



Brick problem: comparing different formalisms

Marta Verweij
Utrecht University
TEC-HQM
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Brick problem

- Apples-to-apples comparison of:
 - ASW-BDMPS
Phys. Rev. D68 014008
 - Opacity expansion:
 - ASW-SH
Phys. Rev. D68 014008
 - WHDG
Nucl. Phys. A784 426

Brick problem

- Apple-to-apple comparison of:
 - ASW-BDMPS
Phys.Rev.D68 014008
 - Opacity expansion:
 - ASW-SH: L/λ determined by T
Phys.Rev.D68 014008
 - WHDG: with $E_{\text{parton}} = 10 \text{ GeV}$ and $E_{\text{parton}} = 100 \text{ GeV}$
Only radiative energy loss
Nucl.Phys.A784 426

See also:

https://wiki.bnl.gov/TECHQM/index.php/Brick_comparison_of_ASW-BDMPS%2C_ASW-SH_and_WHDG

Common scale T

Baier gluon gas

- BDMPS

$$\hat{q} = \frac{\langle k_t^2 \rangle}{\lambda} = \frac{72 \cdot 1.202 \alpha_s^2}{\pi} T^3$$

$$\omega_c = \frac{1}{2} \hat{q} L^2$$

$$R = \frac{2\omega_c^2}{\hat{q}L} = \omega_c L = \frac{1}{2} \hat{q} L^3$$

- WHDG

$$\mu = \sqrt{4\pi\alpha_s} T$$

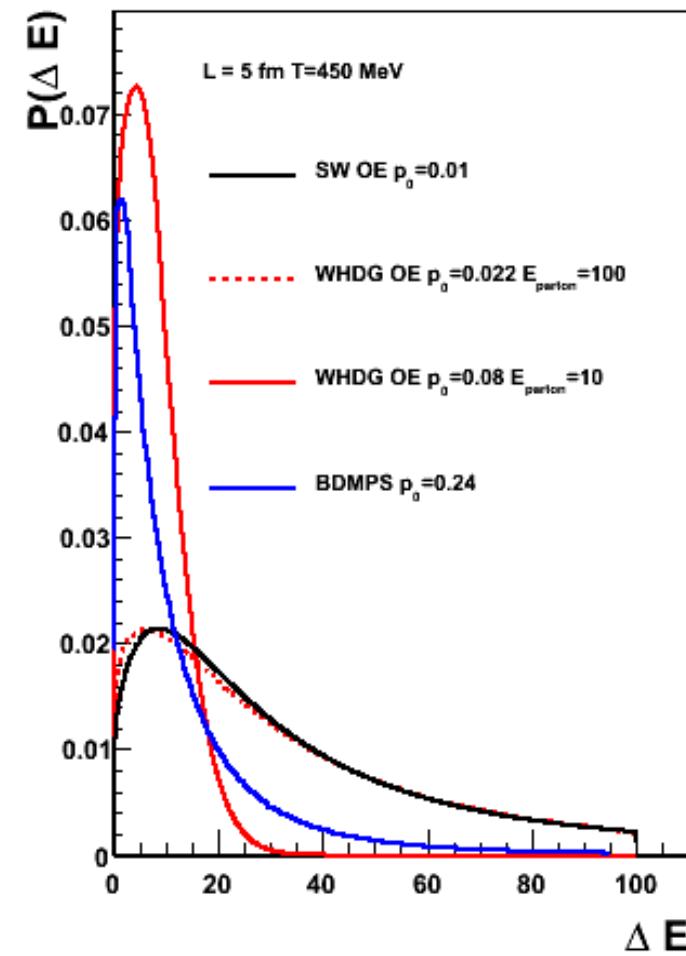
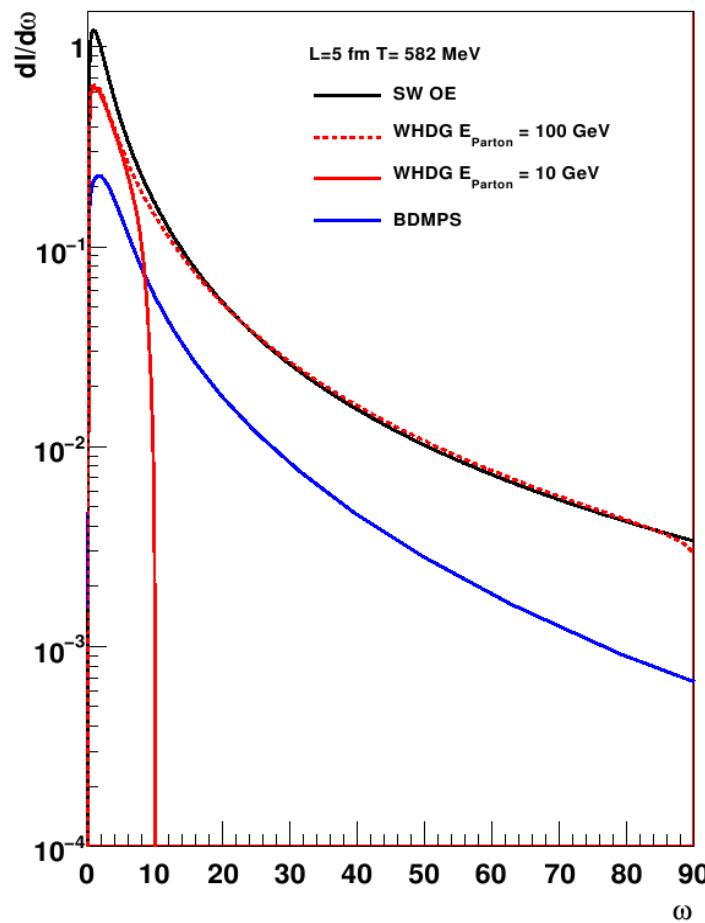
$$\frac{1}{\lambda} = \rho\sigma = \frac{144 \cdot 1.202 \alpha_s}{8\pi^2} T$$

$$\bar{\omega}_c = \frac{1}{2} \mu^2 L = 2\pi\alpha_s T^2 L$$

$$\bar{R} = \bar{\omega}_c L = 2\pi\alpha_s T^2 L^2$$

Brick examples

- Single gluon spectrum
- Energy loss distribution

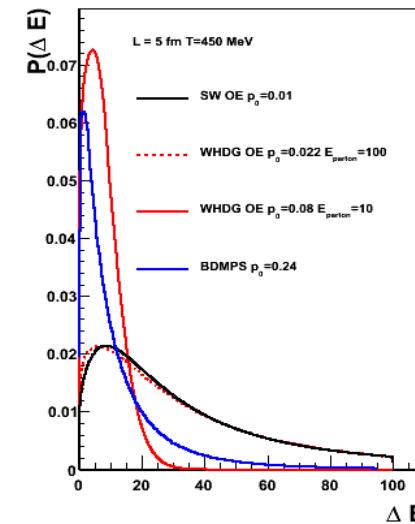
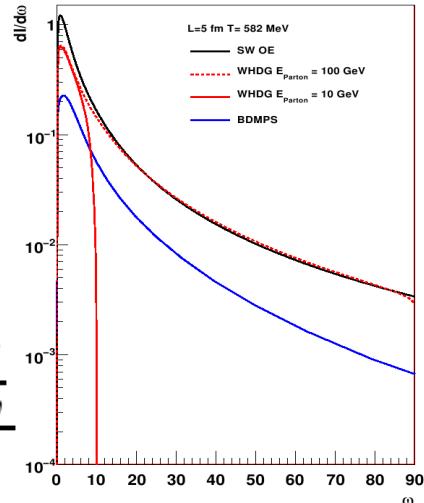


How much energy do we lose?

- Single gluon spectrum
- Energy loss distribution

$$\langle N_{gl} \rangle = \int d\omega \frac{dI}{d\omega}$$

$$\langle \omega \rangle = \frac{\int d\omega \omega \frac{dI}{d\omega}}{\langle N_{gl} \rangle}$$



	$\langle N_{gl} \rangle$	$\langle \omega \rangle$ (GeV)	$\langle \Delta E \rangle$ (GeV)
BDMPS	2.0	13	3.4
WHDG (E=10 GeV)	3.0	3.2	5.4
WHDG (E=100 GeV)	5.3	12	8.1
SWOE	7.2	9.8	9.0

→ for $E_{\text{max}} = 10 \text{ GeV}$

- Large difference between BDMPS and OEs!

Suppression

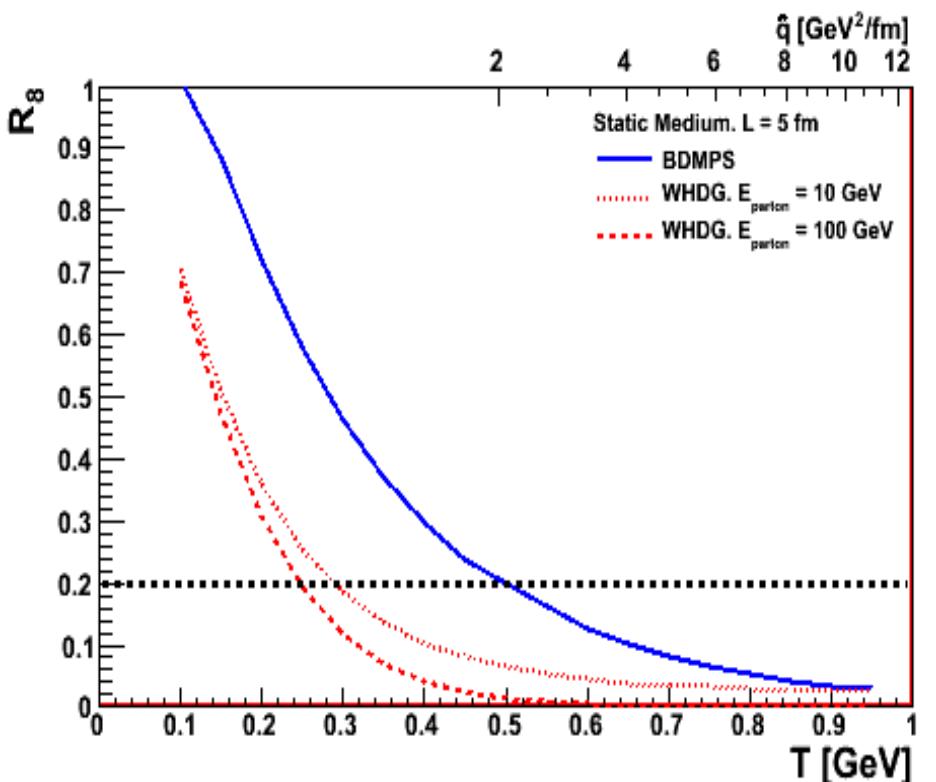
- Eloss depends on L and T of the brick.
- Estimation for R_{AA}

