WHDG Brick and Comparing WHDG to ASW-SH

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With many thanks to Brian Cole, Ulrich Heinz, and Yuri Kovchegov

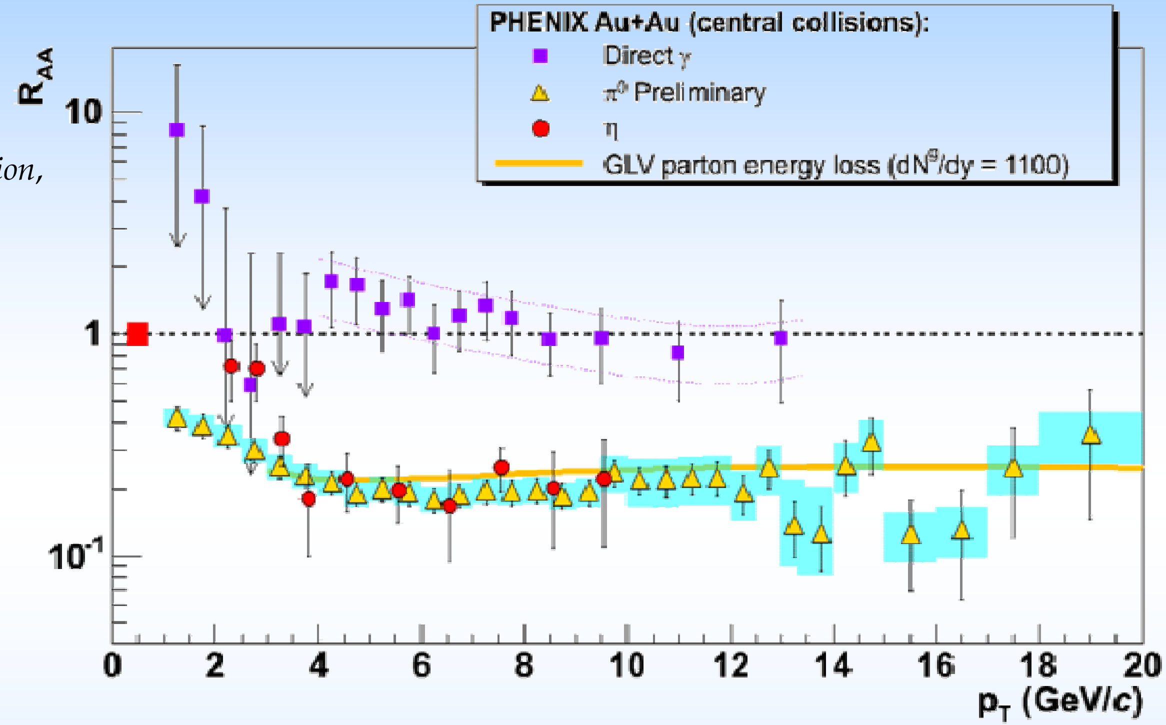
Outline

- Context
- Pedagogy
- (Some) brick results
- Comparing WHDG to ASW-SH
- Pedagogy
- Surprise!
- Conclusions

pQCD Success at RHIC:

(circa 2005)

Y. Akiba *for the PHENIX collaboration*, hep-ex/0510008



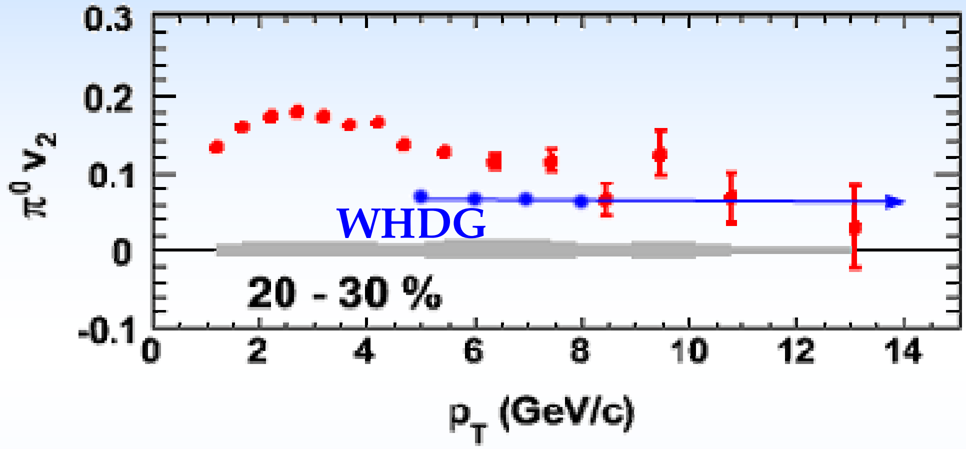
- Consistency: $R_{AA}(\eta) \sim R_{AA}(\pi)$
- Null Control: $R_{AA}(\gamma) \sim 1$
- GLV Prediction: Theory~Data for reasonable fixed L~5 fm and $dN_g/dy \sim dN_\pi/dy$

