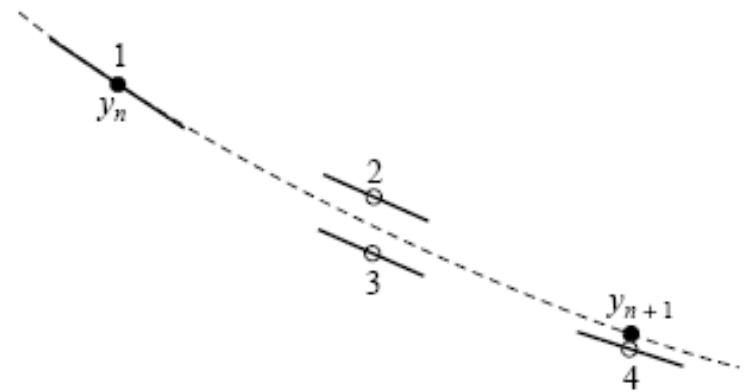
$$k_1 = hf(t_n, u_n)$$

$$k_2 = hf\left(t_n + \frac{h}{2}, u_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_n + \frac{h}{2}, u_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_n + h, u_n + k_3)$$

$$u_{n+1} = u_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5)$$



X-spaces Grid

$$\begin{aligned}\frac{\partial \mathbf{q}(x, Q^2)}{\partial \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \mathbf{P}(z, \alpha_s(Q^2)) \mathbf{q}\left(\frac{x}{z}, Q^2\right) \\ &= \frac{\alpha_s(Q^2)}{2\pi} \mathbf{P}(x, \alpha_s(Q^2)) \otimes \mathbf{q}(x, Q^2)\end{aligned}$$

► PDFs $xq(y = \ln \frac{1}{x}, t) = \sum_{\alpha} w_{\alpha}(y) q_{\alpha}(t),$

$q_{\alpha}(t) \equiv x_{\alpha} q(y_{\alpha}, t)$ Piecewise
interpolating
polynomials

X-spaces Grid

➤ convolution $(P \otimes q)(y, t) = \sum_{\alpha} w_{\alpha}(y) (P \otimes q)_{\alpha}(t),$

$$(P \otimes q)_{\alpha}(t) = \sum_{\beta} P_{\alpha\beta}(t) q_{\beta}(t),$$

$$P_{\alpha\beta}(t) = \int_{e^{-y_{\alpha}}}^1 dz P(z, t) w_{\beta}(y_{\alpha} + \ln z)$$

➤ DGLAP evolution

$$\frac{\partial q_{\alpha}(t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \sum_{\beta} P_{\alpha\beta}(t) q_{\beta}(t)$$

$$\rightarrow \partial_t M_{\alpha\beta}(t) = \frac{\alpha_s(t)}{2\pi} \sum_{\gamma} P_{\alpha\gamma}(t) M_{\gamma\beta}(t) \quad M_{\alpha\beta}(t_0) = \delta_{\alpha\beta}$$

Evolution operator

$$q_{\alpha}(t) = \sum_{\beta} M_{\alpha\beta}(t) q_{\beta}(t_0)$$