$$R_8 = \int_0^1 d\epsilon (1 - \epsilon)^8 P(\epsilon)$$

$$\epsilon = \Delta E / E$$

- $R_{8,WHDG} < R_{8,BDMPS}$
for same medium density

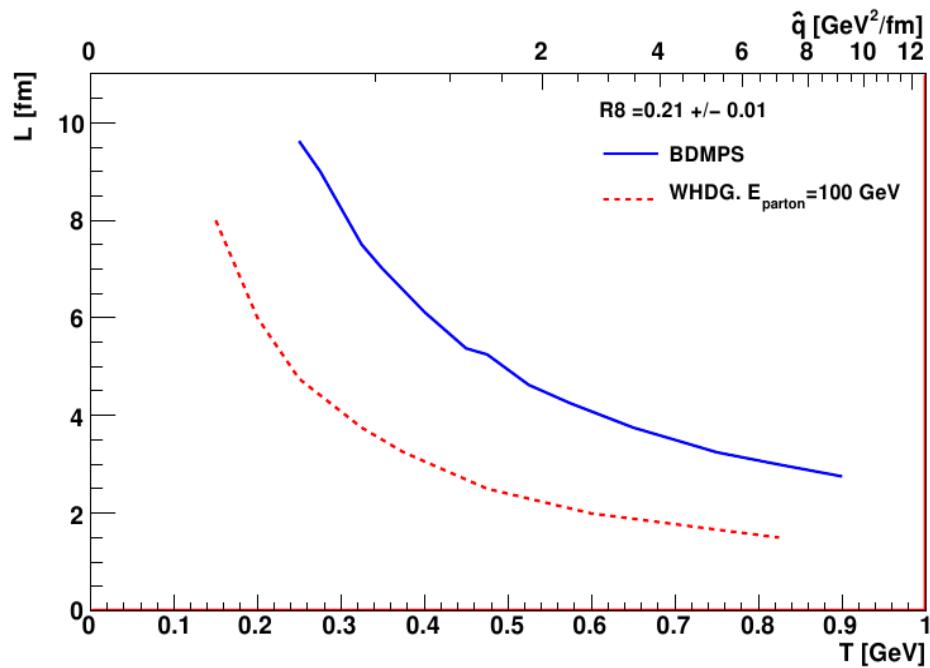
Fixed L = 5 fm



	BDMPS	WHDG E=10	WHDG E=100
T(R8=0.2) (MeV)	500	260	290

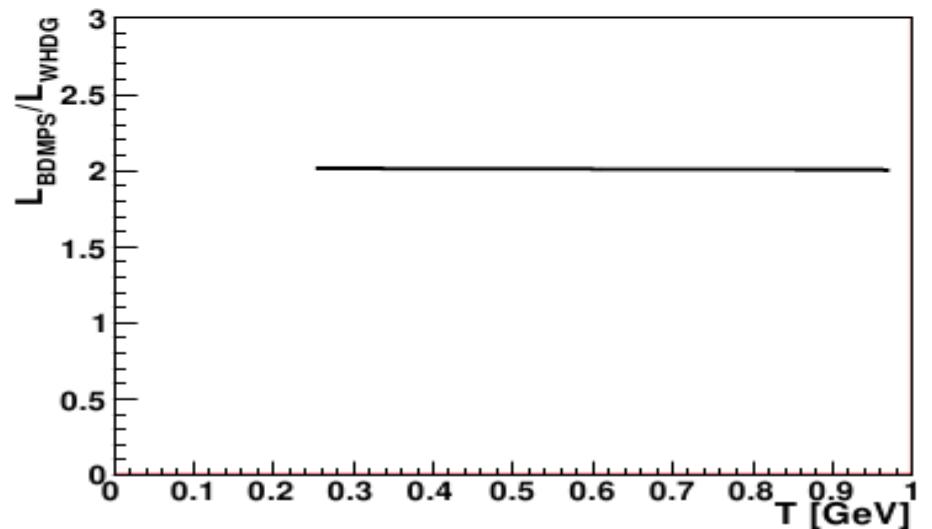
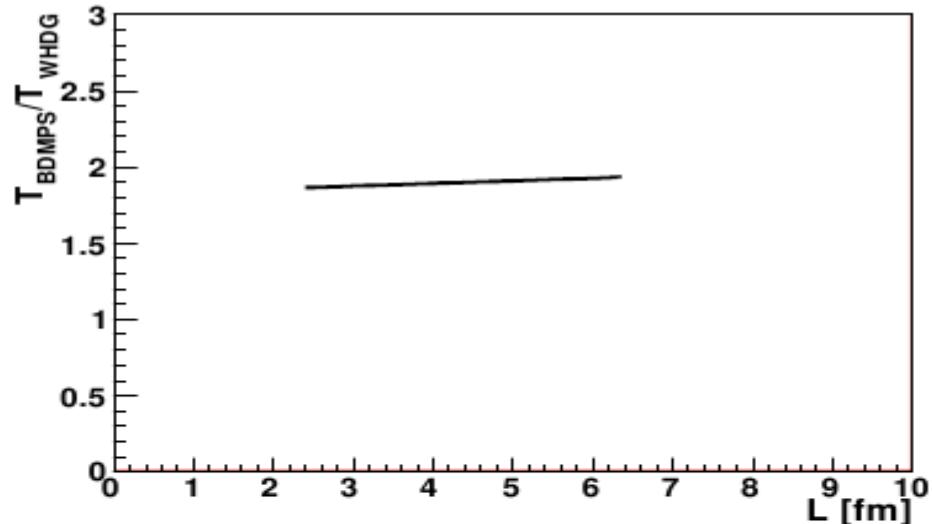
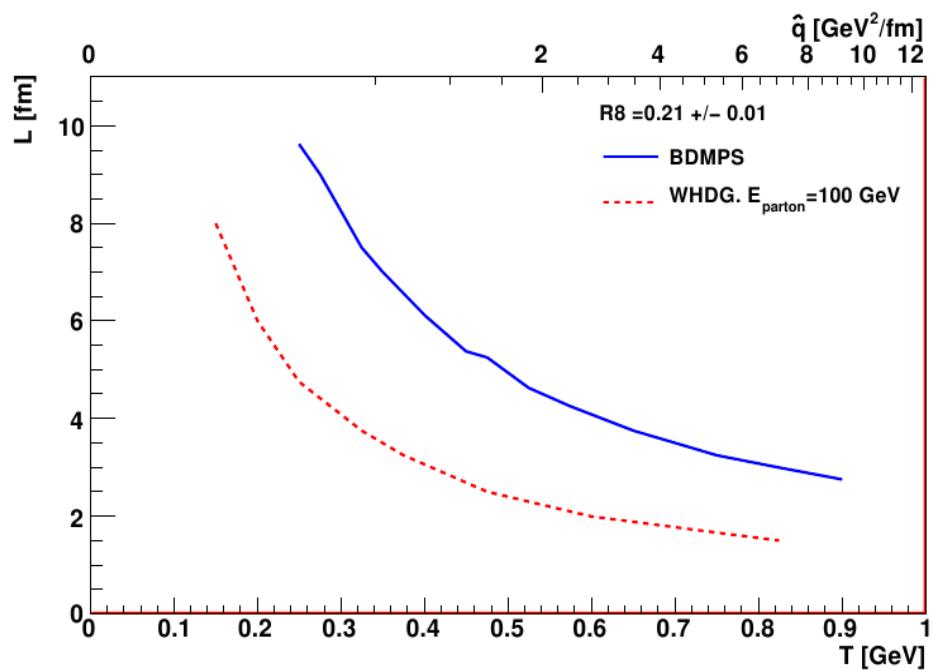
Vary T and L

- Select $R_8 = 0.2$ for all possible combinations of T and L.