Trouble for High-p_T wQGP Picture

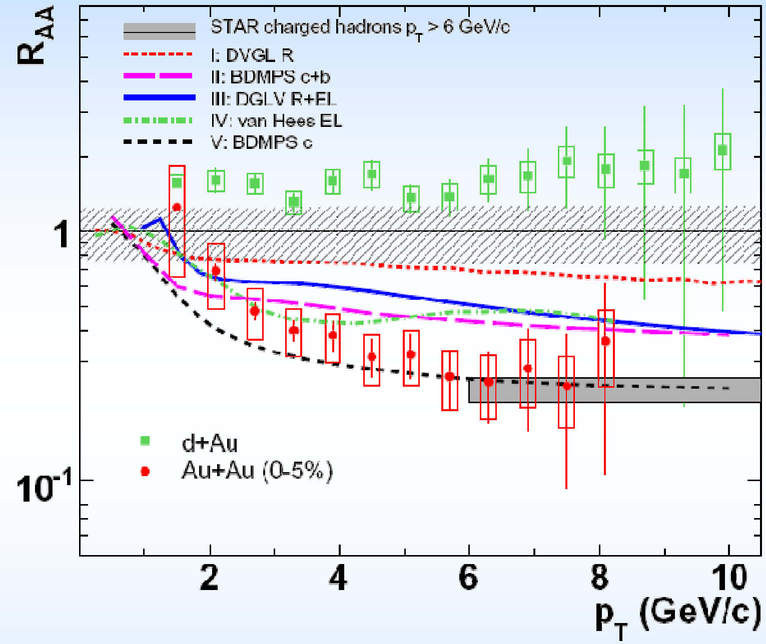
– v_2 too small

– NPE supp. too large

π^0 v_2

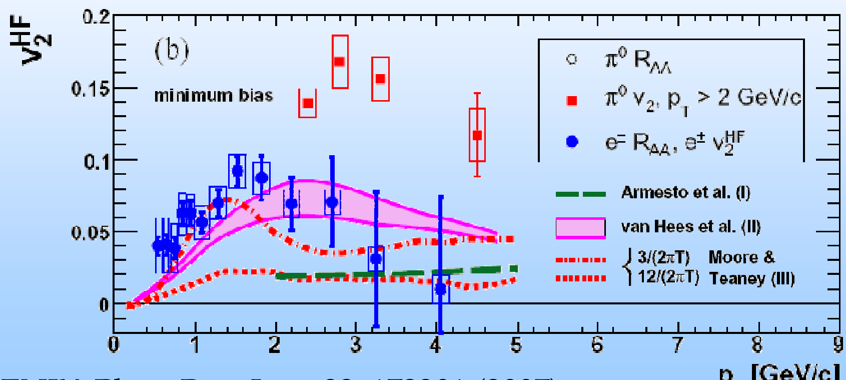


C. Vale, QM09 Plenary (analysis by R. Wei)



STAR, Phys. Rev. Lett. 98, 192301 (2007)

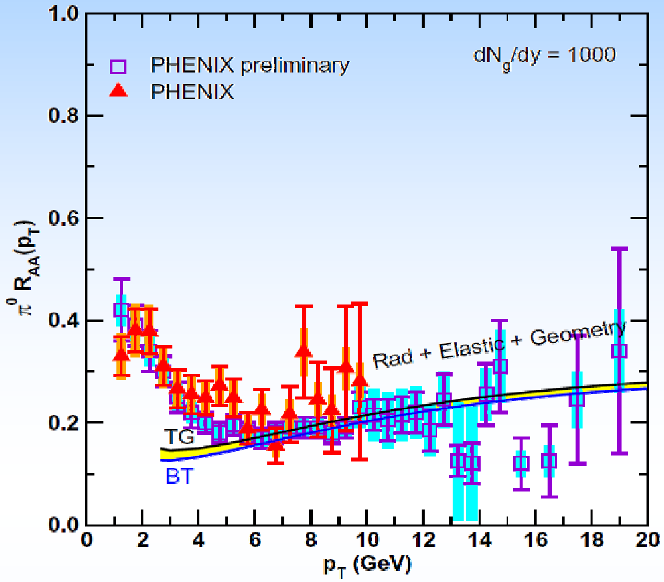
NPE v_2



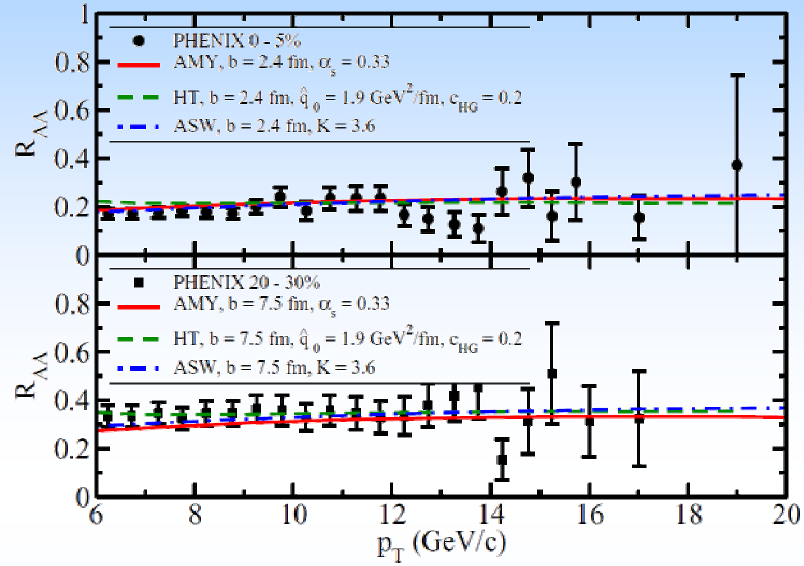
PHENIX, Phys. Rev. Lett. 98, 172301 (2007)

Pert. at LHC energies?

Multiple Models



WHDG, Nucl.Phys.A784:426-442,2007



Bass et al., Phys.Rev.C79:024901,2009

– Inconsistent medium properties

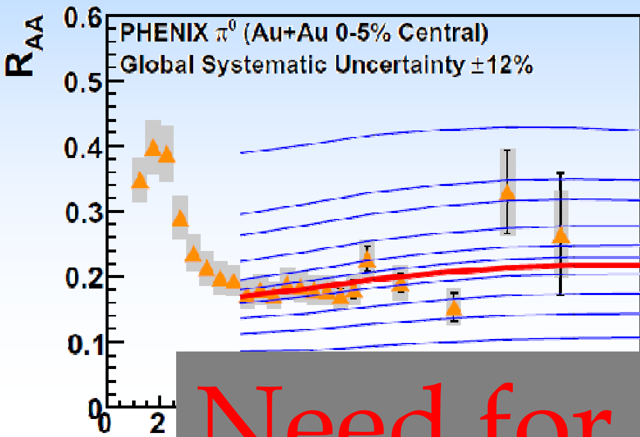
$\hat{q}(\vec{r},\tau)$ scales as	ASW $\hat{q}_0$	HT $\hat{q}_0$	AMY $\hat{q}_0$
$T(\vec{r},\tau)$	10 $\mathrm{GeV}^2/\mathrm{fm}$	2.3 $\mathrm{GeV}^2/\mathrm{fm}$	4.1 $\mathrm{GeV}^2/\mathrm{fm}$
$\epsilon^{3/4}(\vec{r},\tau)$	18.5 $\mathrm{GeV}^2/\mathrm{fm}$	4.5 $\mathrm{GeV}^2/\mathrm{fm}$	
$s(\vec{r},\tau)$		4.3 $\mathrm{GeV}^2/\mathrm{fm}$	

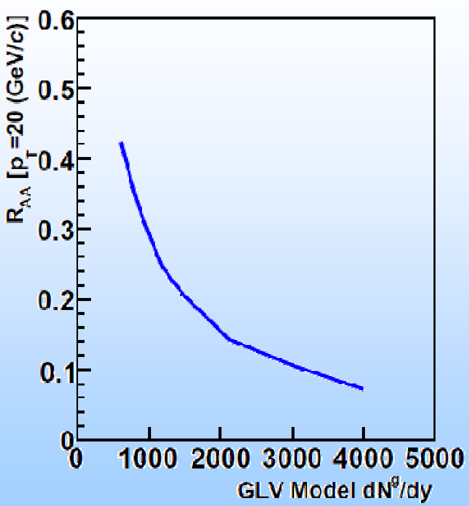
Bass et al.

