Uncertainties in the first generation of parton energy loss models
(BDMPS-Z,ASW,GLV,WHDG,…)

Urs Achim Wiedemann
CERN Physics Department
TH Division

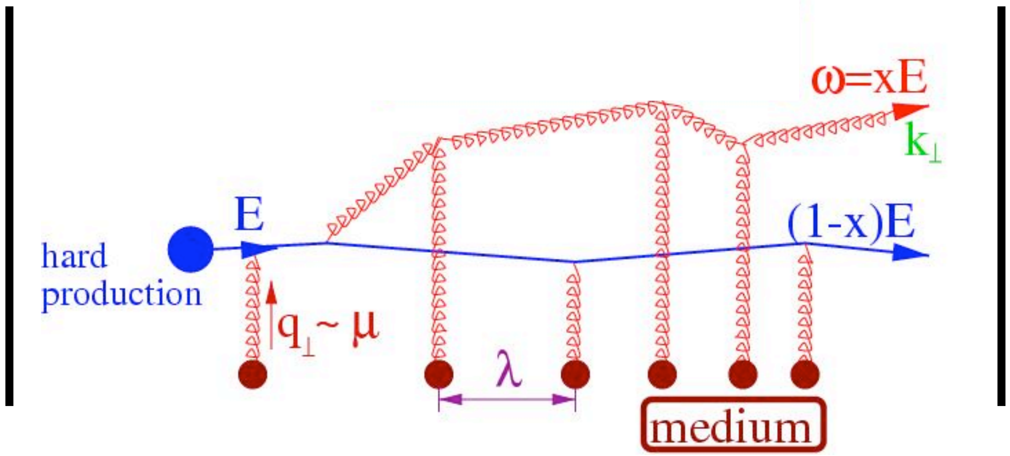
The following does not intend to repeat a solution of the brick problem which we (ASW) posted on the TEC-HQM wiki page.
Rather, the plan is to explain

- what is the reason for the theoretical uncertainties
- what is needed to overcome these uncertainties

3rd TEC-HQM workshop,
CERN 7 July 09

What is calculated?

$$\frac{dI}{d\ln\omega\, dk_T} =$$



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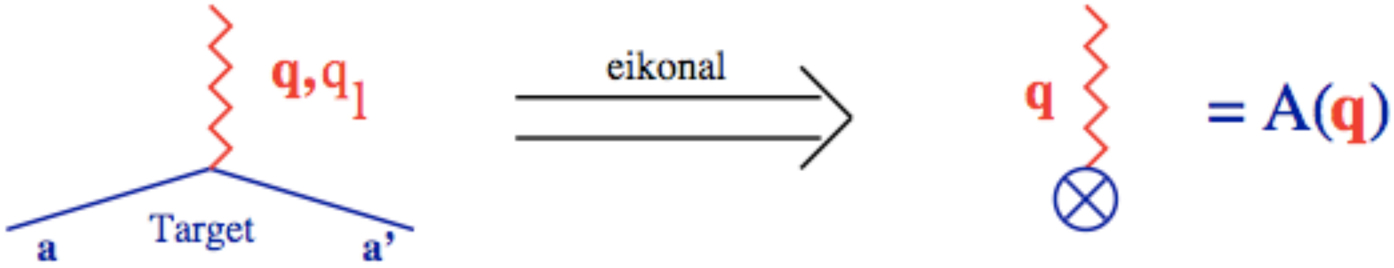
In which approximation?

$$E >> \omega >> |k_T|, |q_i| >> \Lambda_{QCD}$$

For the following, we refer to this as “**eikonal**”

Consequences of this approximation?