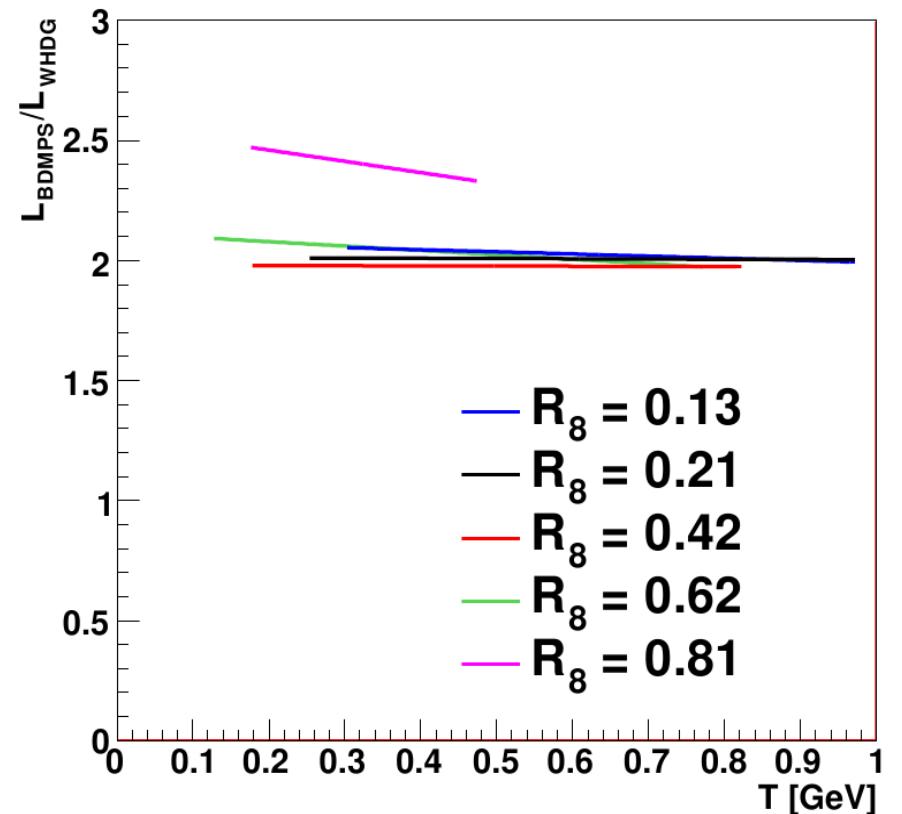
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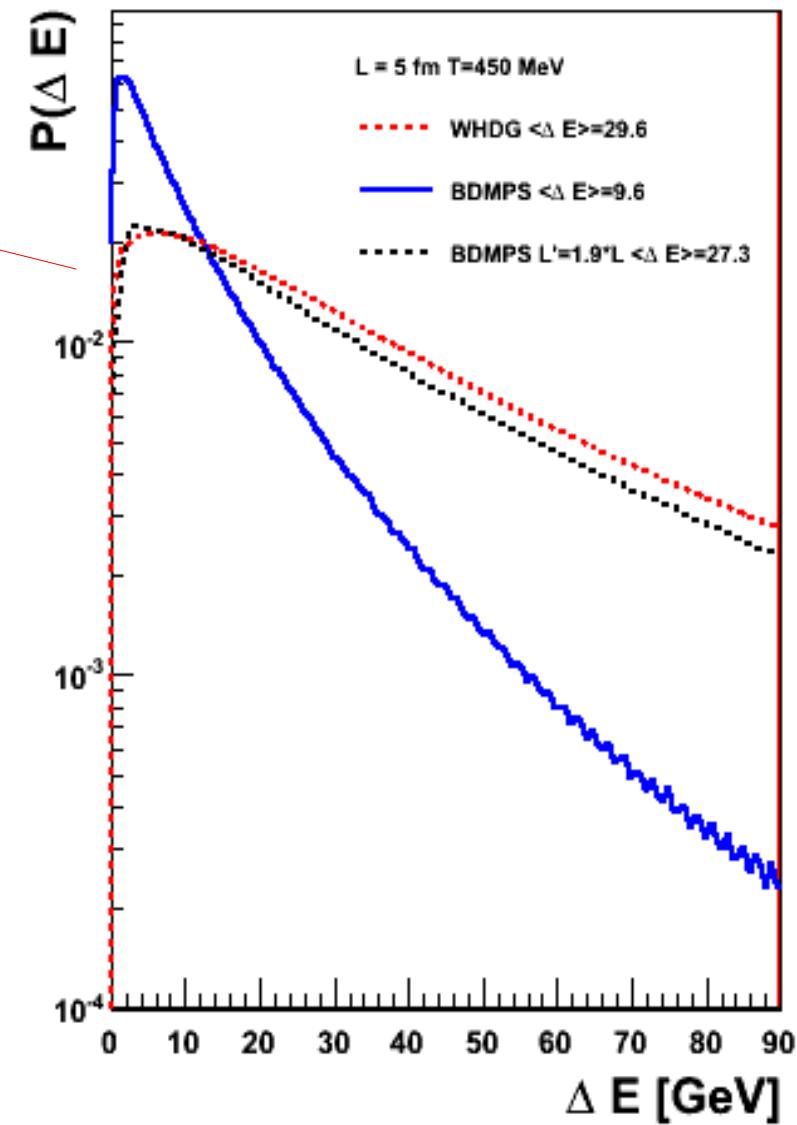
Mapping in L

- BDMPS and WHDG can be mapped by a factor of 2 in L.
- Holds for large range in T and R_8



Back to $P(\Delta E)$

- With mapping in L energy loss probability distributions of WHDG and BDMPS are very similar.



Conclusions brick

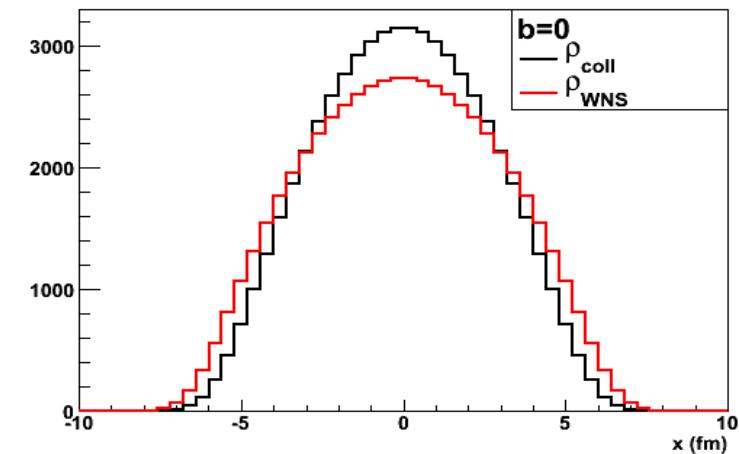
- Opacity expansions lose more energy than BDMPS.
- Difference between BDMPS and OEs in general, is much larger than difference between OEs.
- Large difference in T between BDMPS and WHDG ($E > 100$ GeV) equal to a mapping in L:
$$L_{BDMPS}/L_{WHDG} = 2$$

“Realistic” Geometry

- Parton spectrum: LO pQCD T. Renk
- Energy loss: BDMPS and WHDG
- Fragmentation: KKP

“Realistic” Geometry

- Parton spectrum: LO pQCD T. Renk
- Energy loss: BDMPS and WHDG
 - Optical Glauber:
 - Density profile of medium
 - Collisional scaling:
$$\rho_{coll} = k_{coll} \times T_A T_B$$
 - Participant scaling:
$$\rho_{WNS} = k_{WNS} \times (T_A (1 - e^{-T_B \sigma_{NN}}) + T_B (1 - e^{-T_A \sigma_{NN}}))$$
 - Fragmentation: KKP



Energy loss with BDMPS

- Multiple soft interactions

Ingredients: w_c and R_{eff} or L_{eff} and \hat{q} .

- PQM integral:

$$I_1 = \frac{1}{2} \hat{q}_h L^2 = w_{c,eff}$$

$$I_n = \int_0^\infty u^n \hat{q}(u) du$$

$$L_{eff} = \frac{2I_1}{I_0}$$

$$\propto \int_0^\infty u^n T^3(u) du$$

$$R = w_c L = \frac{1}{2} \hat{q}_h L^3$$

Physical Review D **68** 014008 (2003)

Energy loss with BDMPS

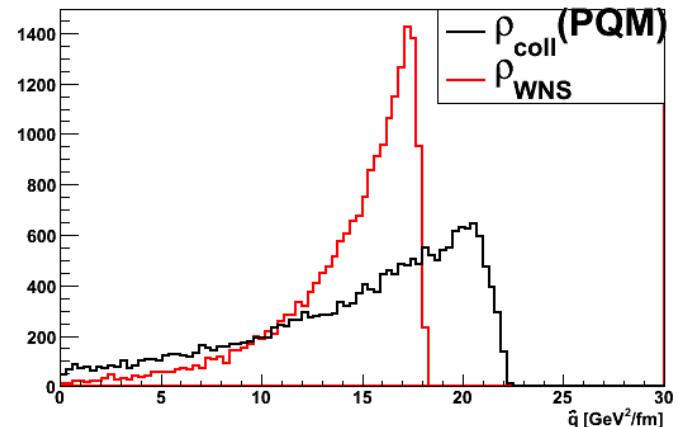
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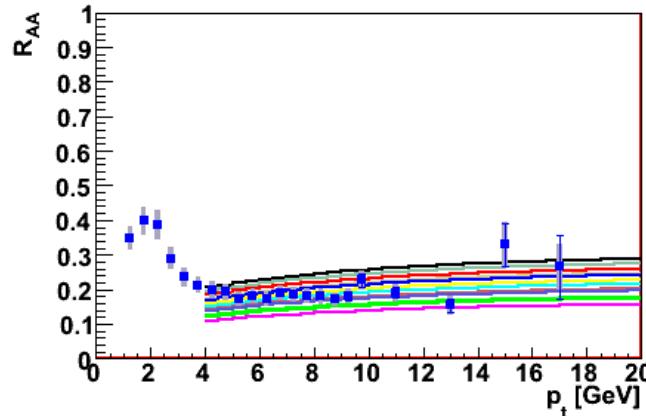
Physical Review D **68** 014008 (2003)

BDMPS + Geometry

Static medium

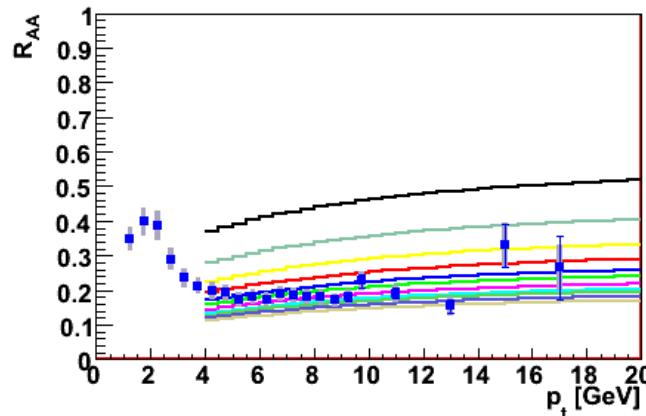
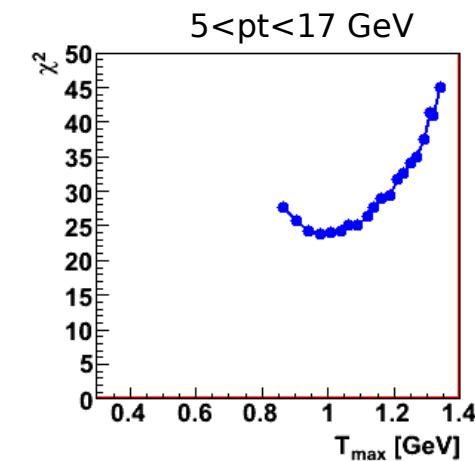
- R_{AA} vs p_t