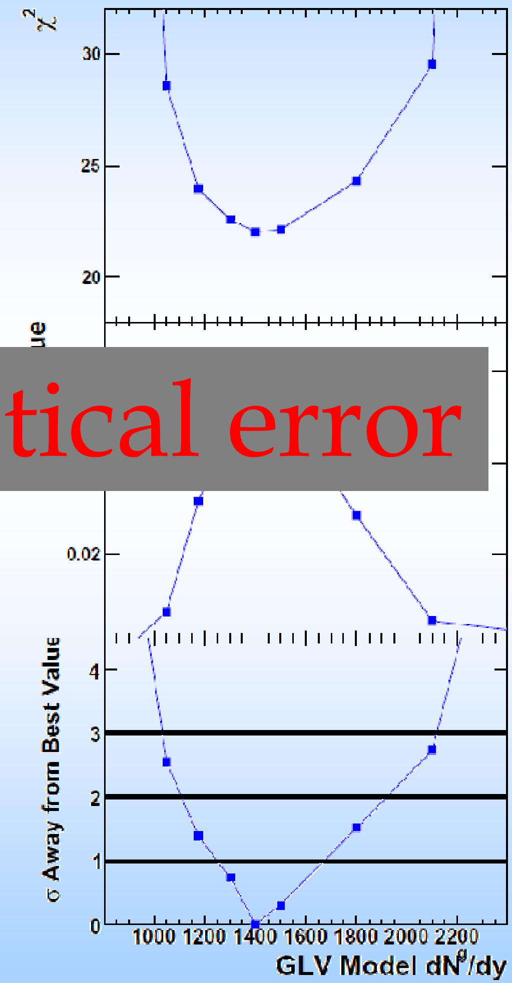
– Distinguish between models

Quantitative Parameter Extraction

- Vary input param.
- Find "best" value







Need for theoretical error

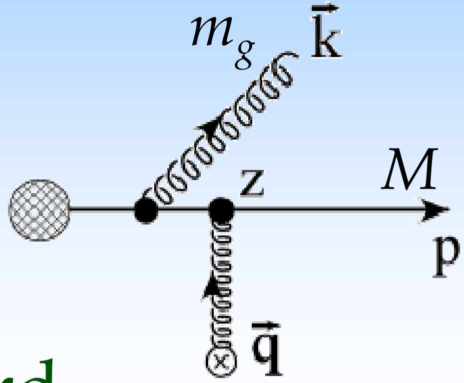
PHENIX, PRC77:064907,2008

pQCD Rad. Opacity Exp. (I)

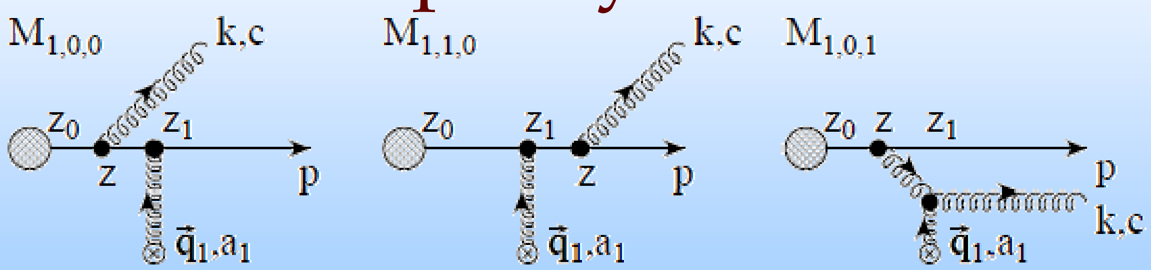
- All orders in L/λ expression for dN_g/dx
    - N_g: number of emitted gluons
    - x: “momentum fraction carried by gluon”
        - Small x regime, $x << 1$
- Assumed length scales: $\mu^{-1} << \lambda << L$
    - Debye screening mass μ
    - Mean free path λ
    - Medium length L
- Localized scattering centers; partons see many centers and radiate coherently (crucial!)

pQCD Rad. Opacity Exp. (II)

- Eikonality assumed
  - $p^+ >> p^-$
  - $k^+ >> k^-$



  – Radiation emitted forward
  – Parent parton continues moving forward
  – Partons move on straight-line paths
- First order in opacity



  – Interference w/ vacuum rad. ***crucial***

FOO dN_g/dx

$$x\frac{dN_g^{\mathrm{GLV}}}{dx} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{\left(\mathbf{q}^2+\mu^2\right)^2}\int\frac{2d^2\mathbf{k}}{\pi}\frac{\mathbf{k}\cdot\mathbf{q}(\mathbf{k}-\mathbf{q})^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^4}\int dz\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2}{2Ex}z\right)\right]\rho(z)$$

$$\rho(z) = \begin{cases} \frac{1}{L}\theta(L-z) \\ \frac{2}{L}\exp(-2z/L) \end{cases}$$

$$x\frac{dN_g^{\mathrm{DGLV}}}{dx} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{\left(\mathbf{q}^2+\mu^2\right)^2}\int\frac{2d^2\mathbf{k}}{\pi}\frac{\mathbf{k}\cdot\mathbf{q}(\mathbf{k}-\mathbf{q})^2-\beta^2\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})}{\left[(\mathbf{k}-\mathbf{q})^2+\beta^2\right]^2(\mathbf{k}^2+\beta^2)}\int dz\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2+\beta^2}{2Ex}z\right)\right]\rho(z)$$

$$\beta^2 = x^2M^2+(1-x)m_g^2$$

- WHDG Rad: μ, M, m_g depend on T

$\mathrm{d}N_g/\mathrm{d}x$ to $P(\varepsilon)$

- Opacity expansion => $\mathrm{d}N_g/\mathrm{d}x$
  - Single gluon emission spectrum
- Approx. multi-gluon fluct. w/ Poisson conv.
  - Prob. to lose mom. frac. ε: $p_f = (1-\varepsilon)p_i$

$$\langle N_g \rangle = \int dx \frac{dN_g}{dx}$$

$$P_n^1 = \frac{e^{-\langle N_g \rangle} \langle N_g \rangle^n}{n!} - \int_0^1 d\epsilon \tilde{P}_n(\epsilon)$$

$$\begin{aligned}
P_0(\epsilon) &= e^{-\langle N_g \rangle} \delta(\epsilon) \\
&= P^0 \delta(\epsilon) \\
P_1(\epsilon) &= e^{-\langle N_g \rangle} \frac{dN_g}{dx}(\epsilon)\theta(1-\epsilon) + P_1^1 \delta(\epsilon - 1) \\
&= \tilde{P}_1(\epsilon) + P_1^1 \delta(\epsilon - 1) \\
&\vdots \\
P_n(\epsilon) &= \frac{1}{n}\int_0^1 dx \tilde{P}_{n-1}(x)\tilde{P}_1(\epsilon - x)\theta(1-\epsilon) + P_n^1 \delta(\epsilon - 1) \\
&= \tilde{P}_n(\epsilon) + P_n^1 \delta(\epsilon - 1),
\end{aligned}$$

$$\boxed{\begin{aligned}
P(\epsilon) &= P^0 \delta(\epsilon) + \sum_{n=1} \tilde{P}_n(\epsilon) + \sum_{n=1} P_n^1 \delta(\epsilon - 1) \\
&= P^0 \delta(\epsilon) + \tilde{P}(\epsilon) + P^1 \delta(\epsilon - 1).
\end{aligned}}$$