A. Strong simplification of the medium



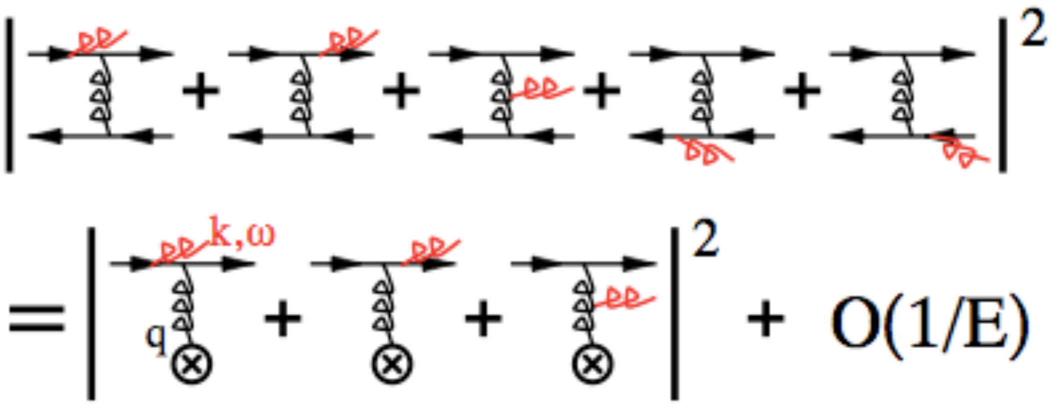
Medium is **recoilless**,

-> no longitudinal momentum transfer,
-> no collisional energy loss
-> static colored scattering centers (Gyulassy-Wang model)

Aside: combining 1st generation radiative energy loss calculations with models of collisional energy loss is ad hoc procedure (collisional models require medium which accepts recoil)

Going beyond eikonal approximation?

Going beyond eikonal approximation requires dramatic refinement in picturing the medium



Beyond leading O(1/E),

-> colored potential scattering is not gauge-invariant (Wang,Plümer,Gyulassy 1993)

-> physics reason: gluon emission off target legs matters

-> target components must be dynamical

Aside: calculations going beyond O(1/E) will allow to calculate radiative and collisional effects on the same footing (i.e. using the same picture of the medium)
Then there will be no sharp distinction between collisional and radiative e-loss since the medium can absorb recoil in inelastic processes.

Consequences of this approximation?

B. Validity in very limited kinematic range

This has been realized early on by many practitioners in the field and has been discussed in much detail in TEC-HQM

-> **$x \ll 1$**
emitted gluon assumed to be much softer than projectile
but dominant contribution to e-loss does not come from soft gluons

-> **$k \ll w$**
emitted gluon assumed to be collinear to projectile
but wide phase space outside the collinear region
(and this is crucial for pt-broadening)

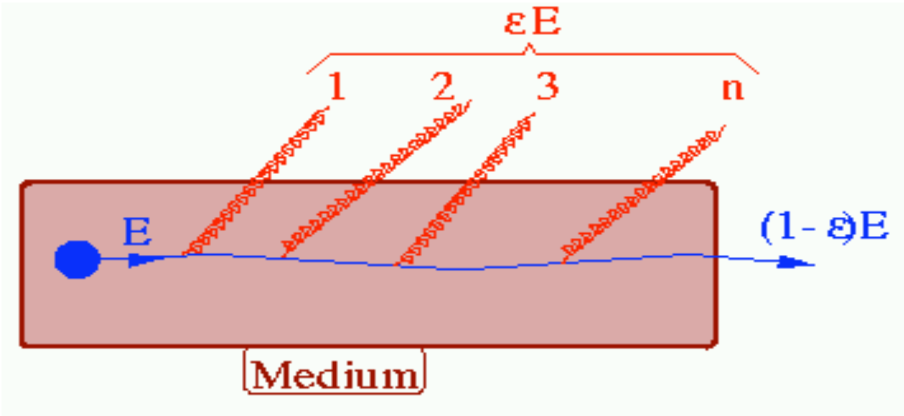
-> no exact E-p conservation
(there is no scattering without longitudinal momentum transfer)

=> $k \sim O(w)$ and exact E-p conservation requires including recoil effects
one may try to guesstimate uncertainties by comparing results between different implementations of eikonal approximation
But: above limitations are direct consequence of eikonal approximation
honest assessment of systematic errors will require to go beyond eikonal approximation

Consequences of this approximation?