Phenix data from *PRL 101 232301 (2008)*

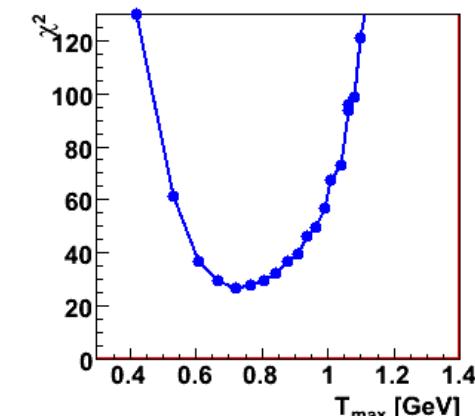


Collisional

- χ^2 analysis to estimate best fit



WNS



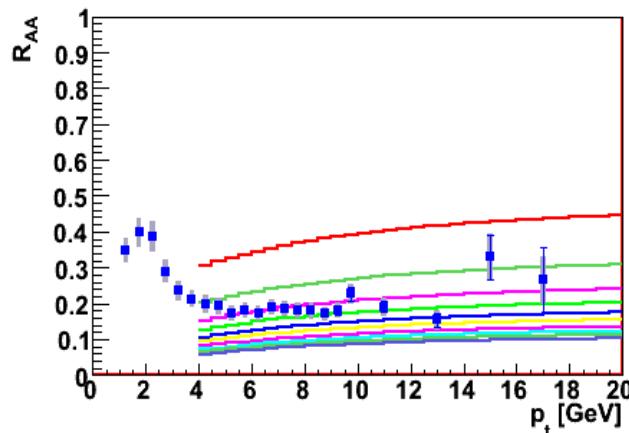
Central collisions 0-5%

T_{max} = temperature in center of medium

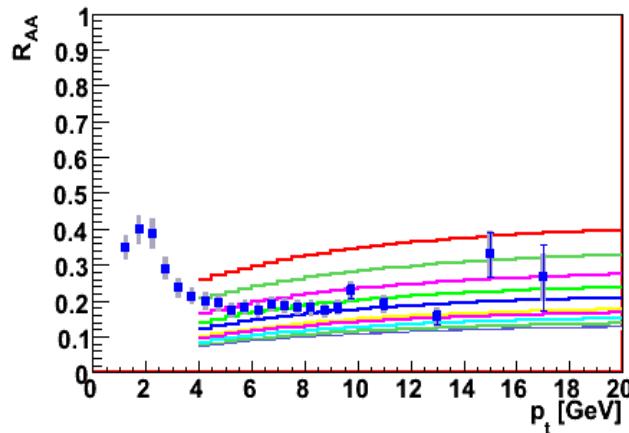
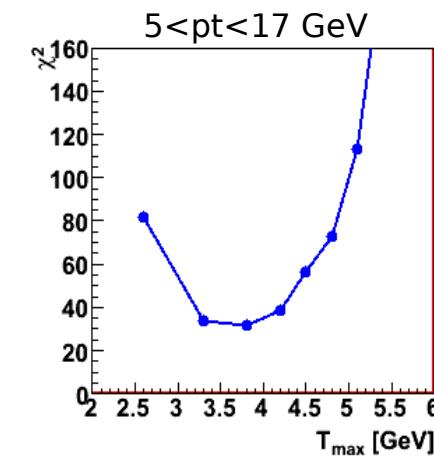
BDMPS + Geometry

Longitudinal expansion
medium which dilutes with $1/\tau$. $\tau_0 = 0.2 \text{ fm}$

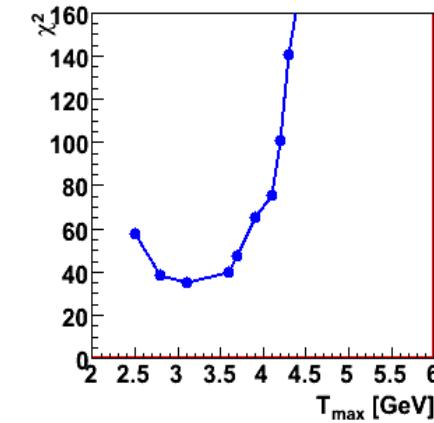
Phenix data from *PRL 101 232301 (2008)*



Collisional



WNS



Central collisions 0-5%

T_{\max} = temperature in center of medium

Opacity Expansion

- Few hard interactions.
- All parameters scale with a power of T:

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L = 2\pi\alpha_s T^2 L$$

$$\frac{1}{\lambda} = \rho\sigma = \frac{144 \cdot 1.202\alpha_s}{8\pi^2} T$$

$$\mu = \sqrt{4\pi\alpha_s} T$$

- Calculation of parameters through

$$J_n^{(m)} = \int_0^\infty u^n T^m(u) du$$

$$J_0^{(2)} = \bar{w}_c$$

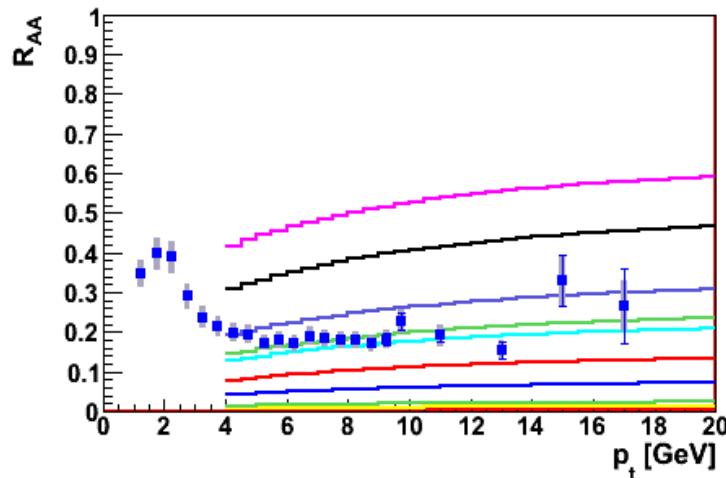
$$J_0^{(1)} = \frac{L}{\lambda}$$

$$\bar{R} \propto \frac{\bar{w}_c^2}{k_t^2} \propto \frac{(\mu^2 L)^2}{\mu^2} = \mu^2 L^2 = J_0^{(2)} \frac{J_0^{(1)}}{J_1^{(1)}}$$

WHDG + Geometry

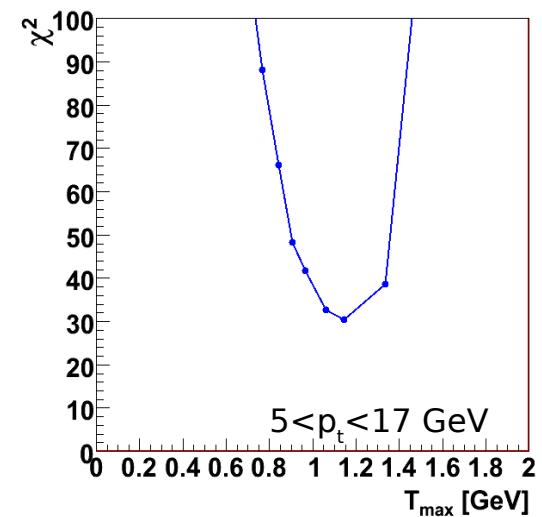
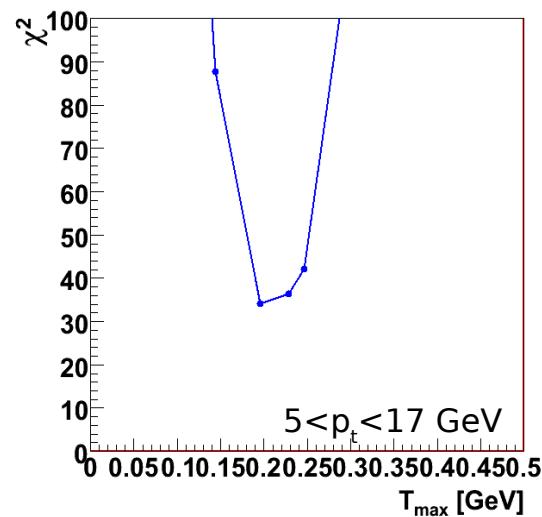
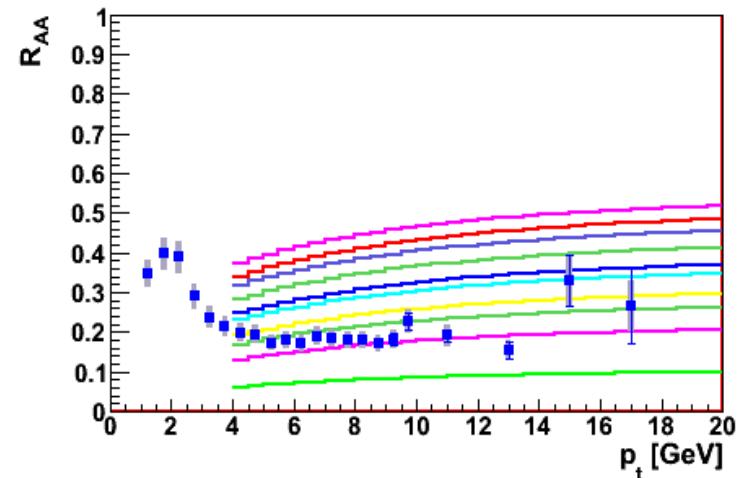
Phenix data from PRL 101 232301 (2008)