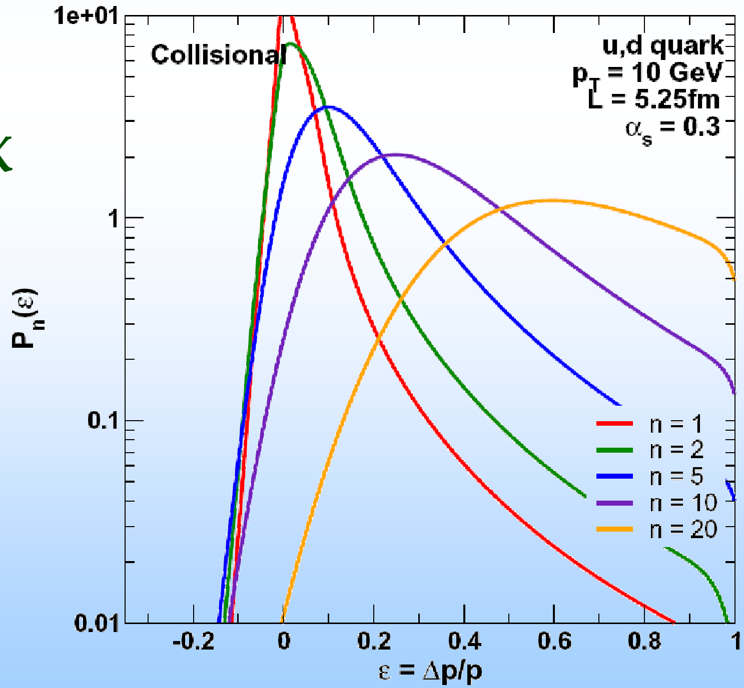
  - Assumes ***incoherent emission*** of non-Abelian gluons

WHDG Collisional Loss

- Gaussian distribution
  - Mean loss for light quarks:
    - Braaten-Thoma, PRD44, 2625 (1991)
- Width given by Fluctuation-Dissipation theorem

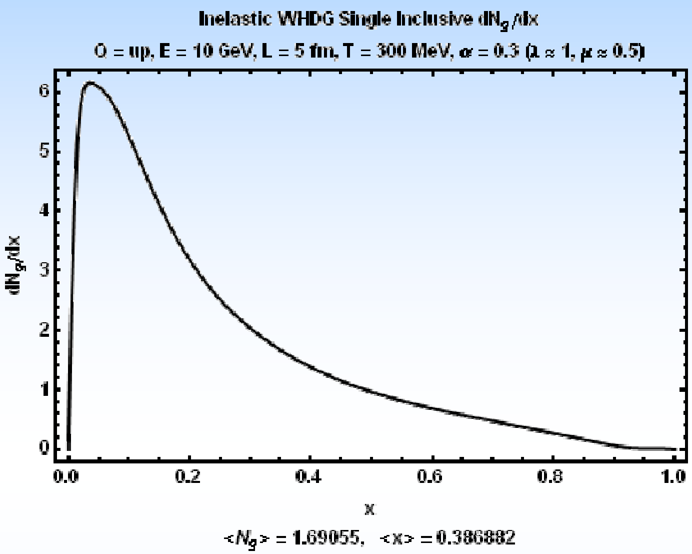
$$\sigma = (2/p) \int dp/dz\, T(z)\, dz$$

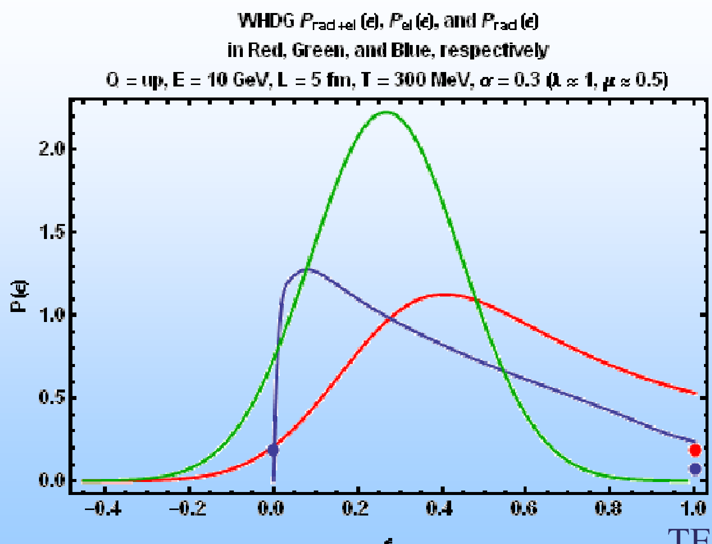
- Poisson conv. not well approx by Gaussian for realistic, small num of scatterings
  - See Simon Wicks' thesis