C. Iteration of eikonal gluon emissions does not account for energy degradation of propagating projectile

Baier, Dokshitzer, Mueller, Schiff, JHEP (2001)



$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} d\omega_i \frac{dI(\omega_i)}{d\omega_i} \right] \delta\left(\Delta E - \sum_{i=1}^{n} \omega_i \right) e^{-\int_0^\infty d\omega \frac{dI}{d\omega}} = p(\Delta E / \omega_c) + p_0 \delta(\Delta E)$$

In eikonal approximation, hard projectile will propagate forever, rather than getting stuck in the medium.

The ASW *ad-hoc solution* is to declare that the probability $\int_E^\infty d\Delta E \; P(\Delta E)$

Is the probability of getting stuck

(in a full calculation, this would not be needed)

Given these uncertainties, rooted in the eikonal approximation, what to do?

$\Rightarrow$ Recall the correct qualitative physics included in
1st generation parton energy loss models

$\Rightarrow$ Delineate where/to what extent these models are quantitatively applicable
This motivates the brick problem, which is the simple lab experiment for quantifying uncertainties.

$\Rightarrow$ Most important: develop a 2nd generation framework
which is free from the deficiencies listed above and includes
-> exact energy-momentum conservation
-> correct treatment of quantum interference
-> dynamical description of recoil effects
(I'll argue tomorrow that a MC provides the natural setting for this, and the all MC presentations tomorrow morning will document progress on some/all of these points.)

Basic e-loss result in eikonal approximation

One of several versions to write the medium-modified gluon distribution
Emitted from a colored projectile in the eikonal approximation

$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2\mathrm{Re} \int_0^\infty dy \int_y^\infty d\bar{y} \int du\, e^{-ik_T u} e^{\left[-\frac{1}{2}\int_y^\infty d\xi\, n(\xi)\sigma(u)\right]}$$

$$\times \frac{\partial}{\partial u} \cdot \frac{\partial}{\partial s} K(s=0, y; u, y \mid \omega)$$

Path integral:

$$K(s, y; u, \bar{y} \mid \omega) = \int_{s=r(y)}^{u=r(\bar{y})} Dr \exp\left[\int_y^{\bar{y}} d\xi \left\{\left(\frac{i\omega}{2}\dot{r}^2\right) - \frac{1}{2} n(\xi)\sigma(r)\right\}\right]$$

Assumptions about the medium:
- strength of single static scattering potential

$$\sigma(r) = 2\int \frac{dq}{(2\pi)^2} |A(q)|^2 \left(1 - \exp[iq.r]\right)$$

- density of such scattering centers

$$n(\xi)$$

Two approximations to do numerics with the result

1. Saddle point approximation

$$n(\xi)\sigma(r) \cong \frac{1}{2}\boxed{\hat{q}(\xi)}r^2$$

BDMPS transport coefficient

$$K\left(s,y;u,\bar{y} \mid \omega\right) = \int_{s=r(y)}^{u=r(\bar{y})} \mathcal{D}r \exp\left[\int_y^{\bar{y}} d\xi \left\{\boxed{\left(\frac{i\omega}{2}\dot{r}^2\right)} - \frac{1}{4}\boxed{\hat{q}(\xi)}r^2\right\}\right]$$

Target average includes Brownian motion:

2. Opacity Expansion

Expand integrand in density of scattering centers times dipole cross section

$$K(\underline{s},y;\underline{u},\bar{y}) = K_0(s;u) - \int d\underline{r}\, d\xi\, K_0(\underline{s},y;\underline{r},\xi)\boxed{n(\xi)\sigma(\underline{r})}K_0(\underline{r},\xi;\underline{u},\bar{y}) + \ldots.$$

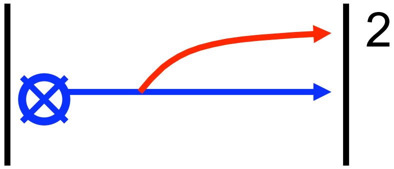
Physically, large transverse momentum tails of static scattering centers are dropped in saddle point approximation but kept in opacity expansion.