Static medium



WNS scaling

Expanding medium



Central collisions 0-5%

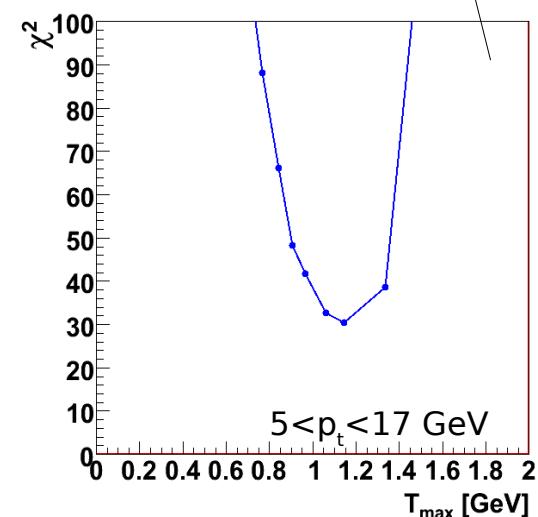
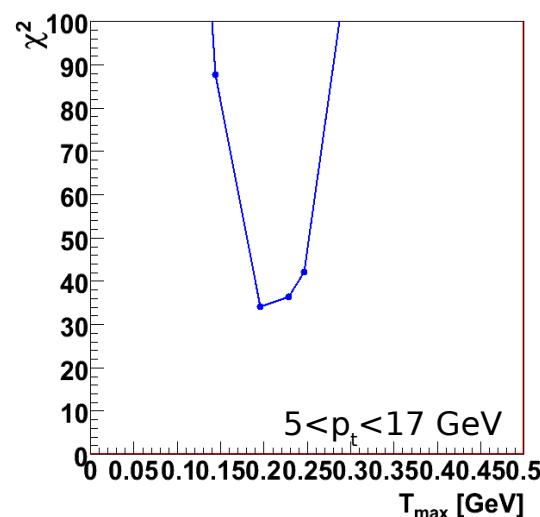
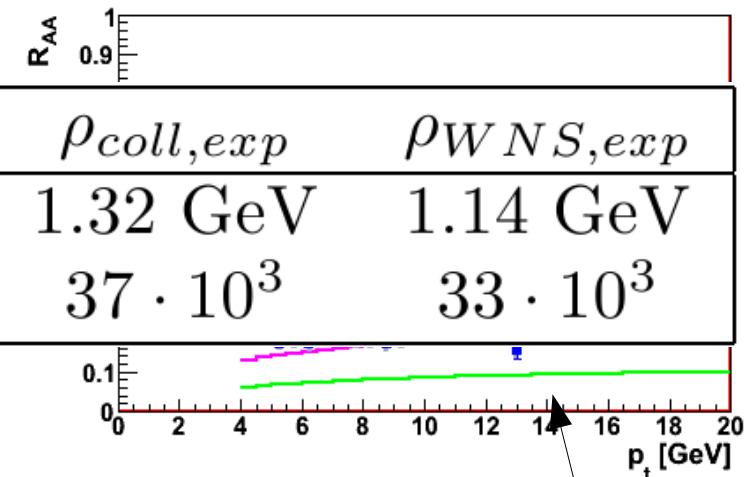
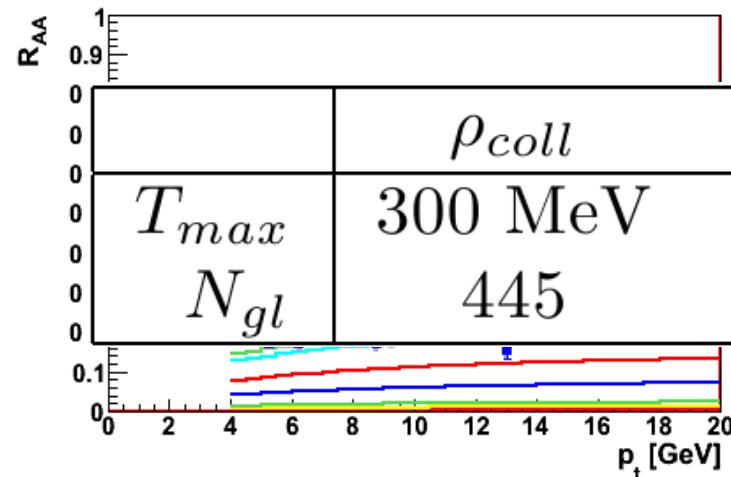
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WHDG + Geometry

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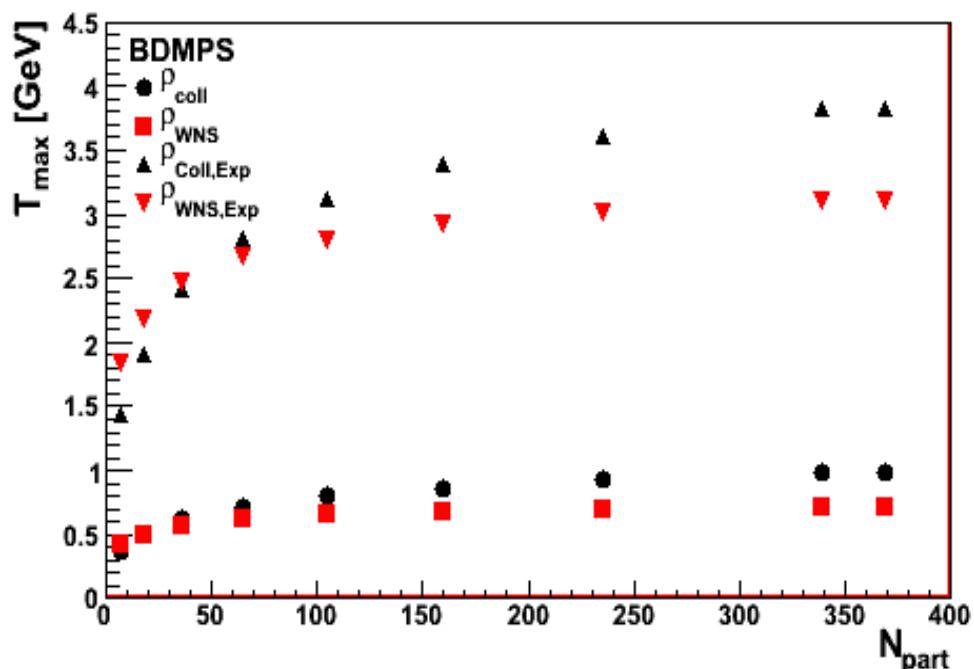
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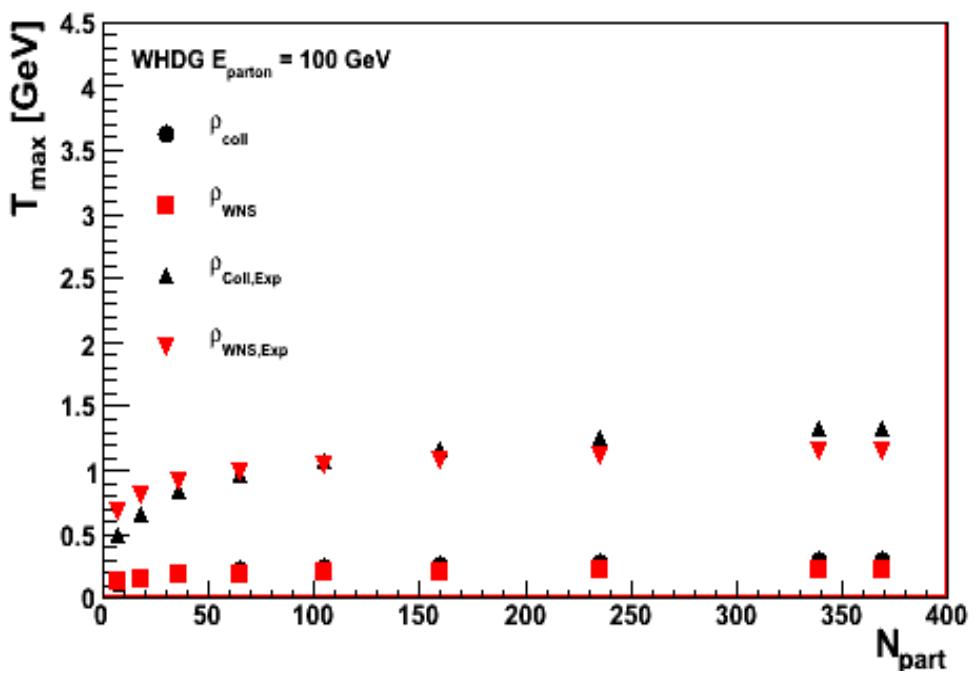


BDMPS vs WHDG (1)

BDMPS



WHDG



Predicted medium densities with common scale T directly comparable.

Large differences between predicted T_{\max} for both models.

BDMPS vs WHDG (1)

Central collisions

- BDMPS:

	Static medium	Expanding medium		
	ρ_{coll}	ρ_{WNS}	$\rho_{coll,exp}$	$\rho_{WNS,exp}$
T_{max}	975 MeV	720 MeV	3.8 GeV	3.1 GeV
N_{gl}	$14 \cdot 10^3$	$8.3 \cdot 10^3$	$89 \cdot 10^4$	$66 \cdot 10^4$

- WHDG ($E_{\text{parton}} = 100 \text{ GeV}$):

	Static medium	Expanding medium		
	ρ_{coll}	ρ_{WNS}	$\rho_{coll,exp}$	$\rho_{WNS,exp}$
T_{max}	300 MeV	229 MeV	1.32 GeV	1.14 GeV
N_{gl}	445	266	$37 \cdot 10^3$	$33 \cdot 10^3$

BDMPS vs WHDG (1)