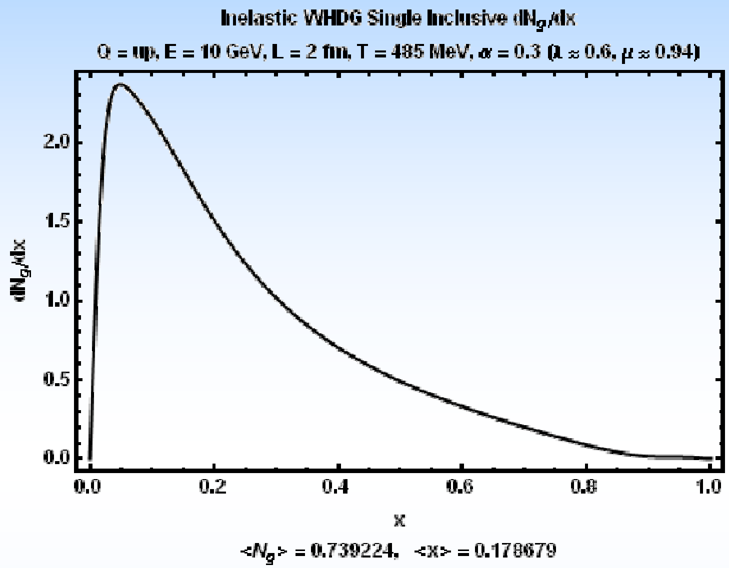
Typical Results

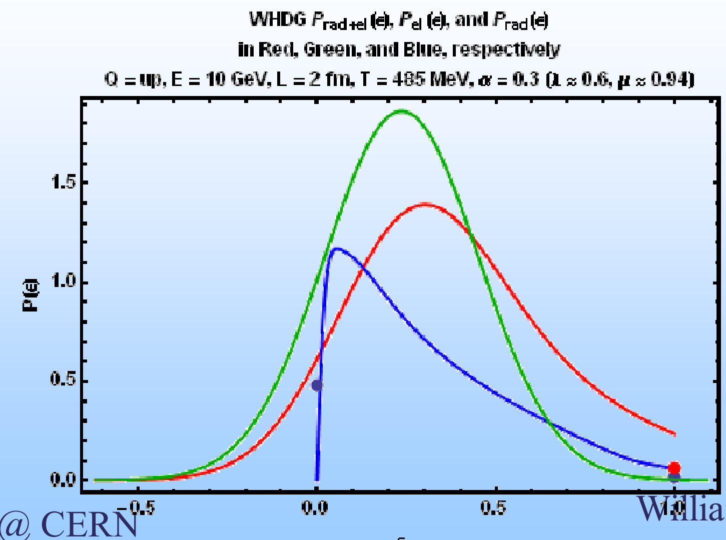
- Original Brick





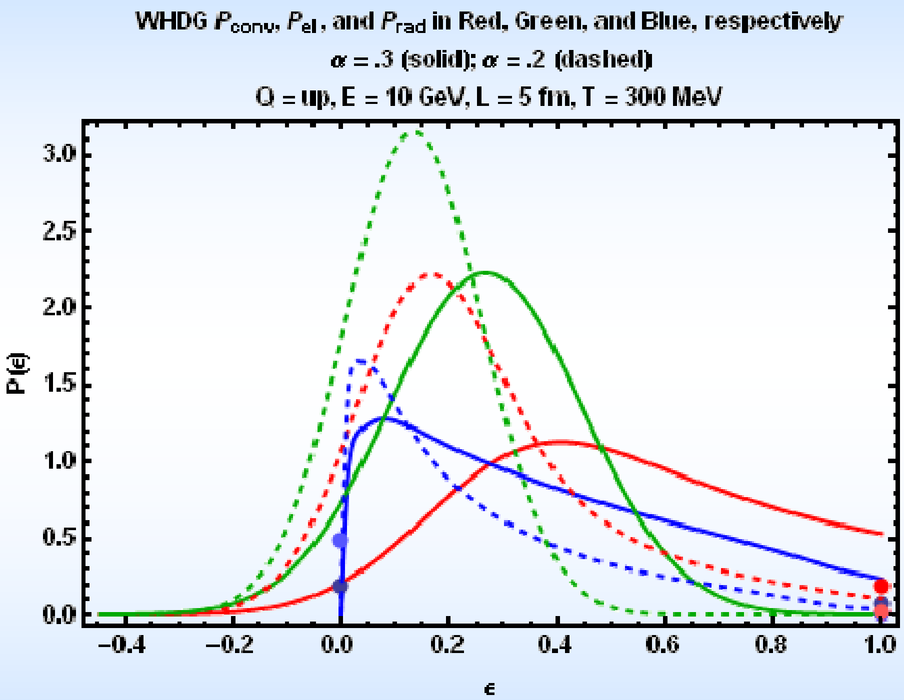
- Wiedemann Brick $\langle \varepsilon \rangle$ = .4



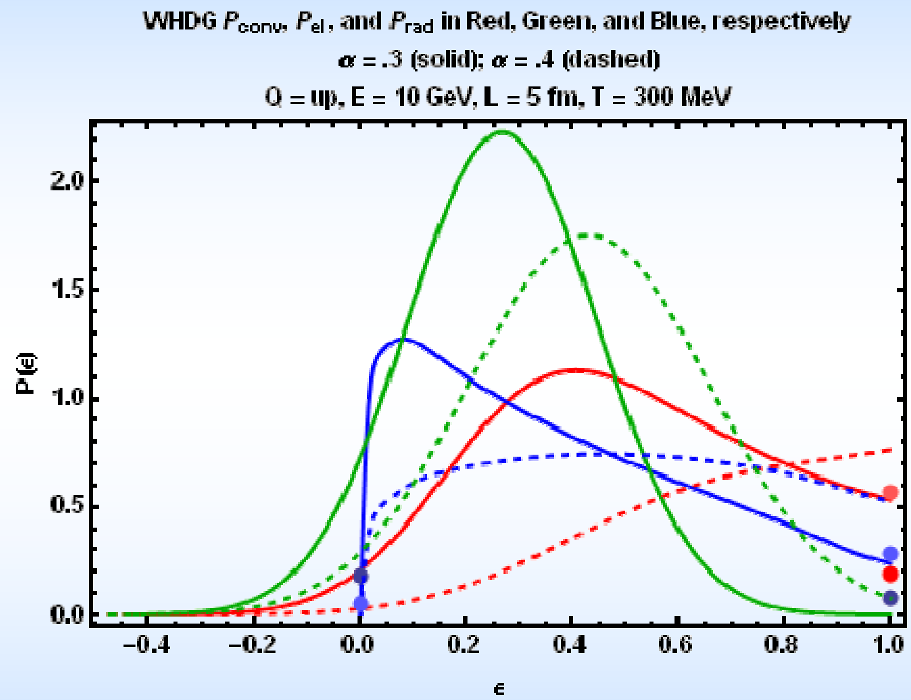


Running α_s?