Qualitative physics contained

A. The calculation establishes (in the eikonal approximation) the dominant medium-modification of vacuum parton branching (i.e. of virtuality evolution)

To zeroth order, there is no medium (vaccum case), and one finds:

$$\omega \frac{dI^{(0)}}{d\omega\, dk_T} = \frac{\alpha_s C_R}{\pi^2} H(k_T) =$$

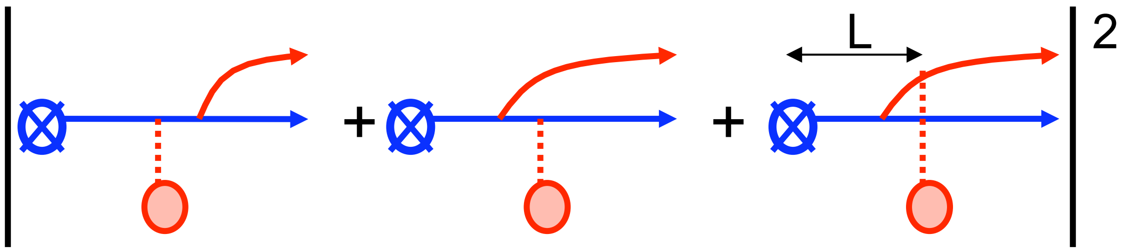


$$, \qquad H(k_T) = \frac{1}{k_T^2}$$

Note that this is the dominant $1/\underline{k}^2$ piece of the DGLAP parton shower,
But amputated to leading order in x=w/E due to eikonal approximation.

To first order in opacity, there is a generally complicate interference between <u>vacuum radiation</u> and <u>medium-induced</u> radiation.

$$\omega \frac{dI^{(1)}}{d\omega\, dk_T} =$$



<u>This is the expression, analyzed in the ASW-SH / WHDG analysis</u>

Understanding limitations and physics contained in N=1

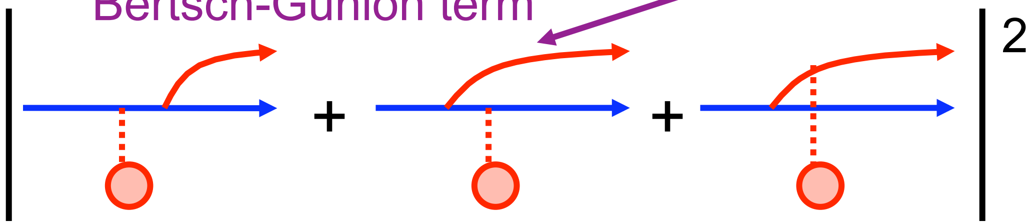
Consider parton cascade limit $L \to \infty$. It contains three contributions:

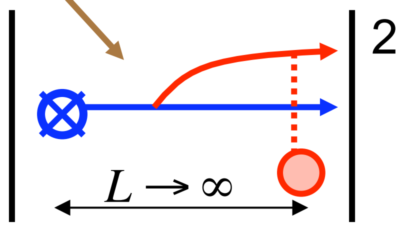
1. **Probability conservation** of medium-independent vacuum terms.
2. **Transverse phase space** redistribution of vacuum piece.
3. **Medium-induced gluon radiation** of quark coming from minus infinity

$$\overset{nL=const}{\lim_{L\to\infty}} \omega \frac{dI^{(1)}}{d\omega\, dk_T} = -\boxed{w_1} H(k_T) + nL\int_{q_T} dq_T \left[\boxed{R(q_T,k_T)} + \boxed{H(q_T+k_T)}\right]$$