Central collisions

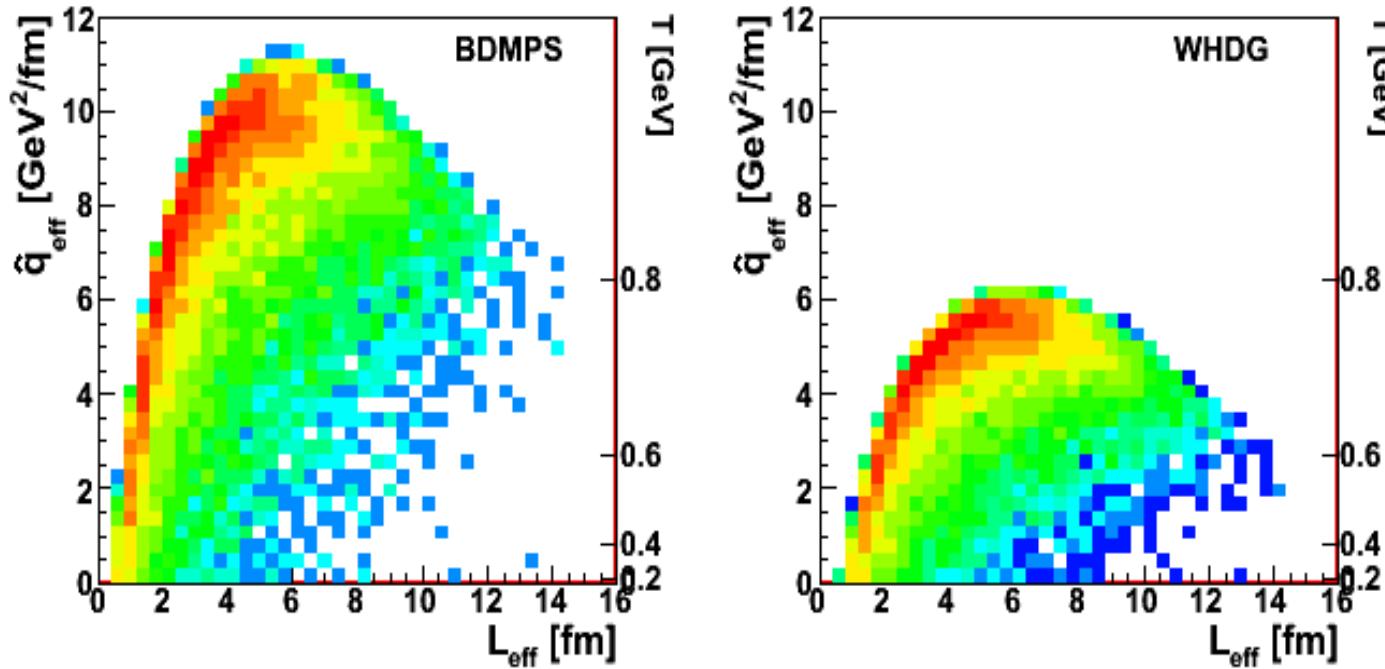
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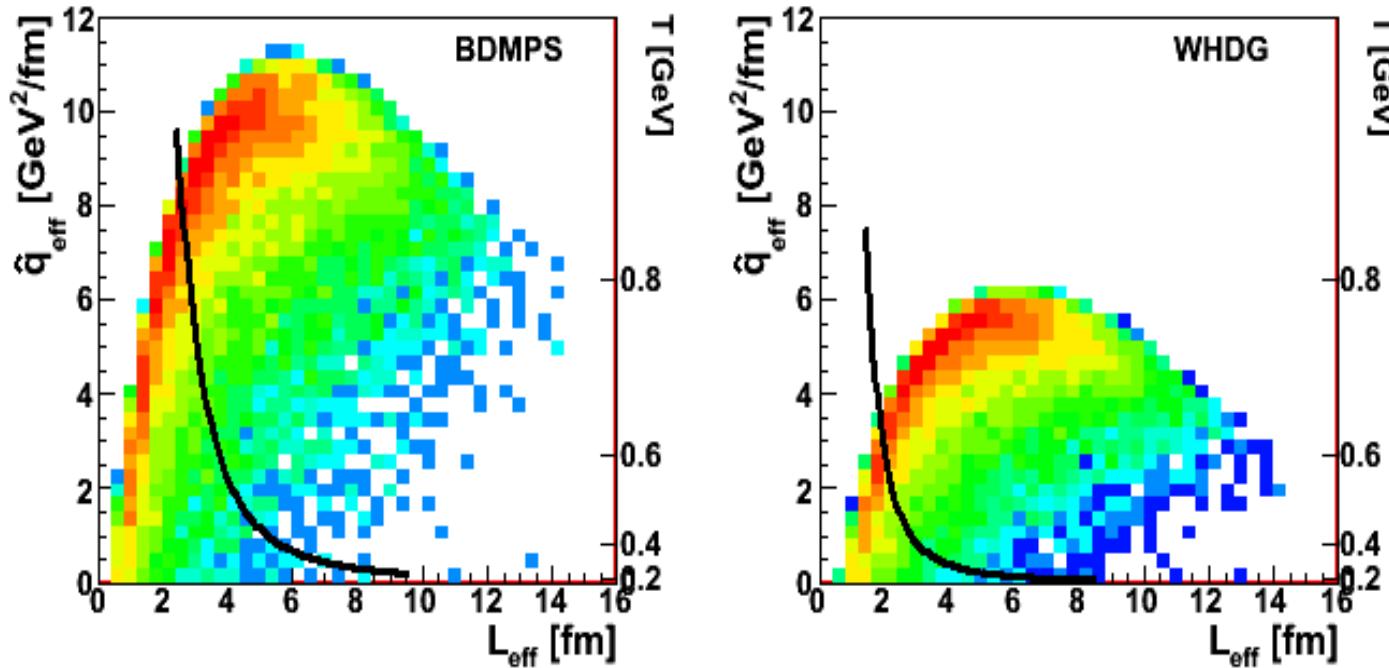
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BDMPS vs WHDG (2)



- Scattered background: medium density for best fits collisional scaling in static medium.

BDMPS vs WHDG (2)

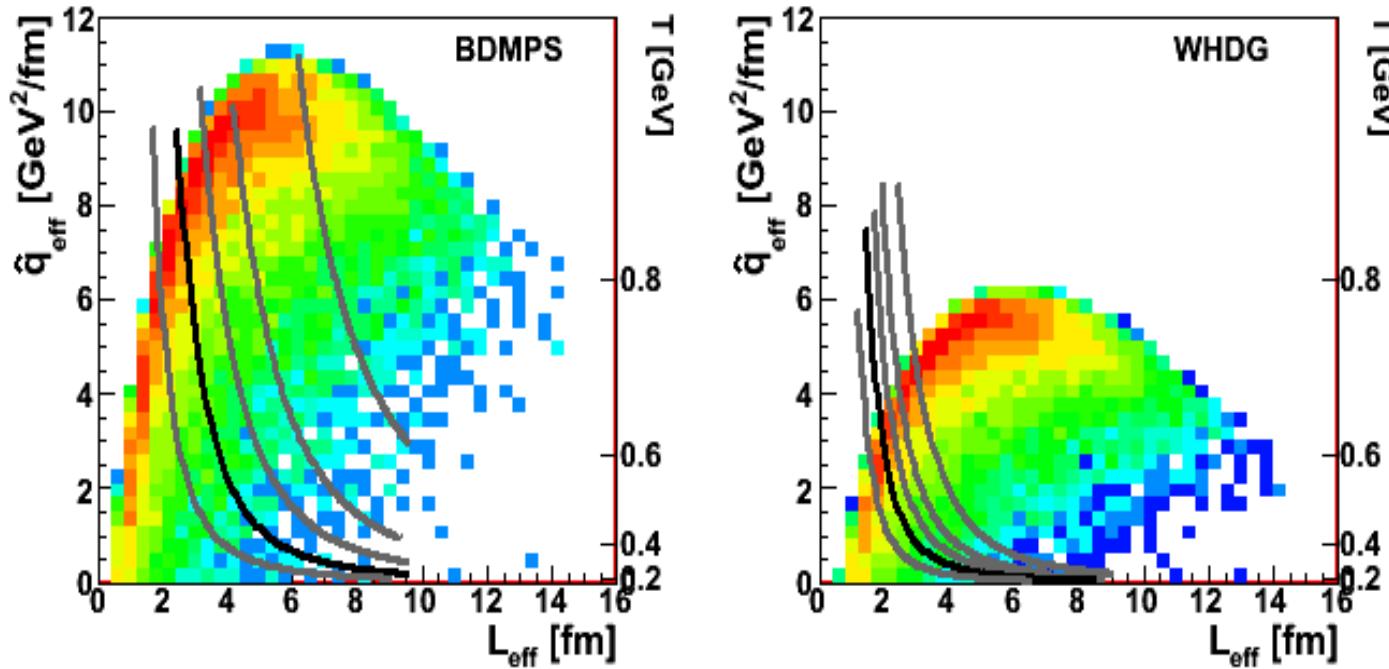


$$\hat{q} = \frac{\langle k_t^2 \rangle}{\lambda} = \frac{72 \cdot 1.202 \alpha_s^2}{\pi} T^3$$

$$\hat{q} = 2\bar{w}_c \frac{L}{\lambda} / L^2 = \frac{\mu^2}{\lambda}$$

- Scattered background: medium density for best fits collisional scaling in static medium.
- Solid line: brick contour for $R_8 = 0.2$
- Small step in $L \rightarrow$ large step in medium density.

BDMPS vs WHDG (2)

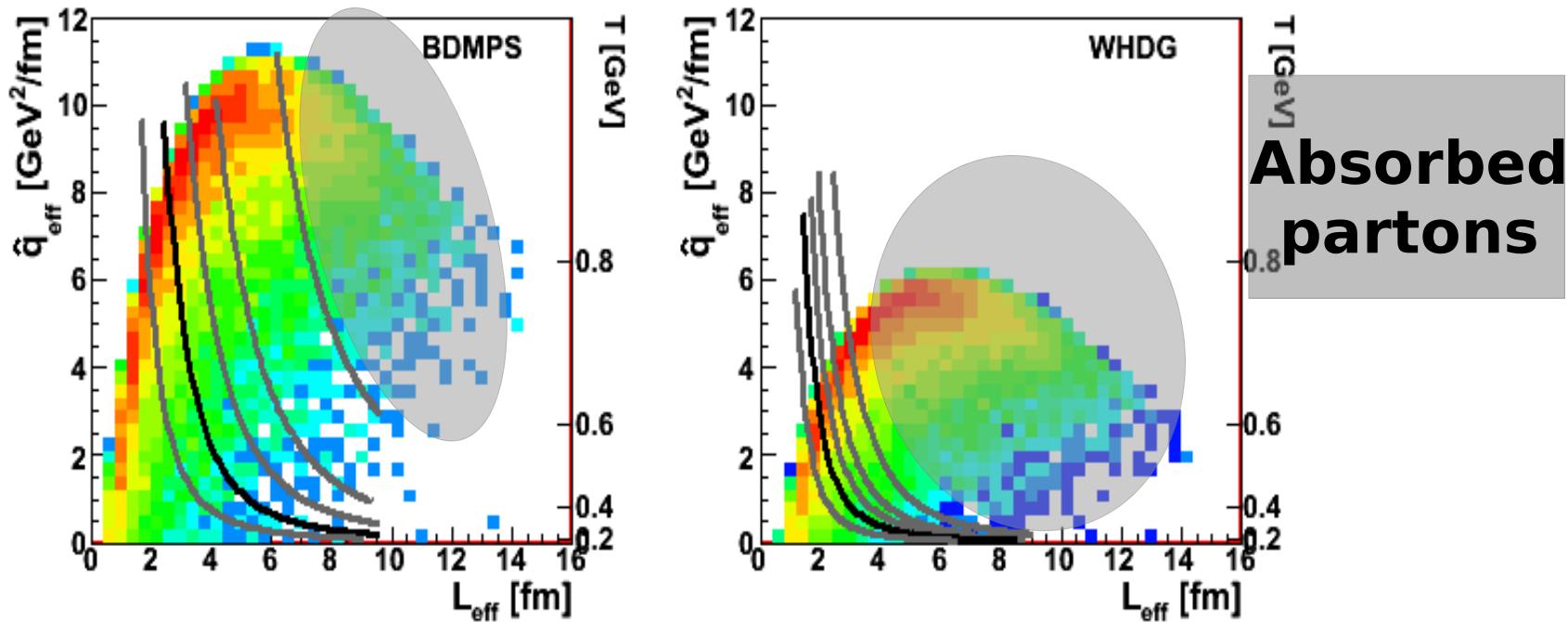


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- Solid black lines: brick contours for fixed R_8 .
- From left to right: $R_8 = 0.4, 0.2, 0.1, 0.05, 0.01$
- Small step in $L \rightarrow$ large step in medium density.