– α_s = .2, .3



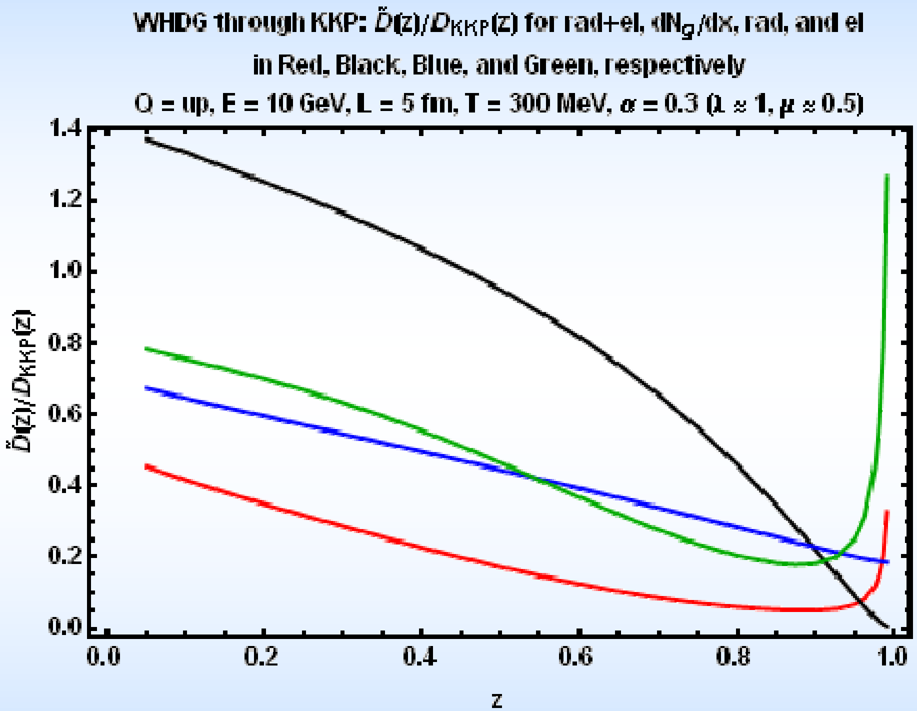
– α_s = .3, .4

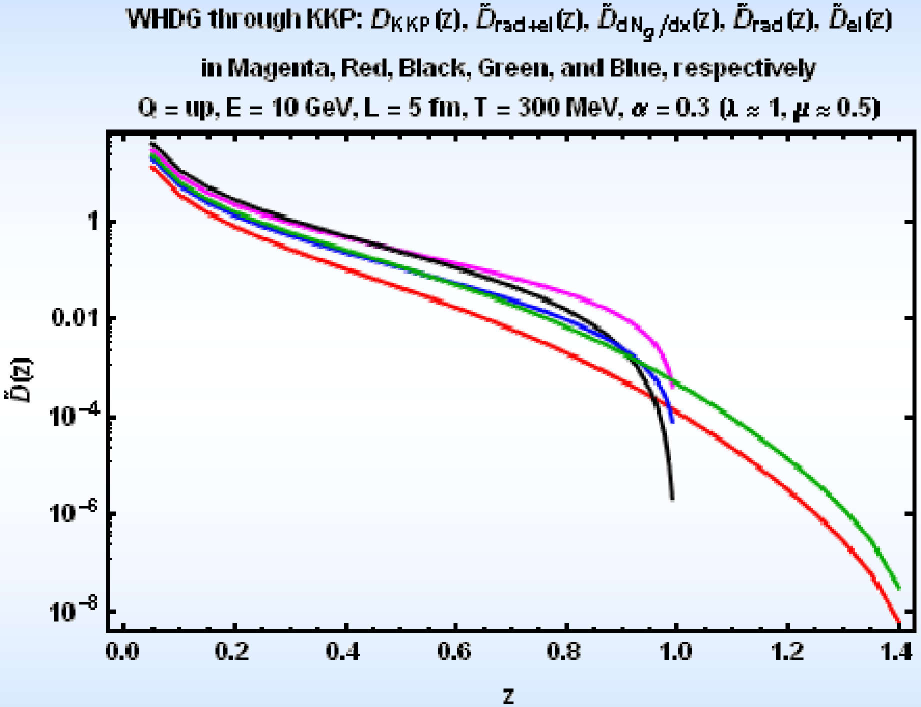


– Not surprisingly, changes in α_s make ***huge*** difference to P(ε)

WHDG thru KKP

– Facilitate comparison between WHDG and HT





– Elastic gain => D(z > 1) > 0

FOO dN_g/dx

$$x\frac{dN_g^{\rm GLV}}{dx} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{(\mathbf{q}^2+\mu^2)^2}\int\frac{2d^2\mathbf{k}}{\pi}\frac{\mathbf{k}\cdot\mathbf{q}(\mathbf{k}-\mathbf{q})^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^4}\int dz\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2}{2Ex}z\right)\right]\rho(z)$$

$$\rho(z) = \begin{cases} \frac{1}{L}\theta(L-z) \\ \frac{2}{L}\exp(-2z/L) \end{cases}$$

$$x\frac{dN_g^{\rm DGLV}}{dx} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{(\mathbf{q}^2+\mu^2)^2}\int\frac{2d^2\mathbf{k}}{\pi}\frac{\mathbf{k}\cdot\mathbf{q}(\mathbf{k}-\mathbf{q})^2-\beta^2\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})}{\left[(\mathbf{k}-\mathbf{q})^2+\beta^2\right]^2(\mathbf{k}^2+\beta^2)}\int dz\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2+\beta^2}{2Ex}z\right)\right]\rho(z)$$

$$\beta^2 = x^2M^2+(1-x)m_g^2$$

- WHDG Rad: μ, M, m_g depend on T

$$x\frac{dN_g^{\rm DH}}{dx} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda_{\rm dyn}}\int\frac{d^2\mathbf{q}}{\pi}\frac{\mu^2}{\mathbf{q}^2(\mathbf{q}^2+\mu^2)}\int\frac{2d^2\mathbf{k}}{\pi}\frac{\mathbf{k}\cdot\mathbf{q}(\mathbf{k}-\mathbf{q})^2-\beta^2\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})}{\left[(\mathbf{k}-\mathbf{q})^2+\beta^2\right]^2(\mathbf{k}^2+\beta^2)}\int dz\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2+\beta^2}{2Ex}z\right)\right]\rho(z)$$

$$\omega\frac{dI^{\rm ASH-SH}}{d\omega} = \frac{4\alpha_sC_R}{\pi}(n_0L)\gamma\int_0^\infty \tilde{q}d\tilde{q}\left[\frac{\tilde{q}^2-\sin\tilde{q}^2}{\tilde{q}^4}\right]\left(\frac{1}{\gamma+\tilde{q}^2}-\frac{1}{\sqrt{(\kappa^2+\tilde{q}^2+\gamma^2)^2-4\kappa^2\tilde{q}^2}}\right)$$

$\gamma = \tilde{\omega}_c/\omega$, $\tilde{\omega}_c = \frac{1}{2}\mu^2L$, $\kappa = \sqrt{\omega L/2}$, and $n_0L = L/\lambda$

Differences

- WHDG Rad
  - $m_g = \mu/\sqrt{2}$
  - $M = \mu/2$
  - $k_{\max} = 2\,x\,(1\text{-}x)\,E$
  - $\rho_{\exp}(z)$
  - $L/\lambda(T)$
  - $q_{\max} = \sqrt{(3\,\mu\,E)}$
  - $\alpha_s = .3$
- ASW-SH
  - $m_g = 0$
  - $M = 0$
  - $k_{\max} = x\,E$
  - $\rho_{\mathrm{theta}}(z)$
  - $L/\lambda = 1$
  - $q_{\max} = \infty$
  - $\alpha_s = 1/3$

Where Did k_{max}'s Come From? (I)

- DGLV
  - Light cone momenta

$$P = (E, E, 0, 0) = [E^+, 0, 0]$$

$$k = [x_+ E^+, \frac{\mathbf{k}_\perp^2}{x_+ E^+}, \mathbf{k}_\perp]$$

$$p = [(1 - x_+)E^+, \frac{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{(1 - x_+)E^+}, \mathbf{q}_\perp - \mathbf{k}_\perp]$$

  - Note x_+ def.!
  - Always on-shell

$$x_+ = \frac{x_E}{2}\left(1 + \sqrt{1 - \left(\frac{k_\perp}{x_E E}\right)^2}\right)$$

- ASW-SH
  - 4-momenta

$$P = (E, E, \mathbf{0})$$

$$p = ((1 - x_E)E, \sqrt{((1 - x_E)E)^2 - (\mathbf{q} - \mathbf{k})^2}, \mathbf{q} - \mathbf{k})$$

$$k = (x_E E, \sqrt{(x_E E)^2 - \mathbf{k}^2}, \mathbf{k})$$

  - Note x_E def.!
  - Always on-shell

$$x_E = x_+\left(1 + \left(\frac{k_\perp}{x_+ E^+}\right)^2\right)$$

The same in the eikonal limit!