Rescattering of vacuum term

Bertsch-Gunion term





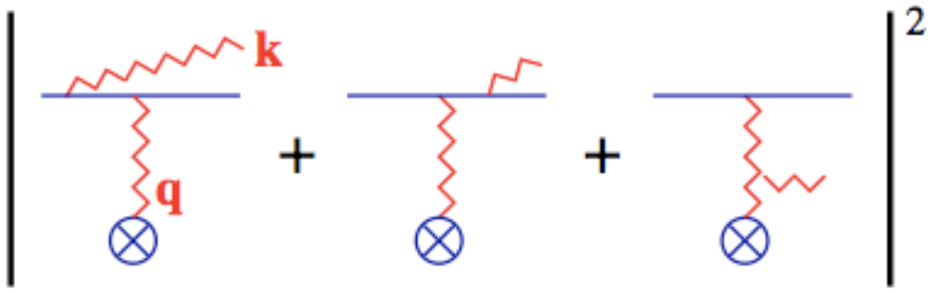
All terms are amputated due to the eikonal approximation!!

- the vacuum term misses the subleading x-terms of the splitting function

- the medium-induced radiation terms …. pto

Eikonal approximation of medium-induced radiation term

In the eikonal approximation, we have



$$= |A(q)|^2 \; R(k;q)$$

The Bertsch-Gunion term $\quad R(k;q) \equiv \dfrac{q^2}{k^2 (k+q)^2}$

depends neither on gluon energy, nor on the energy of the projectile

⇒ Approximation implies violation of E-p-conservation on microscopic level

⇒ (same problem as with other high energy approximations (e.g. BFKL)

Fundamental remedy (explained tomorrow):

- Identify unambiguously the terms A^2R and A^2 in the interference pattern
- Relate A^2R unambiguously to inelastic 2->3 cross section and

  A^2 to elastic 2-> 2 cross section
- Use for inelastic and elastic cross section the known expressions without any kinematic approximation (restores E-p-conservation)
- keep the interference terms as identified in the opacity expansion.

Realizing/Quantifying uncertainties due to E-p approximation

… we found that the small-x gluon emission probabilities are very high. This indicates the importance of multiple gluon radiation which is not contained in the BDMPS-Z formalism. Moreover, the BDMPS-Z formalism is based on the assumption of small transverse gluon momemtum |k|<< w, while we find the main contribution to radiative energy loss for |k|~O(w). Both features question the validity of the BDMPS-Z formalism …