BDMPS vs WHDG (2)



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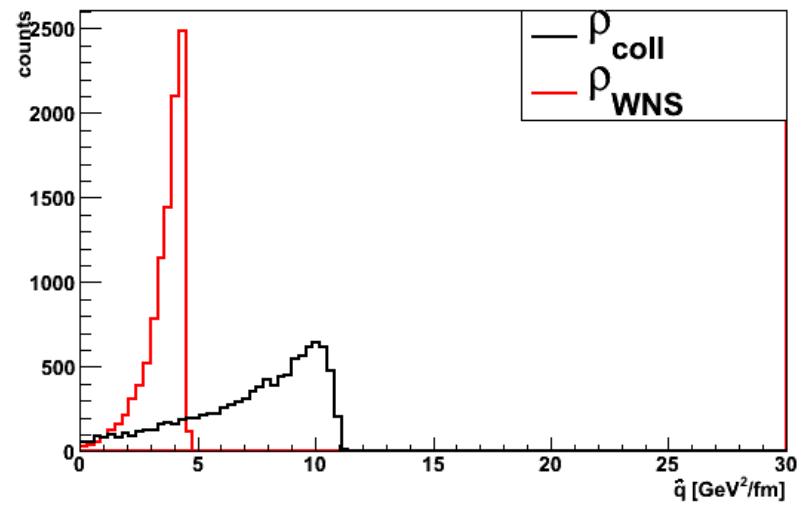
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Conclusion

- Common scale T makes direct comparison of BDMPS and Opacity Expansions possible.
- Differences in p_0 and R_8 very large for BDMPS and OEs.
- Full geometry:
 - Medium is not dominated by one R_8 brick contour.
 - Larger fraction of partons is absorbed in WHDG than in BDMPS.

Backup

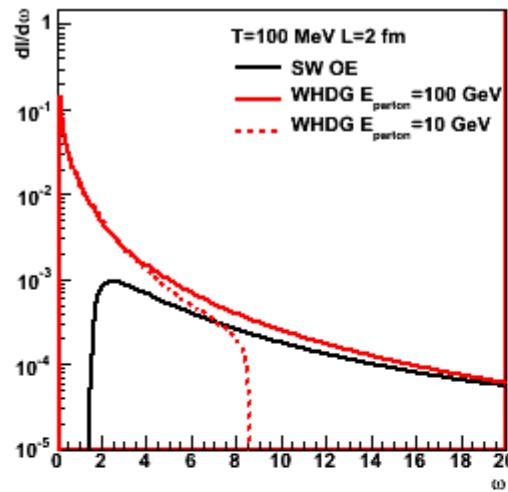
Static BDMPS



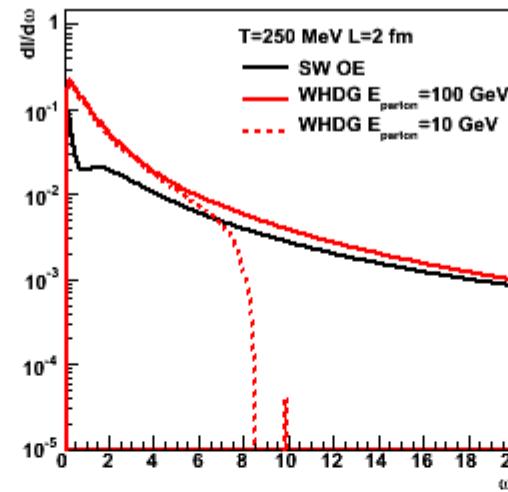
$$\langle \hat{q} \rangle_{\text{coll}} = 7.4 \text{ GeV}^2/\text{fm}$$
$$\langle \hat{q} \rangle_{\text{WNS}} = 3.6 \text{ GeV}^2/\text{fm}$$

SWOE vs WHDG Single Gluon Spectra

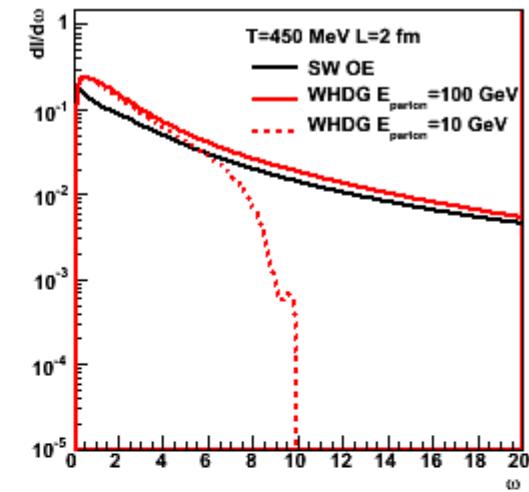
$T = 100 \text{ MeV}$



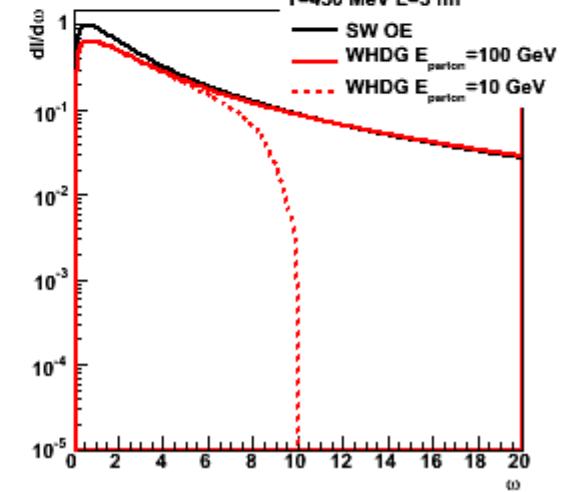
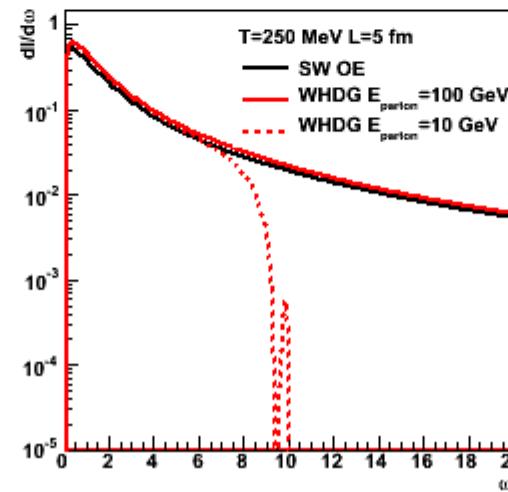
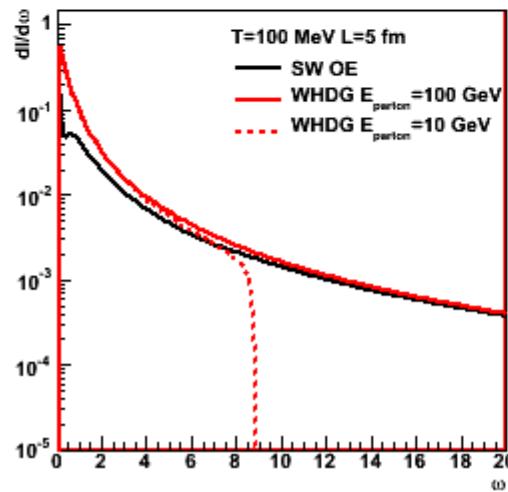
$T = 250 \text{ MeV}$



$T = 450 \text{ MeV}$



$L = 2$



$L = 5$

Single Gluon Spectra

- **BDMPS:**

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2\text{Re} \int_{\xi_0}^{\infty} dy_l \int_{y_l}^{\infty} d\bar{y}_l \int d\mathbf{u} \int_0^{\chi\omega} d\mathbf{k}_{\perp} e^{-i\mathbf{k}_{\perp} \cdot \mathbf{u}} e^{-\frac{1}{2} \int_{\bar{y}_l}^{\infty} d\xi n(\xi) \sigma(\mathbf{u})} \\ \times \frac{\partial}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{u}} \int_{\mathbf{y}=0}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r} \exp \left[i \int_{y_l}^{\bar{y}_l} d\xi \frac{\omega}{2} \left(\dot{\mathbf{r}}^2 - \frac{n(\xi)\sigma(\mathbf{r})}{i\omega} \right) \right]$$

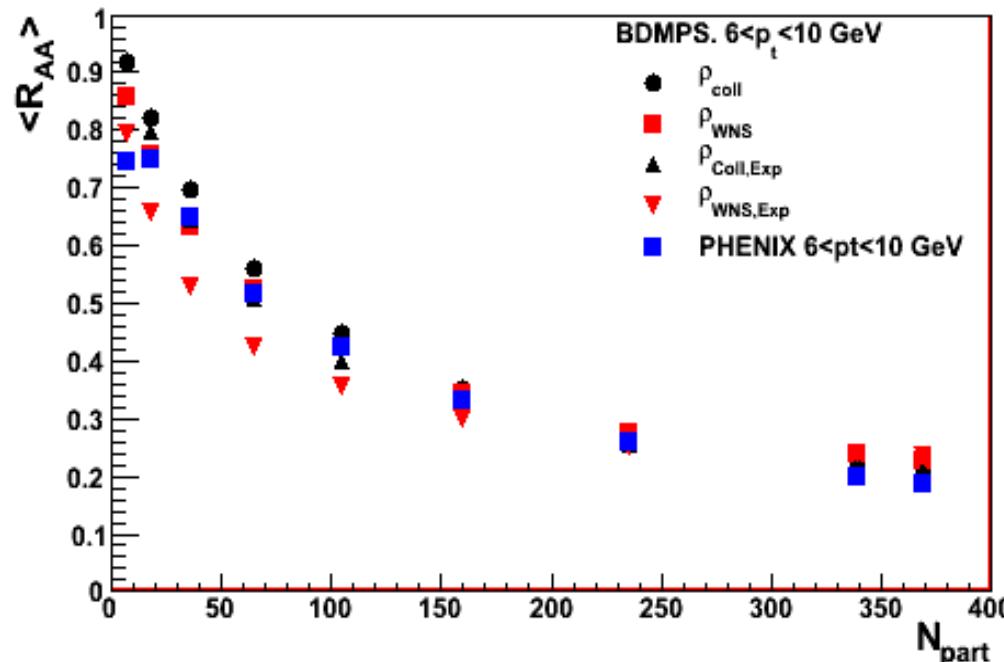
- **ASW-SH:**

$$\omega \frac{dI}{d\omega} = \frac{4\alpha_s C_R}{\pi} (n_0 L) \gamma \int_0^{\infty} \tilde{q} d\tilde{q} \left[\frac{\tilde{q}^2 - \sin \tilde{q}^2}{\tilde{q}^4} \right] \times \left(\frac{1}{\gamma + \tilde{q}^2} - \frac{1}{\sqrt{(\kappa^2 + \tilde{q}^2 + \gamma^2)^2 - 4\kappa^2 \tilde{q}^2}} \right)$$