Where Did k_{max}'s Come From? (II)

- DGLV
  - Light cone momenta

$$P = (E, E, 0, 0) = [E^+, 0, 0]$$

$$k = [x_+ E^+, \frac{\mathbf{k}_\perp^2}{x_+ E^+}, \mathbf{k}_\perp]$$

$$p = [(1 - x_+)E^+, \frac{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{(1 - x_+)E^+}, \mathbf{q}_\perp - \mathbf{k}_\perp]$$

  - Note x_+ def.!
  - Always on-shell

$$k^+ \gg k^- \;\Rightarrow\; x_+ E^+ \gg k_\perp$$

$$p^+ \gg p^- \;\Rightarrow\; (1 - x_+)E^+ \gg |\mathbf{q}_\perp - \mathbf{k}_\perp| \approx k_\perp$$

  - $k_T < x_+ E^+ = 2\, x_+ E$
  - Forward travel

- ASW-SH
  - 4-momenta

$$P = (E, E, \mathbf{0})$$

$$p = ((1 - x_E)E, \sqrt{((1 - x_E)E)^2 - (\mathbf{q} - \mathbf{k})^2}, \mathbf{q} - \mathbf{k})$$

$$k = (x_E E, \sqrt{(x_E E)^2 - \mathbf{k}^2}, \mathbf{k})$$

  - Note x_E def.!
  - Always on-shell

$$k^z > 0 \;\Rightarrow\; x_E E > k_\perp$$

$$p^z > 0 \;\Rightarrow\; (1 - x_E)E > |\mathbf{q}_\perp - \mathbf{k}_\perp| \approx k_\perp$$

  - $k_T < x_E\, E$
  - Forward travel

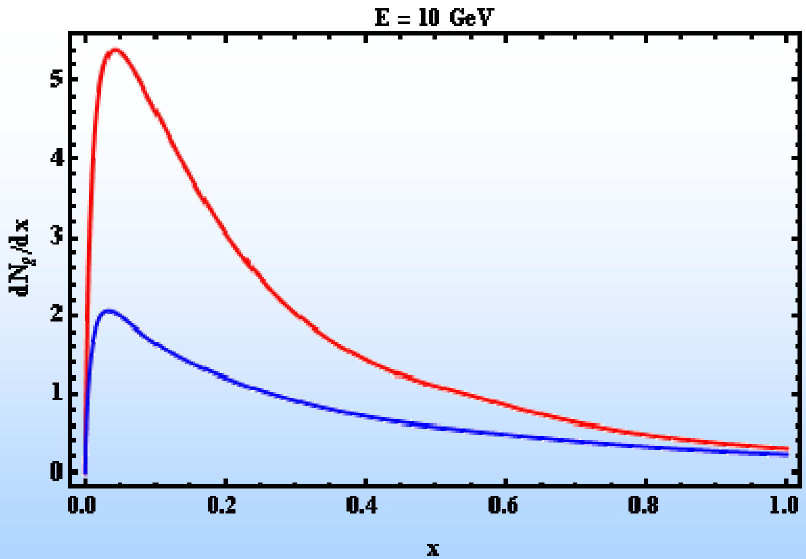
Same physics: cutoff when gluons radiated at 90^o

Compare Apples to Apples

- Differences must be due to non-eikonality

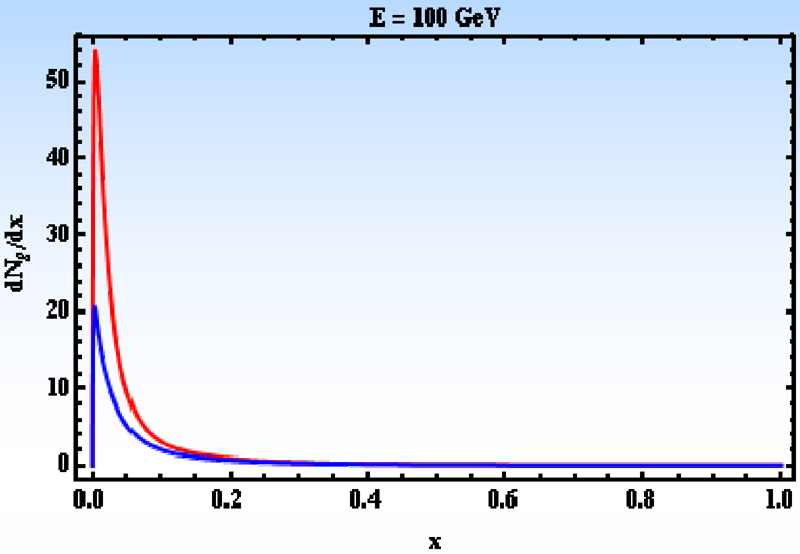
$$\frac{dN^{\mathrm{ASW-SH}}}{dx_E}(x_E) = \int_0^{q_{\max}} dq \int_0^{x_E E} dk \frac{dN^{\mathrm{ASW-SH}}}{dx_E dq dk}(x_E; k, q)$$