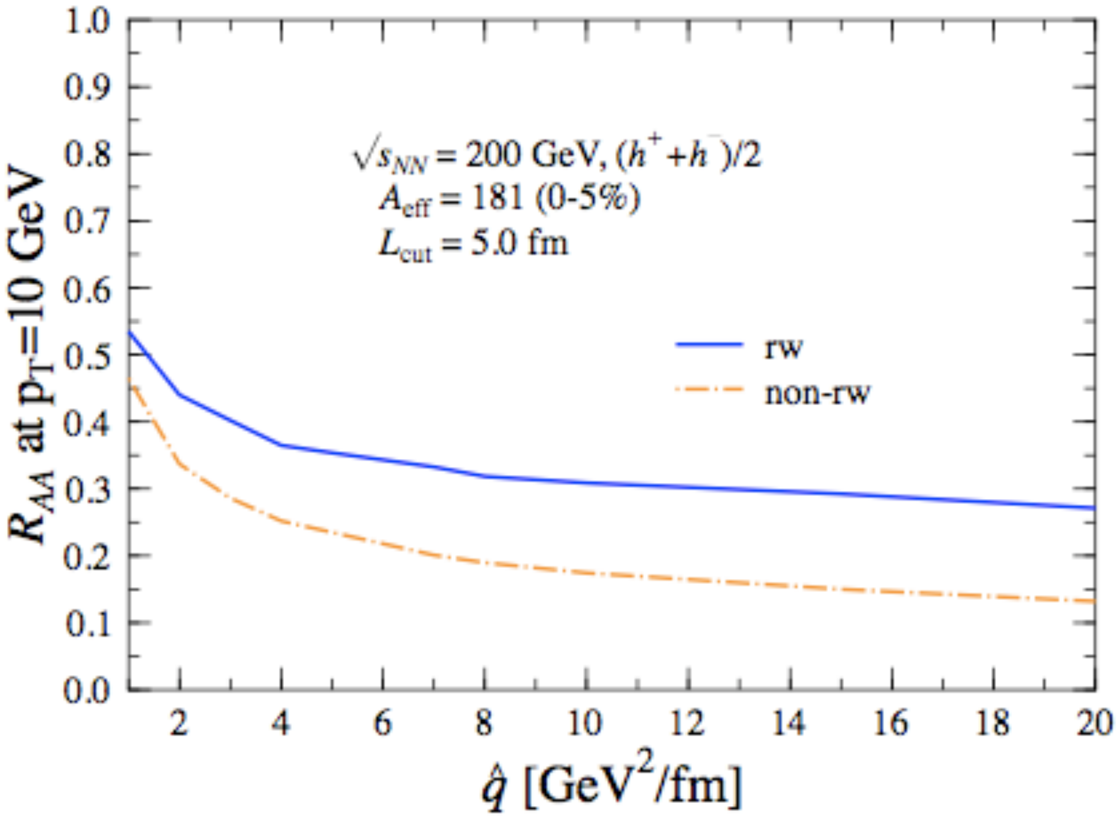
… However, the physical origin of the qualitatively novel effects discussed here [interference of vacuum and medium radiation, broadening of gluon radiation] appear to be very generic. Even outside the BDMPS-Z formalism, expect that … leave similar, quantitatively important traces.

Last paragraph of UAW, NPA 690:731 (2001)

What did we (ASW and others) do about the known uncertainties?

- First we worked (2001-2003) mainly on other potentially dominant uncertainties, such as static medium versus medium of decreasing density
- Realizing qualitatively that uncertainties are more severe if projectile energy is smaller (i.e. if eikonal approximation is more doubtful)
  - try to associate error bars by mainly qualitative considerations, and study of cut-off dependencies (these are the yellow bands in our publications since 2004)
    (Not as sophisticated as what may be needed, but)

Early attempts of assigning uncertainties



Eskola, Honkanen, Salgado, Wiedemann, NPA747; 511, 2005.

Two different (crude!) procedures of handling $\int_E^\infty d\Delta E \; P(\Delta E)$

result for R_{AA}~0.2 in values of qhat ~ 7 GeV^2/fm or qhat much larger

Assessing uncertainties within ASW eikonal approximation

Assess uncertainty of kt~O(w) by doing kt-integral

$$\frac{dI}{d\ln\omega} \equiv \int_0^{\boxed{\chi\omega}} dk \frac{dI}{d\ln\omega\, dk}$$

Quenching weights of ASW are tabulated as functions of

$$\omega_c \equiv \frac{1}{2}\hat{q}L^2 \qquad R \equiv \frac{1}{2}\hat{q}\boxed{\chi^2}L^3$$

Salgado & Wiedemann, PRD68: 015008, 2003.

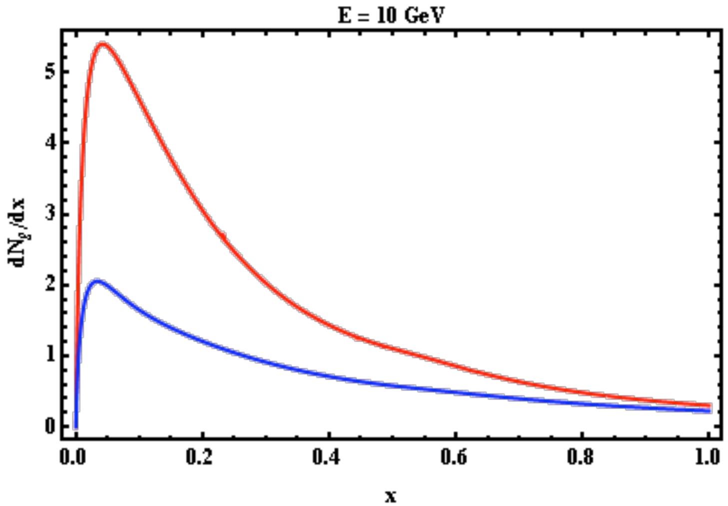
For data comparisons, our default value was always chi= 1.