- **WHDG:**

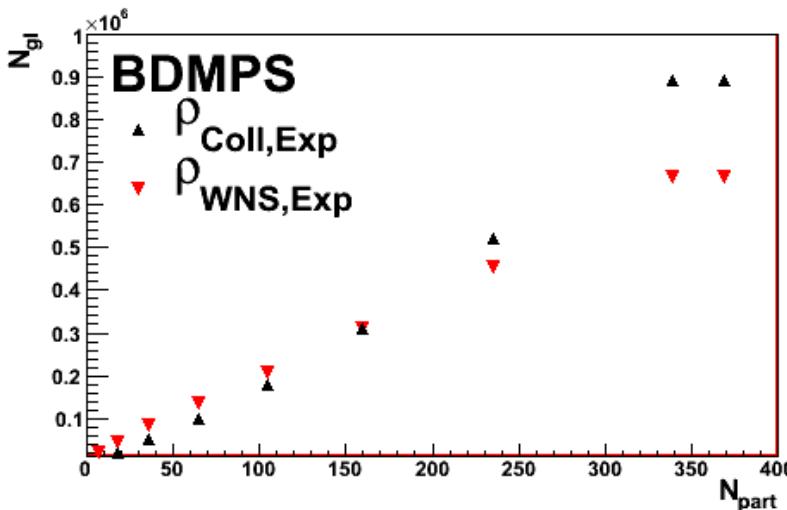
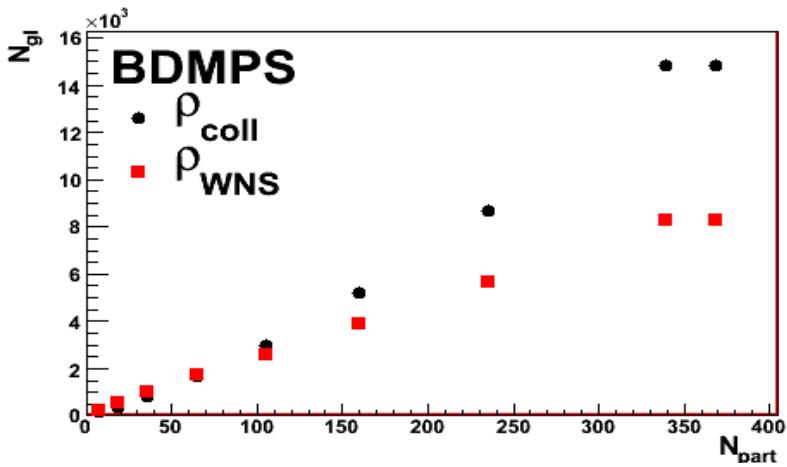
$$x \frac{dN_g}{dx} = \frac{C_R \alpha_s}{\pi} \frac{L}{\lambda} \int_0^{q_{\max}^2} \frac{2q^2 \mu^2 dq^2}{(4xE\hbar c/L)^2 + (q^2 + \beta^2)^2} \\ \times \int_0^{k_{\max}^2} \frac{dk^2}{k^2 + \beta^2} \frac{k^2(k^2 - q^2 + \mu^2) - \beta^2(k^2 - q^2 - \mu^2)}{((k - q)^2 + \mu^2)^{3/2} ((k + q)^2 + \mu^2)^{3/2}}$$

RAA BDMPS



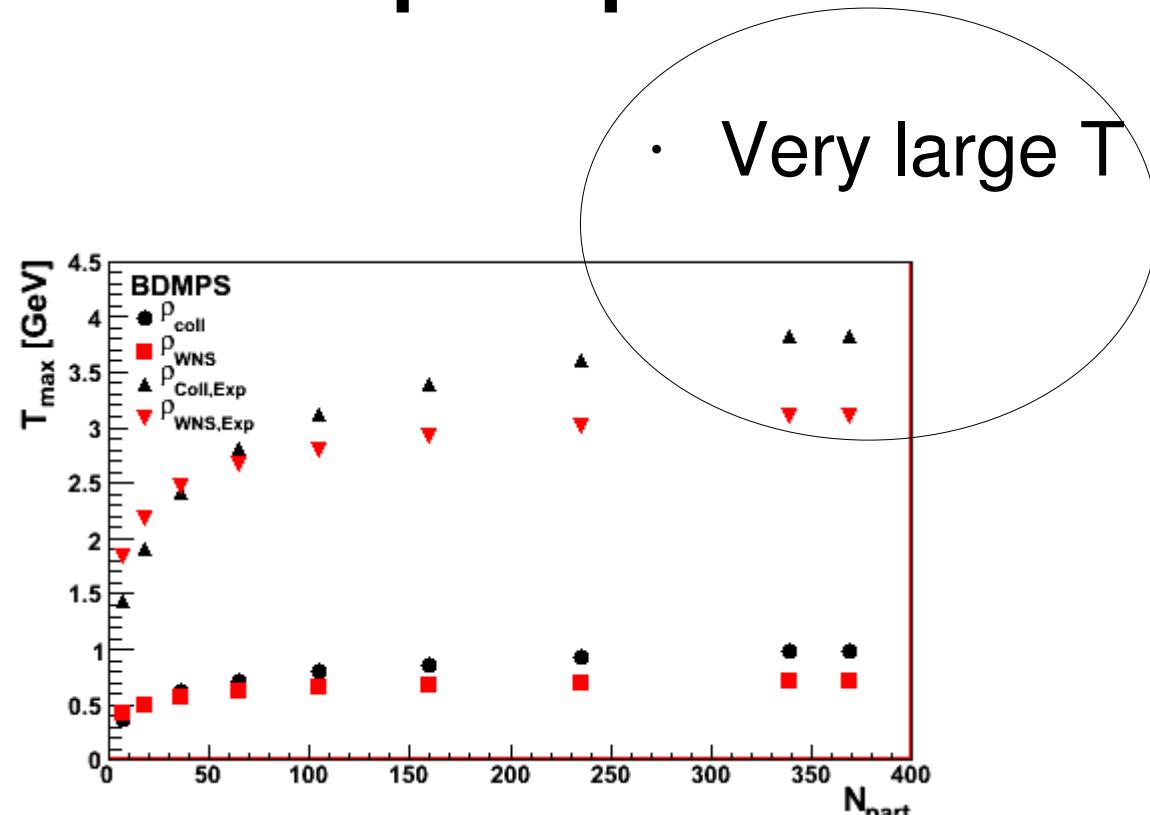
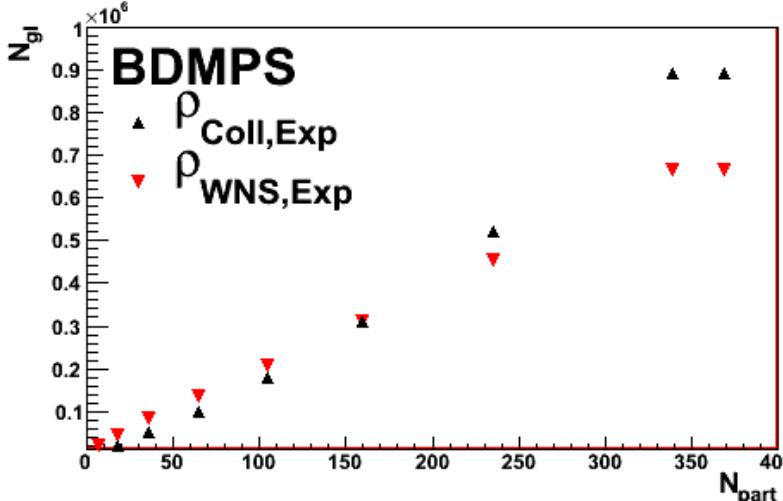
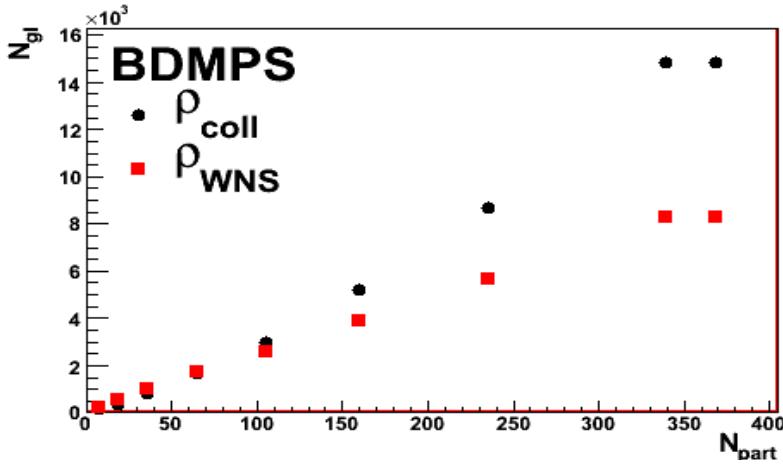
- Agreement between data and model.
Best correspondence with static WNS geometry.
- But very large temperatures needed.

BDMPS medium properties



0-5%

BDMPS medium properties



0-5%	ρ_{coll}	ρ_{WNS}	$\rho_{coll,exp}$	$\rho_{WNS,exp}$
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