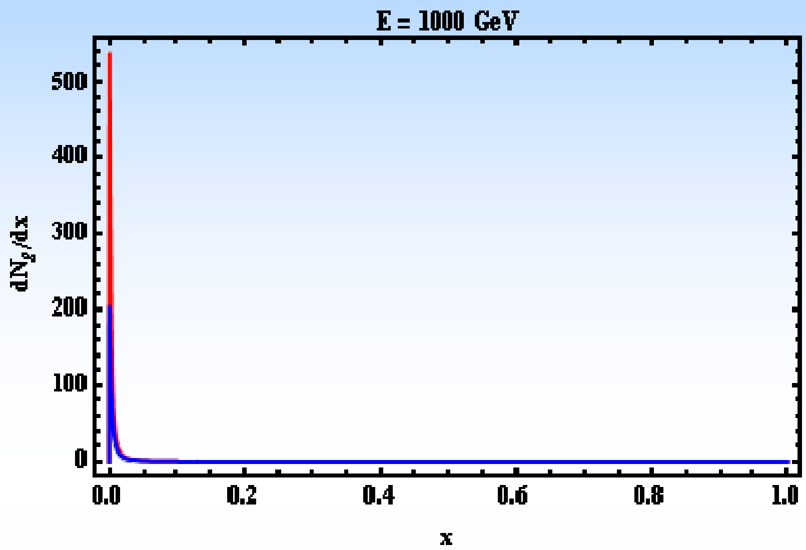
$$\frac{dN^{\mathrm{GLV}}}{dx_E}(x_E) = \int_0^{q_{\max}} dq \int_0^{x_E E} dk \frac{dx_+}{dx_E}(x_E; k) \frac{dN^{\mathrm{GLV}}}{dx_+ dq dk}(x_+(x_E); k, q)$$



- GLV(x_E) in red; ASW-SH(x_E) in blue

Large E Limit





– GLV(x_E) in red; ASW-SH(x_E) in blue
– For most values of x, naïve interpretation holds
  - What's going on at small x?

Interpretation

- Physically:
  - Typical $k_T \sim \mu$ => typ. $\omega \sim \mu$
    - System wants to radiate lots of glue at $x \sim \mu / E$
    - *BUT*, this is right at our k_T cutoff:
      - $k_T \sim \mu < \mu \sim k_{T,\,max}$: the system will always take advantage of all the "phase space" we give it
- Analytically:
  - More natural way to write dN_g/dx

$$\frac{dN_g}{dx} = \frac{4\alpha_s C_R L}{\pi\lambda}\int d\bar{q}\,\frac{\bar{q}^3}{\bar{q}^4 + (4x/\bar{\gamma})^2}\left(\frac{1}{\bar{q}^2+1} - \frac{1}{\sqrt{\left[(\bar{k}_{\max}-\bar{q})^2+1\right]\left[(\bar{k}_{\max}+\bar{q})^2+1\right]}}\right)$$

$\bar{\gamma} = \mu^2 L/E$, $\bar{q} = q/\mu$, $\bar{k} = k/\mu$, and $\bar{k}_{\max} = \# x E/\mu$

$$x_{\max} = \frac{\mu}{E}\ln\left(\frac{\mu^2 L}{E}\right)$$

Consequences

- At large energies, $<N_g>$ is E ind.
  - Large *irreducible* systematic uncertainty for some observables
  - For T = 485 GeV, L = 2 fm, α_s = 0.3:
    - $<N_g> \approx 1$, $k_{\mathrm{max}} = x\, E$
    - $<N_g> \approx 2$, $k_{\mathrm{max}} = 2\, x\, E$
  - Note that R_{AA} becomes insensitive to details of k_{max} (goes to 1)

Conclusions

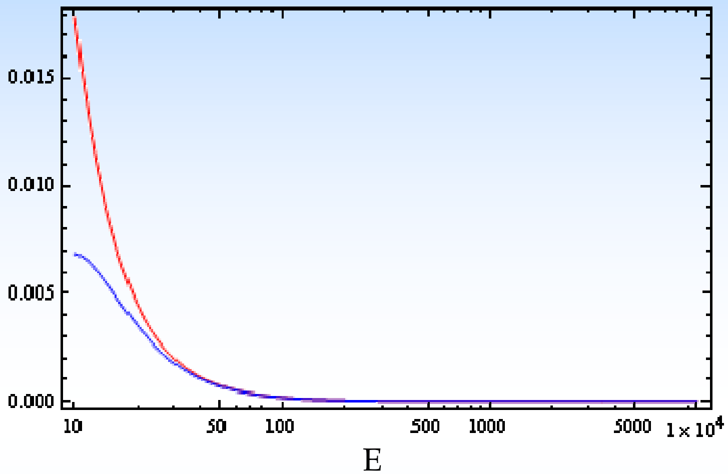
- Current phen. comparisons of pQCD to data unsatisfactory
- WHDG not oversuppressed

- Opacity expansion suffers from large systematic errors
  - Strong dependence on $k_{\text{max}} = \# \; x \; E$
    - # is not specified by framework
  - Similar dependence on IR cutoff, m_g
  - Irreducible?
- Consequences for other models, parameter extractions?

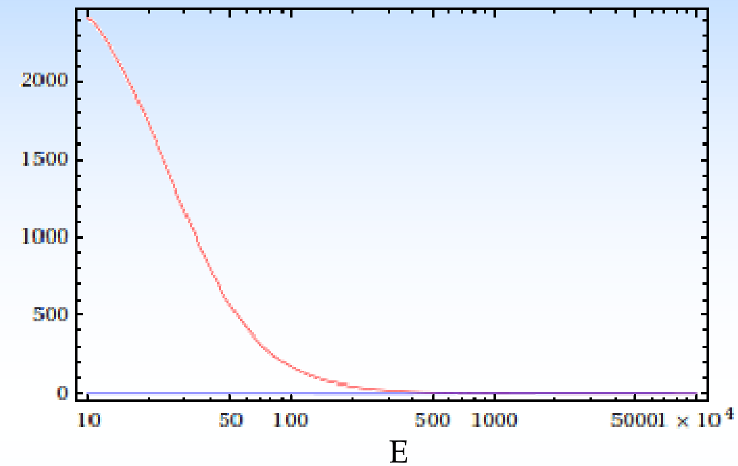
Supplement

Eikonality Sets in for all x

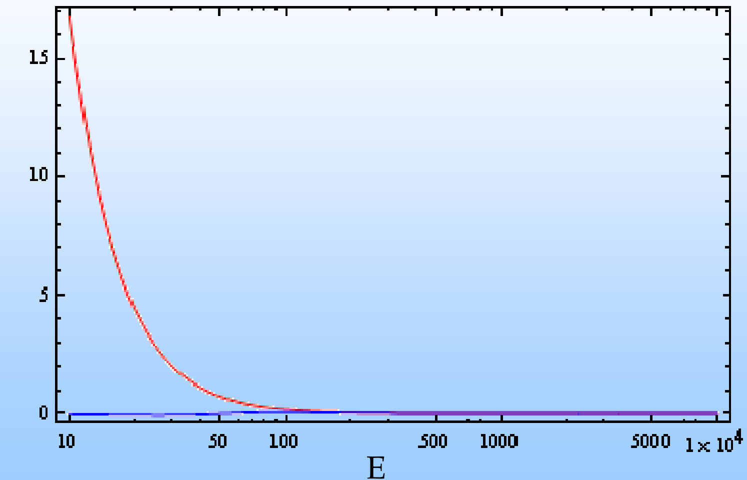
X = .1



X = .001



X = .01



X = .00001