But chi-dependence allows one to vary the upper cut-off of the kt-integration by using the existing tabulated quenching weights of ASW.

Comment on TECHQM-effort of assigning uncertainty

William & Brian pointed out that there within the eikonal approximation a freedom of interpreting the x-dependence of the integrand as a **light-cone x_+** or **cartisian x_E** .

The consequence is a factor ~2 difference in their 'apples-to-apples comparison

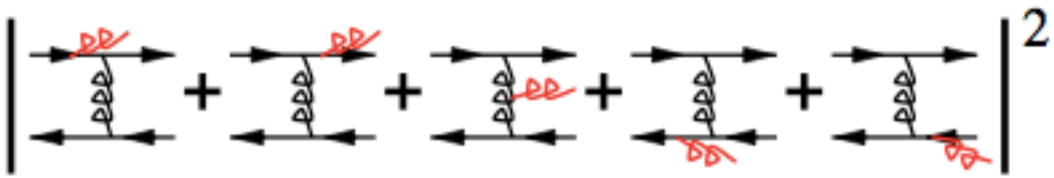


The basic reason for this ambiguity is that the radiative cross section in the eikonal approximation is

$$|A(q)|^2 R(k;q) \equiv |A(q)|^2 \frac{q^2}{k^2 (k+q)^2}$$

and does not depend on gluon energy w and projectile energy E.

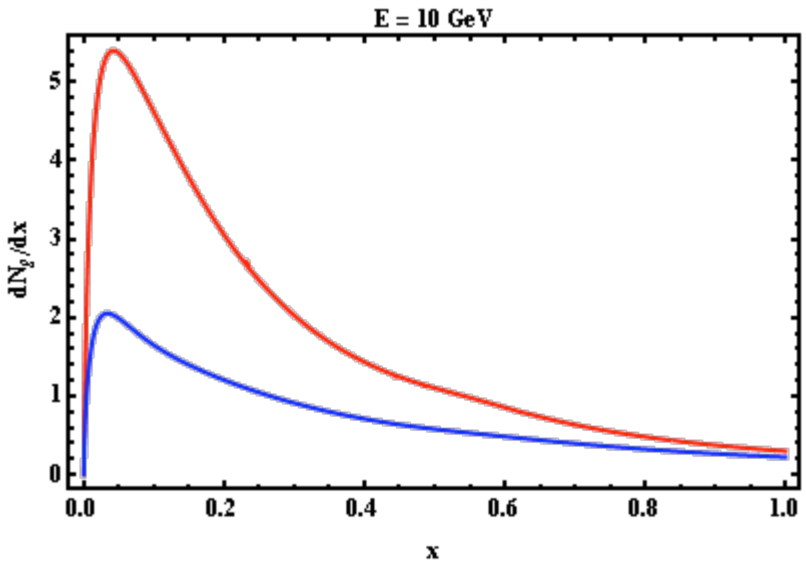
Beyond the eikonal approximation, the gluon radiation x-section does depend on w and E, and this will remove any ambiguity about which x to choose.



Within eikonal approximation

Can a TECHQM critical assessment of the uncertainties in qhat be based on expanding on these two plots?

William & Brian's
Apples-to apples comparison
Interpreting integrand as function
of x_E (blue) or x_+ (red)



Marta's key plot today:
(input may require further discussion)